Numerical relativity and hyperbolicity in teleparallel gravity

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Outline

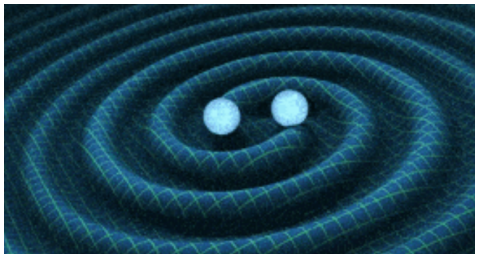
1. 3+1 formalism and hyperbolicity in GR
2. Metric teleparallel framework
3. 3+1 formulation in teleparallel
4. Hyperbolicity in TEGR and beyond
5. Conclusions

Acknowledgments

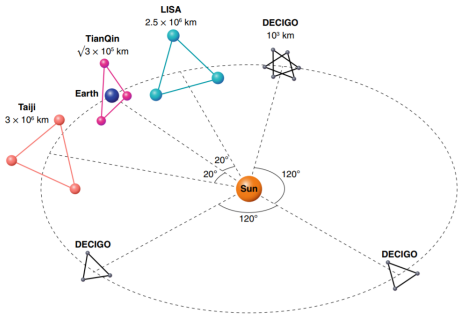
Work in collaboration with Cheng Cheng, Marie Femke Jaarma, Laxmipriya Pati, Armin van de Venn, among others



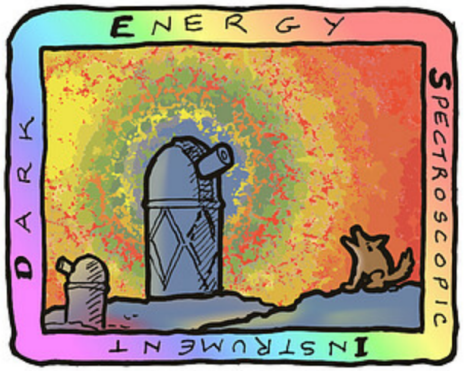
Motivation



Strong gravity is nonlinear, needs numerical relativity to be accurately described

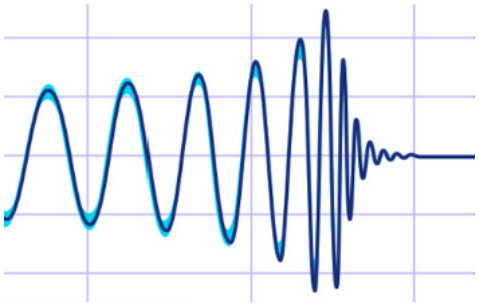


Third-generation gravitational wave detectors by 2035s, evidence of physics beyond GR



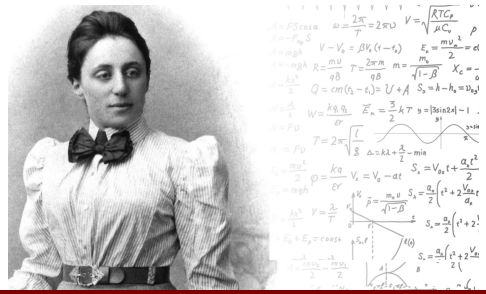
Cosmological tensions H_0 and DESI variable dark energy results suggest GR would need modifications

Motivation



To create GW templates for alternative or modified gravities

AI as valuable tool for managing code (and even symbolic tensorial computations)



Theoretical understanding of the well-posedness of metric-affine and teleparallel theories

3+1 decomposition

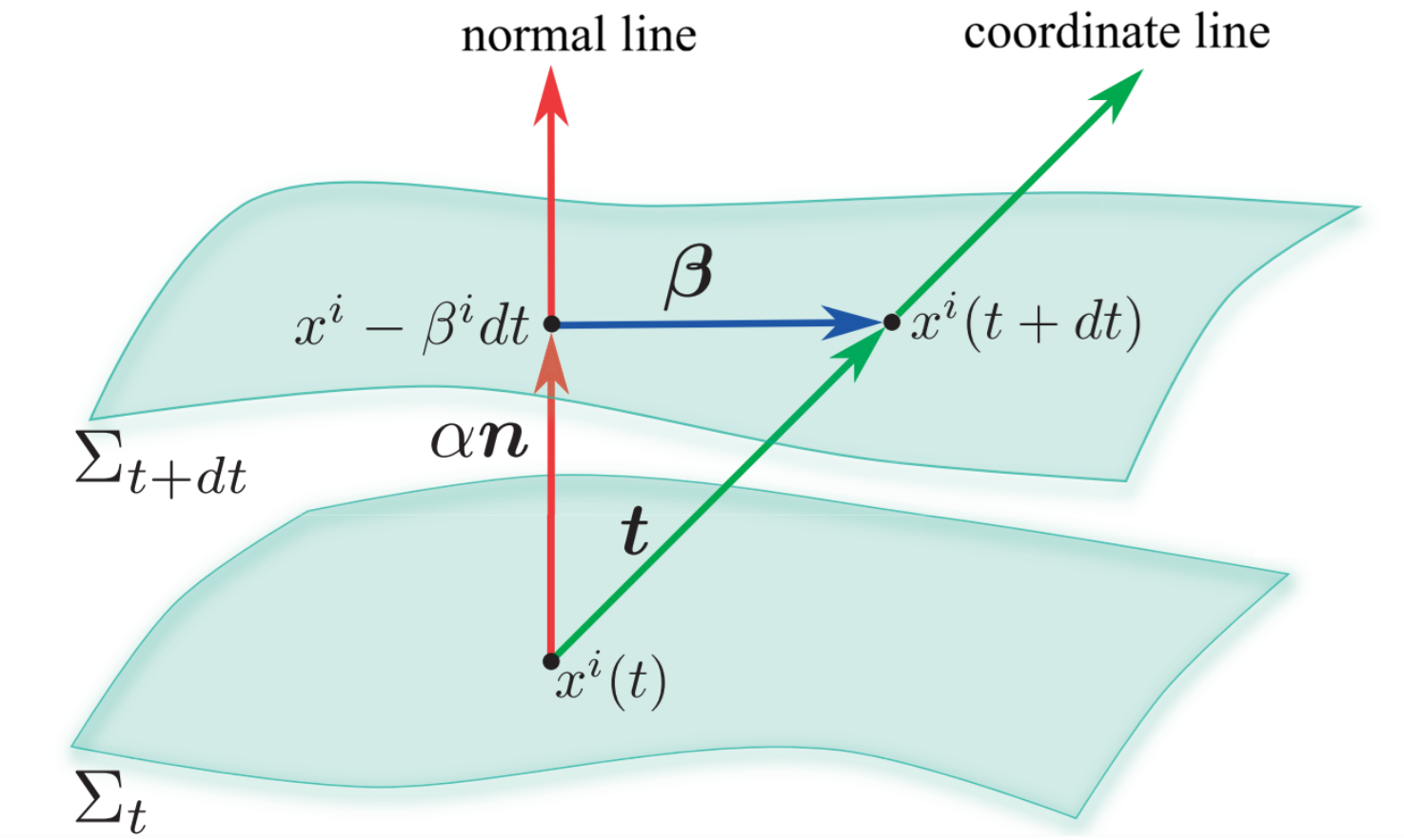


Figure: Space-time foliation and ADM variables

ADM metric

The metric of the foliated spacetime corresponds to

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \gamma_{ij}\beta^i\beta^j & \beta^i \\ \beta^j & \gamma_{ij} \end{pmatrix}, \tag{1}$$

while its inverse is

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{\alpha^2} & \frac{\beta^i}{\alpha^2} \\ \frac{\beta^j}{\alpha^2} & \gamma^{ij} - \frac{\beta^i\beta^j}{\alpha^2} \end{pmatrix}. \tag{2}$$

It will also be useful to define the normal vector in coordinates such that

$$n_\mu = (-\alpha, \vec{0}), \quad n^\mu = \left(\frac{1}{\alpha}, -\frac{\beta^i}{\alpha}\right), \tag{3}$$

where we have raised the index in n^μ with the inverse metric $g^{\mu\nu}$.

ADM equations

The most general form for the ADM evolution equations are

$$\dot{\gamma}_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i, \tag{4}$$

$$\dot{K}_{ij} = - D_iD_j\alpha + \alpha({}^{(3)}R_{ij} - 2K_{ik}K^k{}_j + KK_{ij}) \tag{5}$$

$$+ \beta^k D_k K_{ij} + K_{ik}D_j\beta^k + K_{jk}D_i\beta^k. \tag{6}$$

These equations are complemented with the Hamiltonian and momenta constraints (in vaccuum)

$${}^{(3)}R + K^2 - K_{ij}K^{ij} = 0, \tag{7}$$

$$D_j(K^{ij} - \gamma^{ij}K) = 0. \tag{8}$$

Hyperbolicity

1D first-order system of equations for u_i

$$\partial_t u_i + \sum_j M_{ij} \partial_x u_j = q_i(u), \tag{9}$$

for $i \in \{1, \ldots, N_u\}$, N_u the total number of fields, M_{ij} is the principal symbol or Jacobian matrix, with eigenvalues λ_i and eigenvectors v_i. Eq.(9) is

- **hyperbolic** if all λ_i are real
- **strongly hyperbolic** if the all λ_i are real, and M_{ij} admits a complete set of v_i
- **weakly hyperbolic** if λ_i are real but no full set of v_i
- **symmetric hyperbolic** if M itself is symmetric

A simple proof of weak hyperbolicity in GR

The linearized ADM equations look like

$$\begin{aligned} \partial_t A_x + \partial_x (K_{xx} + K_{yy} + K_{zz}) &= 0 \\ \partial_t K_{xx} + \partial_x A_x + \partial_x (D_{xyy} + D_{xzz}) &= 0 \\ \partial_t K_{jj} + \partial_x D_{xjj} = 0, \qquad & \partial_t K_{xj} = 0 \\ \partial_t K_{yz} + \partial_x D_{xyz} = 0, \qquad & \partial_t D_{xjk} + \partial_x K_{jk} = 0 \end{aligned} \tag{10}$$

This is a system of thirteen variables

$$u = \{A_x, K_{xx}, K_{yy}, K_{zz}, K_{xy}, K_{xz}, K_{yz}, D_{xx}, D_{yy}, D_{zz}, D_{xy}, D_{xz}, D_{yz}\} \tag{11}$$

The principal symbol M_{ij} has eigenvalues $+1, -1, 0$ with multiplicity $4, 4, 5$, respectively, but only nine total eigenvectors. This means that the system is weakly hyperbolic. It can be made strongly hyperbolic by conveniently adding linearized constraints.

[Alcubierre, Oxford, 2008]

Hypotheses

Can tetrad formalism and its gauge freedom affect hyperbolicity properties?

What is the role of tetrads in numerical relativity with matter coupling? (fermionic matter, Dirac stars [Alcubierre (2025), 2503.03918])

Outline

1. 3+1 formalism and hyperbolicity in GR
2. Metric teleparallel framework
3. 3+1 formulation in teleparallel
4. Hyperbolicity in TEGR and beyond
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Metric and linear connection

- Metric and linear connection are two separate entities. The metric can be defined from the tetrad θ and cotetrad e

$$g_{\mu\nu} = \theta^a{}_\mu \theta^b{}_\nu \eta_{ab}, \qquad \eta_{ab} = g_{\mu\nu} e_a{}^\mu e_b{}^\nu .$$

- The most general linear connection is related to the spin connection ω and the tetrad as

$$\Gamma^\rho{}_{\mu\nu} = e_a{}^\rho \partial_\nu \theta^a{}_\mu + e_a{}^\rho \omega^a{}_{b\nu} \theta^b{}_\mu$$

- In the metric affine framework $\Gamma^\rho{}_{\mu\nu}$ is free but in teleparallelism it satisfies some conditions.

Geometric properties of the linear connection

$$R^{\alpha}{}_{\beta\mu\nu}(\Gamma) = \partial_\mu \Gamma^{\alpha}{}_{\nu\beta} - \partial_\nu \Gamma^{\alpha}{}_{\mu\beta} + \Gamma^{\alpha}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\beta} - \Gamma^{\alpha}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\beta}$$

curvature

$$T^{\rho}{}_{\mu\nu} = 2\Gamma^{\rho}{}_{[\mu\nu]}$$

torsion

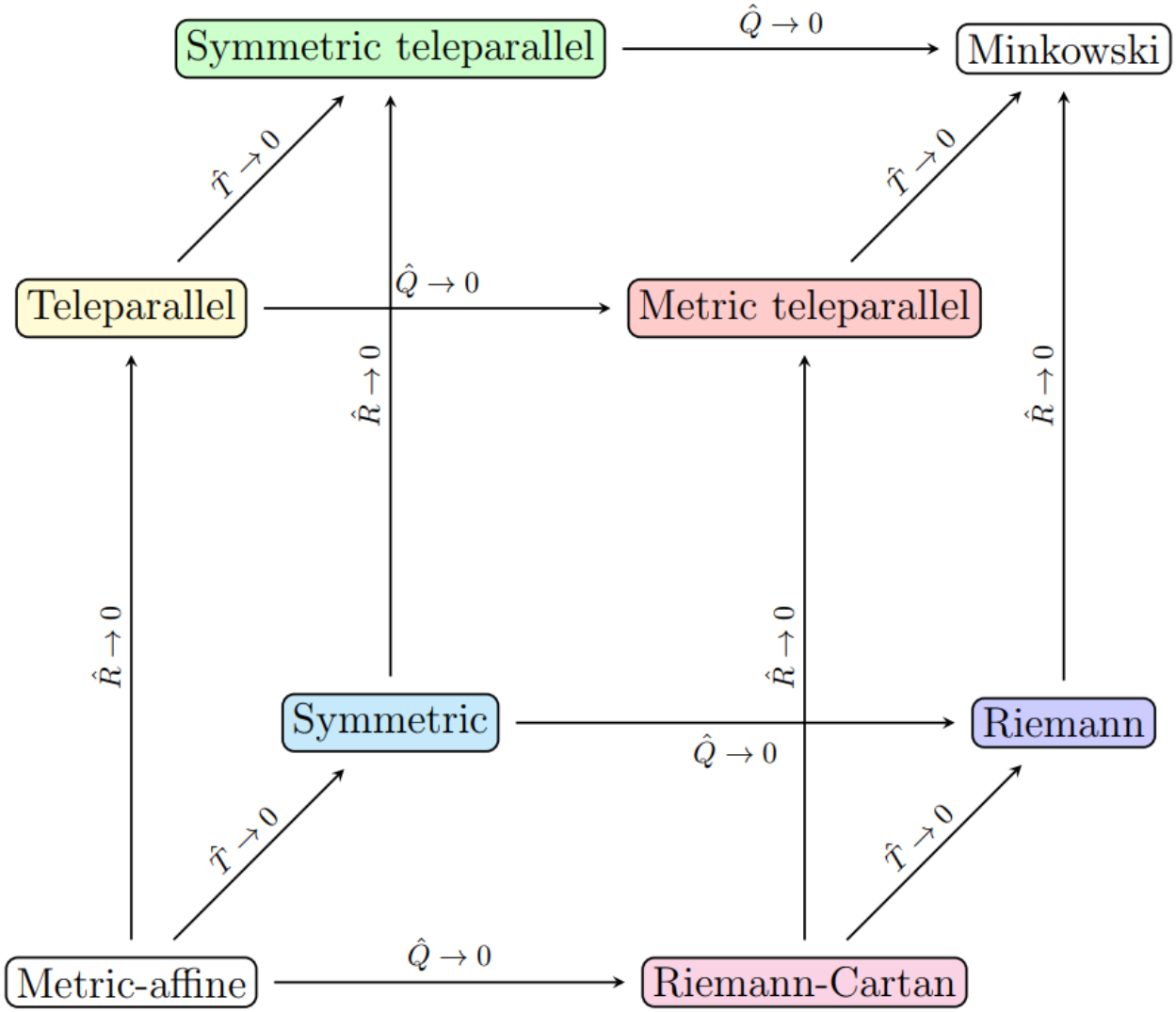
$$Q_{\rho\mu\nu} = \nabla_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - \Gamma^{\beta}{}_{\mu\rho} g_{\beta\nu} - \Gamma^{\beta}{}_{\nu\rho} g_{\mu\beta}$$

nonmetricity

Conversely, the most general linear connection can be written in terms of three pieces containing these elements

$$\Gamma^{\rho}{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} + K^{\rho}{}_{\mu\nu}(T) + L^{\rho}{}_{\mu\nu}(Q)$$

Metric-affine geometry



[Bahamonde, Dialektopoulos, et al. (2023)]

Teleparallel framework

The condition of vanishing curvature $R^{\alpha}{}_{\beta\mu\nu}(\Gamma) \equiv 0$ is satisfied by the most general connection parameterized by an element $L^{\mu}{}_{\nu}$ of the general linear group $GL(4,\mathbb{R})$ as

$$\Gamma^{\alpha}{}_{\mu\nu} = (L^{-1})^{\alpha}{}_{\lambda}\partial_{\mu}L^{\lambda}{}_{\nu}$$

$$\omega^{a}{}_{b\mu} = \Lambda^{a}{}_{c}\,\partial_{\mu}(\Lambda^{-1})^{c}{}_{b}$$

$$\Gamma^{\rho}{}_{\mu\nu} = \frac{\partial x^{\rho}}{\partial \xi^{\sigma}}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial \xi^{\sigma}}{\partial x^{\nu}}\right)$$

metric tele

$$Q_{\rho\mu\nu} \overset{!}{=} 0$$

symmetric tele

$$T^{\rho}{}_{\mu\nu} \overset{!}{=} 0$$

Metric teleparallel

The **teleparallel equivalent of general relativity (TEGR)** is written in terms of the torsion scalar

$$\mathbb{T} = \frac{1}{4} T_{\rho\mu\nu} T^{\rho\mu\nu} + \frac{1}{2} T_{\rho\mu\nu} T^{\mu\rho\nu} - T_\mu T^\mu, \tag{12}$$

therefore its Lagrangian is $L = \det(\theta^a_\mu)\mathbb{T}$. It is satisfied

$$\overset{\circ}{R} = -\mathbb{T} + 2\overset{\circ}{\nabla} T^\mu, \tag{13}$$

which makes TEGR equivalent to GR once the boundary term is integrated out under the action integral.
In the so-called covariant formulation of teleparallel gravity, $T^\rho{}_{\mu\nu}(\theta^a{}_\mu, \Lambda^a{}_b)$. A simultaneous Lorentz transformation on θ and Λ

$$\theta'^a{}_\mu = \Lambda^a{}_b \theta^b{}_\mu \,, \qquad \omega'^a{}_{b\mu} = \Lambda^a{}_c (\Lambda^{-1})^d{}_b \, \omega^c{}_{d\mu} + \Lambda^a{}_c \, \partial_\mu (\Lambda^{-1})^c{}_b \tag{14}$$

leaves the action unchanged.

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ADM split of the tetrad

We propose a tetrad decomposition as

$$\theta^A{}_i\theta^B{}_j\eta_{AB} = \gamma_{ij}, \qquad \theta^A{}_0 = \alpha\xi^A + \beta^i\theta^A{}_i$$

where the normal vector ξ^A with Lorentz indices must satisfy the following conditions

$$\eta_{AB}\xi^A\xi^B = \xi_A\xi^A = -1, \qquad \eta_{AB}\xi^B\theta^A{}_i = \xi_A\theta^A{}_i = 0, \tag{15}$$

so the decomposition correctly reproduces the ADM metric.
We decompose the inverse tetrad components $e_A{}^\mu$ as

$$e_A{}^0 = -\frac{1}{\alpha}\xi_A, \qquad e_A{}^i = \theta_A{}^i + \xi_A\frac{\beta^i}{\alpha}, \tag{16}$$

where the object $\theta_A{}^i = \eta_{AB}\gamma^{ij}\theta^B{}_j$ (different from $\theta^A{}_i$!).

ADM split of the TEGR Lagrangian

The 3+1 decomposition of the torsion tensor ${T^A}_{\mu\nu}$ consists in the following components

$$ {T^A}_{0i} = -{T^A}_{i0} = \partial_0 {\theta^A}_i - \partial_i {\theta^A}_0 + {\omega^A}_{B0}{\theta^B}_i - {\omega^A}_{Bi}{\theta^B}_0, \tag{17} $$

$$ {T^A}_{ij} = \partial_i {\theta^A}_j - \partial_j {\theta^A}_i + {\omega^A}_{Bi}{\theta^B}_j - {\omega^A}_{Bj}{\theta^B}_i \tag{18} $$

Since the TEGR Lagrangian has the form

$$ L = \frac{1}{2\kappa}\theta H_{AB}{}^{\mu\nu\rho\lambda} {T^A}_{\mu\nu} {T^B}_{\rho\lambda} \tag{19} $$

the 3+1 decomposition can be written as

$$ \mathcal{L} \propto M^{ij}_{AB} {T^A}_{0i} {T^B}_{0j} - N^{il}_{AB} {T^A}_{0i} {T^B}_{kl} + {}^3\mathbb{T}. \tag{20} $$

Summary of Hamilton's equations in TEGR

Canonical var.	Conj. momenta	Primary c.	Hamilton's eq.
Lapse (α)	${}^{\alpha}\pi = 0$	Yes	Hamiltonian c.
Shift (β_i)	${}^{\beta}\pi_i = 0$	Yes	Momenta c.
Spatial tetrad ${\theta^A}_i$	${\pi_A}^i = \partial L/\partial\dot{\theta}^A{}_i$	No	
	${}^{\mathcal{V}}C^i = 0$	Yes	$\dot{\theta}^A{}_i = \frac{\partial H}{\partial \pi^i_A}$
	${}^{\mathcal{A}}C^{ij} = 0$	Yes	$\dot{\pi}^i_A = -\frac{\partial H}{\partial \theta^A_i}$
Lorentz matrices ${\Lambda^A}_B$	$\mathsf{C}^{AB} = 0$	Yes	$\dot{\Lambda}^A_B = \frac{\partial H}{\partial P^B_A}$

$$C^{AB} = P_D^{[A}\eta^{B]C}{\Lambda_C}^D + {\pi_C}^i\eta^{C[B}{\theta^{A]}}_i \tag{21}$$

$${}^{\mathcal{V}}C^i = {}^{\mathcal{V}}\pi^i\kappa/\sqrt{\gamma} + {T^B}_{jk}\gamma^{ik}\gamma^{jl}{\theta^A}_l\eta_{AB} \tag{22}$$

$${}^{\mathcal{A}}C^{ij} = {}^{\mathcal{A}}\pi^{ij}\kappa/\sqrt{\gamma} - \frac{1}{2}\gamma^{ik}\gamma^{jl}{T^B}_{kl}\xi_B \tag{23}$$

[Blixt, Guzman and Pati (2022)] [Golovnev and Guzman (2021)]

Degrees of freedom and momenta

The counting of degrees of freedom in covariant TEGR goes in a different way when compared to GR

- In the Weitzenböck gauge (only tetrad):

$$16 \times 2 - (1 + 3 + 6) \times 2 - 4 \times 2 = 4 \tag{24}$$

- In the covariant formulation (tetrad and connection):

$$22 \times 2 - (1 + 3 + 6 + 6) \times 2 - 4 \times 2 = 4 \tag{25}$$

We have presented a relation between the GR and TEGR momenta

$$\pi_A{}^i = \pi^{ij}\theta^B{}_j\eta_{AB} + {}^{\mathcal{V}}\pi^i\xi_A + {}^{\mathcal{A}}\pi^{ji}\theta^B{}_j\eta_{AB} \tag{26}$$

with

$$\pi^{ij} = \frac{\partial L}{\partial \dot{\gamma}_{ij}}, \qquad \pi_A{}^i = \frac{\partial L}{\partial \dot{\theta}^A{}_i}. \tag{27}$$

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Hyperbolicity in TEGR

In TEGR the linearized Hamilton's equations look like

$$\dot{\theta}^a{}_i = \frac{1}{2}\left[\pi_b{}^j(\delta^a_j\delta^b_i - \delta^a_i\delta^b_j) + \pi_b{}^j\delta^a_k\delta^b_l\delta_{ij}\delta^{kl}\right] \tag{28}$$

and

$$-\dot{\pi}_a{}^i = \frac{2}{\kappa}H_{ca}{}^{[mn][il]}\partial_l T^c{}_{mn}. \tag{29}$$

Unlike GR, we don't see spatial derivatives of lapse. This is consequence of different boundary terms.

[Blixt, Guzman and Pati, 2210.07971] [C. Cheng, M. J. Guzman, 2602.01830]

Hyperbolicity in TEGR

We expand the tetrad at the linear level

$$\begin{aligned} \theta^0{}_0 &= 1 + h^0{}_0, & \theta^0{}_i &= h^0{}_i, \\ \theta^a{}_0 &= h^a{}_0, & \theta^a{}_i &= \delta^a_i + h^a{}_i. \end{aligned} \tag{30}$$

The momenta $\pi_a{}^i$ are first order in tetrad perturbations. The system can be written in TEGR in terms of the vector

$$u = (\pi_x{}^i, \pi_y{}^i, \pi_z{}^i, \pi_0{}^i, D^x{}_i, D^y{}_i, D^z{}_i, D^0{}_i), \tag{31}$$

where $D^a{}_i := \partial_x h^a{}_i$, $D^0{}_i = \partial_x h^0{}_i$, as

$$\partial_t u_i + \sum_i M_{ij} \partial_x u_j = 0.$$

Principal symbol in TEGR

$M_x =$ 

[C. Cheng, M. J. Guzman, 2602.01830]

Non-hyperbolic system

Eigenvalues are

$$\begin{aligned} \lambda_1 &= i, && \text{with multiplicity} \quad 2, \\ \lambda_2 &= -i, && \text{with multiplicity} \quad 2, \\ \lambda_3 &= 0, && \text{with multiplicity} \quad 20. \end{aligned} \tag{32}$$

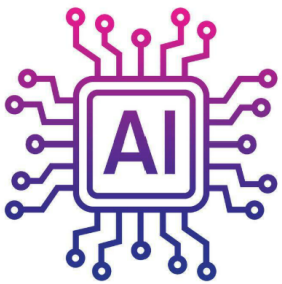
Eigenvectors: 4 complex, 6 null (trivial), 12 real nontrivial.

Hyperbolicity is lost :(

Can we redefine variables and/or add constraints to solve this?
[C. Cheng, M. J. Guzman, 2602.01830]

Strong hyperbolicity (in one block)

$$
\begin{aligned}
F &:= \pi_y{}^z + \pi_z{}^y, \\
G &:= \pi_y{}^y + \pi_z{}^z, \\
H &:= \pi_y{}^y - \pi_z{}^z, \\
Q &:= D^y{}_z + D^z{}_y, \\
T &:= D^y{}_y + D^z{}_z, \\
R &:= D^y{}_y - D^z{}_z, \\
C &:= G + D^0_x
\end{aligned}
$$



[C. Cheng, M. J. Guzman, 2602.01830]

$$
\begin{aligned}
\dot{D}^x_x + \frac{1}{2}\partial_x C &= 0, \\
\dot{C} + \partial_x D^x{}_x &= 0,
\end{aligned}
$$

eigenvalues $\pm\lambda_1 = 1/\sqrt{2}$, $\rightarrow$ **strongly hyperbolic :)**

$$
\begin{aligned}
\dot{F} + \partial_x Q &= 0, \\
\dot{Q} - \partial_x F &= 0,
\end{aligned}
$$

$$
\begin{aligned}
\dot{H} + \partial_x R &= 0, \\
\dot{R} - \partial_x H &= 0.
\end{aligned}
$$

eigenvalues $\lambda = \pm i$ **:(**

Discussion

- In the 1D case, tetrad perturbations $h^A{}_\mu(x)$ can be recast as strongly hyperbolic system via gauge fixing of ill-behaved sector
- In the 3D case $h^A{}_\mu(x, y, z)$ also shows principal symbol with imaginary eigenvalues
- Tetrad gauge freedom presents both possibilities and problems

Interest in tetrad formalism in numerical relativity

- **Estabrook–Robinson–Wahlquist (1996):** impose Nester's special orthonormal-frame conditions to turn the tetrad form of vacuum GR into a closed first-order symmetric-hyperbolic system.

$$\text{tetrad } \theta^a \quad \Longrightarrow \quad \text{connection var } (\mathrm{K}, \mathrm{N}, \mathbf{a}, \boldsymbol{\omega}) \quad \Longrightarrow \quad \text{Weyl var } (\mathrm{A}, \mathrm{B})$$

$$\text{vacuum tetrad eqns} + \text{Nester frame gauge} = 34 \text{ evol eqns} + 30 \text{ constr}$$

- Later, **Buchman and Bardeen (2005)** adapt formalism for numerical relativity.
- **Peshkov, Olivares and Romenski (2022)** show that torsion can be promoted to independent electric- and magnetic-type variables, producing a balance-law formulation particularly suited to first-order numerical methods

Next step: time-dependent spherical symmetry

- We want to reproduce the metric

$$ds^2 = -\alpha^2 dt^2 + A^2(dr + \beta dt)^2 + r^2 B^2 d\Omega^2, \tag{33}$$

where $\alpha = \alpha(t, r)$, $A = A(t, r)$, $B = B(t, r)$, and $\beta = \beta^r(t, r)$ is the radial component of the shift vector.

- A naive choice would be the almost-diagonal tetrad

$$\theta^A{}_\mu = \begin{pmatrix} \alpha & 0 & 0 & 0 \\ A\beta & A & 0 & 0 \\ 0 & 0 & rB & 0 \\ 0 & 0 & 0 & rB\sin(\theta) \end{pmatrix} \tag{34}$$

with $\omega^a{}_{b\mu} = 0$. However, this choice does not reproduce the Minkowski limit asymptotically.

- We need to provide a spherically symmetric spin connection

$$\omega^{\hat{r}}{}_{\hat{\theta}\theta} = -1, \quad \omega^{\hat{r}}{}_{\hat{\phi}\phi} = -\sin(\theta), \quad \omega^{\hat{\theta}}{}_{\hat{\phi}\phi} = -\cos(\theta). \tag{35}$$

[A. van de Venn, M. J. Guzman, M. Jaarma, L. Pati, 26xx.xxxx]

Time-dependent spherical symmetry

- If we insist in working with $\omega^{a}{}_{b\mu} = 0$, then we must rotate away the non-zero spin connection, which rotates the tetrad so we obtain

$$\theta^{A}{}_{\mu} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin\vartheta\cos\phi & \cos\vartheta\cos\phi & -\sin\phi \\ 0 & \sin\vartheta\sin\phi & \cos\vartheta\sin\phi & \cos\phi \\ 0 & \cos\vartheta & -\sin\vartheta & 0 \end{pmatrix}}_{\mathcal{M}(\vartheta,\phi)} \underbrace{\begin{pmatrix} \alpha & 0 & 0 & 0 \\ A\beta & A & 0 & 0 \\ 0 & 0 & rB & 0 \\ 0 & 0 & 0 & rB\sin(\vartheta) \end{pmatrix}}_{\mathcal{F}(t,r)}, \qquad R \tag{36}$$

- Other possibilities to explore in the future are a boosted tetrad

$$\theta^{a}{}_{\mu} = \begin{pmatrix} \alpha ch\psi + A\beta sh\psi & Ash\psi & 0 & 0 \\ \alpha sh\psi + A\beta ch\psi & Ach\psi & 0 & 0 \\ 0 & 0 & R & 0 \\ 0 & 0 & 0 & R\sin\vartheta \end{pmatrix} \tag{37}$$

[A. van de Venn, M. J. Guzman, M. Jaarma, L. Pati, 26xx.xxxx]

Spherical symmetry in tetrad

Canonical momenta $\beta = 0$ with naive almost-diagonal tetrad:

$$\pi_A{}^i = \begin{pmatrix} \frac{2rB\sin\theta\,(B + rB_r)}{\kappa A} & \frac{A\cos\theta}{\kappa} & 0 \\ -\frac{2r^2 B\sin\theta\, B_t}{\alpha} & 0 & 0 \\ 0 & -\frac{r\sin\theta\,(BA_t + AB_t)}{\alpha} & 0 \\ 0 & 0 & -\frac{r\,(BA_t + AB_t)}{\alpha} \end{pmatrix}. \tag{38}$$

[A. van de Venn, M. J. Guzman, M. Jaarma, L. Pati, 26xx.xxxx]

The case of STEGR

The $3+1$ split of the $\Gamma\Gamma$ action (STEGR, coincident gauge) gives

$$\begin{aligned}\mathbb{Q} = &-{}^{(3)}\mathbb{Q} - K^{ij}K_{ij} + K^2 - \frac{1}{\alpha}K\partial_i\beta^i - \frac{1}{\alpha^3}\partial_i\beta^i\partial_0\alpha + \frac{1}{\alpha^3}(\partial_i\alpha - \alpha\,{}^{(3)}\Gamma^j{}_{ij})\partial_0\beta^i \\ &+ \frac{1}{\alpha}\left[{}^{(3)}\Gamma^i{}_j{}^j - {}^{(3)}\Gamma^{ji}{}_j\right]\partial_i\alpha + \frac{1}{\alpha^3}(\beta^i\partial_j\beta^j - \beta^j\partial_j\beta^i)\partial_i\alpha + \frac{1}{\alpha^2}({}^{(3)}\Gamma^k{}_{jk}\beta^i\partial_i\beta^j + \partial_i\beta^j\partial_j\beta^i).\end{aligned} \tag{39}$$

Terms can be integrated by parts to give

$$S = -\frac{1}{2\kappa}\int d^4x\sqrt{\gamma}\alpha\left[-{}^{(3)}\mathbb{Q} - D_i(Q^i - \tilde{Q}^i) + K^2 - K^{ij}K_{ij}\right], \tag{40}$$

where it has been defined the 3 dimensional nonmetricity scalar as

$${}^{(3)}\mathbb{Q} = {}^{(3)}\Gamma^i{}_{ij}\,{}^{(3)}\Gamma^{jk}{}_k - {}^{(3)}\Gamma^i{}_{jk}\,{}^{(3)}\Gamma^j{}_i{}^k \tag{41}$$

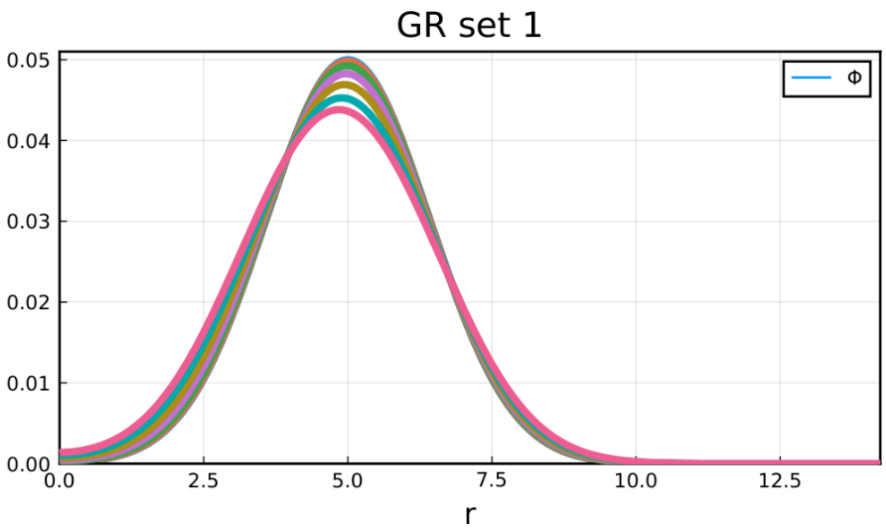
Without any further action, this action is equivalent to the Einstein-Hilbert action of GR, since it can be proved that

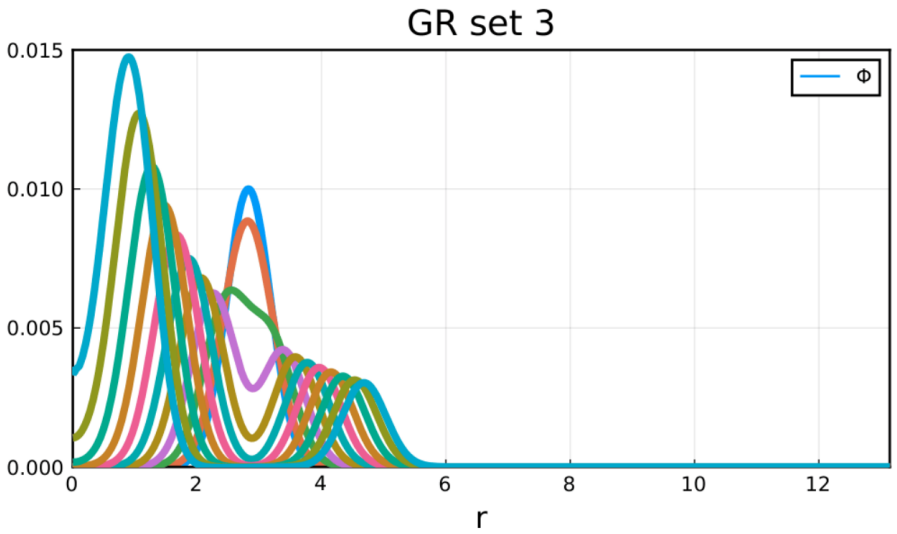
$$\boxed{{}^{(3)}\mathbb{R} = -{}^{(3)}\mathbb{Q} - D_i(Q^i - \tilde{Q}^i).}$$

[F. D'Ambrosio, M. Garg, L. Heisenberg, S. Zentarra, 2007.03261]

First simulations in GR

Evolution of a self-interacting scalar field in a spherically symmetric spacetime





[Marie Femke Jaarma, 2026, MSc thesis]

Numerical metric-affine?

- Affine connection, action built from quadratic curvature, torsion and /or nonmetricity tensors.
- More complicated geometric definitions

$$\check{a}_\mu := n^\rho \nabla_\rho n_\mu, \quad \hat{a}^\mu := n^\rho \nabla_\rho n^\mu$$

- Generalized Gauss-Codazzi-Ricci equations
- Practical examples.

[A. van de Venn, M.J. Guzman, work in progress]

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2. Metric teleparallel framework
3. 3+1 formulation in teleparallel
4. Hyperbolicity in TEGR and beyond
5. Conclusions

Conclusions

- Tetrads and connection introduce extra dynamical variables but also more constraints, so the number of degrees of freedom is unaltered.
- Linearized TEGR Hamilton's equations can be recast as strongly hyperbolic + isolated problematic gauge sector
- Work in progress in spherical symmetry in TEGR also shows gauge choice matters, possible nontrivial role of shift vector β^i
- STEGR does not provide additional insights
- Numerical relativity in the metric-affine framework is mostly unexplored

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Thank you! Aitäh!

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