UNIVERSIDAD DE LA FRONTERA

Symmetry-Driven k-essence Cosmological Dynamics

Andrés Lueiza-Colipí, Nikolaos Dimakis, Genly Leon, Andronikos Paliathanasis

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Some Open Questions in Cosmology

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Question

Can a single field, selected by a geometric rule, address some of these issues at once?

k-essence Theory: A Generalization of a Usual Scalar Field

Canonical scalar field

$$S_\phi = \int d^4x\sqrt{-g}\left(\underbrace{\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi}_{X} - V(\phi)\right)$$

k-essence

$$S_k = \int d^4x\sqrt{-g}\,F(\phi, X)$$

k-essence can account for the **inflationary epoch**, **dark energy** and **a dark matter component**, and it can even **unify** them.

Selection of the Theory via Scaling Symmetry

Theory under consideration

$$S = \int d^4x\sqrt{-g}\left[f_1 R + f_2(\phi, X)\right], \qquad X = \tfrac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi$$

A simple function: the inverse squared potential

$$f_2(\phi, X) = \frac{1}{\phi^2}\,F(X), \qquad \text{has the scaling symmetry} \qquad \xi = N\frac{\partial}{\partial N} + \frac{a}{3}\frac{\partial}{\partial a} + \phi\frac{\partial}{\partial\phi}$$

Particular choice: power-law k-essence

$$\boldsymbol{F(X) = \alpha X + \beta X^n \implies f_2(\phi, X) = \frac{1}{\phi^2}(\alpha X + \beta X^n)}$$

The Minimal Symmetry-Driven Model

Model

$$f_1 = f_0, \qquad f_2(\phi, X) = \alpha X + \beta_n(\phi) X^n, \quad \beta_n(\phi) = \beta e^{(n-1)\phi}$$

- Canonical kinetic term αX plus a power law correction; the exponential coupling is fixed by the symmetry.
- β can be absorbed by shifting ϕ: only its sign matters.
- $n = 0, 1$ reduce to a canonical scalar field.
- Free parameters: n and $k \equiv 2\sqrt{f_0/(3\alpha)} > 0$.
- The system has an exact vacuum solution, that can be obtained via the Noether charge and corresponds to a power-law expansion

Dynamical Analysis

$$x = \frac{\sqrt{\alpha}\,\dot{\phi}}{2\sqrt{3f_0}\,H}$$

$$y = \frac{(2n-1)}{6f_0H^2}\beta_n(\phi)X^n$$

$$\boldsymbol{\Omega_m = 1 - x^2 - y,}$$

$$\boldsymbol{w = x^2 + \frac{y}{2n-1}}$$

System ($' = d/d\ln a$)

The field equations in these dimensionless variables are

$$\boldsymbol{A(x,y)x' = P(x,y)}$$

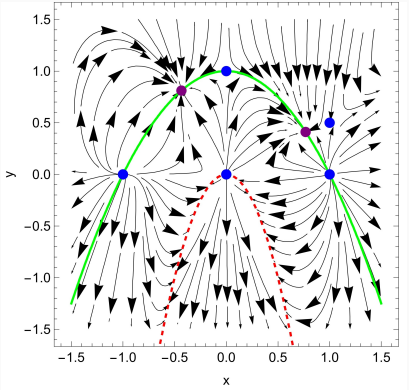
$$\boldsymbol{A(x,y)y' = Q(x,y)}$$

with $A(x,y) = x^2 + ny$.

$A(x,y) = 0$ is the impasse surface: there the behavior of the vector field reverts.

The cases $n = 1, 0, \frac{1}{2}$ are excluded from our analysis.

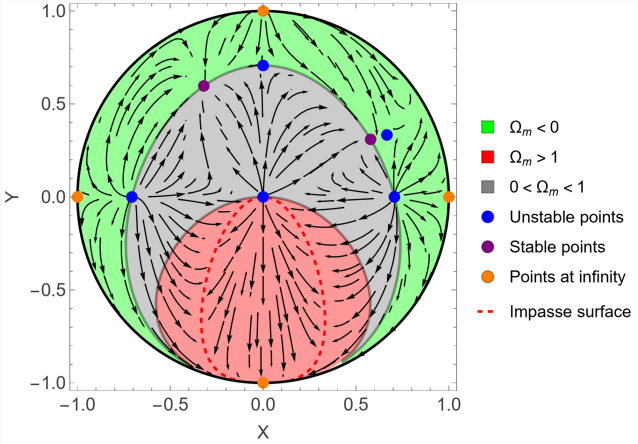
Phase Portrait



Phase space portrait for the values $n = 1/4, k = 1$

- The green curve is the $\Omega_m = 0$ boundary and is never crossed by trajectories.
- In red is plotted the impasse surface, its stability depends on n, k.

Poincaré Compactification: Studiying the Infinity



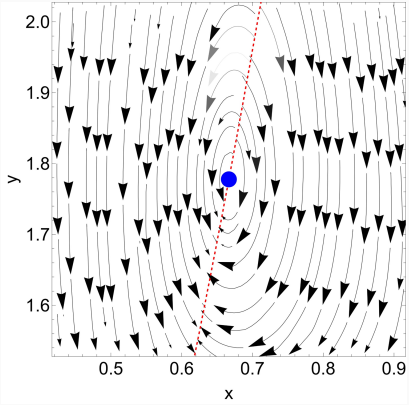
Poincaré disc $X = \rho\cos\theta$, $Y = \rho\sin\theta$, $n = 1/4$, $k = 1$.

- Change of variables

$$x = \frac{\rho\cos\theta}{\sqrt{1-\rho^2}}, y = \frac{\rho\sin\theta}{\sqrt{1-\rho^2}}$$

- The system can evolve from a physically allowed region to a nonphysical $\Omega_m > 1$ region, which constrains the sign of β.

The Properties of the Impasse Surface $x^2 + ny = 0$

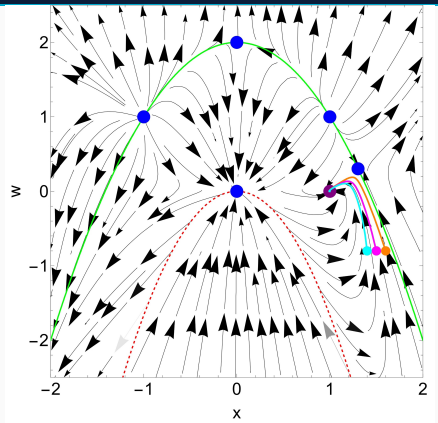


Impasse surface for the values $n = -1/4, k = 1$.

The impasse surface attractive/repulsive properties are described by the flux near the surface

- The blue point is the boundary between the attractive and repulsive region.
- Near the point the phase space behaves as "half-circles".
- The position of such point is determined by the parameters n, k.

Describing the Early Universe: From Inflation to Dark Matter



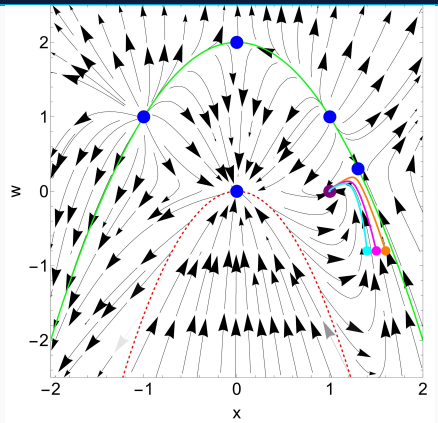
Phase space portrait for the values

$n = 3/4,\ k = 1$

INF

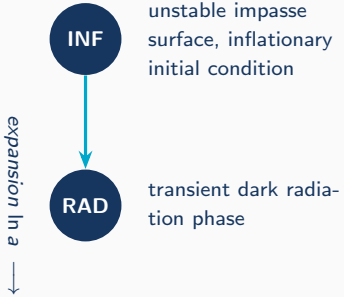
unstable impasse surface, inflationary initial condition

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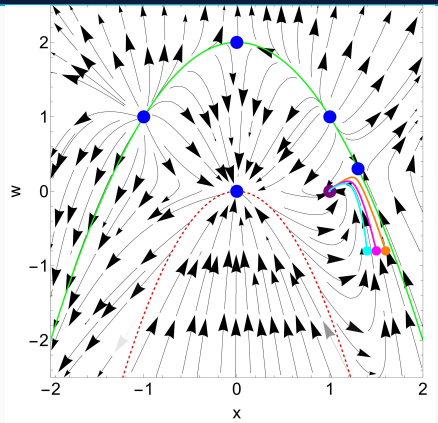


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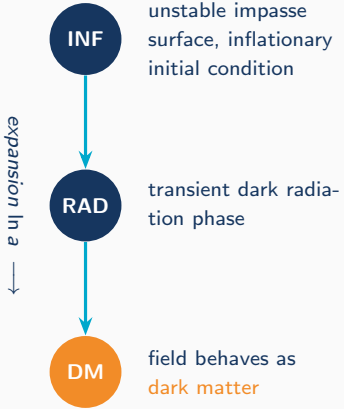


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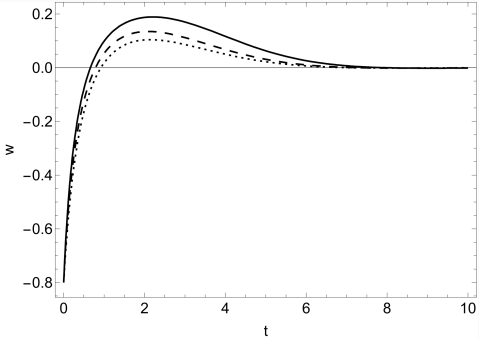


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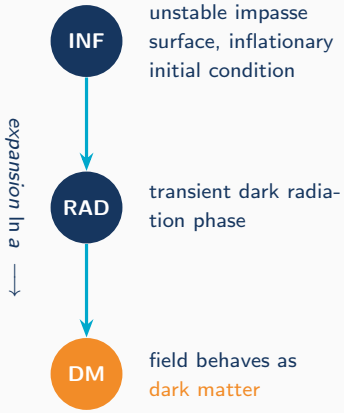


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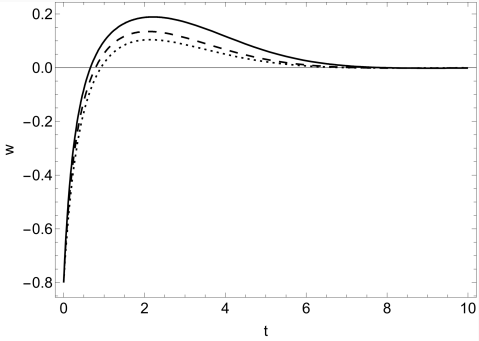


Evolution of the EoS for the values

$n = 3/4,\ k = 1$

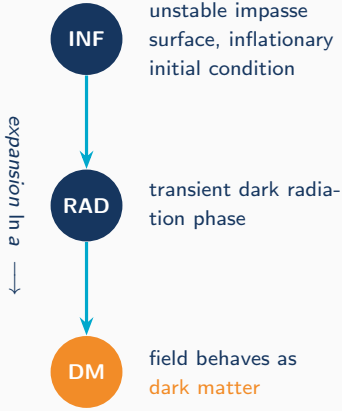


Describing the Early Universe: From Inflation to Dark Matter



Evolution of the EoS for the values

$n = 3/4,\ k = 1$



unstable impasse surface, inflationary initial condition

transient dark radiation phase

field behaves as dark matter

Realizable for

$0 < k < \sqrt{3}$

$1 - k^2/2 < n \leq 7/2 + 20/(k^2 - 8)$

Conclusions

- The model $f_2 = \alpha X + \beta e^{(n-1)\phi} X^n$ presents a scaling symmetry; the system is Liouville integrable.
- The phase space contains stiff, scaling and dark-matter-like epochs, plus a rich impasse surface and singular behavior at infinity.
- The model describes inflation with a natural exit to a matter-dominated era, the inflaton evolves as a dark matter source.

Thank you

a.lueiza01@ufromail.cl

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