Slowly rotating stars in light cone coordinates

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Motivation

- Hartle–Thorne: slow rotation in a pressure-adapted chart.
- Radiation is extracted at future null infinity, $\mathscr{I}^+$.
- Goal: one coordinate system from the regular centre to $\mathscr{I}^+$.
- Why not transform Hartle–Thorne? Affine-null continues into the dynamical regime (critical collapse); HT coordinates do not. Transformation is algebraically intractable and still requires re-expansion and re-matching.
- Gauge and regularity control (vertex, surface matching, Bondi-frame Geroch–Hansen moments) are transparent from the start.
- Same infrastructure (enthalpy, surface series, matching) carries over to the time-dependent problem.

Affine-null metric

$$ds^2 = -W\,du^2 - 2\,du\,d\lambda + R^2 h_{AB}(dx^A - W^A du)(dx^B - W^B du)$$

- u =const: outgoing null hypersurfaces.
- λ: affine parameter along the generators.
- Stationary + axisymmetric $\Rightarrow$ independent of u, ϕ.

Slow-rotation/quasi-spherical approximation

Expand the metric and stress-energy in a formal parameter ε:

$$g_{ab} = g_{ab}^{\mathrm{TOV}} + g_{ab}^{(1)}\varepsilon + g_{ab}^{(2)}\varepsilon^2 + O(\varepsilon^3),$$

$$T_{ab} = T_{ab}^{\mathrm{TOV}} + T_{ab}^{(1)}\varepsilon + T_{ab}^{(2)}\varepsilon^2 + O(\varepsilon^3).$$

- ε^0: static spherical TOV star on null rays.
- ε^1: frame dragging $\omega = \Omega - W^{\phi}_{[1]}$.
- ε^2: $\ell = 0$ mass correction + $\ell = 2$ deformation.

Principal equations

- Background: TOV system for (r, m, p).
- First order: second-order ODE for ω (or first-order system with ϖ).
- Second order: master equation for ψ (proportional to the Weyl scalar Ψ_0), split into homogeneous + particular.

Exterior and matching

- Vacuum exterior solved with associated Legendre Q_ℓ^m.
- Match ψ and φ at the surface $\Rightarrow C_Q^\gamma$, C_P^γ.
- Komar integrals: $K_m = m_0 + \delta m\, \varepsilon^2$, $K_L = J\varepsilon$.

Quadrupole

Newman–Penrose Ψ_2 in a Bondi frame:

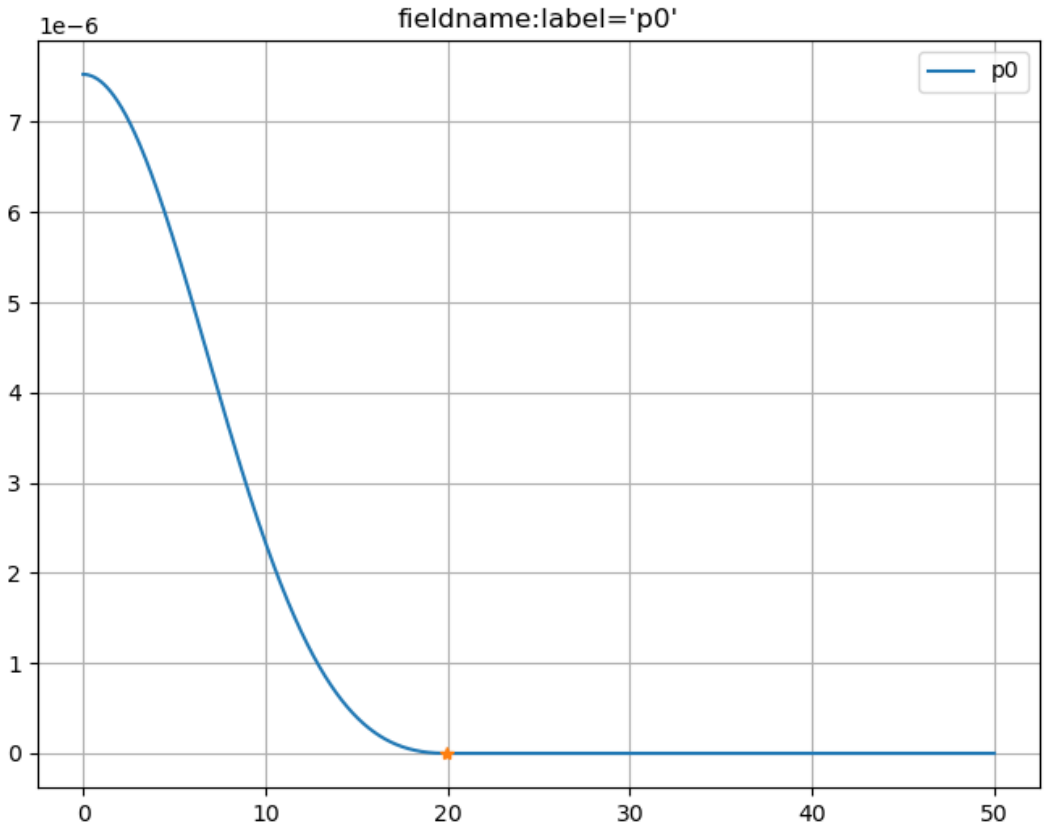
$$Q = -\left(\frac{J^2}{m_0} - \frac{4\, C_Q^\gamma\, m_0^4}{5(A_1^r)^2}\right).$$

- Kerr limit: $C_Q^\gamma = 0 \Rightarrow Q = -J^2/M$.

Numerical status

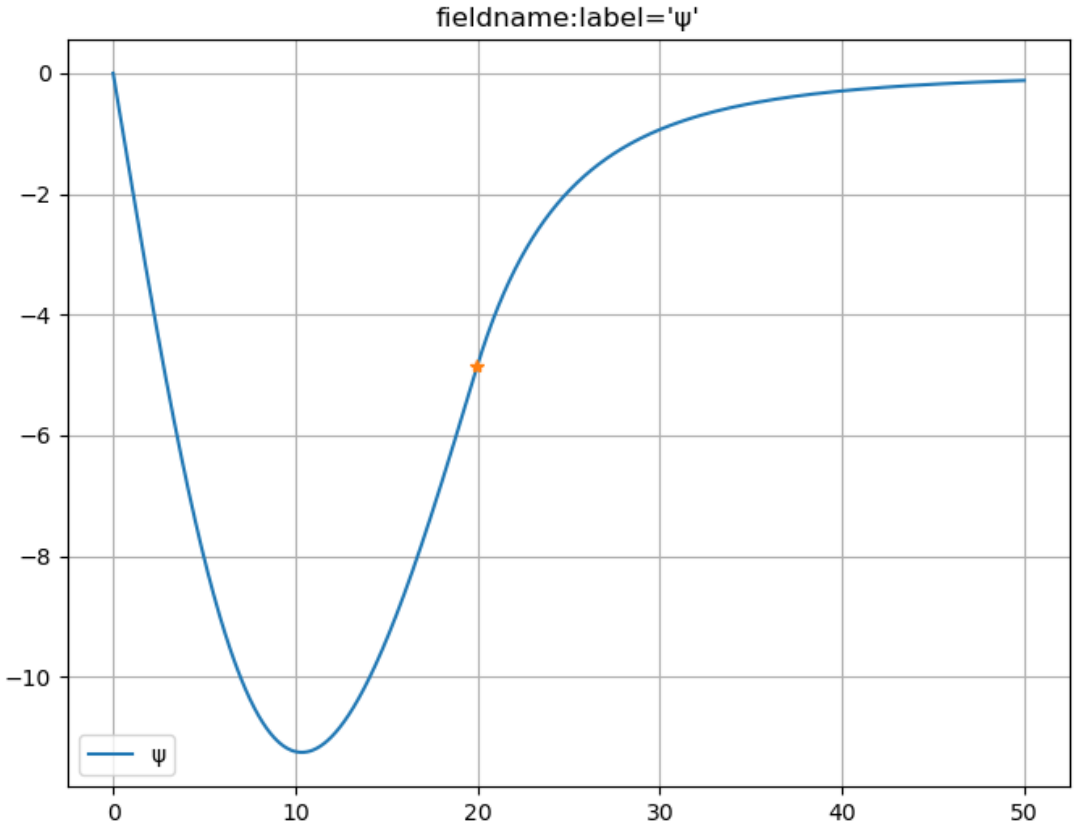
- Enthalpy formulation removes divergent de/dp.
- Surface series in $z = p^{(n-1)/n}$ for the last shell.
- Residual in $\ell = 2$ functions reduced (constrained by the field equations; not all integration constants can be matched).

Pressure profile



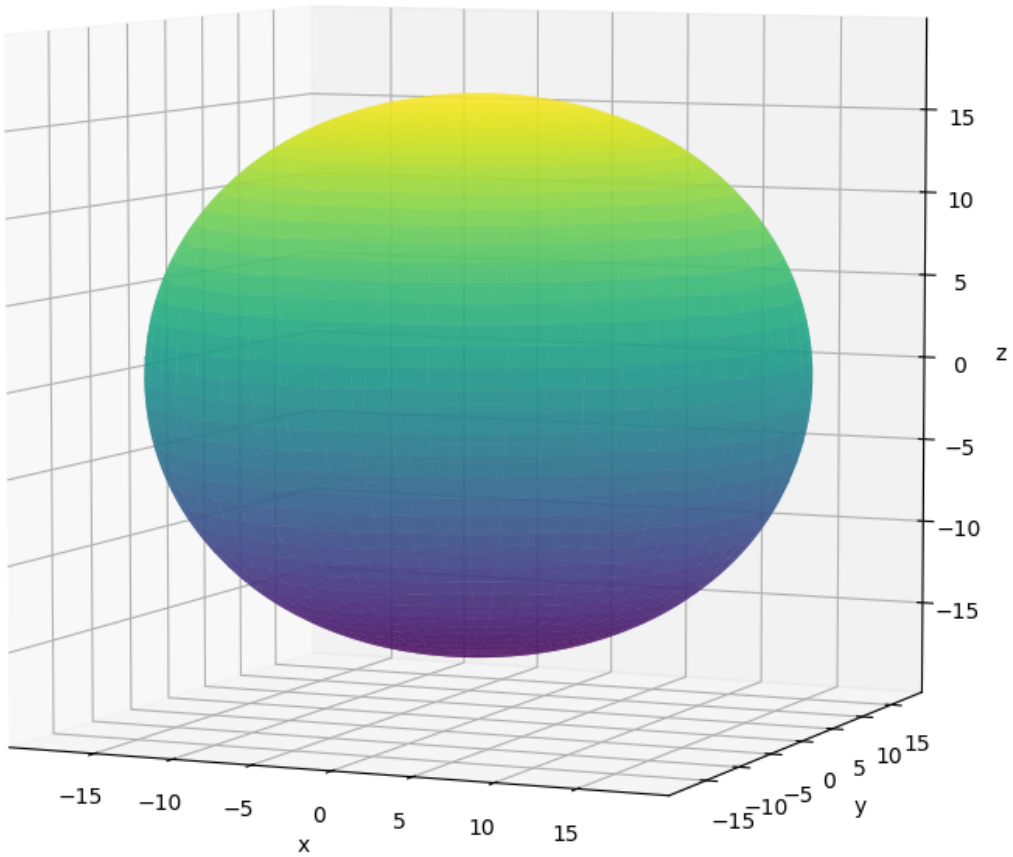
Pressure $p_0(\lambda)$ for a polytrope ($n = 1.9$, $C = 100$).
The star surface is marked by the orange star where $p = 0$.

Matching of ψ



Master function $\psi(\lambda)$ (interior numerical + exterior analytic). Orange star marks the stellar surface where continuity of ψ and φ is imposed.

Oblate surface



Proper-distance surface at moderate spin ($\Omega_{\text{phys}} = 0.06$).
The polar radius is smaller than the equatorial radius (oblate).

Outlook

- Extract J, δm, C^{γ}_{Q} for a sequence of central densities.
- Characteristic evolution of the matched data to $\mathscr{I}^{+}$.

Thank you

Questions?