Primordial Gravitational Waves from Scalar Backreaction in Axion SU(2) Inflation

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COSMOMAG Project

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Why Axions in the Early Universe?

1. They are theoretically well motivated and welcomed in many extensions of the Standard Model

2. They Naturally Couple to Gauge Fields

3. During Inflation they prevent loop corrections to the inflaton mass because of the shift symmetry

4. Support rich Phenomenology (Chiral GW, Non-Gaussianities, PBH…)

The Spectator Model

$$\mathscr{L} = \sqrt{-g}\left[\underbrace{\frac{M_p^2}{2}R}_{\textbf{Gravity}} - \underbrace{\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]}_{\textbf{Inflaton}} - \underbrace{\left[\frac{1}{2}(\partial\chi)^2 + U(\chi) + \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}\right]}_{\textbf{Spectator and Gauge Sector}} + \underbrace{\frac{\lambda\chi}{4f}F^a_{\mu\nu}\tilde{F}_a^{\mu\nu}}_{\textbf{Chern-Simons}} \right]$$

$$F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{abc}A^b_\mu A^c_\nu$$

The Model Parameters: (f, g, λ)

The Background

The Inflaton Potential is left unspecified in this work

Isotropic Configuration

$$A_0^a = 0, \quad A_i^a = \delta_i^a \, a(t) \, Q(t)$$

Axion Potential

$$U(\chi) = \mu^4 \left[1 + \cos\left(\frac{\chi}{f}\right)\right],$$

$$\ddot{\chi} + 3H\dot{\chi} + U_\chi + \frac{3g\lambda}{f} Q^2 \left(\dot{Q} + HQ\right) = 0$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right)Q + gQ^2 \left(2gQ - \frac{\lambda\dot{\chi}}{f}\right) = 0$$

The attractor regime

$$m_Q \equiv \frac{gQ}{H}, \quad \Lambda \equiv \frac{\lambda Q}{f}$$

$\Lambda \gg 2$

$\Lambda \gg \sqrt{3}/m_Q$

We can neglect the accelerations

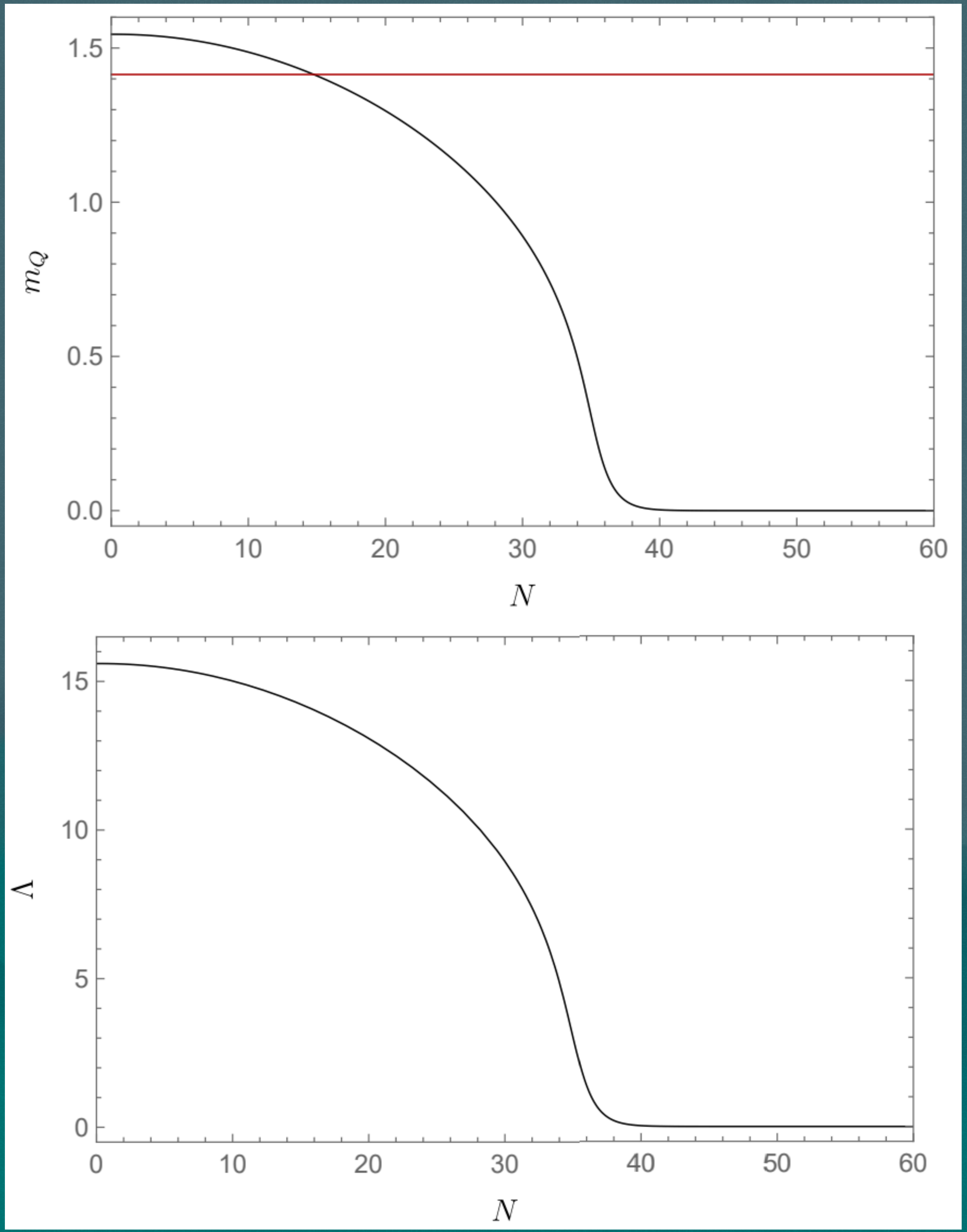
$$Q \simeq \left(\frac{-f\,U_\chi}{3g\lambda H}\right)^{1/3} \qquad \xi \simeq m_Q + 1/m_Q\,.$$

Important Functions

$$m_Q \equiv \frac{gQ}{H}, \quad \Lambda \equiv \frac{\lambda Q}{f}$$

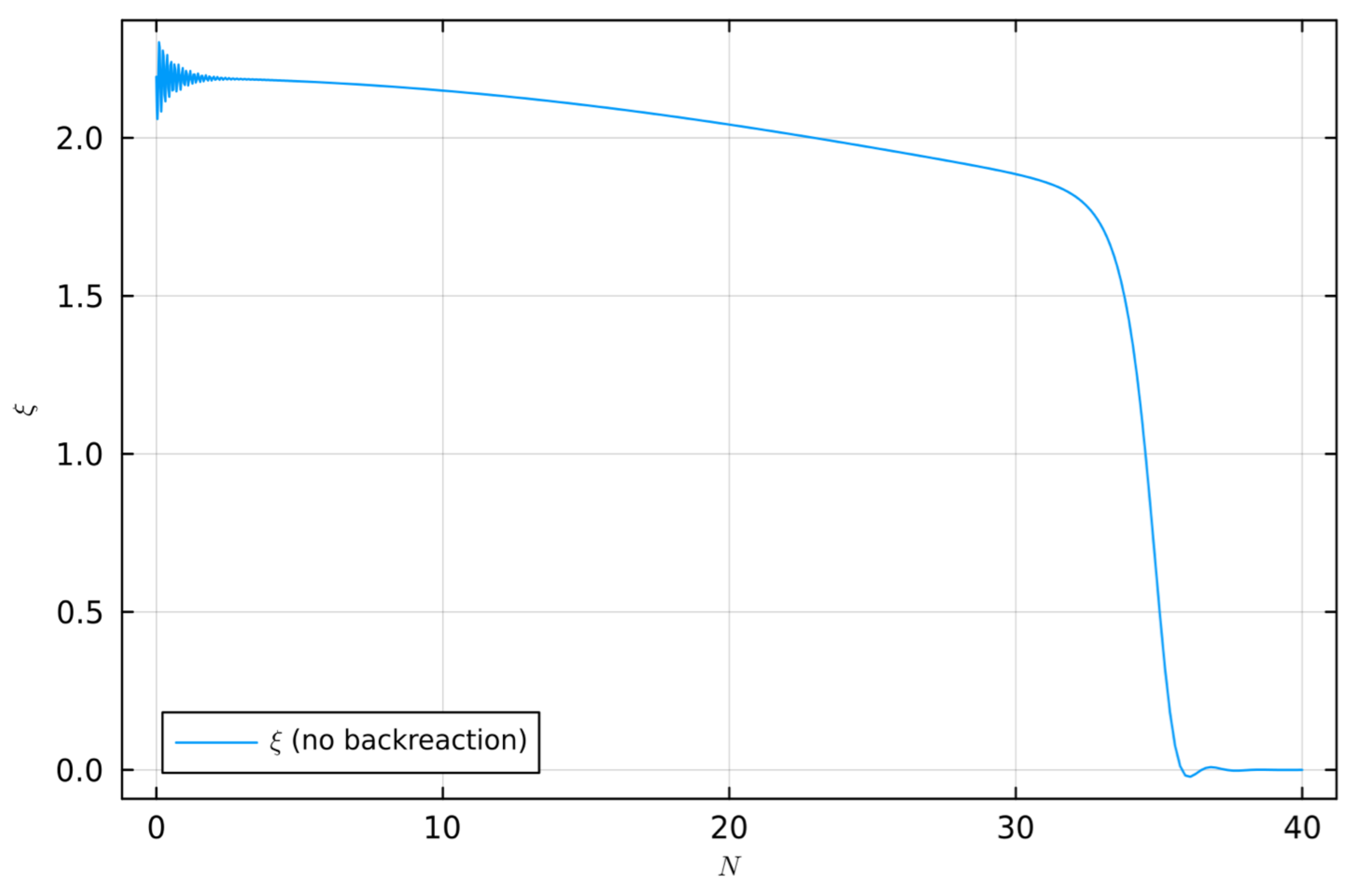
At some point, mQ drops below the critical value

And triggers a tachyonic instability INSIDE the Horizon



The Particle Production Function

$$\xi \equiv \frac{\lambda \dot{\chi}}{2Hf}$$



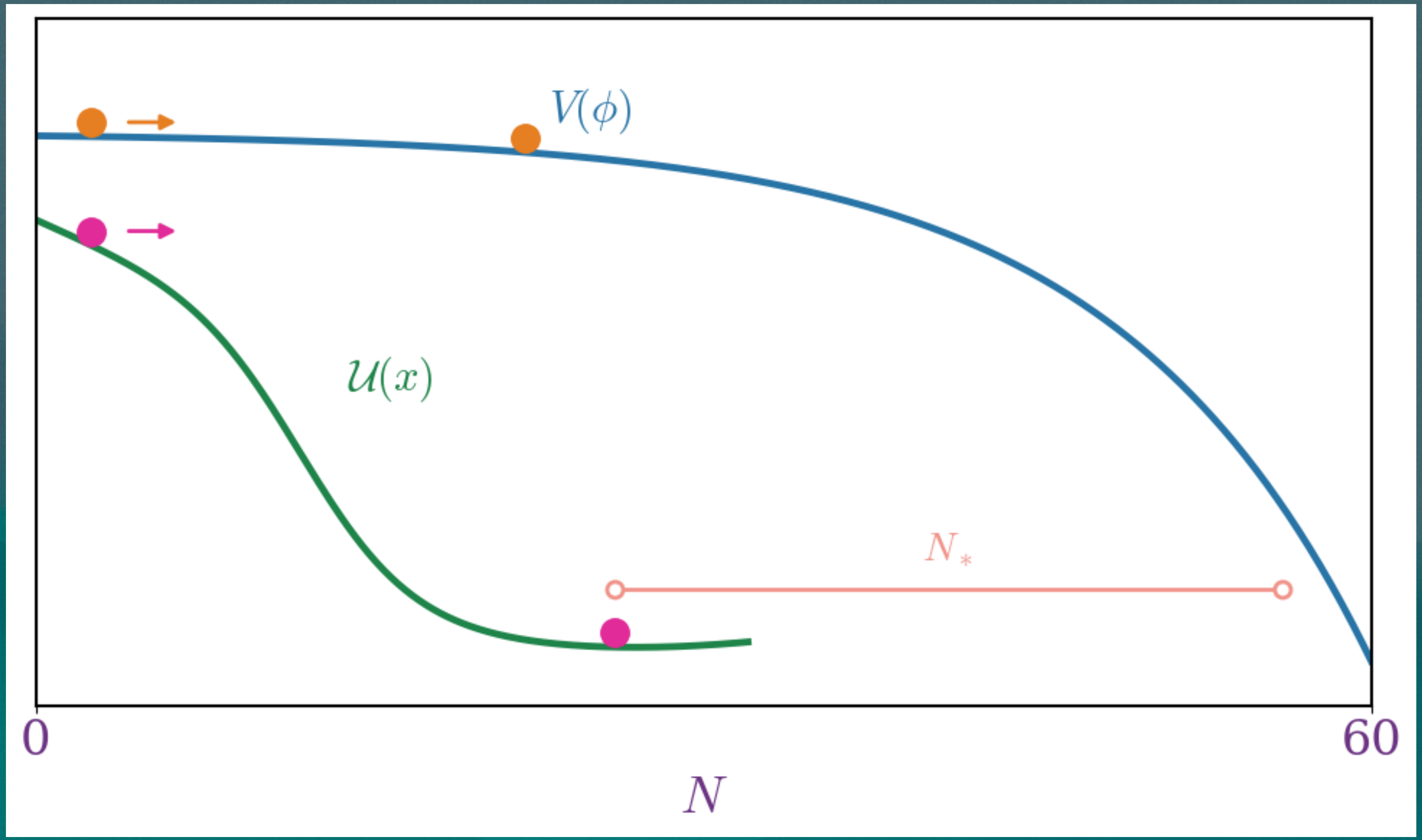
It goes to zero rapidly in absence of the backreaction

Our Set-up: Decay of the axion-gauge sector during inflation

$$P_{\zeta_\chi}/P_{\zeta_\phi}\Big|_{\text{end}} = \mathrm{e}^{-3N_*}P_{\zeta_\chi}/P_{\zeta_\phi}\Big|_{\text{decay}}\,.$$

So, we avoid large non-Gaussianity

And direct sourcing to the Power Spectrum



The perturbations park

SCALARS

$$\Big(\underbrace{X}_{Axion}, \underbrace{Z, \phi}_{Gauge}, \underbrace{\Phi}_{Inflaton} \Big)$$

TENSORS

$$\Big(\underbrace{T_{R/L}}_{Gauge}, \underbrace{\psi_{R/L}}_{GW} \Big)$$

We expand the Lagrangian and obtain the equations of motion...

Me

The EoM

$$
\begin{aligned}
&\ddot{\hat{X}} + H\dot{\hat{X}} + \left[-2+\epsilon_H+3\eta_X+\frac{k^2\left(k^2+m_Q^2(2+\Lambda)\,a^2H^2\right)}{a^2H^2\left(k^2+2m_Q^2\,a^2H^2\right)}\right]H^2\hat{X} + \frac{\sqrt{2}m_Q^2\Lambda\,aH}{\sqrt{k^2+2m_Q^2\,a^2H^2}}H\dot{\hat{\Phi}} \\
&+\frac{m_Q\Lambda\left(k^4+3m_Q^2\,k^2a^2H^2+4m_Q^4\,a^4H^4\right)}{aH\left(k^2+2m_Q^2\,a^2H^2\right)^{3/2}}\sqrt{\frac{2\epsilon_E}{\epsilon_B}}H^2\hat{\Phi} - \sqrt{2}m_Q\Lambda\,H\dot{\hat{Z}} - 2m_Q^2\Lambda\sqrt{\frac{2\epsilon_E}{\epsilon_B}}\,H^2\hat{Z} = 0,
\end{aligned}
$$

$$
\begin{aligned}
&\ddot{\hat{\Phi}} + H\dot{\hat{\Phi}} + \left[\frac{6m_Q^4\,k^2\,a^2H^2}{\left(k^2+2m_Q^2\,a^2H^2\right)^2}\frac{\epsilon_E}{\epsilon_B}+\frac{k^4+2m_Q(3m_Q-\xi)k^2\,a^2H^2+4m_Q^4\,a^4H^4}{\left(k^2+2m_Q^2\,a^2H^2\right)\,a^2H^2}\right]H^2\hat{\Phi} \\
&-\frac{2(m_Q-\xi)\sqrt{k^2+2m_Q^2\,a^2H^2}}{aH}H^2\hat{Z} - \frac{\sqrt{2}m_Q^2\Lambda\,aH}{\sqrt{k^2+2m_Q^2\,a^2H^2}}H\dot{\hat{X}} \\
&+\frac{\sqrt{2}m_Q^2\Lambda\,aH}{\sqrt{k^2+2m_Q^2\,a^2H^2}}\left[1+\frac{k^4}{m_Q\left(k^2+2m_Q^2\,a^2H^2\right)\,a^2H^2}\sqrt{\frac{\epsilon_E}{\epsilon_B}}\right]H^2\hat{X} = 0,
\end{aligned}
$$

$$
\begin{aligned}
&\ddot{\hat{Z}} + H\dot{\hat{Z}} + \left[\frac{k^2}{a^2H^2}+2m_Q\left(m_Q-2\xi\right)\right]H^2\hat{Z} + \sqrt{2}m_Q\Lambda\,H\dot{\hat{X}} - \sqrt{2}m_Q\Lambda\,H^2\hat{X} \\
&-\frac{2\left(m_Q-\xi\right)\sqrt{k^2+2m_Q^2\,a^2H^2}}{aH}H^2\hat{\Phi} = 0.
\end{aligned}
$$

Linear Strong Backreaction

$$\ddot{\chi} + 3H\dot{\chi} + U_\chi + \frac{3g\lambda}{f} Q^2\left(\dot{Q} + HQ\right) + B_\chi^{\mathrm{BR}} = 0$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right)Q + gQ^2\left(2gQ - \frac{\lambda\dot{\chi}}{f}\right) + B_Q^{\mathrm{BR}} = 0$$

$$B_\chi^{\mathrm{BR}} = \frac{V^{(3)}(\chi)}{2a^2}\int \frac{d^3k}{(2\pi)^3}\left|\hat{X}(\tau,k)\right|^2,$$

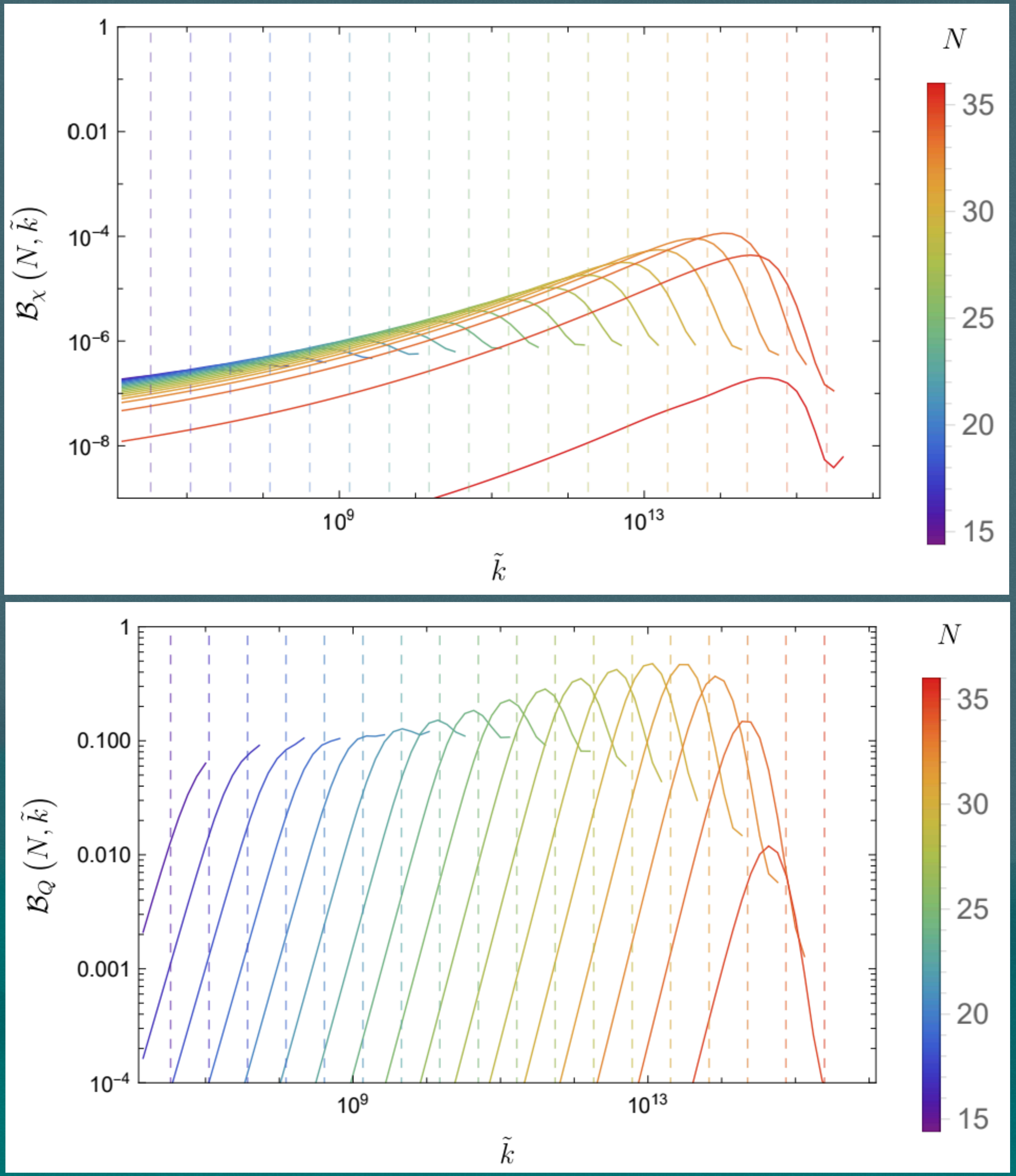
$$B_Q^{\mathrm{BR}} = \frac{2gH\Lambda^2 m_Q}{3a^2}\int \frac{d^3k}{(2\pi)^3}\,\frac{k^2\left(k^2 + a^2H^2m_Q^2\right)}{\left(k^2 + 2a^2H^2m_Q^2\right)^2}\left|\hat{X}(\tau,k)\right|^2,$$

WE DISREGARD THE TENSOR BACKREACTION. IT IS NEGLIGIBLE AS LONG AS $m_Q \leq \sqrt{2}$

The New Sources

$$\frac{B_\chi}{U_\chi} \equiv \int d\ln\tilde{k}\, \mathscr{B}_\chi\left(N,\tilde{k}\right),$$

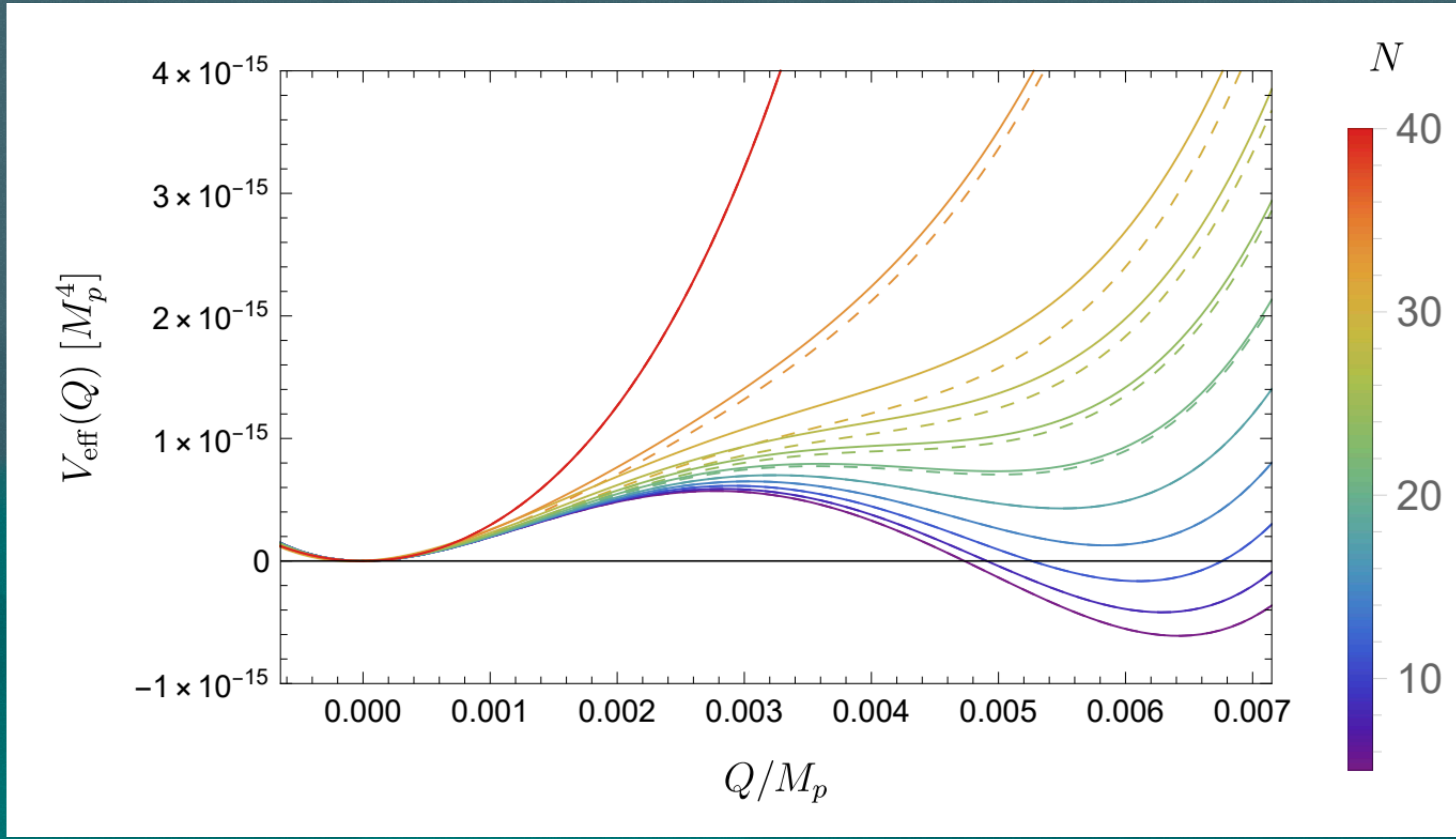
$$\frac{B_Q}{2H^2Q + gQ^2\left(2gQ - \frac{\lambda\dot{\chi}}{f}\right)} \equiv \int d\ln\tilde{k}\, \mathscr{B}_Q\left(N,\tilde{k}\right) \;,$$



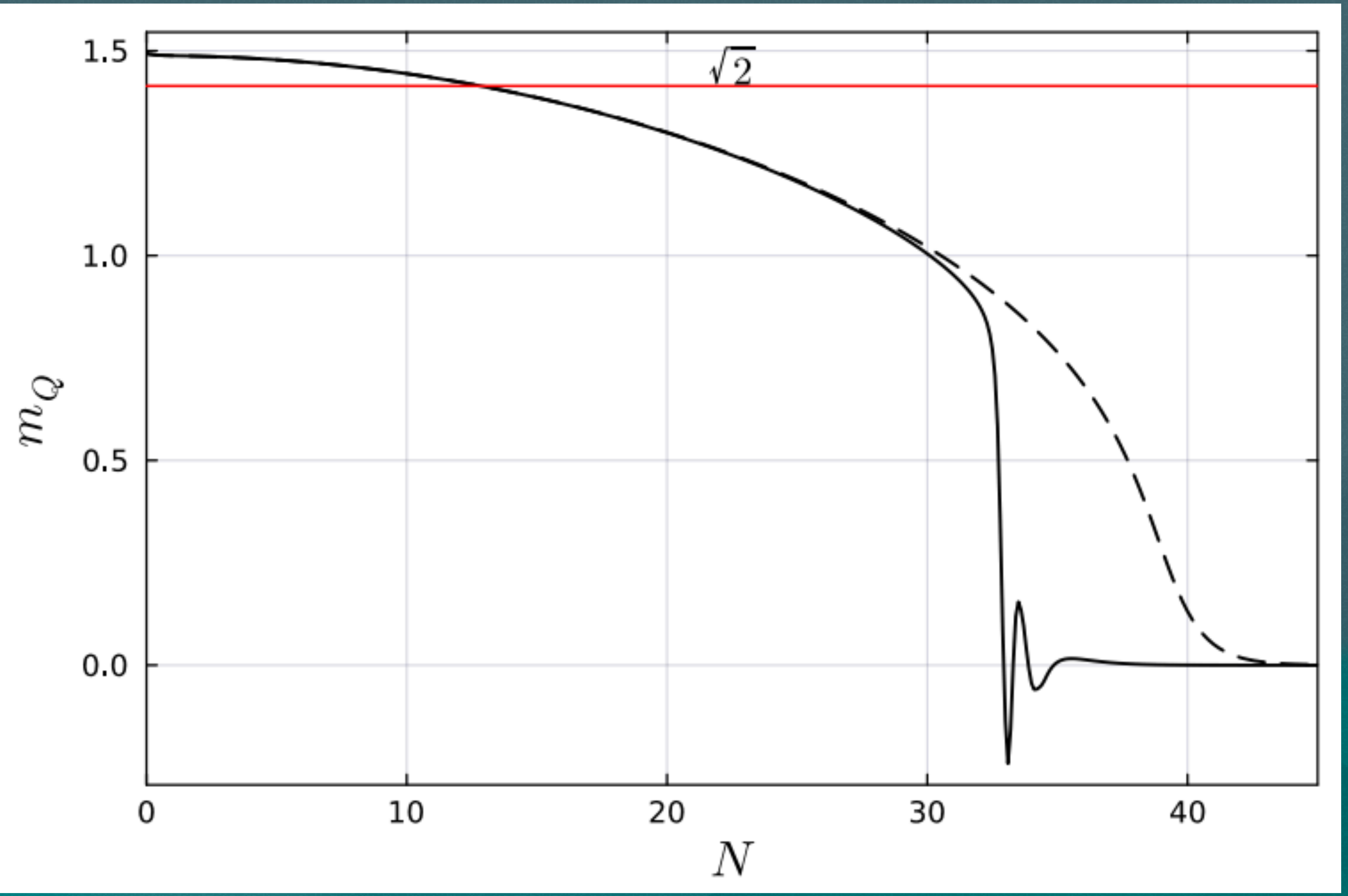
The Effective Potential

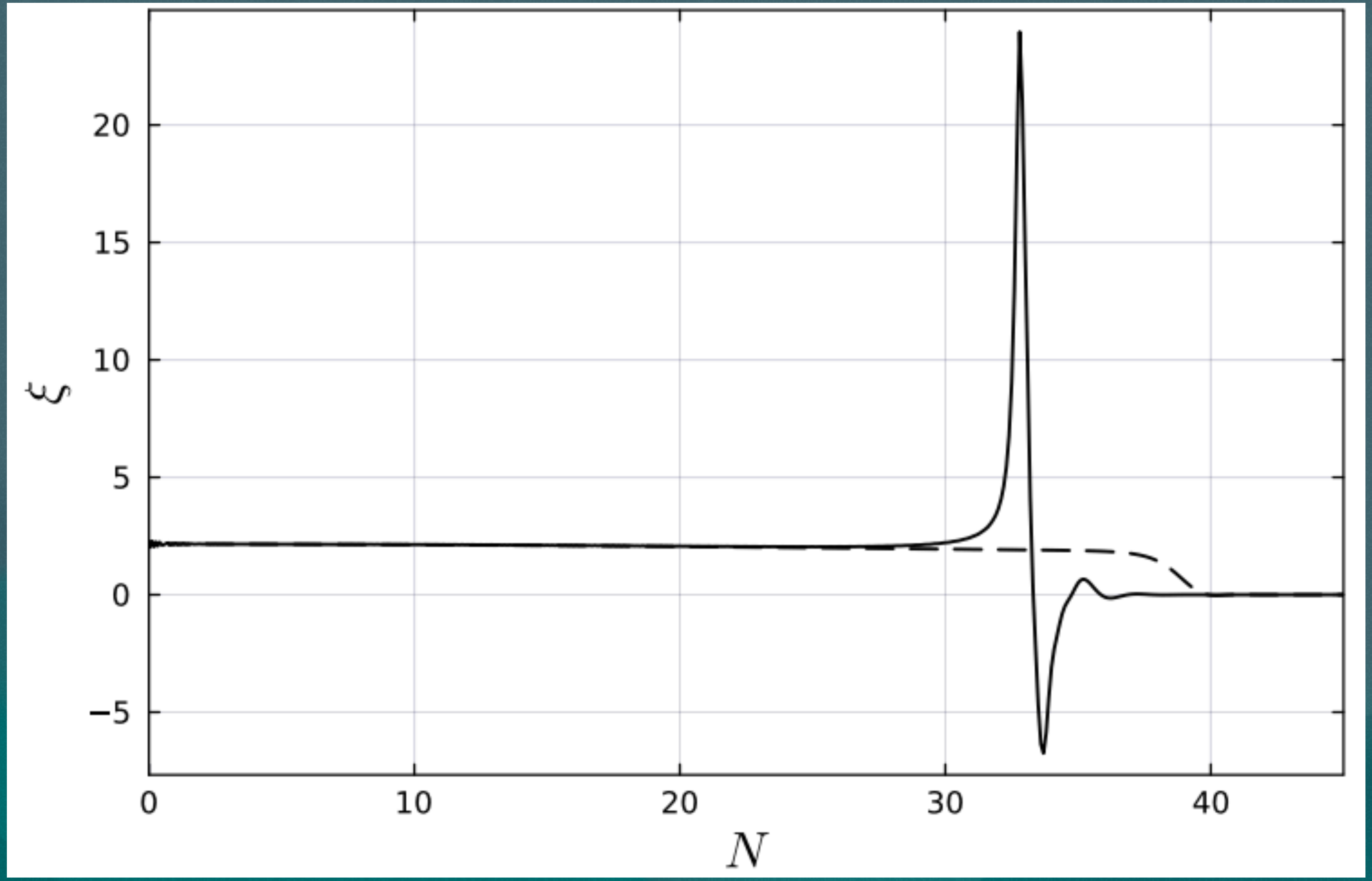
$$V_{\rm eff}(Q) \equiv H^2 Q^2 - \frac{\lambda\, g\, \dot{\chi}}{3f} Q^3 + \left(\frac{1}{2} g^2 + \beta(t)\right) Q^4 \,,$$

$$\beta(t) \equiv \frac{g^2\lambda^2}{6f^2a^2} \int \frac{d^3k}{(2\pi)^3} \frac{k^2\left(k^2 + a^2H^2m_Q^2\right)}{\left(k^2 + 2a^2H^2m_Q^2\right)^2} \left|\hat{X}(\tau,k)\right|^2 .$$

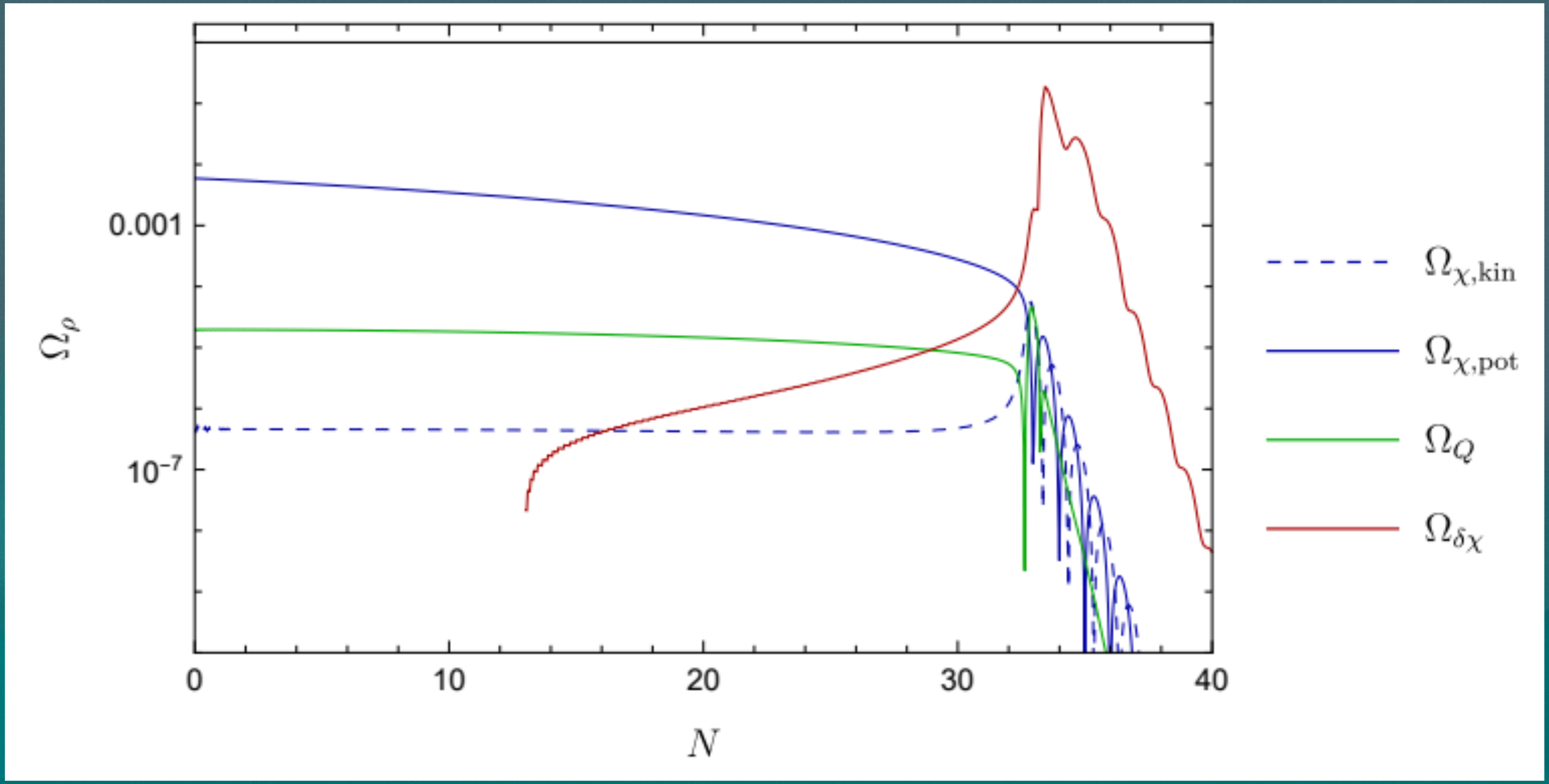


The Effect of the Scalar Instability Backreaction





The Energy Densities



Tensor and Gws

$$\delta g_{ij} = a^2 h_{ij}, \qquad \delta A_i^a = a\, t_i^a$$

$$h_{R,L} \equiv \left(h_+ \pm h_\times\right)/\sqrt{2} \qquad t_{R,L} \equiv \left(t_+ \pm t_\times\right)/\sqrt{2}$$

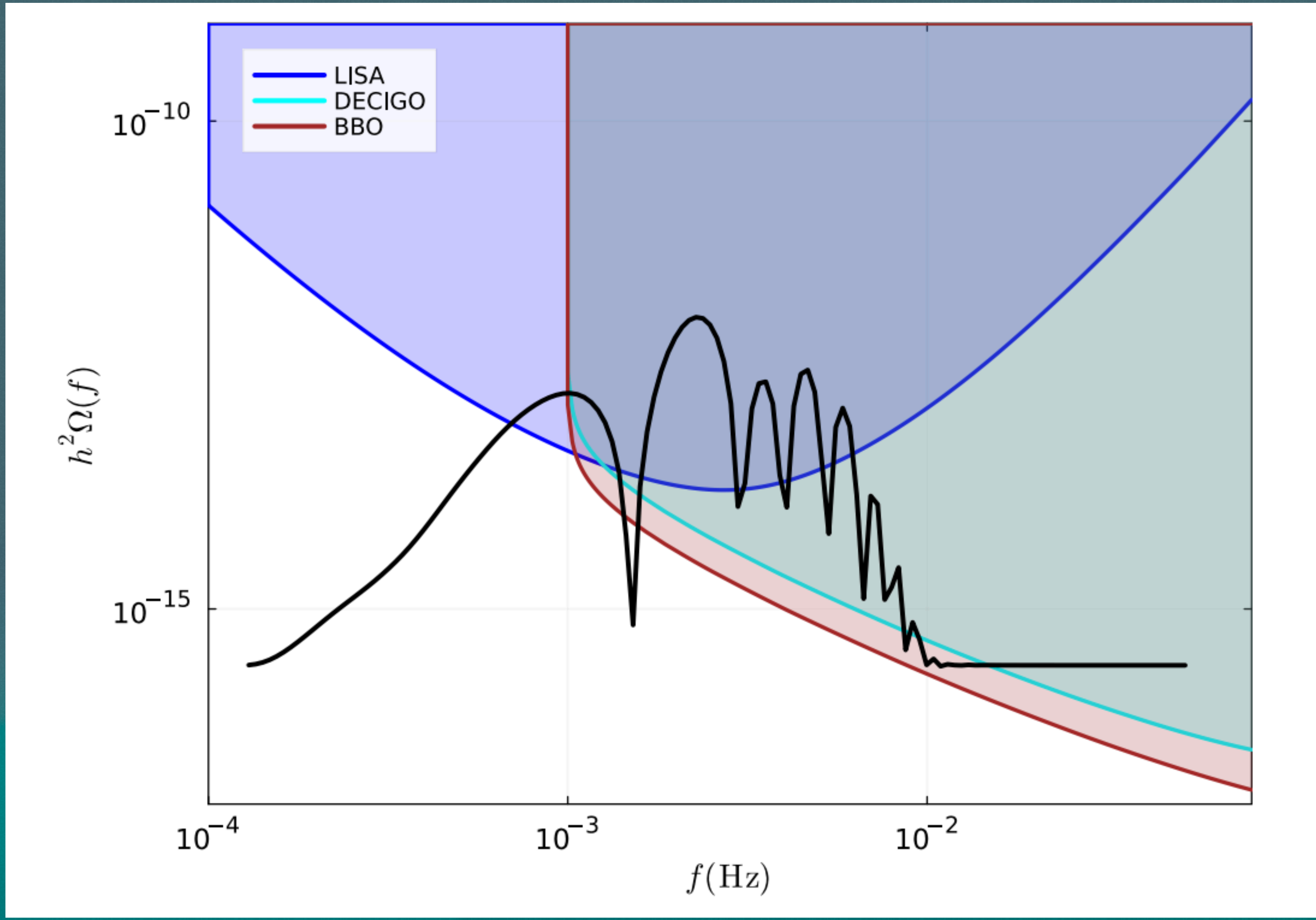
$$h_{R,L} = \frac{\sqrt{2}}{M_{Pl} a}\psi_{R,L}, \qquad t_{R,L} = \frac{1}{\sqrt{2}a} T_{R,L}\,.$$

$$\ddot{T}_{L,R} + H\dot{T}_{L,R} + \left\{\frac{k^2}{a^2} + 2H^2\left[m_Q\xi \pm \frac{k}{aH}\left(m_Q + \xi\right)\right]\right\} T_{L,R} =$$

$$+2H\sqrt{\epsilon_E}\,\dot{\psi}_{L,R} - 2H^2\left[\sqrt{\epsilon_B}\left(m_Q - 2\xi \mp \frac{k}{aH}\right) + \sqrt{\epsilon_E}\right]\psi_{L,R}$$

$$\ddot{\psi}_{L,R} + H\dot{\psi}_{L,R} + \left(\frac{k^2}{a^2} - 2H^2\right)\psi_{L,R} = -2H\sqrt{\epsilon_E}\,\dot{T}_{L,R} + 2H^2\sqrt{\epsilon_B}\left(m_Q \mp \frac{k}{aH}\right) T_{L,R}\,.$$

Peaks and oscillations in the GW spectrum at LISA scale



$$\lambda = 100, \quad f = 3.847 \cdot 10^{-2}\, M_p, \quad g = 4 \cdot 10^{-3}$$

$$\mu = 1.5 \cdot 10^{-3}\, M_p, \quad H = 1.69 \cdot 10^{-5}\, M_p, \quad \chi_{\text{in}} = \frac{\pi f}{2}$$

THANK YOU

… and see you soon in Stockholm! :)