Emergence of cosmic structure from Planckian discreteness

Gabriel León

In collaboration with G. R. Bengochea (IAFE–UBA, CONICET) and A. Perez (Univ. Aix-Marseille)

Instituto de Astrofísica de La Plata (CONICET)
Universidad Nacional de La Plata, Argentina

3rd International Joint Meeting on Cosmology and Gravitation
(Universidad de Tarapacá, Arica, Chile)

Outline

1. Standard Inflation and its conceptual challenges
2. Inflation from Planckian discreteness: our proposal
3. Generation of perturbations
4. Energy transfer hypothesis and the primordial spectrum
5. Observational consequences
6. Conclusions and outlook

Inflation: The Standard Picture

Well-established successes:

- An early cosmological epoch characterized by an accelerated expansion of the Universe
- *Historically:* Solves horizon, flatness problems.
- *Today:* Predicts nearly scale-invariant spectrum of perturbations, consistent with *Planck* 2018.

The standard inflationary picture:

- Inflaton field: $\phi = \phi_0(\eta) + \delta\phi(\mathbf{x}, \eta)$
- Metric: $g_{ab} = g^{(0)}_{ab} + \delta g_{ab} \rightarrow$ Prim. curv. pert. $\mathcal{R}(x, \eta)$
- Initial state: Bunch–Davies vacuum
- **"Quantum fluctuations" $\rightarrow$ seeds of structure**

Inflation: The Standard Picture

Well-established successes:

- An early cosmological epoch characterized by an accelerated expansion of the Universe
- *Historically:* Solves horizon, flatness problems.
- *Today:* Predicts nearly scale-invariant spectrum of perturbations, consistent with *Planck* 2018.

The standard inflationary picture:

- Inflaton field: $\phi = \phi_0(\eta) + \delta\phi(\mathbf{x}, \eta)$
- Metric: $g_{ab} = g^{(0)}_{ab} + \delta g_{ab} \rightarrow$ Prim. curv. pert. $\mathcal{R}(x, \eta)$
- Initial state: Bunch–Davies vacuum
- **"Quantum fluctuations" $\rightarrow$ seeds of structure**

Three Notions of "Fluctuations"

A crucial source of confusion in inflationary cosmology:

I. Quantum Uncertainties

Indefiniteness of a quantum quantity

E.g., width of the ground-state Gaussian in a harmonic oscillator

Not stochastic!

II. Stochastic Fluctuations

Variation across an ensemble of outcomes

E.g. the height of people here

Requires that each element has a *definite* value

III. Space-time Variations

Changes of a quantity from one point to another in an extended system

E.g. the temperature in this classroom

Physical inhomogeneity

Standard inflation treat all three equally! This confusion affects the **"explanation" of structure formation**.

Three Notions of "Fluctuations"

A crucial source of confusion in inflationary cosmology:

I. Quantum Uncertainties	II. Stochastic Fluctuations	III. Space-time Variations
Indefiniteness of a quantum quantity	Variation across an ensemble of outcomes	Changes of a quantity from one point to another in an extended system
E.g., width of the ground-state Gaussian in a harmonic oscillator	E.g. the height of people here	E.g. the temperature in this classroom
Not stochastic!	**Requires that each element has a *definite* value**	**Physical inhomogeneity**

Standard inflation treat all three equally! This confusion affects the **"explanation" of structure formation**.

Three Notions of "Fluctuations"

A crucial source of confusion in inflationary cosmology:

I. Quantum Uncertainties	II. Stochastic Fluctuations	III. Space-time Variations
Indefiniteness of a quantum quantity	Variation across an ensemble of outcomes	Changes of a quantity from one point to another in an extended system
E.g., width of the ground-state Gaussian in a harmonic oscillator	E.g. the height of people here	E.g. the temperature in this classroom
Not stochastic!	**Requires that each element has a *definite* value**	**Physical inhomogeneity**

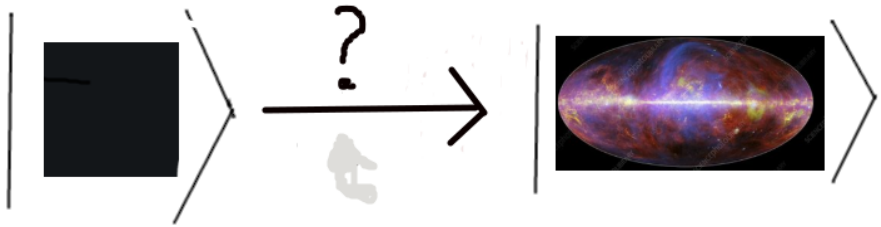
Standard inflation treat all three equally! This confusion affects the **"explanation" of structure formation**.

Three Notions of "Fluctuations"

A crucial source of confusion in inflationary cosmology:

I. Quantum Uncertainties	II. Stochastic Fluctuations	III. Space-time Variations
Indefiniteness of a quantum quantity	Variation across an ensemble of outcomes	Changes of a quantity from one point to another in an extended system
E.g., width of the ground-state Gaussian in a harmonic oscillator	E.g. the height of people here	E.g. the temperature in this classroom
Not stochastic!	**Requires that each element has a *definite* value**	**Physical inhomogeneity**

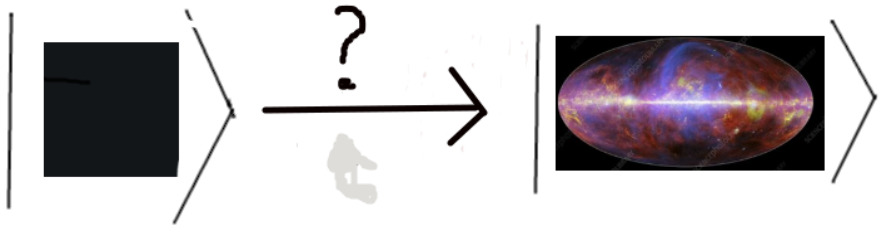
Standard inflation treat all three equally! This confusion affects the **"explanation" of structure formation**.



Quantum fluctuations *can be* turned into classical density fluctuations by something **like a measurement**, i.e. by collapsing the wavefunction corresponding to the initial symmetric state (the BD vacuum)

The measurement or collapse is necessary, otherwise the expectation value of T_{ab} is homogeneous and isotropic

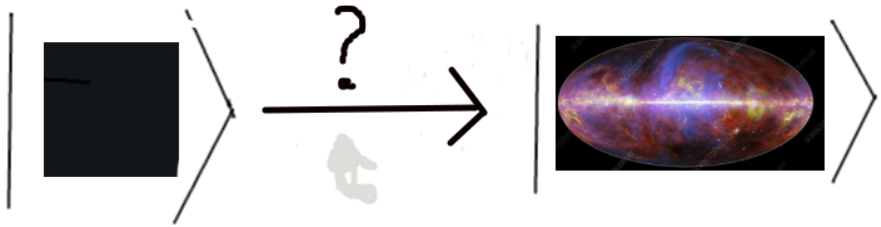
$$\langle \mathrm{BD}|\hat{T}_{ab}(t,\mathbf{x})|\mathrm{BD}\rangle = \langle \mathrm{BD}|\hat{T}_{ab}(t,\mathbf{x}+\mathbf{r})|\mathrm{BD}\rangle \qquad (1)$$



Quantum fluctuations *can be* turned into classical density fluctuations by something **like a measurement**, i.e. by collapsing the wavefunction corresponding to the initial symmetric state (the BD vacuum)

The measurement or collapse is necessary, otherwise the expectation value of T_{ab} is homogeneous and isotropic

$$\langle \mathrm{BD}|\hat{T}_{ab}(t,\mathbf{x})|\mathrm{BD}\rangle = \langle \mathrm{BD}|\hat{T}_{ab}(t,\mathbf{x}+\mathbf{r})|\mathrm{BD}\rangle \qquad (1)$$

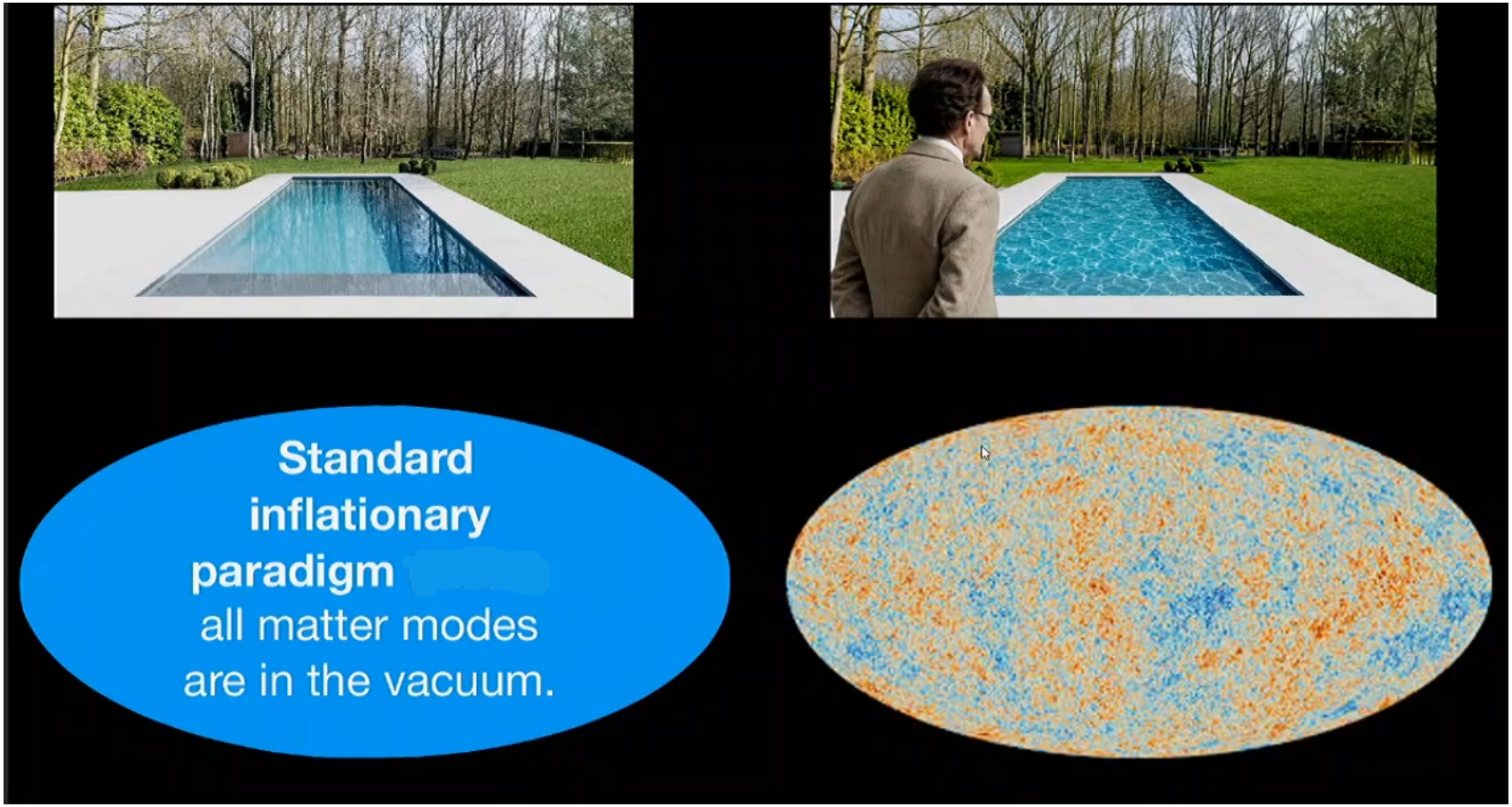


Quantum fluctuations *can be* turned into classical density fluctuations by something **like a measurement**, i.e. by collapsing the wavefunction corresponding to the initial symmetric state (the BD vacuum)

The measurement or collapse is necessary, otherwise the expectation value of T_{ab} is homogeneous and isotropic

$$\langle \mathrm{BD}|\,\hat{T}_{ab}(t,\mathbf{x})|\mathrm{BD}\rangle = \langle \mathrm{BD}|\,\hat{T}_{ab}(t,\mathbf{x}+\mathbf{r})|\mathrm{BD}\rangle \qquad (1)$$

The “cosmic” Schrödinger cat problem



Trans-Planckian problem

- The physical wavelengths of comoving scales which contribute the most to the CMB anisotropies, when traced backward to the start of inflation, were smaller than the Planck length.
- Equivalently, for a typical mode

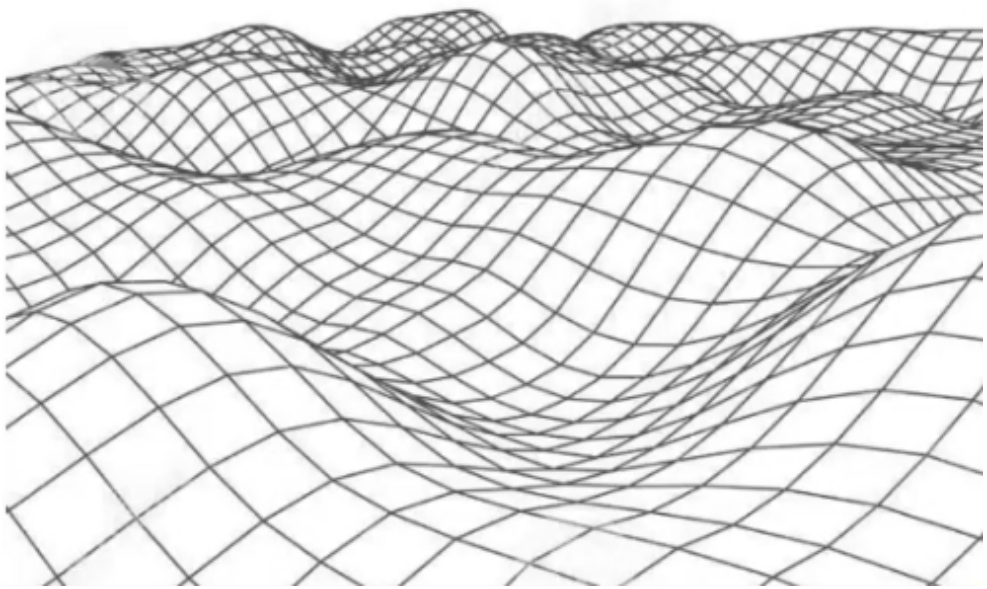
$$k_*^{\rm ph}(a_{\rm ini}) = \frac{k_*}{a_{\rm ini}} = \frac{a_* H_*}{a_{\rm ini}} \qquad \text{with} \qquad \frac{H_*^2}{M_P^2 \epsilon_1^*} \simeq 10^{-9} \tag{2}$$

so if inflation lasts long enough, then $a^* \simeq \exp(N_*) a_{\rm ini} \gg 1$ before horizon exit and

$$k_*^{\rm ph}(a_{\rm ini}) \gg M_P \tag{3}$$

An alternative scenario

Primordial inhomogeneities are born from the interaction between matter fields and a fundamental discreteness of the spacetime (translational symmetry has no meaning at this scale)

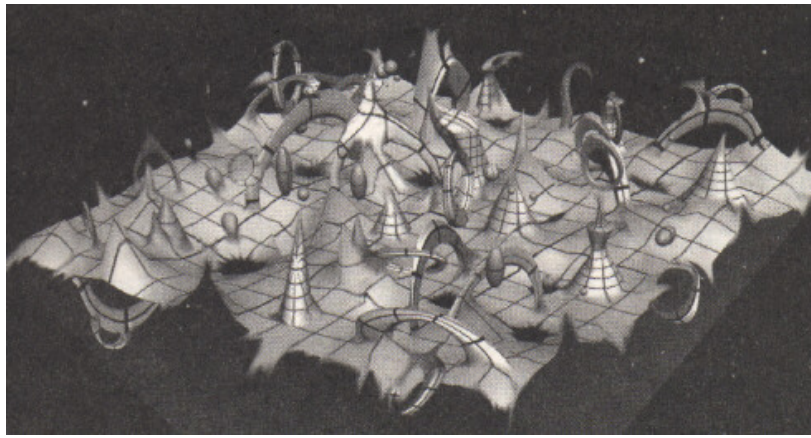


Discreteness should manifest itself in regions of **high curvature**

J. Collins, A. Perez, D. Sudarsky, L. Urrutia and H. Vucetich, Phys. Rev. Lett. **93**, 191301 (2004).

The Planckian discreteness hypothesis

- We assume a quasi-de Sitter stage with $H \sim M_{\rm P}$.
- Microscopic geometric degrees of freedom excite scalar modes as they reach $k/a = M_{\rm P}$.
- The emerging state $|\psi\rangle$ need not share the symmetries of the macroscopic FLRW geometry.



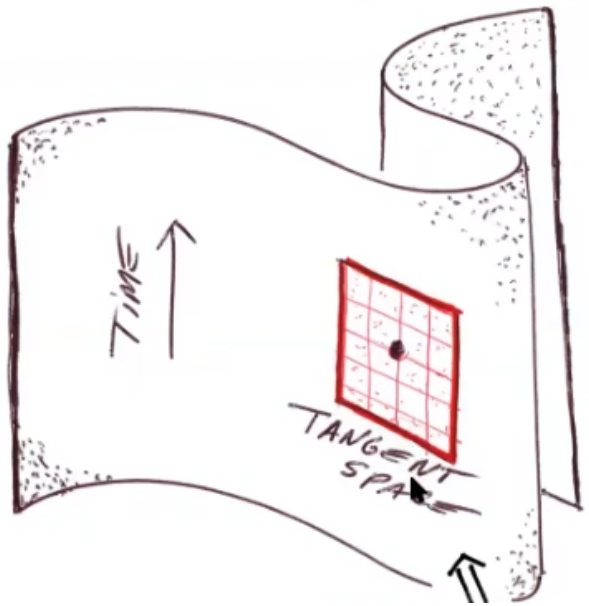
$$\langle\psi|\,\hat{T}_{ab}(t,\mathbf{x})|\psi\rangle \neq \langle\psi|\,\hat{T}_{ab}(t,\mathbf{x}+\mathbf{r})|\psi\rangle$$

Advantages of our proposal

(i) There is no trans-Planckian problem by construction
(ii) The "cosmic Schrödinger cat" problem is bypassed

Conservation of the energy-momentum tensor?

- Recall that $\nabla_a T^{ab} = 0$ is directly linked to the local symmetries.

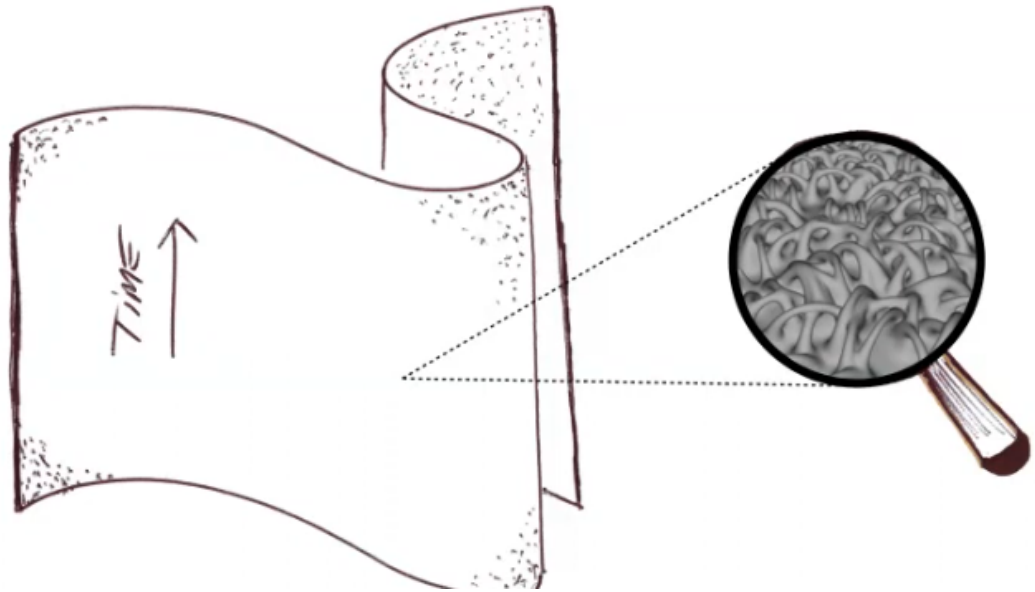


Spacetime is always smooth and continuous

Conservation of the energy-momentum tensor?

- But what if there is a **fundamental granularity** associated with the spacetime.

Conservation of Energy: fails if spacetime is not smooth at the Planck scale



$$\nabla_a T^{ab} \neq 0$$

Modification of the continuity equation

- For matter, we choose a scalar field.
- We model the interaction between the quantum geometry and the scalar field describing by adding a noise source term to the usual field equation.

$$\nabla^a \nabla_a \phi - \partial_\phi V[\phi] = -\xi, \tag{4}$$

  where ξ is the stochastic source.
- Consequently,

$$\nabla^a T_{ab} = -\xi \nabla_b \phi, \tag{5}$$

Simplest model: Slow-roll Inflation at Planckian scale

- We consider single scalar field ϕ (the dominant type of matter).
- ϕ is highly homogeneous; $\phi \simeq \phi_0(t) + \delta\phi(\mathbf{x}, t)$, with $\delta\phi(\mathbf{x}, t) \ll 1$
- Inflation is guaranteed if the field *rolls slowly* down the potential such that

$$\frac{\dot{\phi}^2}{2} \ll V. \tag{6}$$

  Moreover, inflation last long enough provided

$$\ddot{\phi} \ll 3H\dot{\phi} \tag{7}$$

- Those two conditions lead to **slow roll inflation but** $V^{1/4} \simeq M_P$**.**

One realization and its statistical ensemble

The inhomogeneous part of the field $\delta\phi$ is:

$$\delta\phi_{\mathbf{k}} \equiv \langle\psi|\widehat{\delta\phi}_{\mathbf{k}}|\psi\rangle \neq 0.$$

- A particular state $|\psi\rangle$ is inhomogeneous.
- An ensemble of such states can be statistically homogeneous and isotropic, with $\mathbb{E}\{\delta\phi_{\mathbf{k}}\} = 0$.
- $\langle\cdot\rangle$ and $\mathbb{E}[\cdot]$ are different averages.

After the mode is born, the metric responds semiclassically:

$$G_{ab} = M_{\mathrm{P}}^{-2}\langle\psi|\,\widehat{T}_{ab}|\psi\rangle.$$

Evolution of $\delta\phi_k$

Using the perturbed EFE at linear order and choosing the *spatially flat* gauge

$$\delta\phi''_{\mathbf{k}} + 2\mathcal{H}\delta\phi'_{\mathbf{k}} + k^2\delta\phi_{\mathbf{k}} + \left[a^2\partial^2_{\phi\phi}V - \frac{8\pi G}{a^2}\left(\frac{a^2\phi_0'^2}{\mathcal{H}}\right)'\right]\delta\phi_{\mathbf{k}} = a\xi_{\mathbf{k}}(\eta) \tag{8}$$

where $\xi_{\mathbf{k}}(\eta) = X_{\mathbf{k}}\delta(\eta - \eta_k^P)$

- $X_{\mathbf{k}}$ is a random variable, it represents the excitation of a mode (or "kick") given by the **spacetime discreteness**
- This excitation only occurs at $k/a(\eta_k^P) = M_P$
- After the generation of $\delta\phi_{\mathbf{k}}$, they evolve deterministically according to the standard continuity equation

Planck crossing and Hubble crossing

$$\frac{k}{a(\eta_k^P)} = M_P \qquad \text{Planck crossing,}$$

$$\frac{k}{a(\eta_k^{\rm HC})} = H(\eta_k^{\rm HC}) \qquad \text{Hubble crossing.}$$

- Planck crossing sets the time of the stochastic excitation.
- Hubble crossing compares the mode scale with the expansion rate.
- For exact de Sitter with $H = M_P$, the events coincide.

In quasi-de Sitter

The between these events contributes to a small modification to the scalar spectral index.

Statistics of the excitation

We assume Gaussian random amplitudes with

$$\begin{aligned} \mathbb{E}[X_{\mathbf{k}}] &= 0, \\ \mathbb{E}[X_{\mathbf{k}} X_{\mathbf{q}}] &= P_X(k)\, \delta^{(3)}(\mathbf{k}+\mathbf{q}). \end{aligned}$$

- The delta function expresses statistical translation invariance.
- Dependence on $k = |\mathbf{k}|$ expresses statistical isotropy.
- Reality requires $X_{-\mathbf{k}} = X_{\mathbf{k}}^*$.

Quantity to determine

The spectrum $P_X(k)$ will be obtained from an assumption about the "energy" transferred to the perturbations. **For the moment let us put it in stand by**

The physical scalar perturbation

Define the gauge-invariant Mukhanov–Sasaki variable and curvature:

$$
\begin{aligned}
v &= a\left(\delta\phi + \frac{\phi_0'}{\mathcal{H}}\psi\right), \\
\mathcal{R} &= \frac{v}{z}, \qquad z = a\frac{\phi_0'}{\mathcal{H}} = a\sqrt{2\epsilon_1}\, M_{\rm P}.
\end{aligned}
$$

- $\mathcal{H} = a'/a = aH$, and a prime denotes $d/d\eta$.
- In spatially flat gauge, $\psi = E = 0$ and $v = a\delta\phi$.
- Here $v_{\mathbf{k}}$ describes a specific realization $v_{\mathbf{k}} = a\langle\psi|\delta\phi_{\mathbf{k}}|\psi\rangle$.
- $\mathcal{R}$ represents the primordial curv. perturb.

The Mukhanov–Sasaki equation with a source

In terms of the MS variable, the Eq. of motion with a source becomes

$$\boxed{v_{\mathbf{k}}'' + \left(k^2 - \frac{z''}{z}\right) v_{\mathbf{k}} = a^2 X_{\mathbf{k}} \delta(\eta - \eta_k^P).} \tag{9}$$

At first order in slow roll,

$$\frac{z''}{z} \simeq \frac{\nu^2 - \frac{1}{4}}{\eta^2}, \qquad \nu = \frac{3}{2} + \epsilon_1 + \frac{\epsilon_2}{2}.$$

The solution to the MS is therefore

$$\boxed{v_{\mathbf{k}}(\eta) = a^2(\eta_k^P) X_{\mathbf{k}} G_k(\eta, \eta_k^P), \qquad \eta > \eta_k^P.} \tag{10}$$

The MS solution

$$\boxed{v_{\mathbf{k}}(\eta) = a^2(\eta_k^P) X_{\mathbf{k}} G_k(\eta, \eta_k^P), \qquad \eta > \eta_k^P.} \tag{11}$$

- G_k is the *retarded green function* which propagates the "initial kick" through horizon crossing.

$$G_k(\eta, \tau) = \Theta(\eta - \tau)\left(\frac{u_k(\tau)w_k(\eta) - w_k(\tau)u_k(\eta)}{W}\right). \tag{12}$$

- The two independent homogeneous solutions are

$$u_k(\eta) = \sqrt{\frac{-\pi\eta}{2}} J_\nu(-k\eta), \qquad w_k(\eta) = \sqrt{\frac{-\pi\eta}{2}} Y_\nu(-k\eta). \tag{13}$$

With $W = u_k w_k' - u_k' w_k = -1$, and J_ν, Y_ν Bessel's functions of 1st and 2nd. kind.

The primordial curvature spectrum

The dimensionless spectrum is defined by

$$\mathbb{E}[\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{q}}] = \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(k)\,\delta^{(3)}(\mathbf{k}+\mathbf{q}). \tag{14}$$

Each realization corresponds

$$\mathcal{R}_{\mathbf{k}} = \frac{v_{\mathbf{k}}}{z} = \frac{\langle\psi|\widehat{v}_{\mathbf{k}}|\psi\rangle}{z}. \tag{15}$$

Thus the primordial spectrum $\mathcal{P}_{\mathcal{R}}(k)$ involves

$$\mathbb{E}[\langle\psi|\widehat{v}_{\mathbf{k}}|\psi\rangle\langle\psi|\widehat{v}_{\mathbf{q}}|\psi\rangle]\,. \tag{16}$$

Interpretation

The ensemble describes the distribution of primordial inhomogeneous fields produced by the microscopic interaction.

The exact PS

Substituting the MS solution into the spectrum gives

$$\mathcal{P}_{\mathcal{R}}(k,\eta) = \frac{k^3}{2\pi^2}\frac{a^4(\eta_k^{\rm P})G_k^2(\eta,\eta_k^{\rm P})}{2\epsilon_1(\eta)M_{\rm P}^2 a^2(\eta)}P_X(k). \tag{17}$$

- $P_X(k)$ fixes the statistics of mode creation. **How exactly do we obtain this spectrum?**

Energy transfer from microscopic geometry

- We have modeled the interaction with Planckian discreteness as a stochastic source.

$$\nabla^a T_{ab} = -\xi \nabla_b \varphi, \tag{18}$$

- If we take **the ensemble average** and multiply by u^b, we propose

$$\mathbb{E}\{\nabla^a T_{ab}\} u^b = -\mathbb{E}\{\xi \nabla_b \varphi\} u^b \equiv -\mathcal{Q} \tag{19}$$

where $\mathcal{Q} > 0$ is the mean injected energy per unit physical volume and time.

Phenomenological proposal

. Physical motivation for $\mathcal{Q}$:

- If the microscopic statistics are approximately stationary, we expect a nearly steady energy input per physical volume.
- For $H \sim M_{\mathrm{P}}$, we take H to set the characteristic energy, length, and time scales of the transfer.

An effective energy γH transferred within a volume H^{-3} over a time H^{-1} gives

$$\boxed{\mathcal{Q} \sim \frac{\gamma H}{H^{-3} H^{-1}} = \gamma H^5.}$$

where γ is a dimensionless parameter representing the **strength of the energy transfer**. Therefore

$$\mathbb{E}\{\xi \nabla_b \varphi\} u^b = \gamma H^5 \tag{20}$$

- From $\mathbb{E}\{\xi\nabla_b\varphi\}u^b = \gamma H^5$, we have in conformal time coordinates

$$\mathbb{E}\{a^{-1}\xi\delta\phi'\} = \gamma H^5 \tag{21}$$

- Since ξ and $\delta\phi'$ both depend on X_k (recall that $v_k = a\delta\phi_k$), we obtain

$$P_X(k) = \frac{2\pi^2\gamma}{k^3} H^4(\eta_k^P) \tag{22}$$

- We substitute this $P_X(k)$ in the previous equation for the primordial curvature perturbation.

$$\mathcal{P}_\mathcal{R}(k,\eta) = \frac{k^3}{2\pi^2}\frac{a^4(\eta_k^{\rm P})G_k^2(\eta,\eta_k^{\rm P})}{2\epsilon_1(\eta)M_{\rm P}^2 a^2(\eta)} P_X(k). \tag{23}$$

Observable regime

We evaluate the frozen curvature spectrum when $k/(aH) \ll 1$.

FINALLY, The resulting primordial spectrum is

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \frac{\gamma H^2}{M_{\rm P}^2 \epsilon_1} \left(\frac{k}{aH}\right)^{3-2\nu} F_\nu(k), \tag{24}$$

- $F_\nu(k)$ reflects the scale dependence originated from the small difference between: *Planck crossing* $k = a(\eta_k^P) M_P$ and *Hubble (or horizon) crossing* $k = a(\eta_k^{\rm HC}) H(\eta_k^{\rm HC})$
- If we were in exact dS spacetime, then $F_{3/2}(k) =$ constant

The scalar amplitude

We adopt the pivot normalization $H(\eta_\diamond^{\rm P}) = M_P$, so for this pivot mode $k_\diamond = a(\eta_\diamond^P) M_P = a(\eta_\diamond^P) H(\eta_\diamond^P)$. Hence at leading order in the amplitude,

$$\boxed{A_s \simeq C\,\frac{\gamma H_\diamond^2}{M_{\rm P}^2 \epsilon_{1\diamond}}, \qquad C \equiv \frac{\pi}{4} J_{3/2}(1)^2 \simeq 0.0454.}$$

The observed CMB amplitude $A_s \simeq 10^{-9}$ then requires

$$\boxed{\gamma \simeq 10^{-8} \epsilon_{1\diamond}} \qquad (H_\diamond = M_{\rm P}).$$

Physical consequence

A Planckian background scale is compatible with small scalar perturbations when energy transfer to the modes is weak.

The scalar spectral index

By definition,

$$n_s - 1 = \left.\frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}\right|_{k_\diamond}.$$

So retaining the scale dependence of the factor $F_\nu(k)$

$$n_s - 1 = -2\epsilon_{1\diamond} - \epsilon_{2\diamond} + \left.\frac{d \ln F_\nu}{d \ln k}\right|_{k_\diamond}.$$

gives

Result including the $F_\nu(k)$ correction

$$\boxed{n_s - 1 \simeq -2.412\,\epsilon_{1\diamond} - \epsilon_{2\diamond}.}$$

It should be included when performing an observational analysis

What fixes the inflationary scale?

Standard slow-roll result

$$A_s^{\rm std} = \frac{H_\diamond^2}{8\pi^2 M_{\rm P}^2 \epsilon_{1\diamond}}.$$

The CMB amplitude fixes $H_\diamond^2/\epsilon_{1\diamond}$. Together with the standard tensor spectrum, the tensor bound constrains $H_\diamond$.

Present proposal

$$A_s \simeq 0.0454 \frac{\gamma}{\epsilon_{1\diamond}}.$$

The scale is fixed by construction $H_\diamond = M_P$, the CMB amplitude fixes our parameter $\gamma/\epsilon_{1\diamond}$. The tensor sector requires an additional prescription.

Gravitational waves are suppressed

GWs are expected at 2nd order

$$\ddot{h}_{ij} - \frac{\nabla^2 h_{ij}}{a^2} + 3Hh_{ij} = a^{-2}\{32\pi G \partial_i \delta\phi \partial_j \delta\phi\}^{TT} \tag{25}$$

We expect something

$$\mathcal{P}_h(k) \simeq \gamma^2 \frac{H^4}{M_P^4} \tag{26}$$

so

$$\boxed{r_\diamond \simeq \gamma \simeq 10^{-8} \epsilon_{1,\diamond}} \tag{27}$$

Work in preparation

A possible potential

Consider Starobinsky Inflation

$$L = \sqrt{-g}\left(R + \frac{\alpha \ell_p^2}{6} R^2\right) \tag{28}$$

Equivalent to

$$V(\phi) = \frac{3M_P^4}{4\alpha}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\right)^2 \tag{29}$$

- **Standard Inflation:** $\alpha \simeq 10^{10}$
- **Our prosal:** $\alpha \simeq 1$, with $\gamma \simeq 10^{-10}$.

Comparison of the Inflationary frameworks

	Standard account	**Present proposal**
Primordial state	The perfectly symmetric BD vacuum	An inhomogeneous excited state $\|\psi\rangle$
Primordial Spectrum	Quantum two-point function $\rightarrow$ M-problem	Ensemble of classical fields $v_{\mathbf{k}} = \langle \hat{v}_{\mathbf{k}} \rangle \neq 0$
Trans-Planckian problem	Yes	No
Scalar amplitude	CMB data fixes the combination $H^2/(M_P^2 \epsilon_1)$, together with the tensor bound, $H \ll M_P$	CMB data fixes a small $\gamma/\epsilon_1 \ll 1$, but $H \simeq M_P$
Scalar spectral index	$n_s = 1 - 2\epsilon_{1\diamond} - \epsilon_{2\diamond}$	$n_s \simeq 1 - 2.412\epsilon_{1\diamond} - \epsilon_{2\diamond}$
Tensor modes	expected $r = 16\epsilon_{1\diamond}$	Naturally suppressed $r \simeq \gamma$

Conclusions and outlook

- We proposed a physical mechanism for generating a nearly scale-invariant spectrum during inflation based on a steady energy transfer from a fundamental discreteness of the spacetime.
- The calculation connects the excitation of modes at the Planck scale with cosmological observables through their subsequent semiclassical evolution.
- A central theoretical task is to derive the stochastic source and its coupling strength from a specific QG theory, such as LQG.
- Further work should establish the consistency of the emerging quantum state, control its backreaction, and determine the tensor spectrum and higher-order correlations.

Selected references

- G. R. Bengochea, G. León and A. Perez,
  Emergence of cosmic structure from Planckian discreteness.
  Manuscript presented in this talk, arXiv: 2506.15413.
- L. Amadei and A. Perez,
  Planckian discreteness as seeds for cosmic structure,
  Phys. Rev. D **106**, 063528 (2022), arXiv:2104.08881.
- G. R. Bengochea, G. León and A. Perez,
  Is Planckian discreteness observable in cosmology?
  Universe **11**, 139 (2025), arXiv:2405.12534.
- A. Perez and D. Sudarsky,
  Dark energy from quantum gravity discreteness,
  Phys. Rev. Lett. **122**, 221302 (2019), arXiv:1711.05183.

THANK YOU