PUCP

Grupo de Altas Energías

Numerical Reconstruction of Dark-Matter Spikes around Black Holes and Their Connection to Gamma-Ray Observables

David Luzquiños
alonso.luzquinos@pucp.edu.pe
Pontificia Universidad Católica del Perú

Luis Asto
asto.luis@pucp.edu.pe
Pontificia Universidad Católica del Perú

David Choque
dchoque@pucp.edu.pe
Pontificia Universidad Católica del Perú

Johan Casimiro
j.casimiro@pucp.edu.pe
Pontificia Universidad Católica del Perú

Oct, 20th, UTA, Lima

UNIVERSIDAD DE TARAPACÁ
Universidad del Estado

UTA FACULTAD DE CIENCIAS

DEPARTAMENTO DE FÍSICA
UNIVERSIDAD DE TARAPACÁ

COSMOGRAV-UTA



[Gondolo Density]

General Dark Matter Density

$$\rho'(r) = \int_{\underbrace{-\frac{GM_{bh}}{r}\left(1-\frac{4R_S}{r}\right)}_{E'_m}}^{0} dE' \int_{\underbrace{2cR_S}_{L'_c}}^{\underbrace{\sqrt{2r^2\left(E'+\frac{GM_{bh}}{r}\right)}}_{L'_m}} \frac{4\pi L'\, dL'}{\underbrace{r^2 v_r}_{\sqrt{2\left(E'+\frac{GM_{bh}}{r}-\frac{L'^2}{2r^2}\right)}}} f'(E', L')$$

[Gondolo-Silk - 1999]

Initial Halo and Potential

- Power Law

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\gamma}$$

$$M(r) = 4\pi \int_0^r \rho(s) s^2 ds$$

Mass cover equation

- Potential

$$\phi = \underbrace{\frac{4\pi G \rho_0 r_0^2}{(3-\gamma)(2-\gamma)}}\phi_0 \left(\frac{r}{r_0}\right)^{2-\gamma}$$

$$\rho(\phi) = \rho_0 \left(\frac{\phi}{\phi_0}\right)^{-\frac{\gamma}{2-\gamma}}$$

$$f(E) = \frac{1}{\sqrt{8}\,\pi^2} \int_E^\infty \frac{d^2\rho}{d\phi^2} \frac{d\phi}{\sqrt{\phi - E}}$$

- Eddington Inversion

$$f(E) = \frac{\rho_0}{(2\pi\phi_0)^{3/2}} \frac{\Gamma(\beta)}{\Gamma\left(\beta - \frac{3}{2}\right)} \left(\frac{\phi_0}{E}\right)^{\beta} \qquad \beta = \frac{6-\gamma}{2(2-\gamma)}$$

[Initial Conditions]

Adiabatic Invariants

- Angular Action

$$I_\psi = L \;\Rightarrow\; L = L'$$

- Radial Action
  - Radial Orbit

$$I_r\big|_{\tau=0} = \frac{\sqrt{2E}\, r_E}{b}$$

  - Circular Orbit

$$I_r\big|_{\tau=\lambda} = 0$$

$$I_r' = -L' + \frac{GM_{bh}}{\sqrt{-2E'}} \;\Rightarrow\; I_r \simeq \frac{1}{b}\left[-\frac{L}{\lambda} + \sqrt{2\phi_0 r_0^2}\left(\frac{E}{\phi_0}\right)^{\frac{4-\gamma}{2(2-\gamma)}}\right]$$

- Equating $I_r = I_r'$ with $L = L'$

$$X(E', L') = \frac{1}{\sqrt{2\phi_0 r_0^2}}\left[b\left(-L' + \frac{GM_{bh}}{\sqrt{-2E'}}\right) + \frac{L'}{\lambda}\right] \qquad E = \phi_0\, X^{\frac{2(2-\gamma)}{4-\gamma}}$$

$$f'(E', L') = A_\gamma\, X^{-p} \qquad A_\gamma = \frac{\rho_0}{(2\pi\phi_0)^{3/2}} \frac{\Gamma(\beta)}{\Gamma\left(\beta - \frac{3}{2}\right)} \qquad p = \frac{6-\gamma}{4-\gamma}$$

[Final Density]

Adiabatic Map and Final Distribution

$$f'(E', L') = A_\gamma \, X^{-p}$$

$$\rho'(r) = \int_{E'_m}^{0} dE' \int_{L'_c}^{L'_m} \frac{4\pi L' \, dL'}{r^2 v_r} f'(E', L')$$

$$x = -\frac{rE'}{GM_{bh}} \qquad l = \frac{L'}{\sqrt{2GM_{bh}r}} \qquad \eta = \frac{4R_S}{r}$$

$$\rho'(r) = D_\gamma \, r^{-\frac{p+3}{2}} \mathcal{J}(\eta) \qquad D_\gamma = \frac{\rho_0}{2^{p+1}\sqrt{\pi}} \frac{\Gamma(\beta)}{\Gamma\left(\beta - \frac{3}{2}\right)} r_0^p \left(\frac{\phi_0}{GM_{bh}}\right)^{\frac{p-3}{2}}$$

$$\mathcal{J}(\eta) = \frac{b^{-p}}{2} \sum_{n=0}^{\infty} \frac{(p)_n}{n!} \kappa_\gamma^n \, B\left(\tfrac{p+n}{2} + 1, \tfrac{1}{2}\right) \left[B\left(\tfrac{n}{2} + 1, \gamma_{sp} + \tfrac{n}{2}\right) - B\left(\eta; \tfrac{n}{2} + 1, \gamma_{sp} + \tfrac{n}{2}\right)\right]$$

$$\gamma_{sp} = \frac{p+3}{2} = \frac{9 - 2\gamma}{4 - \gamma}$$

[Final Density]

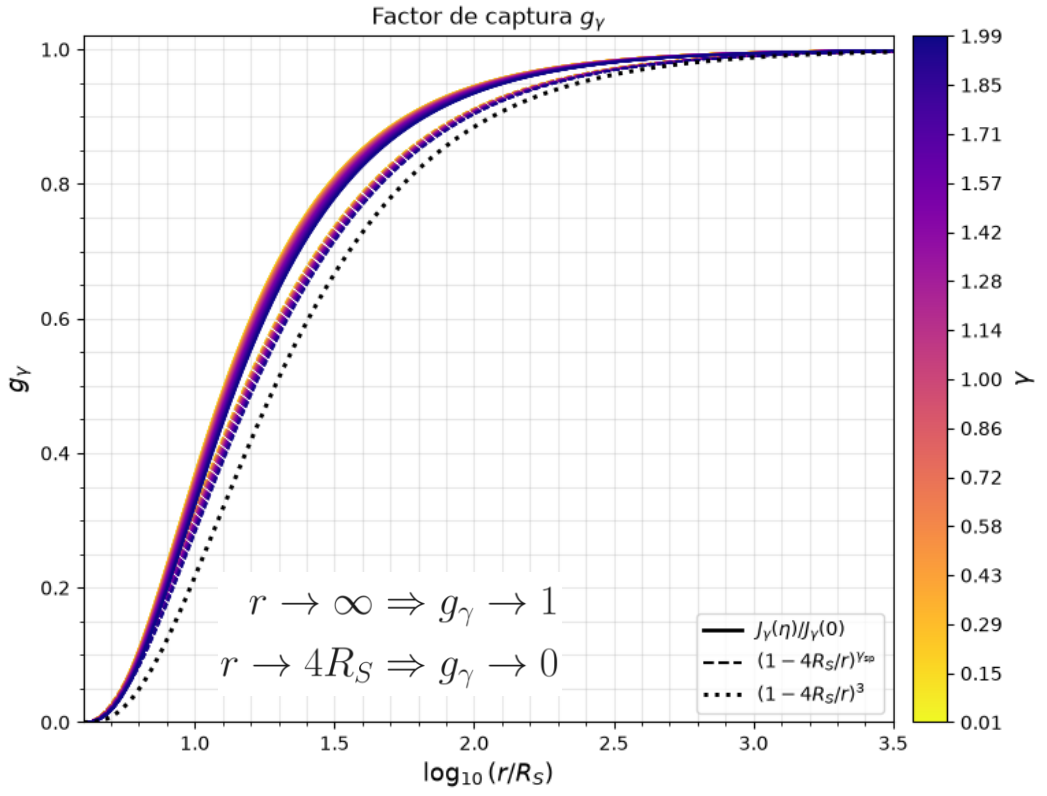
Capture function

- Capture Function

$$g_\gamma(\eta) = \frac{\mathcal{J}(\eta)}{\mathcal{J}(0)} \qquad \eta = \frac{4R_S}{r}$$

- Zeroth order (n=0)

$$g_\gamma^{(0)}(r) = \left(1 - \frac{4R_S}{r}\right)^{\gamma_{sp}}$$



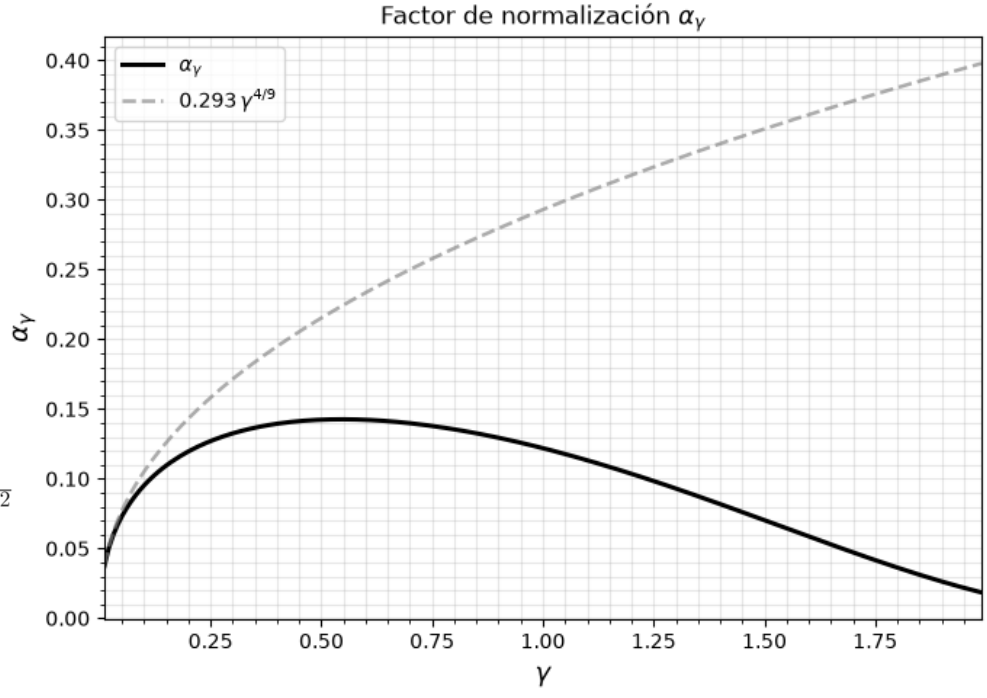
[Final Density]

Spike Radius

$$D_\gamma \mathcal{J}(0) R_{sp}^{-\gamma_{sp}} = \rho_R = \rho_0 \left(\frac{R_{sp}}{r_0}\right)^{-\gamma}$$

$$R_{sp} = \alpha_\gamma r_0 \left(\frac{M_{bh}}{\rho_0 r_0^3}\right)^{\frac{1}{3-\gamma}}$$

$$\alpha_\gamma = \left[\mathcal{J}(0) \frac{2^{\frac{10-2\gamma}{4-\gamma}} \sqrt{\pi}\, \Gamma(\beta)}{\Gamma\left(\beta - \frac{3}{2}\right)} \left(\frac{(3-\gamma)(2-\gamma)}{4\pi}\right)^{\frac{3-\gamma}{4-\gamma}}\right]^{\frac{4-\gamma}{(3-\gamma)^2}}$$



Final DM Density Profile

$$R_{sp} = \alpha_\gamma \, r_0 \left(\frac{M_{bh}}{\rho_0 r_0^3} \right)^{\frac{1}{3-\gamma}}$$

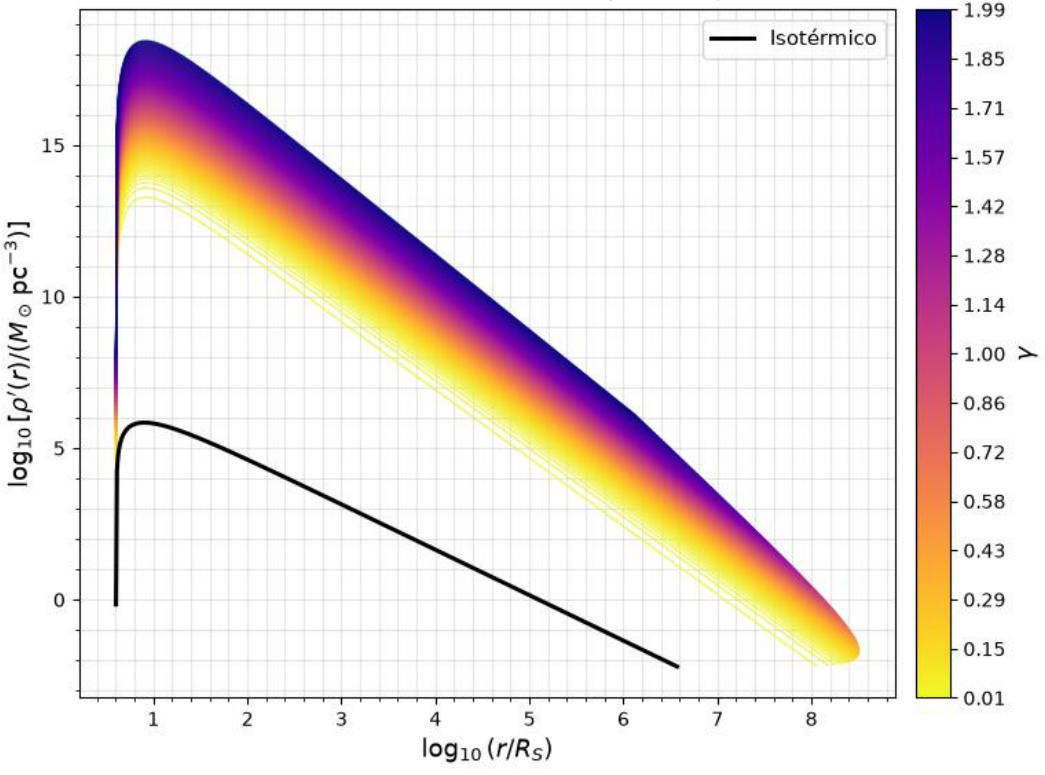
$$\rho'(r) = \rho_R \, g_\gamma(\eta) \left(\frac{R_{sp}}{r} \right)^{\gamma_{sp}} \frac{9-2\gamma}{4-\gamma}$$

$$g_\gamma(\eta) = \frac{\mathcal{J}(\eta)}{\mathcal{J}(0)}$$

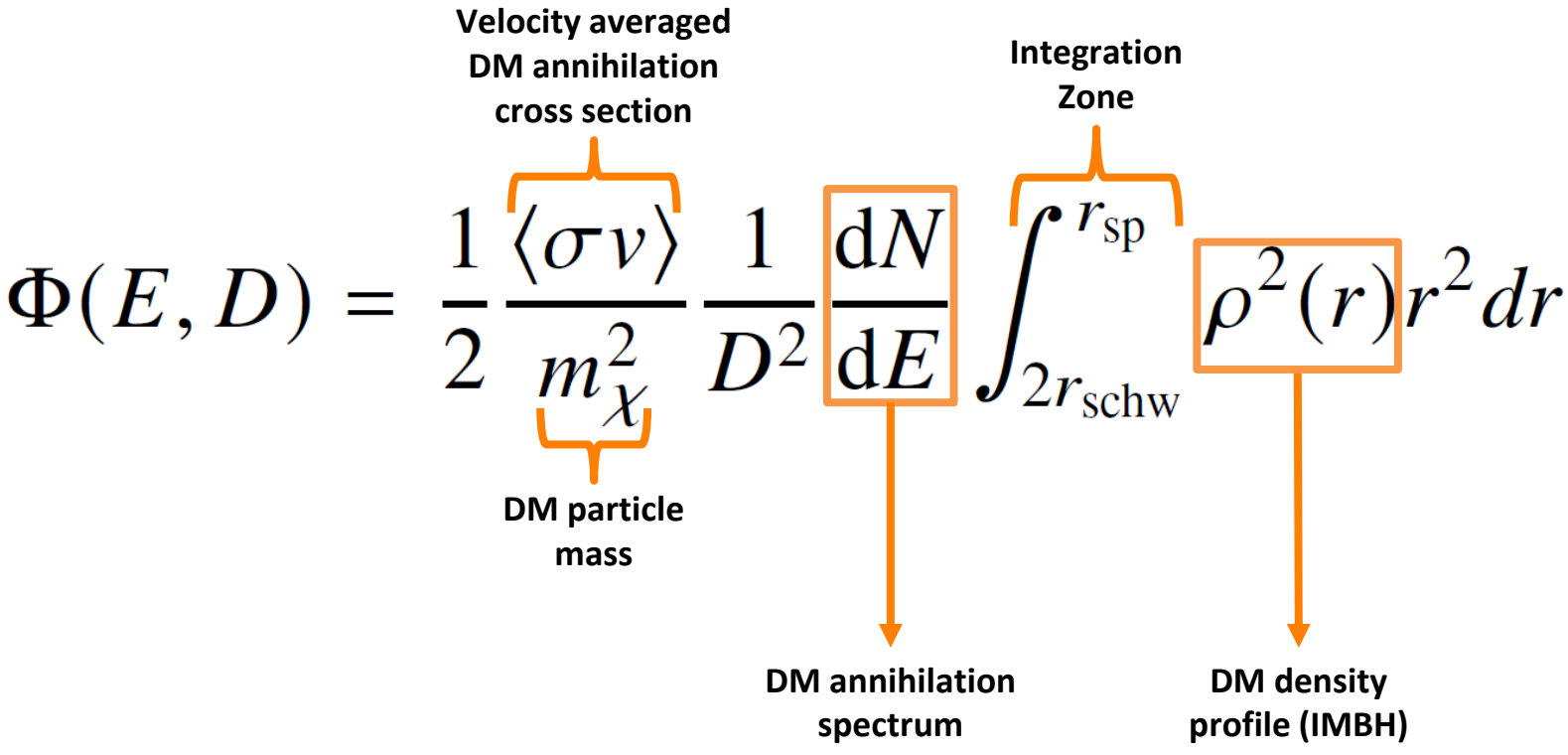
[Final Density]

Final DM Density Profile

$$\rho'(r) = \rho_R \, g_\gamma(\eta) \left(\frac{R_{sp}}{r} \right)^{\gamma_{sp}}$$

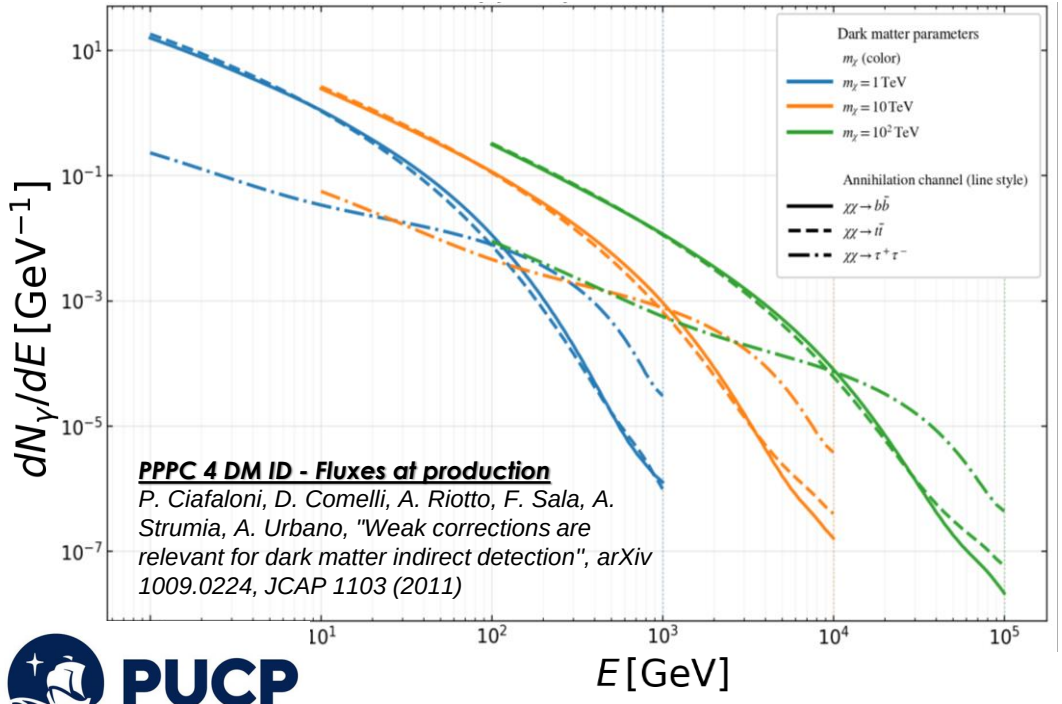


[Flux]
Gamma-ray Flux from DM Annihilation



[Flux] Dark Matter annihilation spectrum from PPPC 4 DM ID

$$\Phi(E, D) = \frac{1}{2}\frac{\langle \sigma v \rangle}{m_\chi^2}\frac{1}{D^2}\frac{dN}{dE}\int_{2r_{schw}}^{r_{sp}} \rho^2(r) r^2 dr$$

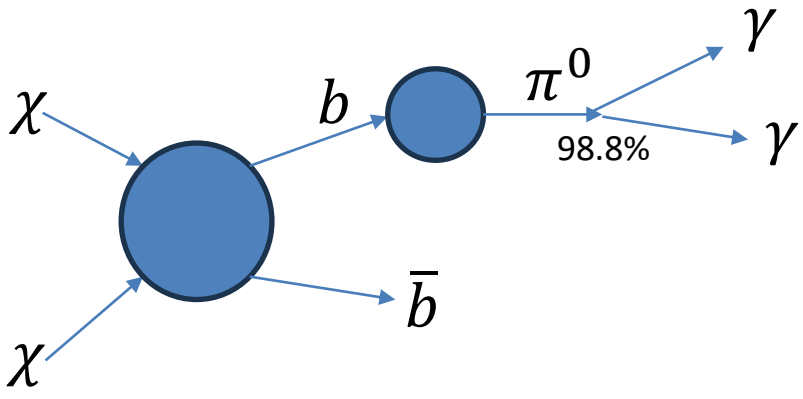


PPPC 4 DM ID - Fluxes at production

P. Ciafaloni, D. Comelli, A. Riotto, F. Sala, A. Strumia, A. Urbano, "Weak corrections are relevant for dark matter indirect detection", arXiv 1009.0224, JCAP 1103 (2011)

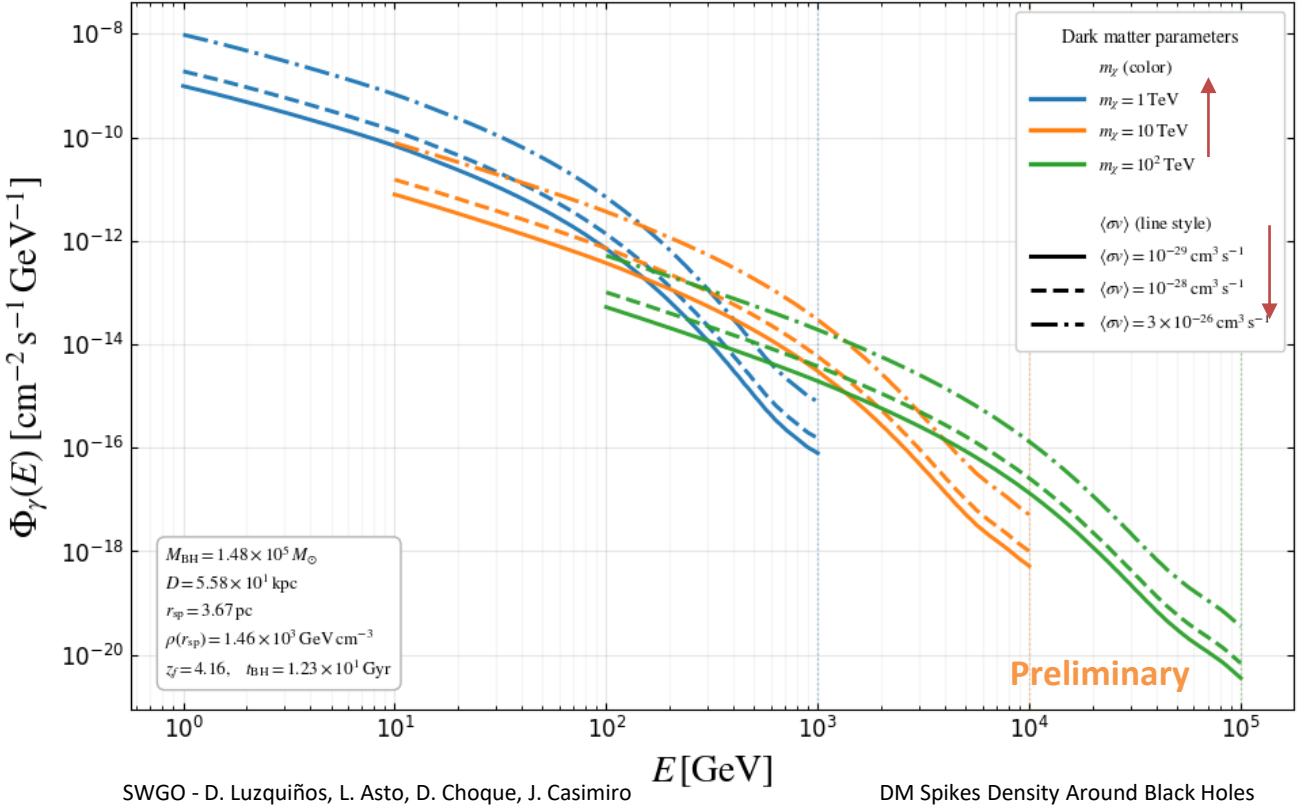
$b\bar{b}$ - Annihilation channel

$$\chi\chi \to b\bar{b} \to \gamma\gamma$$



[Flux] Gamma-ray Flux from DM Annihilation in IMBH Spike

$$\Phi(E,D) = \frac{1}{2}\frac{\langle\sigma v\rangle}{m_\chi^2}\frac{1}{D^2}\frac{\mathrm{d}N}{\mathrm{d}E}\int_{2r_{\mathrm{schw}}}^{r_{\mathrm{sp}}}\rho^2(r)r^2dr \approx \frac{\mathrm{d}N}{\mathrm{d}E}\frac{\langle\sigma v\rangle}{m_\chi^2 D^2}\rho(r_{\mathrm{sp}})^2 r_{\mathrm{sp}}^3\frac{2\gamma_{\mathrm{sp}}-1}{8\gamma_{\mathrm{sp}}-12}\left(\frac{r_{\mathrm{cut}}}{r_{\mathrm{sp}}}\right)^{3-2\gamma_{\mathrm{sp}}}$$



$[N_\gamma]$ Expected Number of Detected Events & Instrument Response Function (IRFs)

Observation time (s)

Gamma-ray Flux from DM Annihilation

$$N(p,E)\mathrm{d}p\mathrm{d}E = t_{\mathrm{obs}} \int_{E_{\mathrm{true}}} \mathrm{d}E_{\mathrm{true}} \int_{p_{\mathrm{true}}} \mathrm{d}p_{\mathrm{true}} R(p,E|p_{\mathrm{true}},E_{\mathrm{true}}) \times \Phi(p_{\mathrm{true}},E_{\mathrm{true}})$$

Response Function

Response Function

$$R(p,E|p_{\mathrm{true}},E_{\mathrm{true}}) = A_{\mathrm{eff}}(p_{\mathrm{true}},E_{\mathrm{true}}) \times PSF(p|p_{\mathrm{true}},E_{\mathrm{true}}) \times E_{\mathrm{disp}}(E|p_{\mathrm{true}},E_{\mathrm{true}})$$

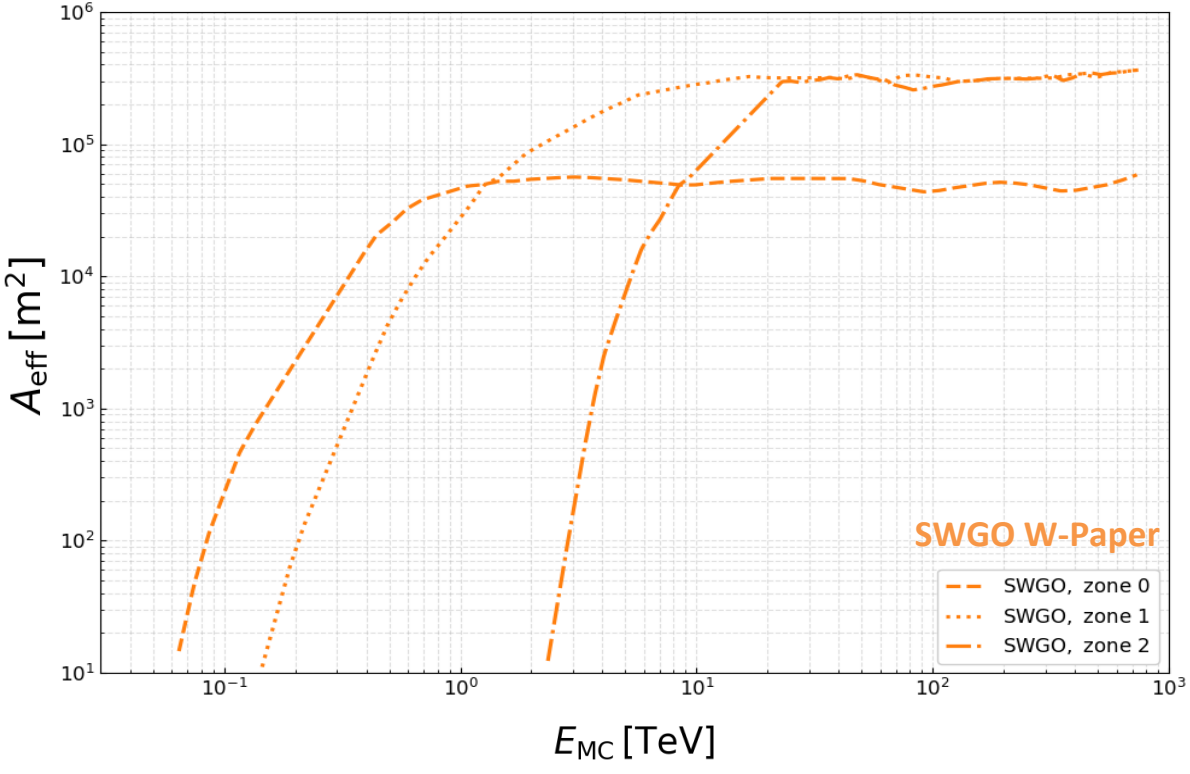
Effective Area

Point Spread Function

Energy Dispersion

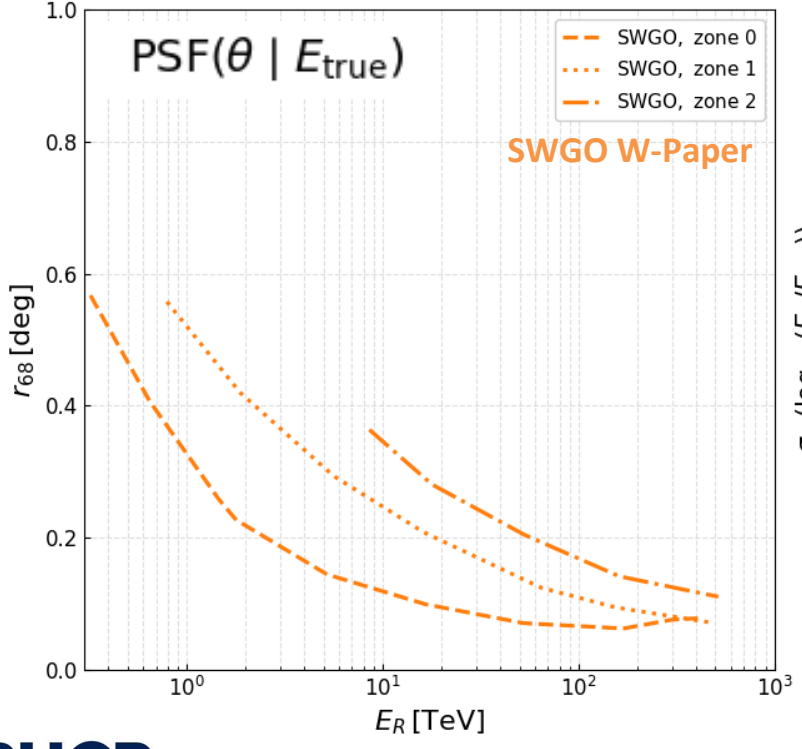
[IRFs] Effective Area

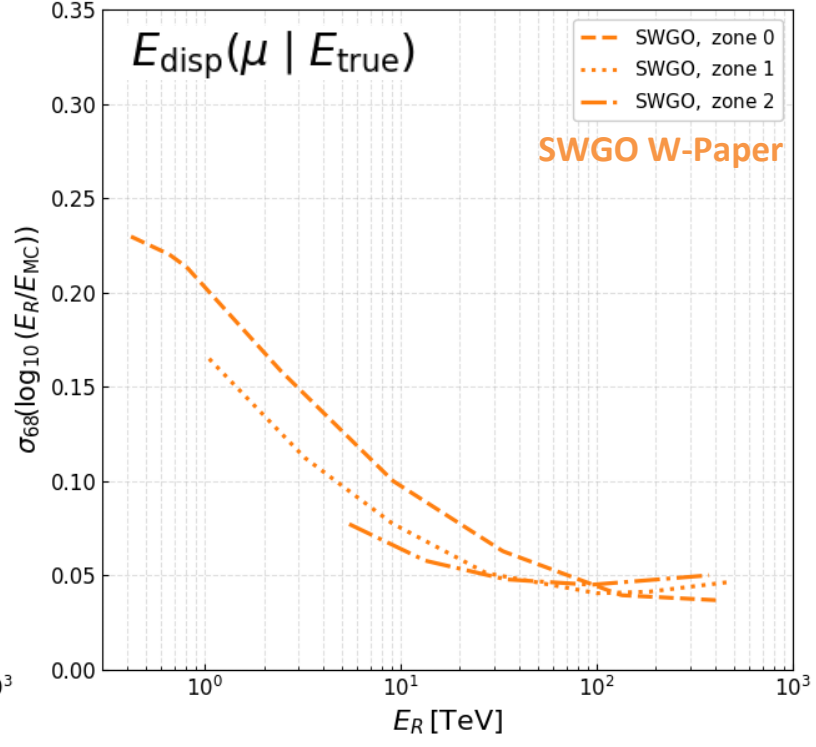
$$R(p, E|p_{\text{true}}, E_{\text{true}}) = A_{\text{eff}}(p_{\text{true}}, E_{\text{true}}) \times PSF(p|p_{\text{true}}, E_{\text{true}}) \times E_{\text{disp}}(E|p_{\text{true}}, E_{\text{true}})$$



[IRFs] Point Spread Function & Energy Dispersion

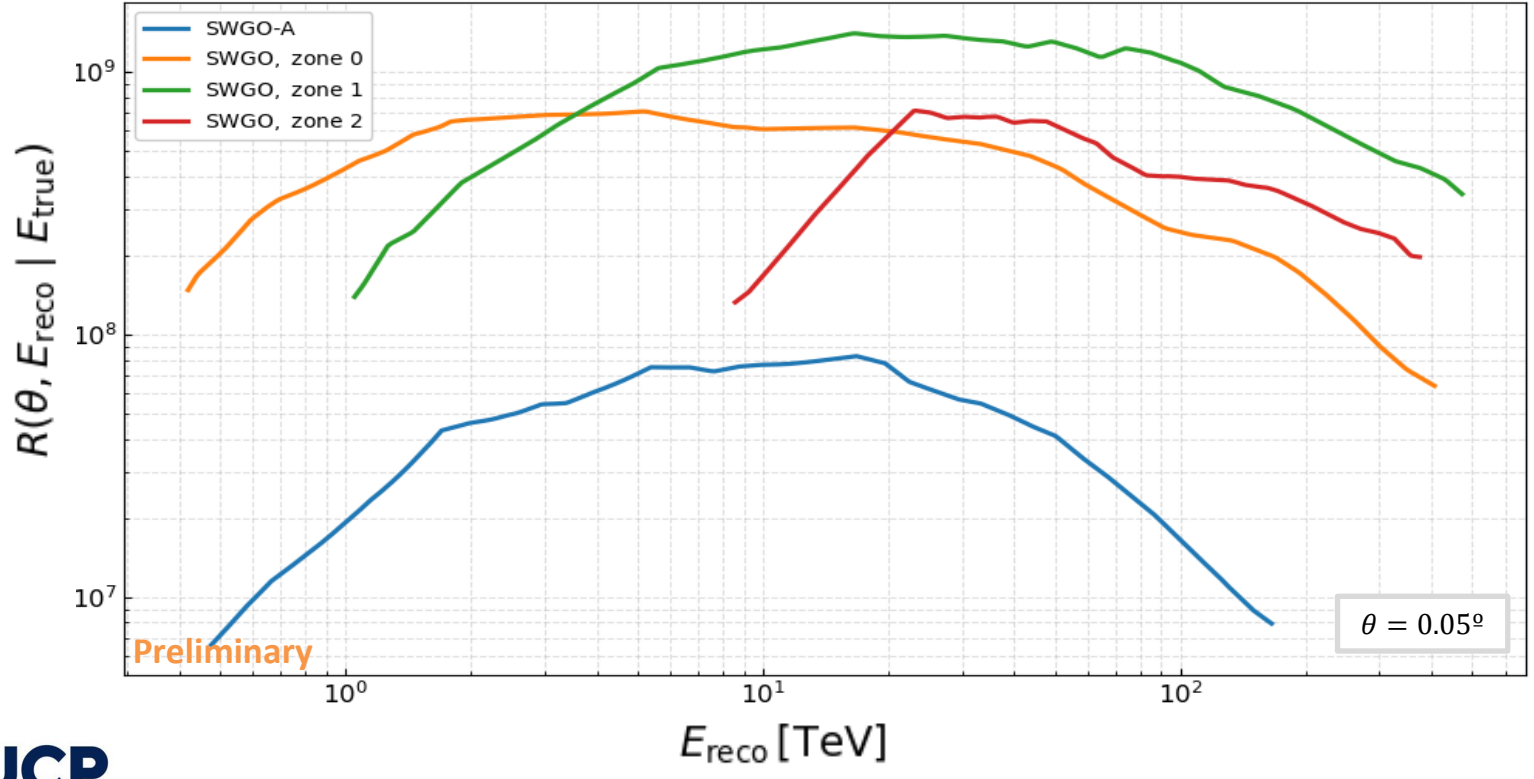
$$R(p, E|p_{\text{true}}, E_{\text{true}}) = A_{\text{eff}}(p_{\text{true}}, E_{\text{true}}) \times PSF(p|p_{\text{true}}, E_{\text{true}}) \times E_{\text{disp}}(E|p_{\text{true}}, E_{\text{true}})$$



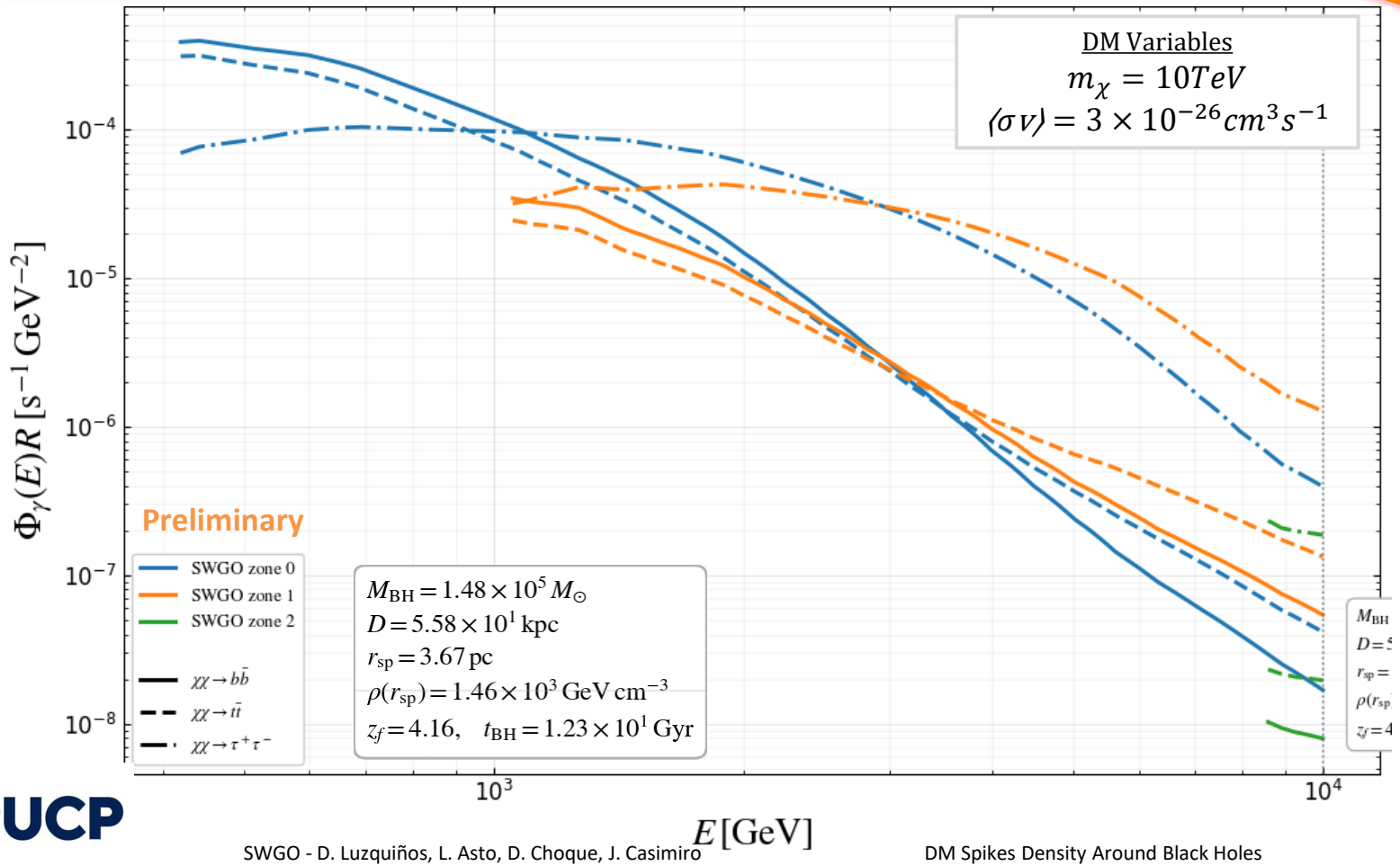


[IRFs] Response Function

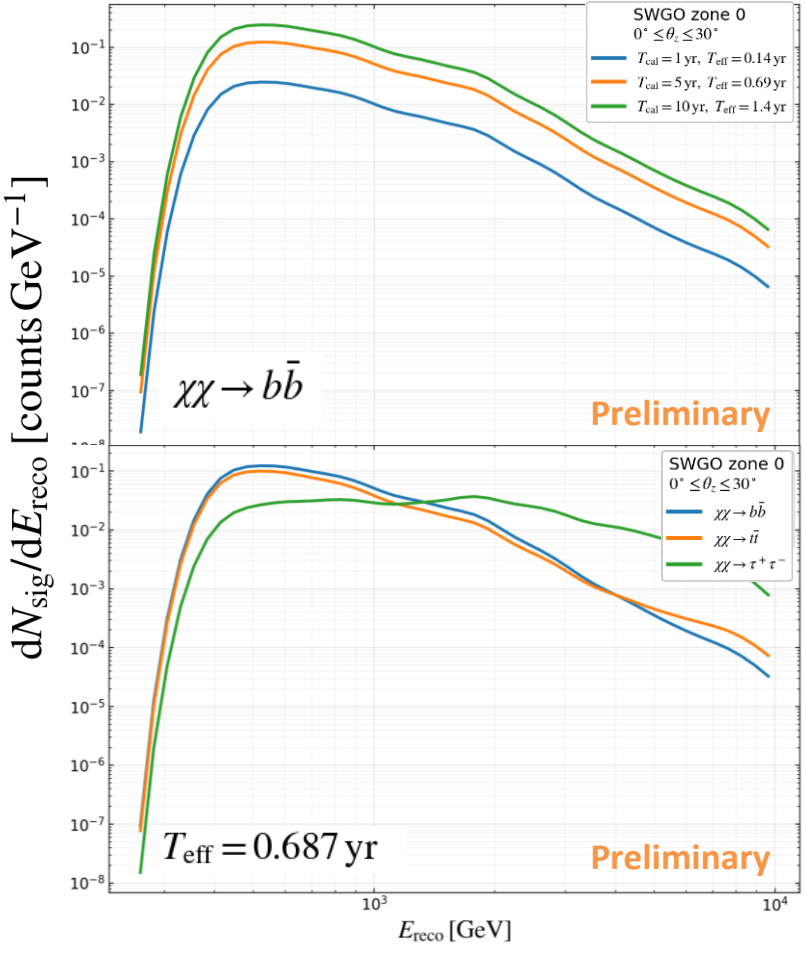
$$R(p, E|p_{\text{true}}, E_{\text{true}}) = A_{\text{eff}}(p_{\text{true}}, E_{\text{true}}) \times PSF(p|p_{\text{true}}, E_{\text{true}}) \times E_{\text{disp}}(E|p_{\text{true}}, E_{\text{true}})$$

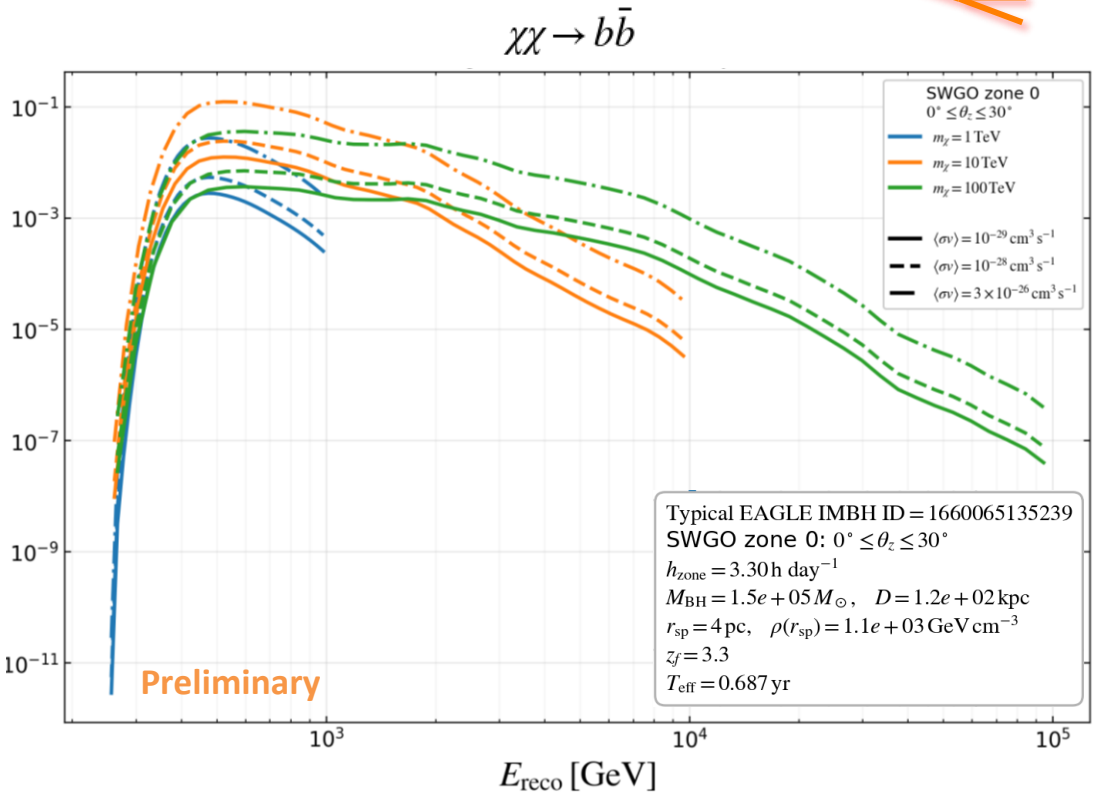


$[N_\gamma]$ Flux weighted by the response function



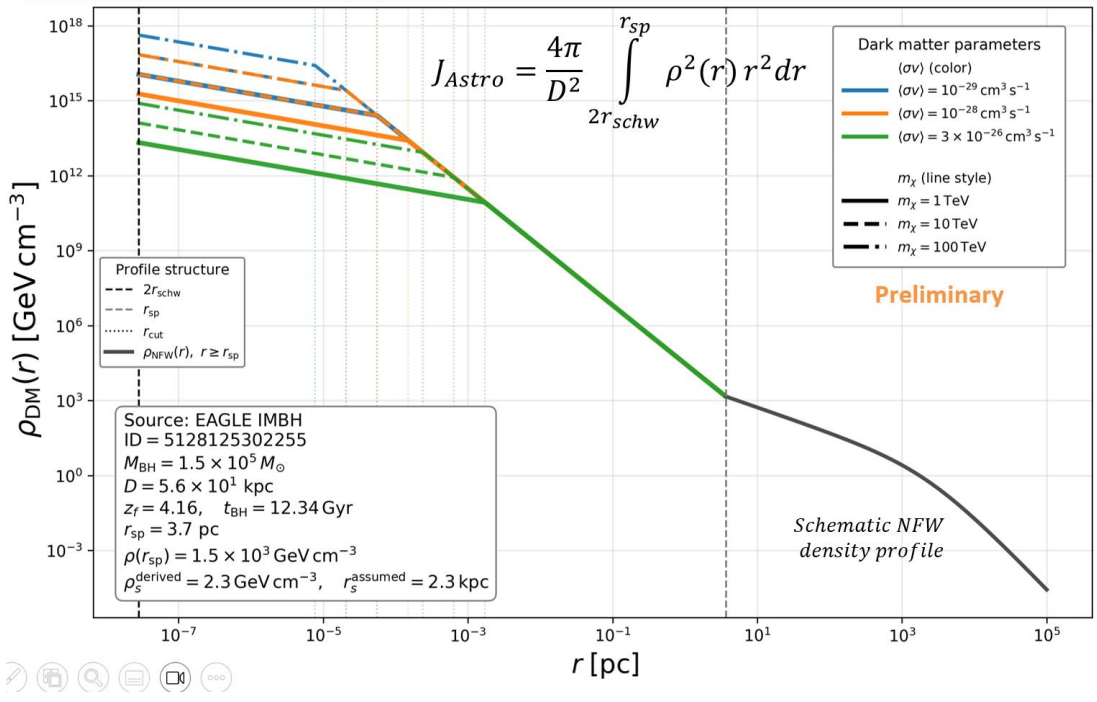
$[N_\gamma]$ Gamma-signal (only) events





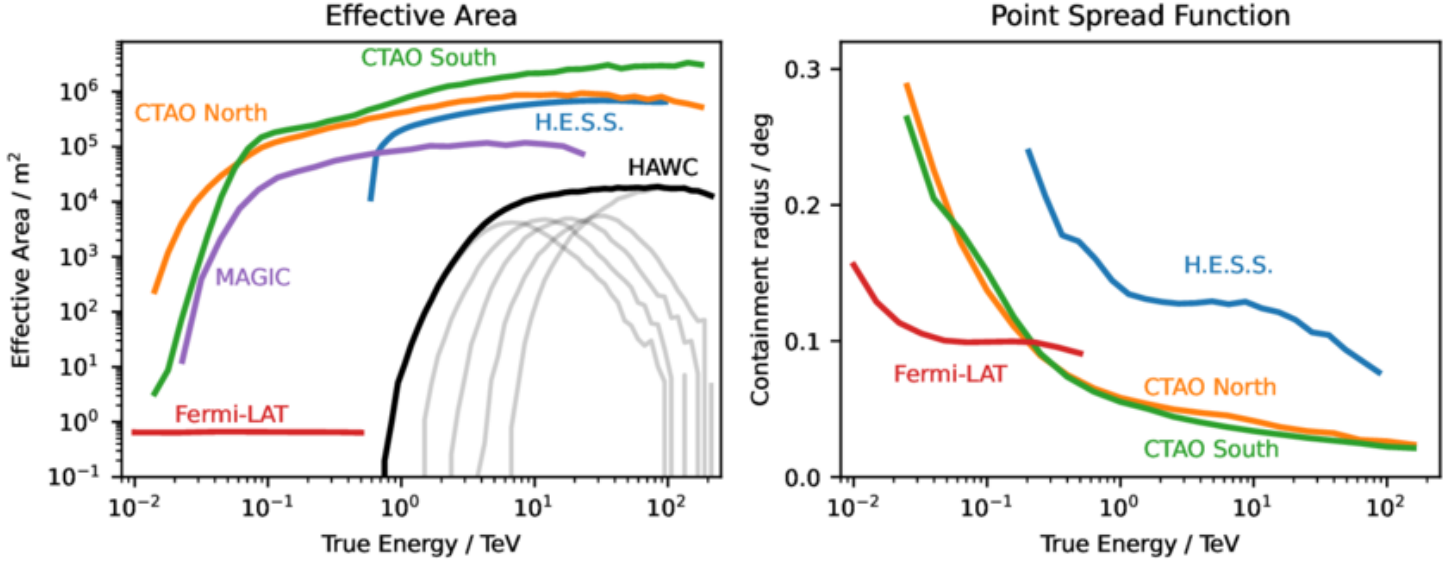
Next Steps

- **Relativistic density construction.** Extend the phase-space integral to the Schwarzschild metric, with the capture radius at $2R_S$ instead of the Newtonian $4R_S$. Quantify how the inner profile and the plateau change.
- **Weak-cusp initial profiles.** Extend the family to shallow cusps and compare them with the cusped and isothermal cases.



Next Steps

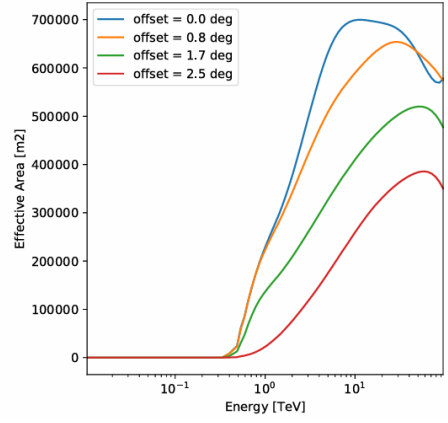
- **Sensitivity of Gamma ray-based experiment.** Use the public instrument response functions (effective area, PSF, energy dispersion) and the background of gamma ray-based experiments.

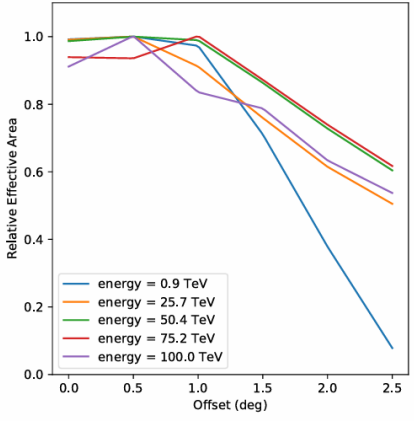


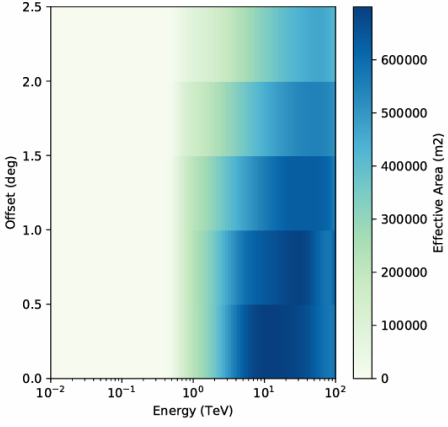
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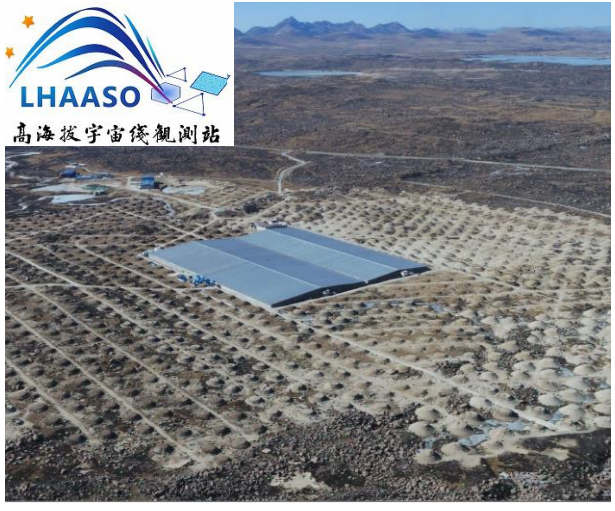


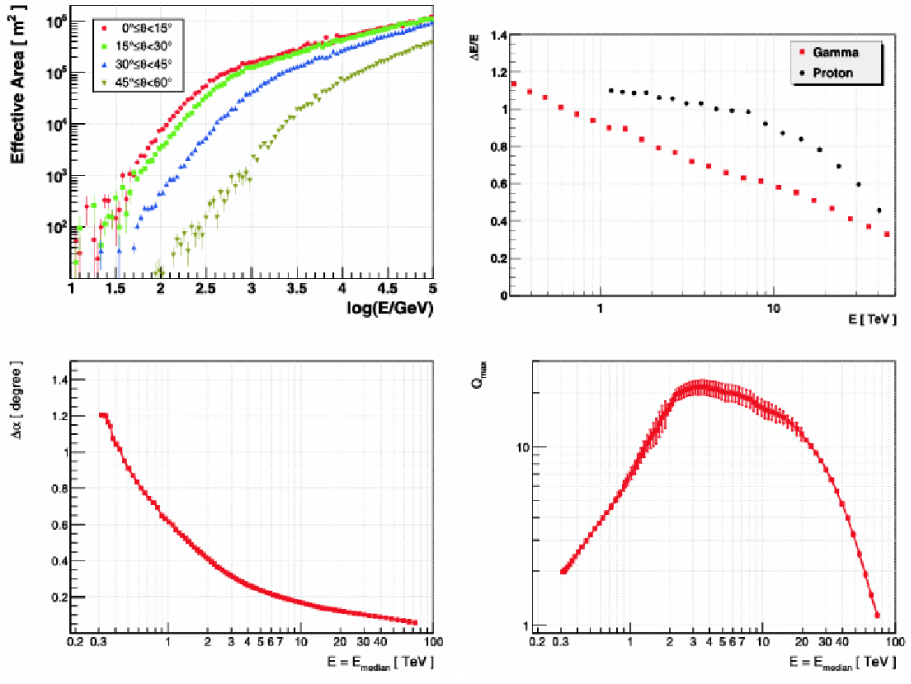




Next Steps

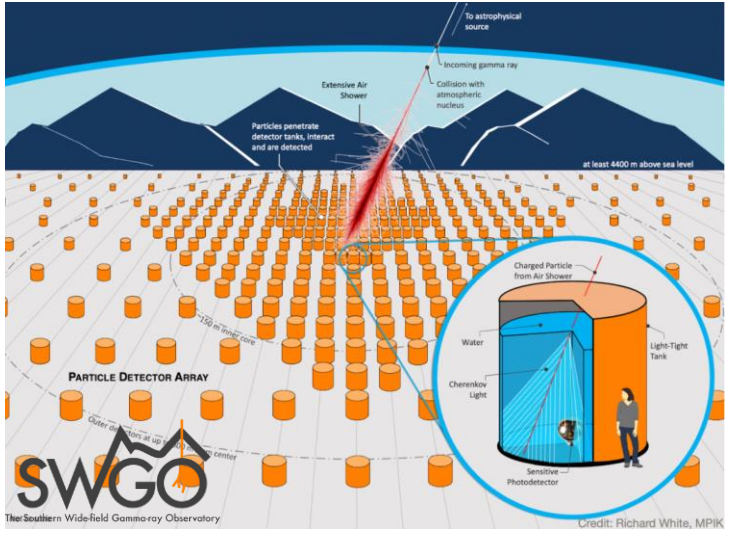
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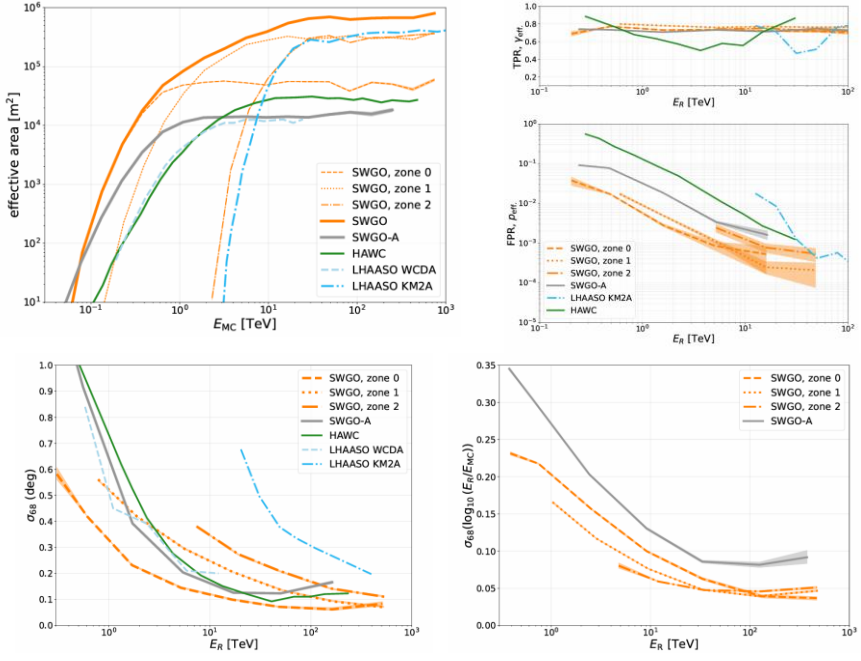




Next Steps

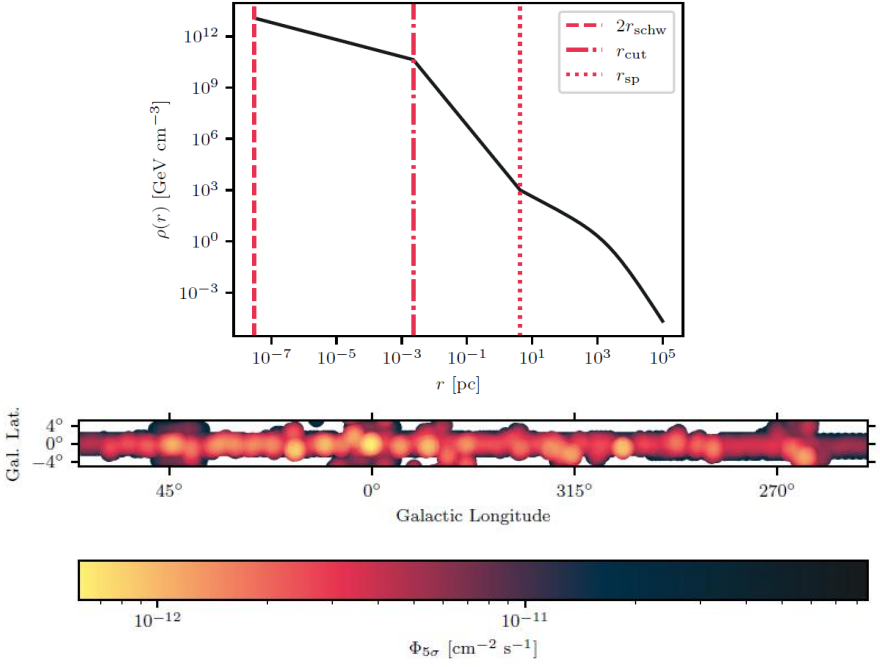
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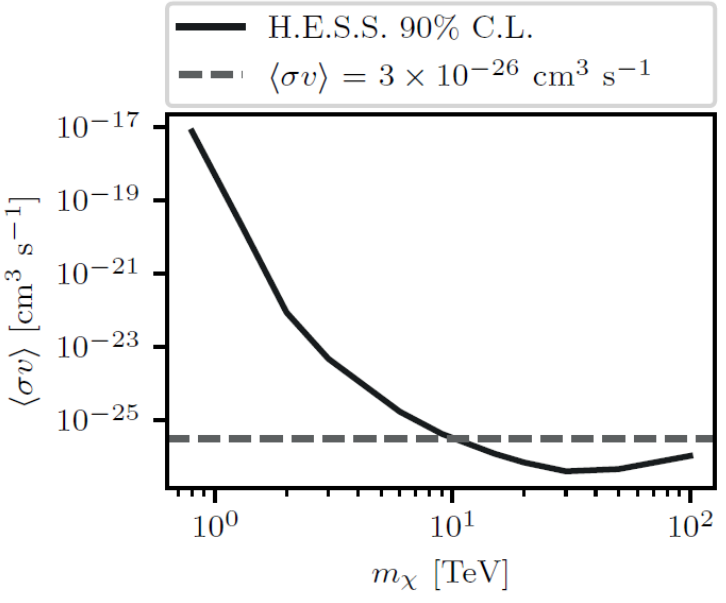


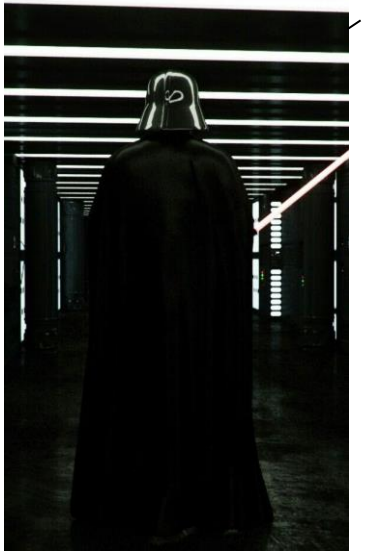


Next Steps

- **Future upper limits.** Convert the sensitivity into projected upper limits on $\langle \sigma v \rangle$ as a function of m_χ,for several annihilation channels and initial slopes.







THANK YOU!