Scalar perturbations in scalar–torsion gravity: toward α-attractors

Reduction in flat gauge and comparison with unitary gauge

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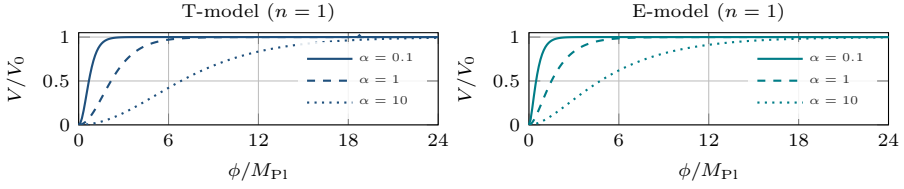
α-attractors: from the kinetic pole to the plateau

A second-order kinetic pole stretches the canonical field and flattens the potential; for $N \gg 1$, the predictions are universal at leading order.

$$\mathcal{L}_{\rm kin} = -\frac{(\partial\varphi)^2}{2\,(1-\varphi^2/\varphi_b^2)^2}, \quad \varphi_b = \sqrt{6\alpha}\,M_{\rm Pl}, \qquad \phi = \varphi_b \,\mathrm{arctanh}\left(\frac{\varphi}{\varphi_b}\right). \tag{1}$$

The pole lies at infinite proper distance: $|\varphi| < \varphi_b$ maps to $-\infty < \phi < \infty$, and a regular potential flattens into a plateau. For example,

$$V_T = V_0 \tanh^{2n}\left(\frac{\phi}{\sqrt{6\alpha}\,M_{\rm Pl}}\right), \quad V_E = V_0\left(1 - e^{-2\phi/(\sqrt{6\alpha}M_{\rm Pl})}\right)^{2n}, \quad n_{\rm s} \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{12\,\alpha}{N^2}. \tag{2}$$



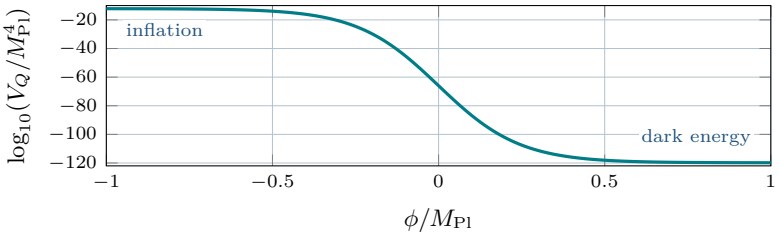
α sets where the plateau begins; for $\alpha = n = 1$, the E-model is the Starobinsky model. N is the number of e-folds.

Ref: Kallosh & Linde (2013); Kallosh, Linde & Roest, JHEP (2013, 2014); Galante et al., PRL (2015).

Quintessence: one field, two epochs

We consider an exponential-type potential motivated by supergravity. After canonicalizing the pole kinetic term,

$$V_Q(\phi) = \lambda M_{\rm Pl}^4 \exp\left[-n \tanh\left(\frac{\phi}{\sqrt{6\alpha}\, M_{\rm Pl}}\right)\right], \qquad V_Q(\mp\infty) = \lambda M_{\rm Pl}^4 e^{\pm n}. \tag{3}$$



Illustrative energy-density ratio: $V_{\rm inf}/V_{\rm DE} \sim 10^{108}$; for $\alpha = 10^{-2}$, $\mu = \sqrt{6\alpha}\, M_{\rm Pl} \simeq 0.245\, M_{\rm Pl}$ sets the tanh field scale. The right tail can support late acceleration if reached. Since V_Q has no minimum, reheating requires an additional non-oscillatory mechanism (not shown).

Ref: Aresté Saló et al., *Phys. Rev. D* 103, 123535 (2021); Kallosh et al., (2026).

Scalar–torsion theory

The tetrad defines the metric $g_{\mu\nu} = \eta_{AB} e^A{}_\mu e^B{}_\nu$, with $e := \det(e^A{}_\mu) = \sqrt{-g}$.

What changes in primordial perturbations if gravity is described by *modified teleparallel gravity*?

In the pure-tetrad formulation, with spin connection $\omega^A{}_{B\mu} = 0$, the torsion tensor is

$$T^\rho{}_{\mu\nu} = e_A{}^\rho(\partial_\mu e^A{}_\nu - \partial_\nu e^A{}_\mu), \quad T_\mu := T^\rho{}_{\rho\mu}, \quad T = \tfrac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \tfrac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - T_\mu T^\mu, \tag{4}$$

$$\mathring{R} = -T + \frac{2}{e}\partial_\mu(eT^\mu), \qquad T_{\mathrm{FLRW}} = 6H^2. \tag{5}$$

$$S = \int \mathrm{d}^4x\, e\,[f(T,\phi) + P(\phi)X], \qquad \text{TEGR: } f = -\tfrac{1}{2}M_{\mathrm{Pl}}^2 T - V(\phi). \tag{6}$$

Convention: $X := -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$. The scalar boost β enters T and imposes a constraint. Covariantly, the tetrad and inertial spin connection transform together.

Today
scalar perturbations of $f(T,\phi) + PX$ in flat gauge

→

Next
choose an α-attractor model; test stability

→

Then
compare slow-roll and numerical spectra

Ref: Zambrano et al., *Int. J. Mod. Phys. D* 35, 2550093 (2026); González-Espinoza and Otálora, *Phys. Lett. B* 809, 135696 (2020); Krššák et al., *Class. Quantum Grav.* 36, 183001 (2019).

Flat gauge: metric and boosted tetrad

ADM metric in flat gauge: $h_{ij} = a^2\delta_{ij}$, $\phi = \phi_0 + Q$,

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t), \qquad N = 1+\alpha, \quad N^i = \partial^i\chi, \quad N_i = a^2\partial_i\chi. \tag{7}$$

Note: here α denotes the lapse perturbation, *not* the attractor parameter.

ADM tetrad: $\bar{e}^0{}_\mu = (N, 0)$, $\bar{e}^a{}_\mu = (N^a, h^a{}_i)$, with $N^a = a\delta^a_i N^i$ and $h^a{}_i = a\delta^a_i$.

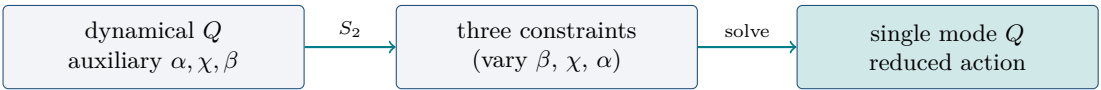
Local boost to second order:

$$\begin{aligned} e^A{}_\mu &= \bar{e}^A{}_\mu + \lambda^A{}_B\bar{e}^B{}_\mu + \tfrac{1}{2}\lambda^A{}_C\lambda^C{}_B\bar{e}^B{}_\mu + \mathcal{O}(\lambda^3), \\ \lambda^A{}_B &= \begin{pmatrix} 0 & \lambda_b \\ \lambda^a & 0 \end{pmatrix}, \qquad \lambda_a = a\partial_a\beta. \end{aligned} \tag{8}$$

Variables

Q: the dynamical mode. α, χ, β: auxiliary variables eliminated by their constraints. In this pure-tetrad parametrization, do not omit β before varying: it contributes to T, and its equation imposes a constraint.

Strategy: from four variables to one mode



We expand $S = S^{(0)} + S^{(1)} + S^{(2)} + \cdots$ around the background.

Evaluating all quantities at $(T, \phi) = (6H^2, \phi_0)$, the background equations are

$$\begin{aligned} f - 12H^2 f_T - \tfrac{1}{2} P\dot{\phi}_0^2 &= 0, \\ P(\ddot{\phi}_0 + 3H\dot{\phi}_0) + \tfrac{1}{2} P_\phi \dot{\phi}_0^2 - f_\phi &= 0, \\ P\dot{\phi}_0^2 &= 4 f_T \dot{H} + 4H\dot{f}_T. \end{aligned} \tag{9}$$

On the background, S_1 vanishes up to a boundary term. Varying $S_2[Q, \alpha, \chi, \beta]$ gives the three constraints.

Quadratic action and constraints

The second-order action, with $\Theta := 3H\,\alpha + \partial^2(\chi + \beta)$, is

$$S_2 = \int \mathrm{d}t\,\mathrm{d}^3x\,a^3 \Bigg\{ \frac{P}{2}\left[\dot{Q}^2 - \frac{(\partial Q)^2}{a^2}\right] + P_\phi \dot{\phi}_0 Q\dot{Q} - P\dot{\phi}_0\,\alpha\,\dot{Q} + A\,\alpha^2 + B\,\alpha\,Q + E\,Q^2 + (4Hf_T\,\alpha + P\dot{\phi}_0 Q)\,\partial^2\chi + 8H^2 f_{TT}\Theta^2 - 4Hf_{T\phi}\,Q\,\Theta \Bigg\}, \tag{10}$$

$$A = 6H^2 f_T + \tfrac{1}{2}P\dot{\phi}_0^2, \qquad B = f_\phi - \tfrac{1}{2}P_\phi\dot{\phi}_0^2, \qquad E = \tfrac{1}{2}f_{\phi\phi} + \tfrac{1}{4}P_{\phi\phi}\dot{\phi}_0^2. \tag{11}$$

For $k \neq 0$ and on the branch $Hf_T f_{TT} \neq 0$, variation with respect to β, χ, and α gives

$$4Hf_{TT}\Theta = f_{T\phi}Q, \quad 4Hf_T\,\alpha + P\dot{\phi}_0 Q = 0 \;\Rightarrow\; \alpha = -\frac{P\dot{\phi}_0}{4Hf_T}Q, \quad \Theta = \frac{f_{T\phi}}{4Hf_{TT}}Q. \tag{12}$$

Physical meaning of the boost constraint

On the branch $f_{TT} \neq 0$, this constraint is equivalent to $\delta f_T = f_{TT}\,\delta T^{(1)} + f_{T\phi}Q = 0$. The branch $f_{TT} = 0$ requires a separate analysis; it cannot be obtained by taking this limit.

Reduced action, effective mass, and equation for Q

After substituting the constraints and integrating by parts, the reduced action is

$$S_f^{(2)} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3 P\left[\dot{Q}^2 - \frac{(\partial Q)^2}{a^2} - M_Q^2 Q^2\right], \tag{13}$$

$$\begin{aligned} M_Q^2 = {} & \frac{1}{a^3 P}\frac{\mathrm{d}}{\mathrm{d}t}\left[a^3\left(P_\phi\dot{\phi}_0 + \frac{P^2\dot{\phi}_0^2}{4Hf_T}\right)\right] - \frac{f_{\phi\phi}}{P} - \frac{P_{\phi\phi}\dot{\phi}_0^2}{2P} + \frac{f_{T\phi}^2}{Pf_{TT}} \\ & - \frac{3P\dot{\phi}_0^2}{4f_T} - \frac{P^2\dot{\phi}_0^4}{16H^2 f_T^2} + \frac{\dot{\phi}_0}{2Hf_T}\left(f_\phi - \tfrac{1}{2}P_\phi\dot{\phi}_0^2\right). \end{aligned} \tag{14}$$

The corresponding field equation is

$$\ddot{Q} + \left(3H + \frac{\dot{P}}{P}\right)\dot{Q} - \frac{\partial^2 Q}{a^2} + M_Q^2 Q = 0. \tag{15}$$

TEGR check and dictionary to unitary gauge

For $f = -\frac{1}{2}M_{\rm Pl}^2 T - V(\phi)$, $P = 1$: $f_{T\phi} \equiv 0$, the boost constraint is trivial (β is pure gauge), and

$$S^{(2)}_{\rm GR} \simeq \frac{1}{2}\int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left[\dot{Q}^2 - \frac{(\partial Q)^2}{a^2} - \left(V_{\phi\phi} - \frac{1}{a^3 M_{\rm Pl}^2}\frac{\mathrm{d}}{\mathrm{d}t}\frac{a^3\dot{\phi}_0^2}{H}\right) Q^2\right] : \tag{16}$$

we recover canonical kinetics, $c_{\rm s} = 1$, and the standard mass, as expected in GR.

For $h_{ij} = a^2 e^{2\zeta}\delta_{ij}$, $\mathcal{R} = \zeta - \dfrac{H}{\dot{\phi}_0}\delta\phi$ is invariant. In flat gauge ($\zeta = 0$) and unitary gauge ($\delta\phi = 0$),

$$\boxed{Q = -\frac{\dot{\phi}_0}{H}\mathcal{R}}. \tag{17}$$

The auxiliary variables α, χ, β transform with $\xi^0 = Q/\dot{\phi}_0$.

This requires $H \neq 0$ and $\dot{\phi}_0 \neq 0$.

Ref: Maldacena, *JHEP* 05 (2003) 013.

Action for $\mathcal{R}$: normalization, sound speed, and mass

With $y \equiv \dot{\phi}_0/H$, the relation $Q = -y\mathcal{R}$ gives

$$S_u^{(2)} = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 Q_\mathrm{s} \left[\dot{\mathcal{R}}^2 - \frac{(\partial\mathcal{R})^2}{a^2} - m_\mathcal{R}^2 \mathcal{R}^2\right], \quad Q_\mathrm{s} = \frac{PX}{H^2}, \quad c_\mathrm{s}^2 = 1, \tag{18}$$

$$m_\mathcal{R}^2 = M_Q^2 + \frac{\ddot{y}}{y} + \left(3H + \frac{\dot{P}}{P}\right)\frac{\dot{y}}{y}. \tag{19}$$

Origin of the mass

- In TEGR (for any $P(\phi)$), $m_\mathcal{R}^2 = 0$; the constant adiabatic solution is recovered in the superhorizon limit.
- If $f_{TT} \neq 0$, in general $m_\mathcal{R}^2 \neq 0$; it vanishes if f_T is constant along the background. The correction relative to TEGR enters through the boost constraint.
- It reproduces the coefficients Q_s, $c_\mathrm{s}^2 = 1$, and the mass m^2 in the unitary-gauge result of González-Espinoza and Otálora.

Ref: González-Espinoza and Otálora, *Phys. Lett. B* 809, 135696 (2020).

Mukhanov–Sasaki equations in both gauges

The canonical variables in the two gauges are the same mode up to a sign. For $P > 0$ and $H\dot{\phi}_0 \neq 0$, define

$$\begin{aligned} z_f &= a\sqrt{P}, \quad v_f = z_f Q, \qquad z = \frac{\dot{\phi}_0}{H} z_f, \quad v = z\mathcal{R}, \\ z^2 &= a^2 P \frac{\dot{\phi}_0^2}{H^2} = 2a^2 Q_{\mathrm{s}}, \qquad v_f = -v. \end{aligned} \tag{20}$$

The canonical equations in conformal time are

$$\boxed{\begin{aligned} v_{f,k}'' + \left(k^2 + a^2 M_Q^2 - \frac{z_f''}{z_f}\right) v_{f,k} = 0, \\ v_k'' + \left(k^2 + a^2 m_{\mathcal{R}}^2 - \frac{z''}{z}\right) v_k = 0. \end{aligned}} \tag{21}$$

The two equations are equivalent because $v_f = -v$. If $m_{\mathcal{R}}^2 = 0$, the standard curvature-mode form $v'' + (k^2 - z''/z)v = 0$ is recovered.

Primes denote derivatives with respect to conformal time η, where $\mathrm{d}\eta = \mathrm{d}t/a$.

Summary and next steps

- The flat-gauge reduction keeps the boost explicit, yields one propagating scalar mode, and reproduces the unitary-gauge result of González-Espinoza and Otálora.
- The quadratic action has $c_{\mathrm{s}}^2 = 1$, $Q_{\mathrm{s}} = PX/H^2$, and an effective mass that can be nonzero beyond TEGR; the expected GR limit is recovered.
- Next, apply the framework to a specified α-attractor potential and torsion coupling, and analyze the background and stability.
- Compute the scalar spectrum and observables in slow-roll, then compare with numerical evolution of the background and perturbation modes beyond slow-roll; include tensor modes to predict r.

thank you!



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