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Nonlinear Scalarization of Reissner-Nordström Black Holes in Scalar-Maxwell-Teleparallel Gravity

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Outline

1. Teleparallel Equivalent of General Relativity
2. Model of Nonlinear Scalarization
3. Decoupled Scalar Dynamics on fixed Reissner-Nordström Background
4. Scalarized Black Hole Solutions
5. Final Remarks

Teleparallel Equivalent of General Relativity (TEGR)

The dynamical fields are the **vierbein** (tetrad fields) $\mathbf{e}_a(x^\mu)$, forming an orthonormal basis $\mathbf{e}_a \cdot \mathbf{e}_b = \eta_{ab}$, which are relate to the spacetime metric $g_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu \,.$$

This theory uses the **Weitzenböck connection** ($\omega^a{}_{b\mu} = 0$), characterized by zero curvature ($R^\rho{}_{\sigma\mu\nu} = 0$) but non-null torsion ($T^\lambda{}_{\mu\nu} \neq 0$):

$$\Gamma^\sigma_{\mu\nu} = e^\sigma_a \partial_\mu e^a_\nu \,.$$

The Torsion, Contorsion and Superpotential are defined directly from this connection as:

$$\begin{aligned}
T^\sigma{}_{\mu\nu} &\equiv \Gamma^\sigma{}_{\mu\nu} - \Gamma^\sigma{}_{\nu\mu} \,, \\
K^\sigma{}_{\mu\nu} &= \frac{1}{2}\left(T_\mu{}^\sigma{}_\nu + T_\nu{}^\sigma{}_\mu - T^\sigma{}_{\mu\nu} \right) \,, \\
S_\sigma{}^{\mu\nu} &\equiv \frac{1}{2}\left(K^{\mu\nu}{}_\sigma + \delta^\mu_\sigma T^\nu - \delta^\nu_\sigma T^\mu \right) \,,
\end{aligned}$$

where $T^\mu = T^{\lambda\mu}_{\ \ \lambda}$ is the torsion trace vector.

[Fundam.Theor.Phys. 173 (2013)]

Teleparallel Equivalent of General Relativity (TEGR)

Those quantities lead to the **Torsion scalar** $\mathcal{T}$ (Weitzenböck invariant):

$$\mathcal{T} = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu} = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}{}^\rho T^{\nu\mu}{}_\nu \,.$$

The TEGR action is given by

$$I = \frac{1}{2\kappa} \int d^4x e \left(\mathcal{T} + \mathcal{L}_m\right),$$

where $\kappa = 8\pi G$, $e = \det(e^a{}_\mu) = \sqrt{-g}$ and $\mathcal{L}_m$ is the matter Lagrangian. The variation with respect to the vierbein yields the equations of motion

$$e^{-1}\partial_\mu(eS_a{}^{\mu\nu}) - e_a{}^\lambda T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} - \frac{1}{4} e_a{}^\nu \mathcal{T} = \frac{\kappa}{2} e_a{}^\rho \Theta_\rho{}^\nu \,,$$

where $\Theta_\rho{}^\nu$ is the energy-momentum tensor.

Equivalence with GR: They differ only by a total boundary divergence: $\mathcal{T} = -R + 2e^{-1}\partial_\nu(eT_\sigma{}^{\sigma\nu})$.

[Fundam.Theor.Phys. 173 (2013)]

Model of Nonlinear Scalarization

In this work, we consider the scalar-Maxwell-torsion model given by the following action

$$I = \int d^4x\, e \left[\left(\frac{1}{2\kappa} + f(\phi) \right) \mathcal{T} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2\kappa} F_{\mu\nu} F^{\mu\nu} \right] ,$$

where ϕ is the scalar field, $f(\phi)$ is a coupling function of the scalar field and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell tensor with $A_\mu dx^\mu = \Psi(r)dt$.

We are interested in **studying nonlinear scalarized black hole solutions** in the model, so the coupling function must follow the conditions

$$\left. \frac{df(\phi)}{d\phi} \right|_{\phi_c} = 0 \Rightarrow \text{GR Solution is solution},$$

$$\left. \frac{d^2 f(\phi)}{d\phi^2} \right|_{\phi_c} = 0 \Rightarrow m^2_{eff} = 0\,.$$

A finite perturbation is needed to activate the nonlinear terms and produce a scalarized phase.

[PhysRevD.106.104064]

Model of Nonlinear Scalarization

Varying the action with respect to the vierbein, the scalar field and the four-potential, the equations of motion respectively are

$$\left(\frac{2}{\kappa}+4f(\phi)\right)\left[e^{-1}\partial_\mu(eS_a{}^{\mu\nu})+T^\rho{}_{\mu a}S_\rho{}^{\mu\nu}-\frac{1}{4}e_a{}^\nu\mathcal{T}\right]-\frac{1}{2}e_a{}^\nu\left[\partial_\mu\phi\partial^\mu\phi-\frac{1}{\kappa}F_{\alpha\beta}F^{\alpha\beta}\right]$$
$$+4f'(\phi)(\partial_\mu\phi)S_a{}^{\mu\nu}+\partial^\nu\phi\partial_a\phi-\frac{2}{\kappa}F^{\nu\lambda}F_{a\lambda}=0\,,$$

$$e^{-1}\partial_\nu(eg^{\mu\nu}\partial_\mu\phi)-f'(\phi)\mathcal{T}=0\,,$$

$$e^{-1}\partial_\nu\left(\frac{2e}{\kappa}F^{\nu\mu}\right)=0\,.$$

In the following we consider the cases

$$f_1(\phi)=\eta\phi^4\qquad f_2(\phi)=\eta\phi^4-\gamma\phi^6\,.$$

where η and γ are constants which shows the strength of the interaction.

Model of Nonlinear Scalarization

In order to find black hole solutions, we assume spherical symmetry. A tetrad that satisfy this condition within the Weitzenböck gauge is

$$e^{a}{}_{\mu} = \begin{pmatrix} 0 & \frac{i}{\sqrt{B(r)}} & 0 & 0 \\ i\sqrt{A(r)}\cos\varphi\sin\theta & 0 & -r\sin\varphi & -r\cos\theta\cos\varphi\sin\theta \\ i\sqrt{A(r)}\sin\theta\sin\varphi & 0 & r\cos\varphi & -r\cos\theta\sin\theta\sin\varphi \\ i\sqrt{A(r)}\cos\theta & 0 & 0 & r\sin^2\theta \end{pmatrix} .$$

Using the relation $g_{\mu\nu} = \eta_{ab} e^{a}_{\mu} e^{b}_{\nu}$, this complex tetrad yields the metric in standard spherical coordinates:

$$ds^2 = A(r)dt^2 - \frac{1}{B(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) .$$

Objective: Investigate the dynamic formation and stability of scalarized RNBH phases.

Background: Fixed RN metric, $A(r) = B(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$. The time-dependent Klein-Gordon equation is given by:

$$\partial_t^2 \phi = n(r, q, r_+) \frac{df_i}{d\phi} + l(r, q, r_+) \partial_r \phi + m(r, q, r_+) \partial_r^2 \phi \,,$$

where $r_+ = M + \sqrt{M^2 - Q^2}$ is the event horizon radius, and $q \equiv \frac{Q}{r_+}$ is the reduced charge.

Numerical Method: Method of lines with a 4th-order Runge-Kutta integration.

Initial State: Gaussian distribution with amplitude A:

$$\phi(t = 0, r) = A \exp\left(-\frac{(r - \overline{r})^2}{2\sigma^2} \right) .$$

Setup Parameters: We set $r_+ = 1$, $q = 0.5$ ($M = 0.625$), $\sigma = 1$, and $\overline{r} = 12 r_+$. The signal is extracted at $r_{sg} = 18 r_+$.

Decoupled Scalar Dynamics on fixed Reissner-Nordström Background

Time Evolution

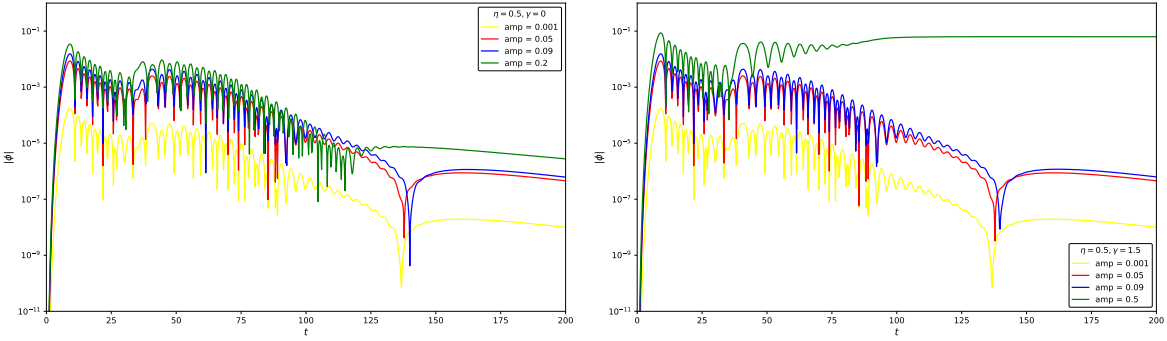


Figure: Time evolution of the scalar field on the RNBH background. Left panel for $f_1(\phi) = \eta\phi^4$. Right panel for $f_2(\phi) = \eta\phi^4 - \gamma\phi^6$. Here $\eta = 0.5$ and $\gamma = 1.5$.

No Scalarization: The system is purely unstable. No stable scalarized phases can be formed for $f_1(\phi) = \eta\phi^4$.

Dynamic Scalarization: For $\gamma > 0$, a threshold perturbation destabilizes the RNBH, dynamically forming a scalarized phase.

We rewrite the time independent Klein-Gordon equation to introduce the effective potential

$$V_{eff}^{(i)}(\phi) = \frac{2\left(q^2 r_+^2 - 2r^2\right)}{r^4} f_i(\phi)\,,$$

By setting $\frac{dV_{eff}^{(i)}(\phi)}{d\phi} = 0$, we find the local extrema of the potential

$$f_1(\phi) = \eta\phi^4 \longrightarrow \phi_c = 0\,,$$

$$f_2(\phi) = \eta\phi^4 - \gamma\phi^6 \longrightarrow \phi_c = \left\{0,\, \pm\sqrt{\frac{2\eta}{3\gamma}}\right\}\,.$$

Decoupled Scalar Dynamics on fixed Reissner-Nordström Background

Effective Potential

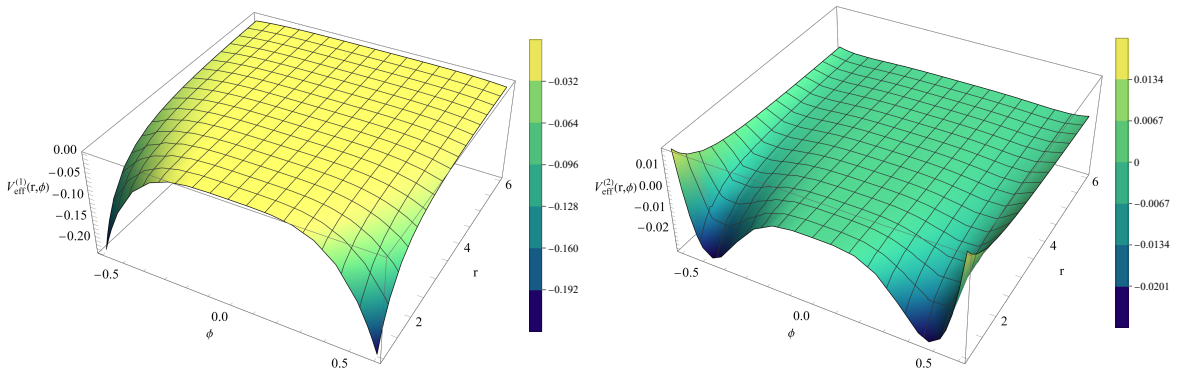


Figure: 3D plot for the effective potential $V_{eff}(r, \phi)$. Left panel for $f_1(\phi) = \eta\phi^4$. Right panel for $f_2(\phi) = \eta\phi^4 - \gamma\phi^6$. Here $\eta = 0.5$ and $\gamma = 1.5$.

Instability: "Inverted bowl" potential causes a nonlinear runaway. Fails to support stable scalar hair.

Stability: The $\gamma\phi^6$ term creates a potential well ("w-shape") that traps the scalar. Enables stable scalarized BHs.

Scalarized Black Hole Solutions
Numerical Setup

We introduce the bounded radial coordinate $z = 1 - r_+/r$ and redefine the variables as:

$$A(z) = za(z), \quad B(z) = zb(z), \quad \phi(z) = (1-z)\tilde{\phi}(z), \quad \Psi(z) = z\psi(z).$$

The ODEs are solved via a shooting method, which requires series expansions of the fields near the event horizon and at spatial infinity.

Event Horizon ($z = 0$)

$$a(z) = a_H + \sum_{n=1}^{10} a_{Hn}(\phi_H, a_H, \gamma, \eta, q) z^n, \qquad b(z) = b_H + \sum_{n=1}^{10} b_{Hn}(\phi_H, a_H, \gamma, \eta, q) z^n,$$

$$\tilde{\phi}(z) = \phi_H + \sum_{n=1}^{10} \phi_{Hn}(\phi_H, a_H, \gamma, \eta, q) z^n, \qquad \psi(z) = \kappa \sqrt{\frac{a_H}{b_H}} q + \sum_{n=1}^{10} \psi_{Hn}(\phi_H, a_H, \gamma, \eta, q) z^n.$$

Spatial Infinity ($z \to 1$)

$$a(z) = 1 + \sum_{n=1}^{10} a_n(\phi_0, b_1, \psi_0, \gamma, \eta, q)(1-z)^n, \qquad b(z) = 1 + \sum_{n=1}^{10} b_n(\phi_0, b_1, \psi_0, \gamma, \eta, q)(1-z)^n,$$

$$\tilde{\phi}(z) = \phi_0 + \sum_{n=1}^{10} \phi_n(\phi_0, b_1, \psi_0, \gamma, \eta, q)(1-z)^n, \qquad \psi(z) = \psi_0 + \sum_{n=1}^{10} \psi_n(\phi_0, b_1, \psi_0, \gamma, \eta, q)(1-z)^n.$$

where $b_H = 1 - \frac{q^2\kappa^2}{1+2\kappa f_2(\phi_H)}$.

Scalarized Black Hole Solutions

Parameter Space: Asymptotically flat scalarized black holes solutions are characterized by a set of eight parameters $\{\phi_H, a_H, \phi_0, b_1, \psi_0, q, \eta, \gamma\}$.

Boundary Constraint: For fixed q, η, and γ, the condition $\phi(z \to 1) \to 0$ restricts the system to specific discrete combinations of the remaining parameters.

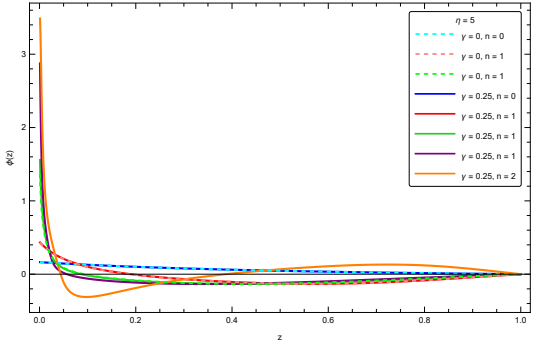
Solution Branches:

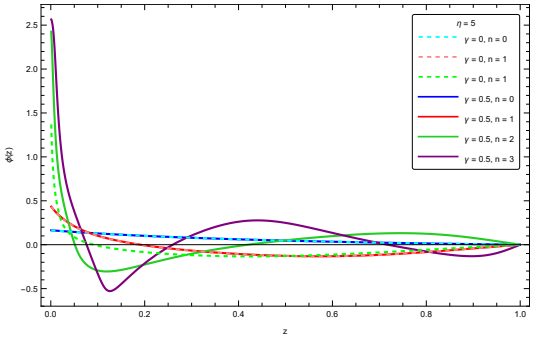
- $\gamma = 0$: Exhibits a countable set of degenerate solutions labeled by the number of nodes n (up to $n_{\max}$).
- $\gamma > 0$: The non-linear interaction breaks the degeneracy, spawning new branches of scalarized solutions.

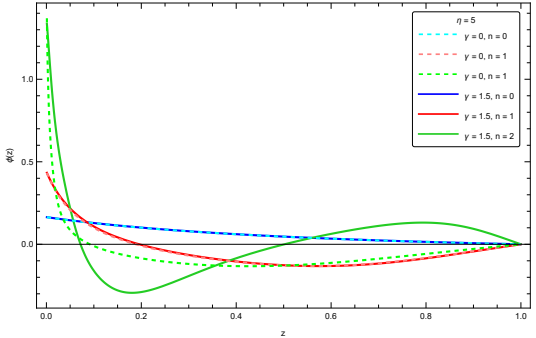
Solutions $\eta = 5$

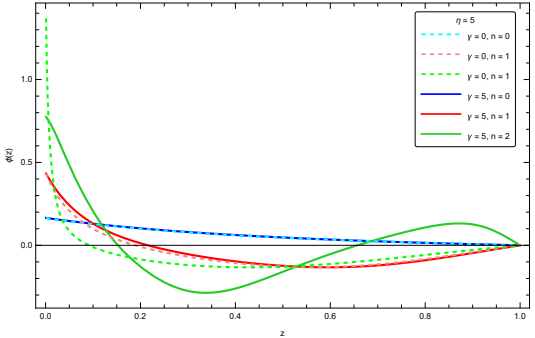
γ	NoS	Nodes (n)
0	3	0, 1, 1
0.25	3+2	0, 1, 1, 1, 2
0.5	3+2	0, 1, 1, 1, 2
1.5	2+2	0, 1, 2, 3
5	1+2	0, 1, 2
10	1+1	0, 1
23	1	0
89	0	–

Scalarized Black Hole Solutions

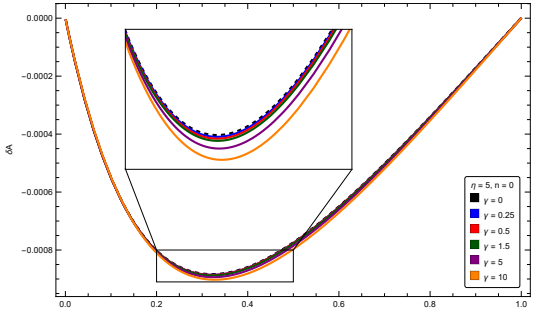


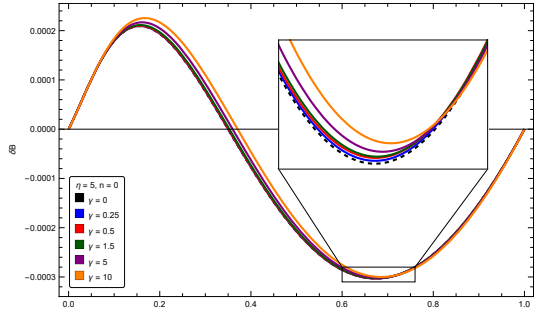


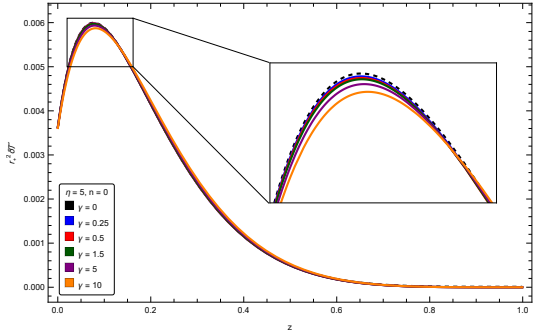




Scalarized Black Hole Solutions







Final Remarks
Summary

Scalarized RNBH Dynamics: We investigated the formation and stability of scalarized Reissner-Nordström black holes using both static boundary analysis and time-domain evolution.

Instability of Quartic Coupling ($f_1(\phi) = \eta\phi^4$):

- Leads to a nonlinear runaway instability, failing to support stable scalar hair.
- Induces an "inverted bowl" effective potential.
- Static Solutions: Yields degenerate solutions.

Stabilization via Higher-Order Terms ($f_2(\phi) = \eta\phi^4 - \gamma\phi^6$):

- The $\gamma\phi^6$ term introduces repulsive feedback, creating a potential well that traps the scalar field.
- Dynamic Formation: A threshold perturbation destabilizes the RN background, allowing the system to dynamically evolve into a stable scalarized phase.
- Static Solutions: Breaks the degeneracy of the $\gamma = 0$ configurations, spawning new discrete branches of scalarized solutions.

- **Thermodynamic Preference:** Compute the thermodynamic quantities of the solutions to rigorously determine whether the scalarized phases are entropically preferred over the standard RNBH.
- **Observational Constraints:** Calculate the angular shadow of the scalarized black holes to restrict the parameter space of the coupling constants (η, γ).
- **Electromagnetic Scalarization:** Extend the theoretical framework by introducing a non-minimal coupling to the electromagnetic sector, $\sim h(\phi)F_{\mu\nu}F^{\mu\nu}$, to explore new dynamical branches.

Final Remarks
Acknowledgments

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