STOCHASTIC INFLATION IN SCALAR-TORSION $f(T,\phi)$ GRAVITY

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Motivation

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Classical and quantum cosmology of $f(R)$ gravity's rainbow in Schutz's formalism

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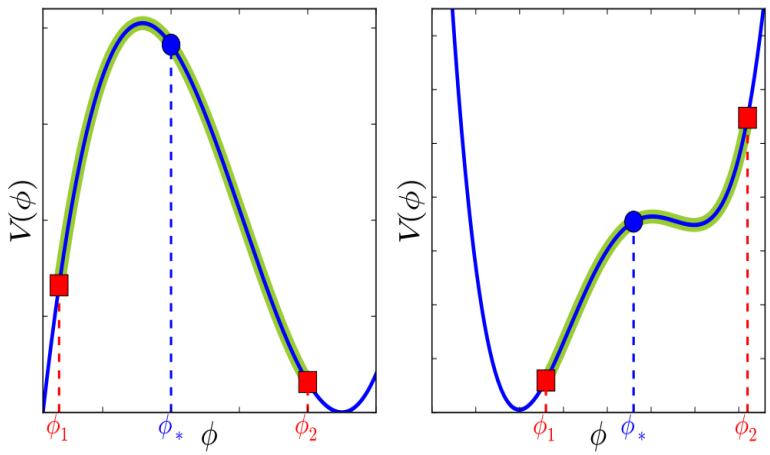
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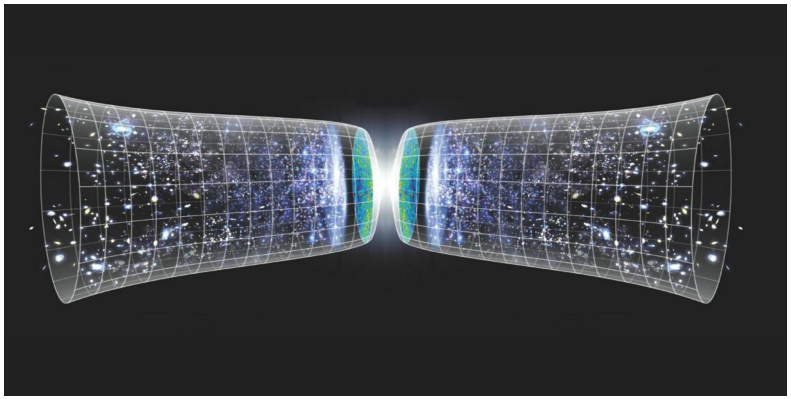
Abstract We investigate the classical and quantum dynamics of $f(R)$ gravity's rainbow in the presence of a perfect fluid, employing Schutz's formalism to establish a well-defined notion of time. In the classical regime, we derive and solve the equations of motion, obtaining both analytical and numerical solutions. Through canonical quantisation, we formulate the Schrödinger–Wheeler–DeWitt (SWD) equation for the quantum model. By solving its eigenfunctions, we construct the wave function of the Universe and obtain analytical solutions in scenarios dominated by stiff matter. Our results highlight the impact of rainbow gravity on quantum evolution, particularly in modifying the structure of the wave function and shaping the transition from the quantum to the classical regime.

1 Introduction

DeWitt demonstrated that renormalization at the one-loop level requires supplementing the Einstein–Hilbert action with higher-order curvature terms [5]. Furthermore, quantum corrections from string theory have shown that the effective low-energy gravitational action naturally includes such higher-order invariants [6–8].

This strong motivation to extend GR has given rise to modified gravity theories, with $f(R)$ gravity being one of the simplest and most extensively studied proposals [9–11]. A major advantage of $f(R)$ gravity is its ability to avoid the Ostrogradski instability, a fundamental issue in most higher-order theories [12]. A prominent example is Starobinsky's R^2 inflation model [13], which explains the Universe's rapid accelerated expansion shortly after the Big Bang. This inflationary period occurred approximately 10^{-34} s after the singularity and lasted between 10^{-33} and 10^{-32} s, while also accounting for the generation of primordial fluctuations [14–





"There was something before the Big Bang and that something is what we will have in our future."

Future singularities in IDE

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I. INTRODUCTION

II. PRELIMINARY NOTIONS

Both the existence of interactions between dark matter and dark energy as well as future cosmological singularities has been widely studied. In the ref. [Beltran] interaction models between the dark components were proposed where there is an exchange of energy flow represented by the Q factor so that when it diverges, it generates future

where $H = \dot{a}/a$. The above equations can be combined to obtain the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{5}$$

Then, considering the barotropic relationship $\omega = p/\rho$, we solve the above equation and obtain

$$\rho(t) \propto a^{-3(1+\omega)} \tag{6}$$

$$H = \frac{2}{3(1+\omega)(t - t_s)} \tag{7}$$

Theoretical Framework

The inflaton $\hat{\phi} = \bar{\hat{\phi}} + \hat{\phi}_s$ splits [Aldabergenov, et al, arXiv:2506.21423] into a classical background and quantum noise (ξ) driving the Langevin dynamics [R. Christie et al, arXiv:2606.12636] (N: *e-folds*):

$$\frac{\partial \bar{\hat{\phi}}}{\partial N} = \hat{\pi} + \hat{\xi}_\phi(N), \quad \frac{\partial \bar{\hat{\pi}}}{\partial N} = -(3 - \epsilon_1)\bar{\hat{\pi}} - \frac{V_{,\phi}(\bar{\hat{\phi}})}{H^2} + \hat{\xi}_\pi(N). \tag{1}$$

Noise amplitude at the coarse-graining scale k_σ [V. Vennin, arXiv:2009.08715]:

$$\langle 0|\hat{\xi}_\phi(N_1)\hat{\xi}_\phi(N_2)|0\rangle \propto |\phi_{k_\sigma}(N_1)|^2 \delta(N_1 - N_2). \tag{2}$$

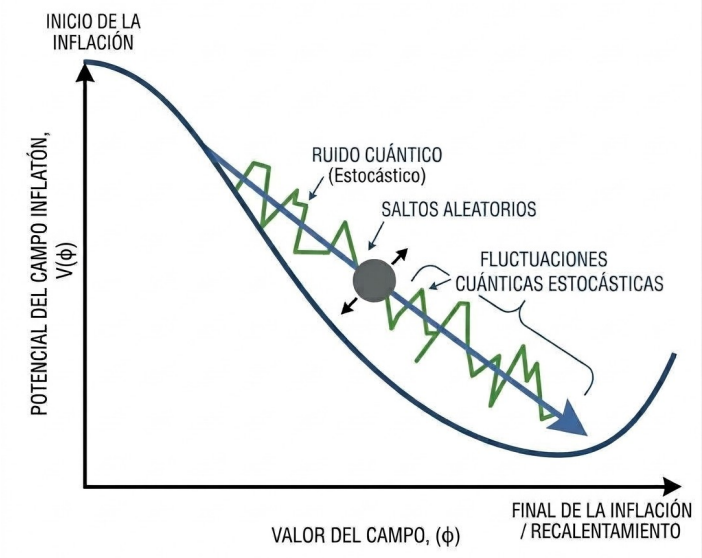


Figure: 1. Stochastic inflation

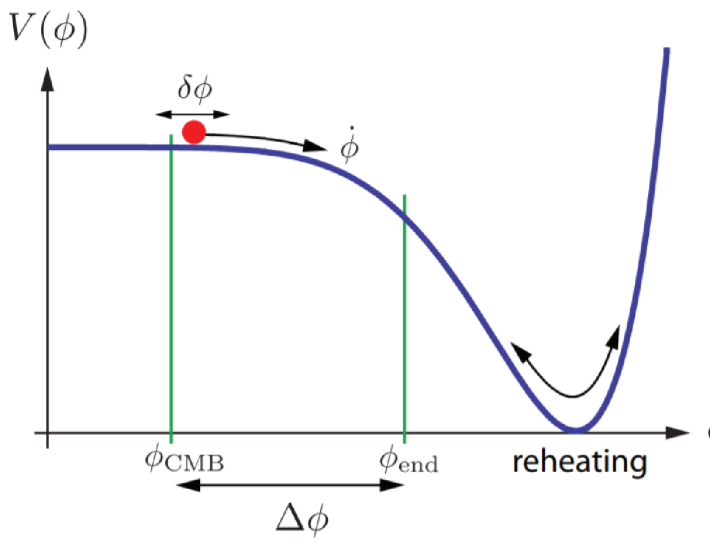


Figure: 2. Slow-roll inflation

Theoretical Framework

The scalar-torsion teleparallel action [Gonzalez, Otalora, P.L. B 809, 135696] is:

$$S = \int d^4x\, e\, [f(T,\phi) + P(\phi)X]. \tag{3}$$

Varying Eq. (3), we obtain the background equations:

$$f(T,\phi) - P(\phi)X - 2Tf_{,T} = 0, \qquad \text{1st Friedmann eq.} \tag{4}$$

$$f(T,\phi) + P(\phi)X - 2Tf_{,T} - 4\dot{H}f_{,T} - 4H\dot{f_{,T}} = 0, \qquad \text{2nd Friedmann eq.} \tag{5}$$

$$-P_{,\phi}X - 3P(\phi)H\dot{\phi} - P(\phi)\ddot{\phi} + f_{,\phi} = 0, \qquad \text{Klein-Gordon eq.} \tag{6}$$

Using e-folds $N = \ln a$, momentum $\Pi \equiv \phi_{,N} = \frac{\dot{\phi}}{H}$ and $\epsilon \equiv -\frac{H_{,N}}{H} = -\frac{\dot{H}}{H^2}$, the background dynamics are:

$$\Pi_{,N} = -(3-\epsilon)\Pi - \frac{P_{,\phi}}{2P}\Pi^2 + \frac{f_{,\phi}}{PH^2}. \tag{7}$$

This is the deterministic drift that must be recovered when the stochastic sources are switched off. In a slow-roll regime,

$$|\Pi_{,N}| \ll 3|\Pi|, \quad \epsilon \ll 1, \quad \left|\frac{P_{,\phi}}{2P}\Pi^2\right| \ll \left|\frac{f_{,\phi}}{PH^2}\right|, \quad \textit{so that} \quad \Pi_{SR} \simeq \frac{f_{,\phi}}{3PH^2}. \tag{8}$$

The sign is consistent with the GR limit because $f_{,\phi} = -V_{,\phi}$.

Cosmological Perturbation Theory

Extra degrees of freedom in MTG are non-dynamical and eliminated via constraints.

$$\mathrm{d}s^2 = -(1+2\alpha)\mathrm{d}t^2 + 2\partial_i\beta\,\mathrm{d}t\,\mathrm{d}x^i + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \quad e^a{}_\mu = \bar{e}^a{}_\mu + \delta e^a{}_\mu \tag{9}$$

In the **spatially flat gauge** ($\psi = 0$), scalar perturbations are entirely encoded in the gauge-invariant field fluctuation ($Q \equiv \delta\phi$). The reduced second-order action:

$$S_2 = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3 P\left[\dot{Q}^2 - \frac{(\partial Q)^2}{a^2} - M_Q^2 Q^2\right]. \tag{10}$$

With the torsion-dependent effective mass parameter:

$$\begin{aligned} M_Q^2 =& \frac{1}{a^3 P}\frac{\mathrm{d}}{\mathrm{d}t}\left[a^3\left(P_{,\phi}\dot{\bar{\phi}} + \frac{P^2\dot{\bar{\phi}}^2}{4Hf_{,T}}\right)\right] - \frac{f_{,\phi\phi}}{P} - \frac{P_{,\phi\phi}\dot{\bar{\phi}}^2}{2P} + \frac{f_{,T\phi}{}^2}{Pf_{,TT}} - \frac{3P\dot{\bar{\phi}}^2}{4f_{,T}} \\ & - \frac{P^2\dot{\bar{\phi}}^4}{16H^2 f_{,T}{}^2} + \frac{\dot{\bar{\phi}}}{2Hf_{,T}}\left(f_{,\phi} - \frac{1}{2}P_{,\phi}\dot{\bar{\phi}}^2\right). \end{aligned} \tag{11}$$

Varying with respect to Q yields the full wave equation of motion (CPT equation):

$$\ddot{Q} + \left(3H + \frac{\dot{P}}{P}\right)\dot{Q} - \frac{\partial^2 Q}{a^2} + M_Q^2 Q = 0. \tag{12}$$

Separate Universe Approach

We decompose the field into the background and first-order perturbation [Jackson, et al, arXiv:2311.03281]:

$$\phi(t,\vec{x}) = \bar{\phi}(t) + \delta\phi(t,\vec{x}). \tag{13}$$

We work in the **spatially flat gauge** ($\psi = 0$), with $Q \equiv \delta\phi$, local lapse perturbation $dt \to (1+\alpha)dt$, and approximating to first order:

$$\delta\dot{\phi} = (\dot{\delta\phi}) - \alpha\dot{\bar{\phi}}, \qquad \delta\ddot{\phi} = (\ddot{\delta\phi}) - \dot{\alpha}\dot{\bar{\phi}} - 2\alpha\ddot{\bar{\phi}}, \qquad \delta X = \dot{\bar{\phi}}(\dot{\delta\phi}) - \dot{\bar{\phi}}^2\alpha. \tag{14}$$

Perturbing the scalar-torsion field equations yields the constraint and dynamical system ($\delta T = 12H\delta H$):

$$\delta T(-f_{,T} - 2Tf_{,TT}) + \delta\phi(f_{,\phi} - P_{,\phi}X - 2Tf_{,T\phi}) - P\delta X = 0, \tag{15}$$

$$\begin{aligned} &-P_{,\phi\phi}\delta\phi X - P_{,\phi}\delta X - 3P_{,\phi}\delta\phi H\dot{\bar{\phi}} - 3P\delta H\dot{\bar{\phi}} - 3PH\delta\dot{\phi} \\ &\quad -P_{,\phi}\delta\phi\ddot{\bar{\phi}} - P\delta\ddot{\phi} + f_{,\phi\phi}\delta\phi + f_{,\phi T}\delta T = 0. \end{aligned} \tag{16}$$

- Non-dynamical variables $(\alpha, \delta H)$ act as algebraic Lagrange multipliers and are integrated out directly.
- Formulates the deterministic classical drift for the coarse-grained Langevin equations.

Separate Universe Approach

Eliminating the non-dynamical constraints yields the local patch equation:

$$\ddot{Q} + \left(3H + \frac{\dot{P}}{P}\right)\dot{Q} + M_Q^2 Q = 0, \tag{17}$$

where the torsion-dependent effective mass is given by:

$$\begin{aligned} M_Q^2 =& \frac{1}{a^3 P}\frac{\mathrm{d}}{\mathrm{d}t}\left[a^3\left(P_{,\phi}\dot{\bar{\phi}} + \frac{P^2\dot{\bar{\phi}}^2}{4Hf_{,T}}\right)\right] - \frac{f_{,\phi\phi}}{P} - \frac{P_{,\phi\phi}\dot{\bar{\phi}}^2}{2P} + \frac{f_{,T\phi}{}^2}{Pf_{,TT}} - \frac{3P\dot{\bar{\phi}}^2}{4f_{,T}} \\ & - \frac{P^2\dot{\bar{\phi}}^4}{16H^2 f_{,T}{}^2} + \frac{\dot{\bar{\phi}}}{2Hf_{,T}}\left(f_{,\phi} - \frac{1}{2}P_{,\phi}\dot{\bar{\phi}}^2\right). \end{aligned} \tag{18}$$

Super-Hubble patches ($k \ll aH$) act as local FLRW universes, causing spatial gradients to vanish in SUA. Comparing with the CPT equation:

$$\underbrace{\left[\ddot{Q} + \left(3H + \frac{\dot{P}}{P}\right)\dot{Q} - \frac{\partial^2 Q}{a^2} + M_Q^2 Q\right]}_{\text{Full CPT Equation}} \xrightarrow{k\to 0} \underbrace{\left[\ddot{Q} + \left(3H + \frac{\dot{P}}{P}\right)\dot{Q} + M_Q^2 Q\right]}_{\text{Separate Universe Equation (SUA)}} \tag{19}$$

This establishes the analytical consistency of the long-wavelength background evolution.

Stochastic Langevin Dynamics

Coarse-graining ($k_\sigma = \sigma a H$) [C. Pattison, et al, JCAP 07, 031] with Bunch-Davies vacuum sets the phase-space Langevin system [V. Vennin, arXiv:2009.08715]:

$$\phi_{c,N} = \Pi_c + \Xi_Q, \quad \Pi_{c,N} = -(3-\epsilon_c)\Pi_c - \frac{P_{,\phi}}{2P}\Pi_c^2 + \frac{f_{,\phi}}{PH_c^2} + \mathcal{C}_\Pi \Xi_\Pi + \mathcal{C}_Q \Xi_Q \quad (20)$$

The symmetric noise correlator $\frac{1}{2}\langle\{\Xi_I(N), \Xi_J(N')\}\rangle = A_{IJ}(N)\delta(N-N')$ yields:

$$A_{QQ} \simeq \frac{H^2}{4\pi^2 P} \implies \delta\phi_q = \frac{H}{2\pi\sqrt{P}}, \quad A_{Q\Pi} \simeq \lambda_Q A_{QQ}, \quad A_{\Pi\Pi} \simeq \lambda_Q^2 A_{QQ} \quad (21)$$

where $\lambda_Q \equiv Q_{k,N}/Q_k$. The diffusion matrix is approximately rank-one on super-Hubble scales.
Diffusion overcomes the classical drift ($\Delta N_{\rm diff} \sim D/A^2$) when:

$$\mathcal{Q}_{\rm diff} \equiv \frac{\delta\phi_q}{\delta\phi_{\rm cl}} = \frac{3H^3\sqrt{P}}{2\pi|f_{,\phi}|} \gtrsim 1 \iff \Delta N_{\rm diff} = \mathcal{Q}_{\rm diff}^2 \quad (22)$$

Teleparallel modifications enter via: drift $f_{,\phi}/(PH^2)$, kick $H/(2\pi\sqrt{P})$, and mass M_Q^2.

Conclusions

- **Equivalence CPT $\leftrightarrow$ SUA:**
  Verified analytically in the super-horizon limit ($k \to 0$) for general scalar-torsion $f(T, \phi)$ gravity.

- **Fokker-Planck and Observables:**
  The stochastic $\delta\mathcal{N}$ framework connects effective mass, kinetic coupling, and spectral corrections to changes in the tensor-to-scalar ratio and non-Gaussian tails.

- **Imprint on Primordial Seeds:**
  First Passage Time statistics ($\langle \Delta\mathcal{N}^2 \rangle$) directly encode torsion corrections into curvature perturbations.

- **Cosmological Bridge:**
  Provides a consistent path from quantum bounce origins to late-time fates via stochastic structure generation.

- **Future Directions:**
  Evaluation of UV correlators for PBH observables via Lindblad open quantum dynamics.


Thank you very much!
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