Discrete Teleparallel Quantum Cosmology

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OUTLINE

Teleparallel Gravity and Gauge Structure

Discrete Teleparallelism

Macroscopic Dynamics and Quantization

Statistical Ensemble and Maximally Loacalized State

Conclusion and Future Outlook

Connection Geometries: Christoffel vs. Weitzenböck

- Standard GR assumes a torsion-free, curvature-inducing **Levi-Civita (Christoffel) connection**:

$$\mathring{\Gamma}^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}) \tag{1}$$

- Teleparallel Gravity utilizes the fundamental tetrads h^a_μ to define the curvature-free, torsion-inducing **Weitzenböck connection** [1]:

$$\Gamma^{\lambda}_{\mu\nu} = \mathring{\Gamma}^{\lambda}_{\mu\nu} + K^{\lambda}_{\mu\nu}. \tag{2}$$

- Spacetime **Torsion** emerges as the antisymmetric part of the Weitzenböck connection:

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu} - \Gamma^{\lambda}_{\mu\nu}. \tag{3}$$

[1] Aldrovandi, R. and Pereira, J. (2013). An Introduction to Teleparallel Gravity. Springer.

Gravity as a Translational Gauge Theory & TEGR

- Teleparallel gravity is a gauge theory of the translation group $\mathbf{T}(4)$ [2].
- The tetrad is the gauge field, containing the translational gauge potential $B^a{}_\mu$: $h^A{}_\mu = \partial_\mu x^A + \omega^A{}_{B\mu} x^B + B^A{}_\mu$.
- In the pure Weitzenböck gauge ($\omega^A{}_{B\mu} = 0$), torsion is precisely the gauge curvature of local translations, mimicking the gauge curvature tensor:

$$T^a{}_{\mu\nu} = \partial_\mu h^a{}_\nu - \partial_\nu h^a{}_\mu = \partial_\mu B^a{}_\nu - \partial_\nu B^a{}_\mu \tag{4}$$

- Contracting torsion with the superpotential yields the **Torsion Scalar** T. It is mathematically equivalent to the Levi-Civita Ricci scalar $\mathring{R}$ up to a boundary divergence [3]:

$$\mathring{R} = -T + 2\nabla_\mu T^{\nu\mu}{}_\nu \; ; \tag{5}$$

$$\mathcal{S}_{\text{TEGR}} = \frac{1}{16\pi G} \int d^4x \, h \, T \; . \tag{6}$$

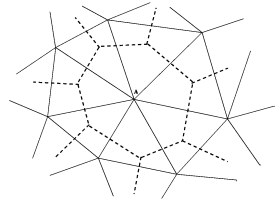
[2] de Andrade, V. C. and Guillen, L. C. T. and Pereira, J. G. (2000). Gravitational Energy-Momentum Density in Teleparallel Gravity. Phys. Rev. Lett. 84, 4533

[3] Maluf, J.W. (2013), The teleparallel equivalent of general relativity. ANNALEN DER PHYSIK, 525: 339-357.

Discrete Teleparallelism and the Burgers Vector

- On a discrete simplicial lattice, torsion is physically equivalent to a structural dislocation concentrated on the 2-surface hinges [4].
- The topological defect is quantified by the **Burgers vector**[5] b^μ, given by the flux of torsion across a dual area bivector $\Sigma^{*\alpha\beta}$:

$$b^\mu = \oint_\Gamma T^\mu_{\alpha\beta}\, d\Sigma^{*\alpha\beta} \implies T^{\mu\nu\rho}_{(d)} T_{(d)\mu\nu\rho} = 24 \frac{b^2_{(d)}}{(\Sigma^*_d)^2} \,. \tag{7}$$



[4] Regge, T (1961). General relativity without coordinates. Nuovo Cim 19, 558571

[5] Kleinert, H. (1989). Gauge Fields in Condensed Matter. World Scientific Publishing Company

The Discrete Teleparallel Action

- Moving around this defect induces a finite translational holonomy:

$$U(b) = \exp(-ib^{\mu} P_{\mu}). \tag{8}$$

- We map the continuous spacetime volume to the discrete invariant volumes of Voronoi polygons:

$$\Omega_d = \frac{1}{6}\Sigma_d \Sigma_d^* \implies \Omega_d T_{(d)} \propto \Sigma_d \left(\frac{b_{(d)}^2}{\Sigma_d^*} \right) \tag{9}$$

- Summing over all hinges d gives the exact discrete TEGR action [6]:

$$\mathcal{S} = \frac{1}{16\pi G} \sum_d \left(\frac{b_{(d)}^2}{\Sigma_d^*} \right) \Sigma_d \tag{10}$$

[6] J G Pereira and T Vargas (2002). Regge calculus in teleparallel gravity. Class. Quantum Grav. 19 4807

Macroscopic FRW Dynamics & Canonical Momenta

- Defining the macroscopic area $\Sigma = \sum_{\text{dis}} \Sigma_d$, macroscopic defect $b = \sum_{\text{dis}} b_d$, and minimal dual area $\Sigma_d^* = l^2 \quad \Longrightarrow \quad \Sigma_d = \gamma l^2$.
- The Hubble expansion induces a relational kinematic mapping for the defect:

$$b = H\Sigma^* \implies b = \Sigma^* \frac{\dot{\Sigma}}{3\Sigma} \quad \text{where} \quad H = \frac{\dot{a}}{a} = \frac{\dot{\Sigma}}{3\Sigma}. \tag{11}$$

- The discrete action reduces to the minisuperspace ADM Lagrangian:

$$L = -\frac{\Sigma^*}{144\pi G}\frac{\dot{\Sigma}^2}{\Sigma} \implies P_\Sigma = -\frac{\Sigma^*}{72\pi G}\frac{\dot{\Sigma}}{\Sigma} \tag{12}$$

- **Mathematical Consequence:** The translational defect is explicitly proportional to the canonical conjugate momentum of the macroscopic spatial area:

$$b = -24\pi G P_\Sigma \tag{13}$$

Polymer Quantization & Effective Friedmann Equation

- The translation group $\mathbf{T}(4)$ is non-compact. On a quantum lattice, continuous momentum operators (b^2) do not exist. We replace them with finite discrete shift operators $\hat{U}_\mu = \exp(i\mu b/l)$ [7].
- The central second-order finite difference operator rigorously yields [8]:

$$b^2 \longrightarrow \frac{1}{(\mu/l)^2}\,\mathrm{Tr}\Big\{\Big[2 - \hat{U}_\mu - \hat{U}_{-\mu}\Big]\Big\} = \frac{4}{(\mu/l)^2}\sin^2\left(\frac{(\mu/l)b}{2}\right) \tag{14}$$

- Applying this to the Hamiltonian defines a critical density $\rho_c = \frac{3}{2\pi G\mu^2 l^2}$.
- Hamilton's equations $\dot{\Sigma} = \{\Sigma, \mathcal{H}\}$ compute the exact area velocity, yielding the bounded **Effective Friedmann Equation**:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \tag{15}$$

[7] Alejandro Corichi, Tatjana Vukasinac, Jose A. Zapata (2006). Polymer quantum mechanics and its continuum limit. Phys.Rev.D 76 044016

[8] Abhay Ashtekar, Tomasz Pawlowski, Parampreet Singh (2006). Quantum Nature of the Big Bang. Phys. Rev. Lett. 96, 141301

Wheeler-DeWitt Equation & The Frozen Formalism

- The classical Super-Hamiltonian features no potential term (manifold is strictly flat):

$$\mathcal{H} = -\frac{36\pi G\Sigma}{\Sigma^*} P_\Sigma^2 = 0 \tag{16}$$

- Quantizing with area eigenvalues $\Sigma_n = nl^2$, i.e, $\hat{\Sigma}\,|\Psi_n\rangle = \Sigma_n\,|\Psi_n\rangle$, the continuous operator $\hat{P}_\Sigma^2 = -\hbar^2\partial^2/\partial\Sigma^2$ becomes a finite difference equation:

$$\frac{36\pi Gn}{l^4}\left(|\Psi_{n+1}\rangle - 2\,|\Psi_n\rangle + |\Psi_{n-1}\rangle\right) = 0 \tag{17}$$

- **The Paradox:** Evaluating the expansion rate on the physical state yields $\langle\Psi|\hat{H}_\Sigma^2|\Psi\rangle \propto \langle\Psi|\hat{P}_\Sigma^2|\Psi\rangle = 0$. The global macroscopic universe appears mathematically stationary [9].

[9] K.V. Kuchar (2011). Time and interpretations of quantum gravity. Int.J.Mod.Phys.D 20 3-86

The Universe as a Statistical Ensemble

- **General Proposal:** We must evaluate the statistical mechanics of the local discrete hinges, modeling the universe as a tensor product of independent cells [10]:

$$|\Psi_{\mathrm{FRW}}\rangle = \bigotimes_{d=1}^{N} |\Psi_n\rangle_d \tag{18}$$

- The local partition function reduces to a Gaussian integral over zero-point torsion fluctuations ($\alpha = \gamma/16\pi G$):

$$Z_d = \int_{-\infty}^{+\infty} \exp\left[-\alpha (b_d^j)^2\right] db_d^j \tag{19}$$

[10] T.Padmanabhan (2009). Thermodynamical Aspects of Gravity: New insights. Rep. Prog. Phys. 73 046901

Local Partition Function & Maximally Loacalized State

- The expansion rate squared is the variance of this localized distribution:

$$\langle H_d^2 \rangle_{\max} = \frac{1}{l_0^4} \frac{\int (b_d^j)^2 e^{-\alpha (b_d^j)^2} db_d^j}{\int e^{-\alpha (b_d^j)^2} db_d^j} = \frac{1}{l_0^4} \left(\frac{1}{2\alpha} \right) \tag{20}$$

$$(H_d)^2_{\max} = \frac{8\pi G}{\gamma l_0^4} \sim 10^{130}\ \mathrm{m}^{-2}\mathrm{s}^{-2} \tag{21}$$

- The maximum value of local torsional defect or local Hubble parameter is associated with the simplex with the size of minimal length scale ($l \sim l_0$).

Ensemble Integration & The Cosmological Constant

- Today, the ensemble is thermalized over a vast spectrum of areas up to the IR causal horizon cutoff $L^2 \sim R_H^2$. We integrate over the geometric phase space:

$$\mathcal{N} = \int_{l_0^2}^{L^2} l^{-4}\, d(l^2) \int_{-\infty}^{+\infty} (b_d^j)^2 e^{-\alpha (b_d^j)^2}\, db_d^j \tag{22}$$

- The area integral evaluates to $\frac{L^2 - l_0^2}{l_0^2 L^2}$:

$$\langle H_d^2 \rangle_{\text{today}} = \langle H_d^2 \rangle_{\max} \cdot \left(\frac{l_0}{L} \right)^2 \approx (H_d)_{BB}^2 \cdot 10^{-122} \tag{23}$$

- **Conclusion:** Does the cosmological constant $\Lambda \approx 3\langle H_d^2 \rangle$ emerge naturally [11] as the residual ensemble average of fundamental teleparallel quantum fluctuations??

[11] T. Padmanabhan, Hamsa Padmanabhan (2014). Cosmological Constant from the Emergent Gravity Perspective. Int. Jour. Mod. Phys. D, Vol. 23, No. 6 1430011

Conclusions: A Unified Teleparallel Cosmology

- **Translational Gauge Geometry:** Mapping the Weitzenböck connection to a simplicial lattice successfully identifies the discrete Burgers vector as the exact canonical conjugate momentum to the macroscopic area.
- **Singularity Resolution:** Polymer quantization of the non-compact continuous translation group inherently bounds the geometric expansion rate. The classical singularity is dynamically replaced by a deterministic **Quantum Bounce**.
- **Emergent Vacuum Energy:** The global Wheeler-DeWitt stationarity (the frozen formalism, $\langle \hat{H}^2 \rangle = 0$) is resolved by transitioning to the local statistical mechanics of independent geometric simplices.
- **Resolution of Λ??** It emerges structurally as the residual thermodynamic ensemble average of the discrete zero-point torsion fluctuations or the local torsional defect, naturally suppressed by the UV/IR phase-space ratio $(l_0/L)^2 \sim 10^{-122}$.

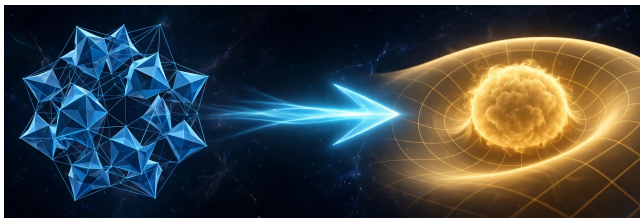
Is it even close to the fundamental theory?

No !!!

Spacetime may **not** be fundamentally discrete ...

We need an effective theory with continuous spacetime[12] for $|x| > l_0$:

$$g_{ij}(x) \sim \begin{cases} g_{ij} & \text{when } |x| > l_0; \\ g_{ij} - \frac{D}{4 l_0^2 x_i x_j} & \text{when } |x| \leq l_0. \end{cases}$$



[12] P. F. de Cordoba, J. M. Isidro and R. Roy (2024). Spacetime metric from quantum-gravity corrected Feynman propagators. Int. J. Geom. Meth. Mod. Phys. **21** (07)

Future Prospects: Phenomenological Compatibility

- **Cosmological Perturbation Theory:** We must derive the perturbed effective holonomic action to calculate the scalar and tensor power spectra. Specifically, we must verify if the discrete translational gauge preserves scale-invariance for scalar perturbations.
- **Modified Dispersion Relations (MDRs):** The granularity of the teleparallel lattice may induce Violation of Lorentz Invariance at ultra-high energies. This must be checked against the propagation speeds of high-frequency Gravitational Waves (GWs).
- **Primordial Non-Gaussianities:** The geometric phase transition from the crushed UV condensate to the thermalized IR ensemble must be analyzed for distinct bispectrum signatures (f_{NL}) and potential anomalous entropy production.

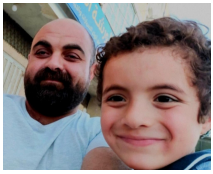
Thank You !!

Question ??
Suggestion ??
Criticism ???

Graphic Content Warning

The following pages contain highly sensitive and graphic depictions of war.

While I regret any distress these images may cause, they represent an inescapable reality that demands to be shown.



Rakan Hamdi Yahya Al-Najjar (Israeli airstrike)
23 May 2025 (10 years)



Omar Ahmed Abdel Naser Shamlakh (Israeli airstrike)
8 Oct 2023 (4 months)



Yaqeen Khader Fathi Hammad (Israeli airstrike)
23 May 2025 (12 years)



Lana Ashraf Yasser Al-Ghusein (Israeli airstrike)
July 2025 (10 years)

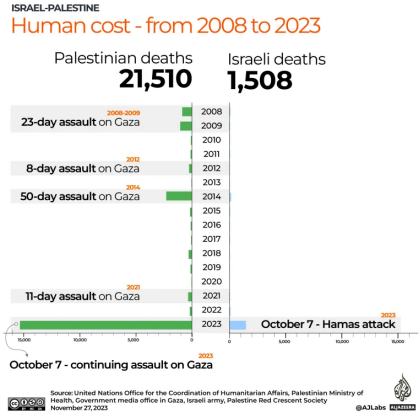


Khaled Mohammed Fathi Jabr (Israeli gunfire)
17 October 2023 (7 years)

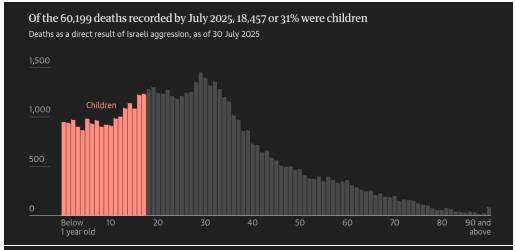


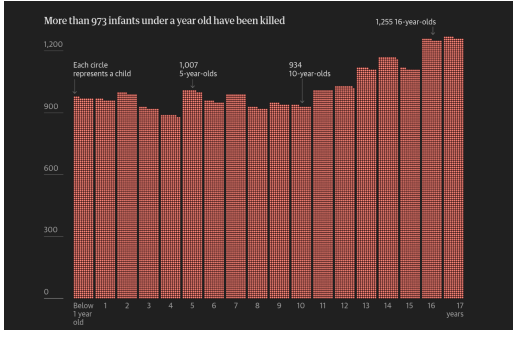
Hind Rami Iyad Rajab (Israeli gunfire)
29 Jan 2024 (5 years)

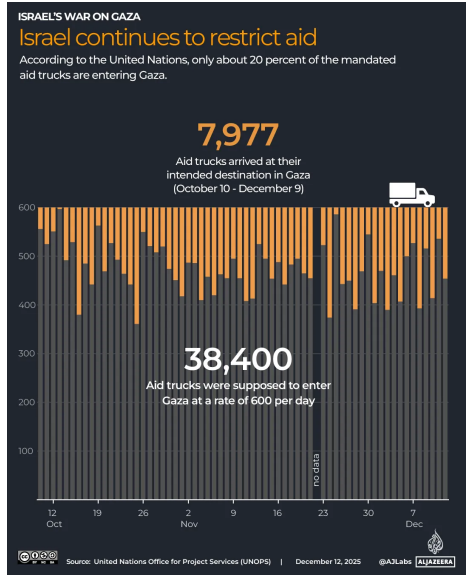

And 21,000 more ...



OCHA United Nations Office for the Coordination of Humanitarian Affairs







source : www.theguardian.com

1917
Pre-British Mandate Palestine

- Palestinian
- Jewish

On October 31, 1917, British forces conquered Palestine from the Ottoman-Turks, ending 1,400 years of Islamic rule over the region.

Before the British Mandate in Palestine, Jews made up around **six percent of the total population**.



Source: McCarthy, The Population of Palestine (1990)

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1947
UN Partition Plan (Resolution 181)

- Arab State | **45%**
- Jewish State | **55%**
- Jerusalem under international control

Following the end of WWII, the newly formed United Nations proposed a plan that would grant **55 percent of historic Palestine to a Jewish state and 45 percent to a non-contiguous Arab one.**

This plan was never implemented on the ground.



Voted For (33)
Australia
Belgium
Bolivia
Brazil
Byelorussian SSR
Canada
Costa Rica
Czechoslovakia
Denmark
Dominican Republic
Ecuador
France
Guatemala
Haiti
Iceland
Liberia
Luxembourg
Netherlands
New Zealand
Nicaragua
Norway
Panama
Paraguay
Peru
Philippines
Poland
Sweden
Ukrainian SSR
Union of South Africa
US
USSR
Uruguay
Venezuela

Voted Against (13)
Afghanistan
Cuba
Egypt
Greece
India
Iran
Iraq
Lebanon
Pakistan
Saudi Arabia
Syria
Turkey
Yemen

Abstained (10)
Argentina
Chile
China
Colombia
El Salvador
Ethiopia
Honduras
Mexico
United Kingdom
Yugoslavia

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1948
Palestinians expelled

- Palestinian
- Israeli
- Armistice "Green" Line (Created in 1949)

Zionist military forces expelled at least **750,000 Palestinians and captured 78 percent of historic Palestine.**

The remaining 22 percent was divided into the West Bank and Gaza Strip.



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1967
Israel occupies Gaza and the West Bank

- Palestinian (under Israeli occupation)
- Israeli
- Egyptian and Syrian land occupied by Israel

During the June 1967 War, Israel occupied all of historic Palestine and expelled a further **300,000 Palestinians** from their homes.

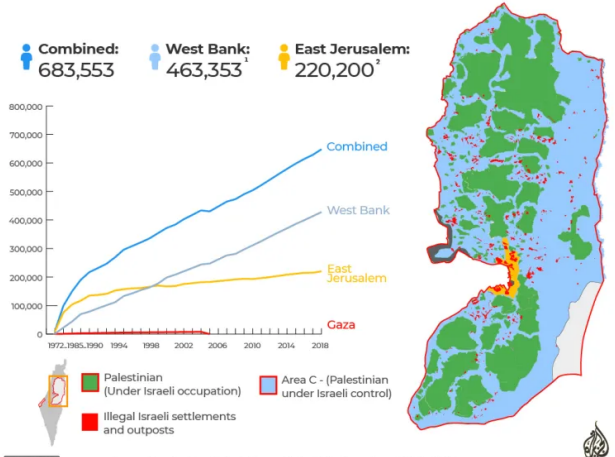


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OCCUPIED WEST BANK

Illegal Israeli settlements

Settlements are Jewish compounds built on Palestinian land. Between 600,000 and 750,000 Israeli settlers live in at least 290 settlements and outposts in the occupied West Bank and East Jerusalem. **Israeli settlements are illegal under international law.**



Foundation for Middle East Peace (1972 - 1999), PeaceNow (2000 - 2018).
Sources: Israel Ministry of Interior (2020)[1], Jerusalem Institute for Policy Research (2020)[2].

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Home » Articles » Mini-Israel in India: A Jewish Enclave or Strategic Infiltration?

Mini-Israel in India: A Jewish Enclave or Strategic Infiltration?

By Syed Hammad Ali / December 29, 2025

Syed Hammad discusses the presence of Israeli soldiers in Northern India, particularly in areas like Kasol and Dharamkot. He focuses on the intertwining of Israeli military interests and Indian tourism, suggesting that these locations have become enclaves for Israeli culture and operations. The strategic significance of these areas for both countries, particularly regarding intelligence and military collaboration, is also emphasized.

Israeli Whites-Only Cafés in India's Himalayan Region Expose the Contradiction at the Heart of Modi's Israel Policy

OCTOBER 6, 2026

A viral report published September 29, 2026, documenting cafés along Himachal Pradesh's so-called "Hummus Trail" that bar Indian customers from entering their own country has produced outrage, official silence, and an awkward political moment for a Hindu nationalist government that has made its embrace of Israel one of its defining diplomatic postures.



El pueblo unido jamás será vencido !!!

Israeli protesters in Tel Aviv demand an end to war on Gaza

Thousands gather demanding an end to the war and the release of Israeli captives in Gaza.

Save Share Add Al Jazeera on Google



A child holds a placard in Tel Aviv, Israel [Ronen Zvulun/Reuters]

10 May 2025

This carnival of protest on the streets of Delhi has shaken Narendra Modi like nothing before

Meena Kandasamy



The singing, dancing and sheer joy in the face of police brutality at the protests led by the Cockroach Janta party show people no longer fear India's regime



Demonstrators and police at a protest march organised by India's Cockroach Janta party, Delhi, 12 October 2026. Photograph: Ritesh Shukla/Getty Images