Gravity-Thermodynamics Connection and Holographic Dark Energy with generalized entropies: Solving cosmological tensions

Emmanuel N. Saridakis

National Observatory of Athens

ARICA 2026

- The history of Astronomy, Cosmology and Gravity is a history of tensions between theoretical predictions and observations

- Astrophysical cosmology has become a precision science with an incredibly huge amount of data

- New Tensions appear.

  Are we approaching New Physics?

Aristotle - 350 BC

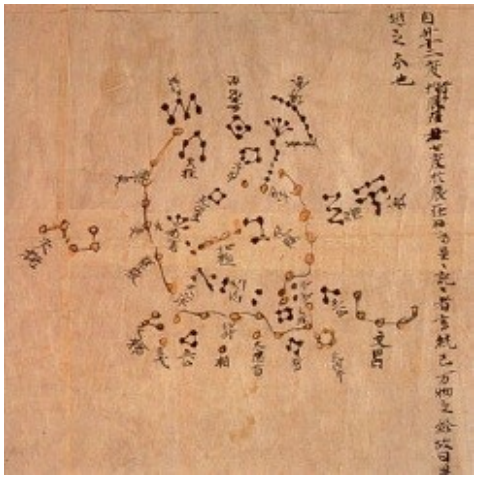
- According to Aristotle heavier bodies fall faster.
- Bodies fall in order to com back to thei “initial state”.





China, India, Persia, Arabic Observations

- Observations in China, India, Persia, e.g. in Maragha in 11th century, started putting into doubt Earth's non-motion, however not geocentrism.



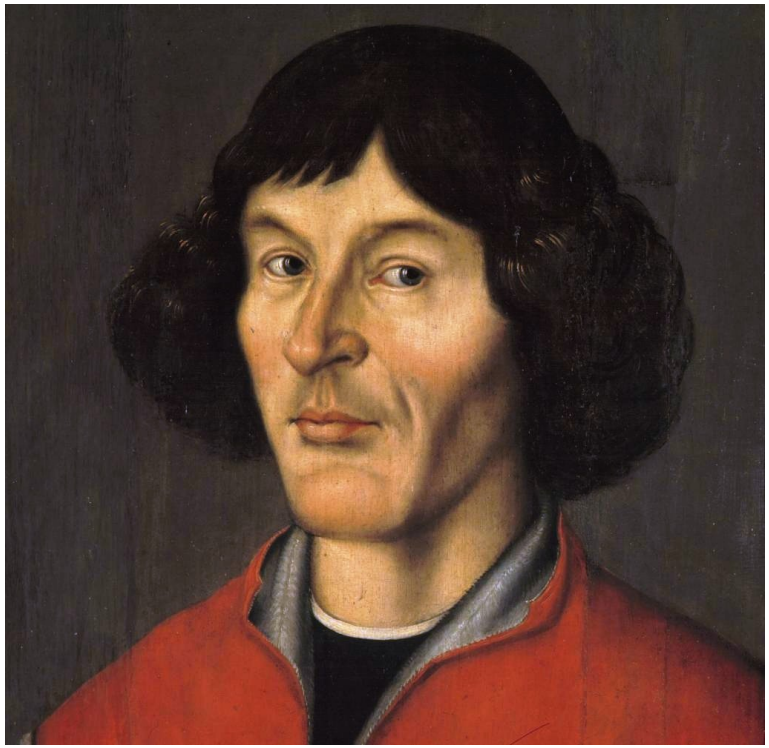
Dunhuang map from the Tang dynasty 710 AD





Copernicus - 1500

- Heliocentrism, (Aristarchus)



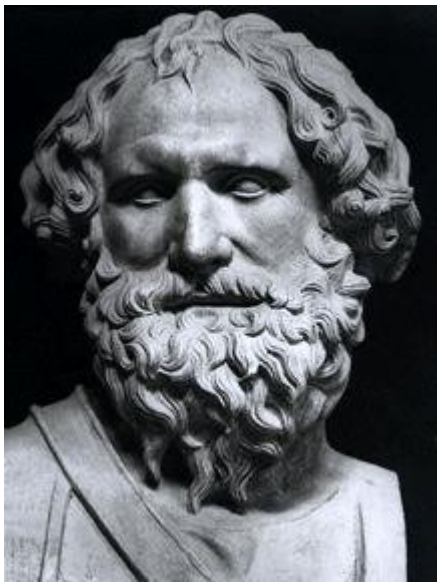


NICOLAI
COPERNICI TO-
RINENSIS DE REVOLVTIONI-
bus orbium cœleſtium,
Libri VI.

IN QVIBVS STELLARVM ET FI-
XARVM ET ERRATICARVM MOTVS, EX VETE-
ribus atq; recentibus obſeruationibus, reſtituit hic autor.
Præterea tabulas expeditas luculentasq; addidit, ex quI-
bus eoſdem motus ad quoduis tempus Mathe-
matum ſtudioſus facillime calcu-
lare poterit.

ITEM, DE LIBRIS REVOLVTIONVM NICOLAI
Copernici Narratio prima per M. Georgium Ioachi-
mum Rheticum ad D. Ioan. Schone-
rum ſcripta.

Cum Gratia & Priuilegio Cæſ. Maieſt.
BASILEAE, EX OFFICINA
HENRICPETRINA.



5

Brahe, Kepler- 1600

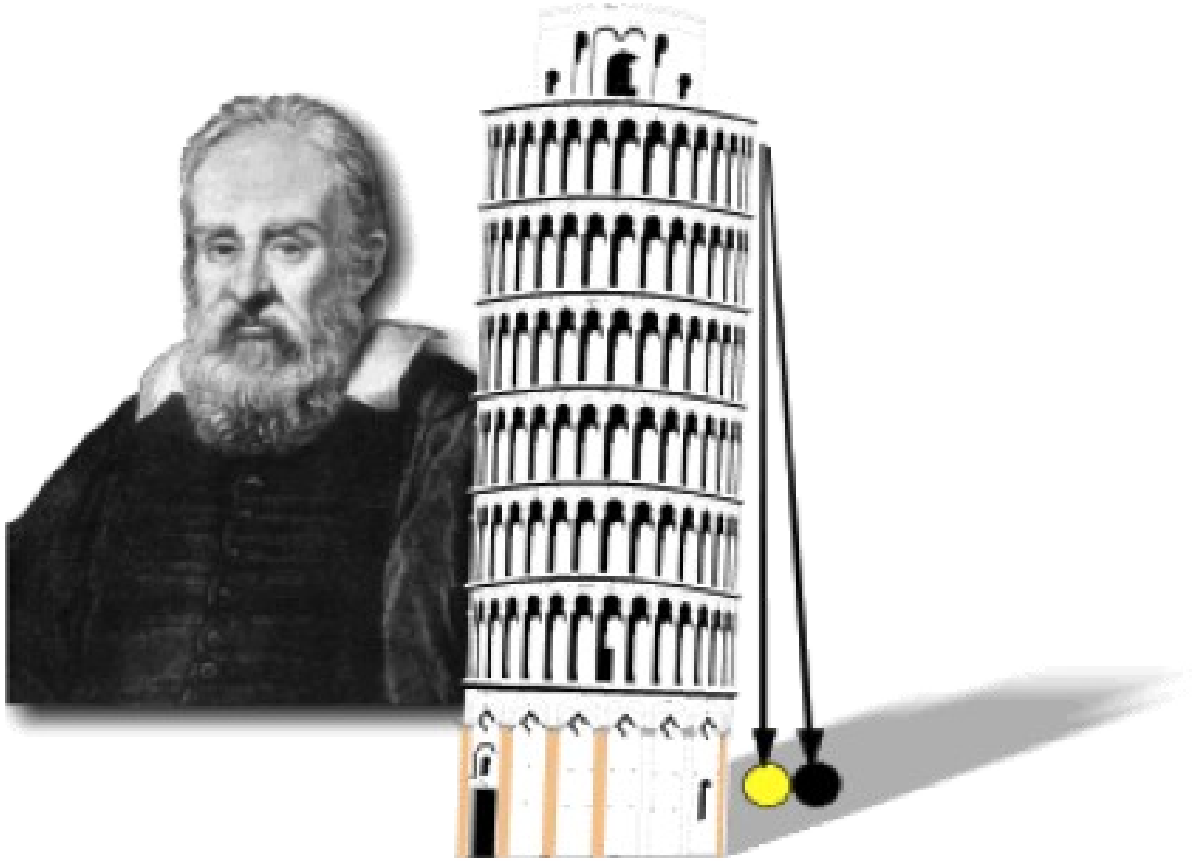
- Heliocentrism, elliptical Orbits





Galileo - 1600

- Bodies fall with the same speed, independently from their weight.



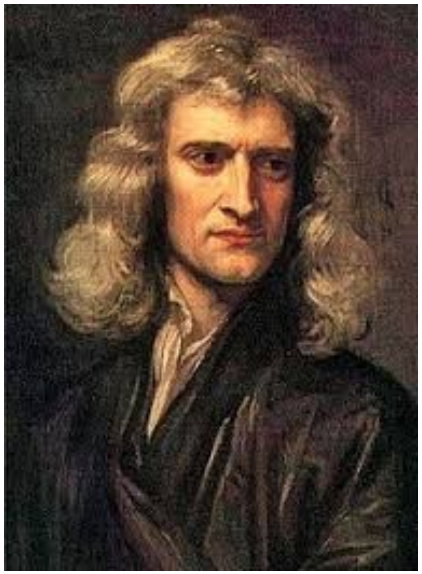
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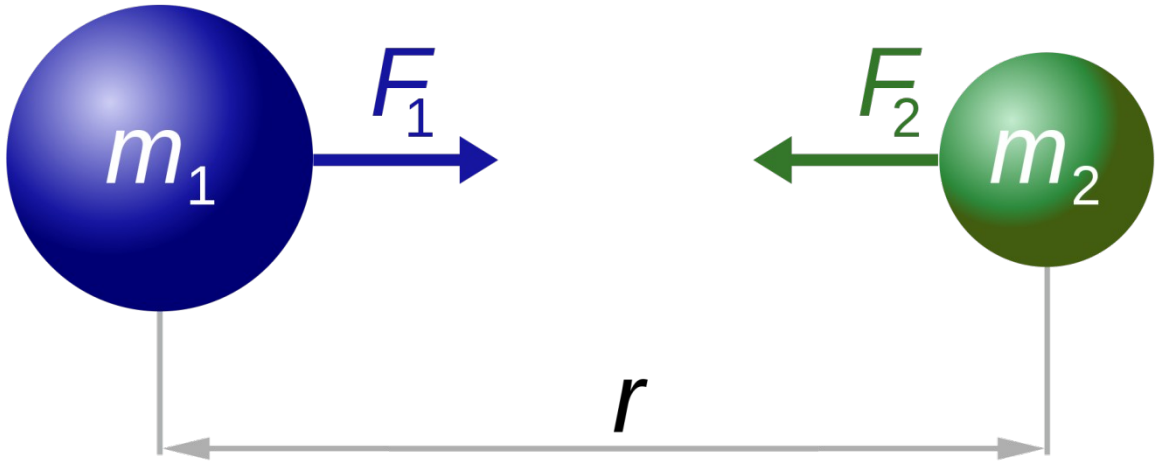
Newton - 1700

- Law of Universal Gravitation:

  All bodies (either apples or planets) attract mutually.

  First time that gravity is related to astronomy

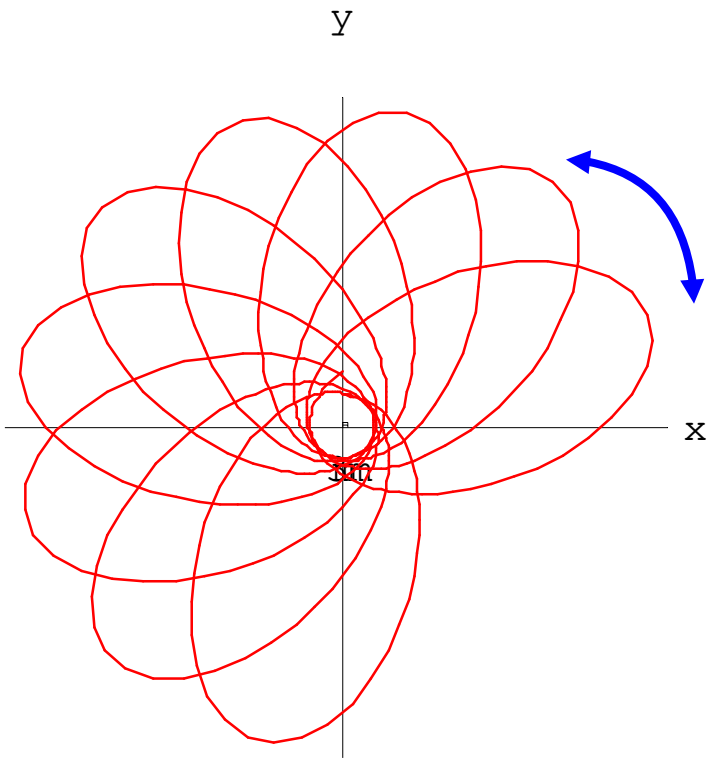


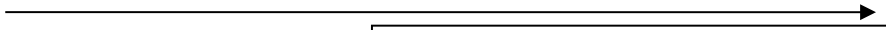


$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Mercury periliheimum - 1859

- *The true orbits of planets, even if seen from the SUN are not ellipses. They are rather curves of this*



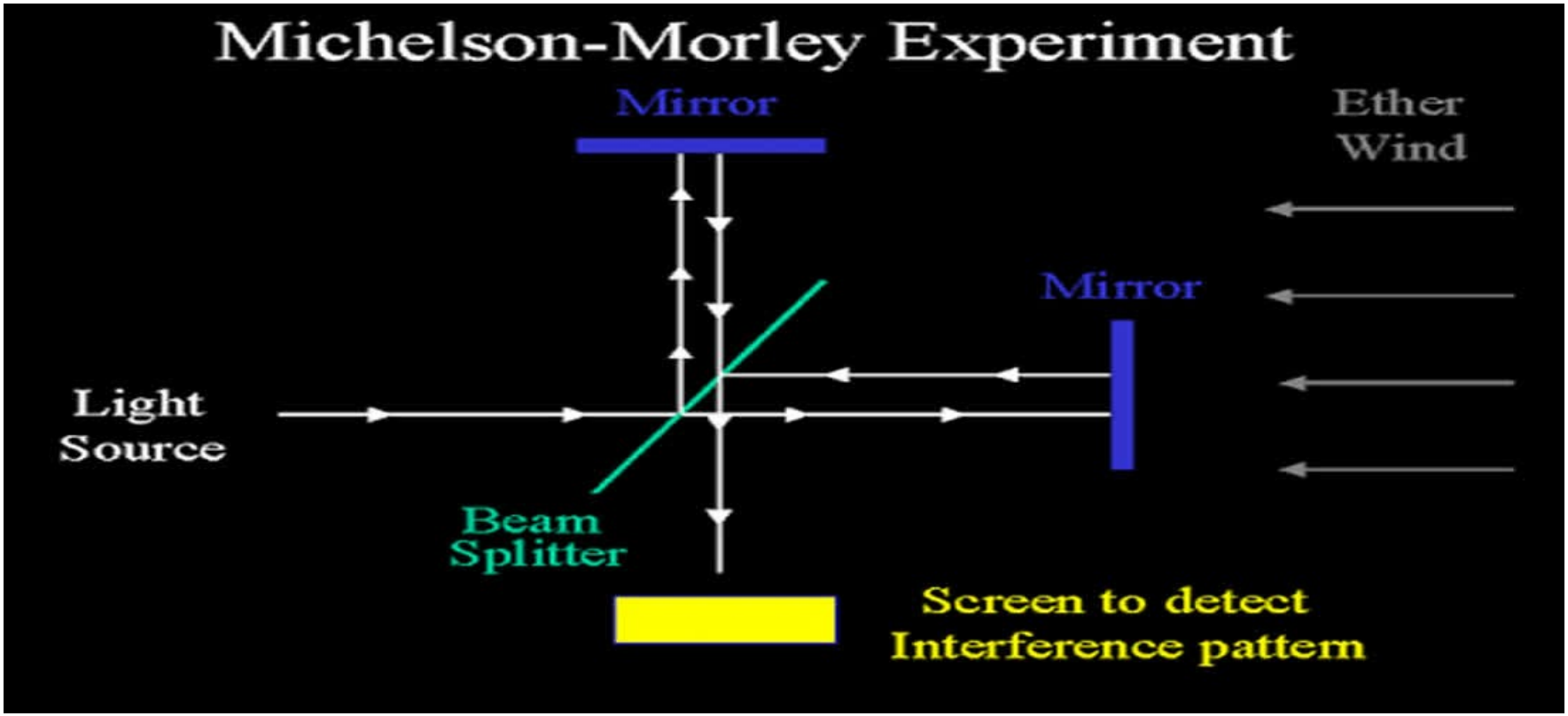


This angle is the perihelion advance, predicted by G.R.

For the planet Mercury it is

$$\Delta\varphi = 43'' \, of \;\; arc \;\; per \;\; century$$

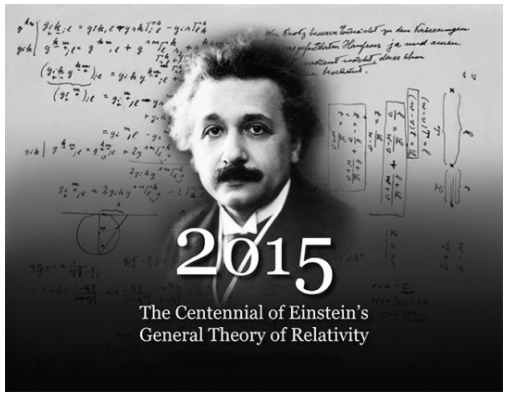
Michelson–Morley experiment - 1887



General Relativity

- Einstein 1915: **General Relativity**:



energy-momentum source of spacetime Curvature

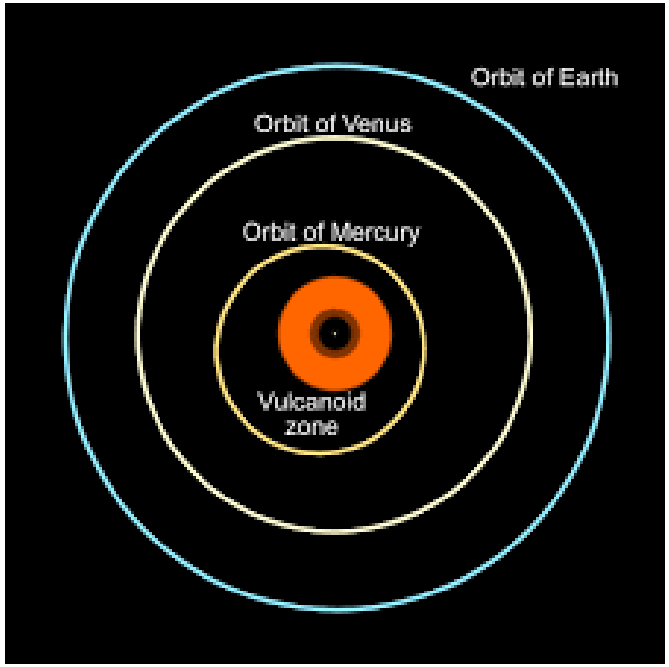
$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R-2\Lambda\right]+\int d^4x L_m\left(g_{\mu\nu},\psi\right)$$

$$\Rightarrow R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+g_{\mu\nu}\Lambda=8\pi G T_{\mu\nu}$$

with $$T^{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g_{\mu\nu}}$$

60 years of tension



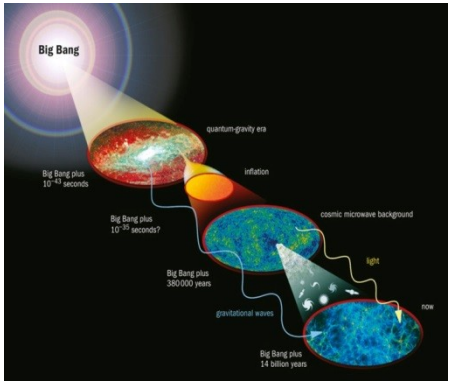


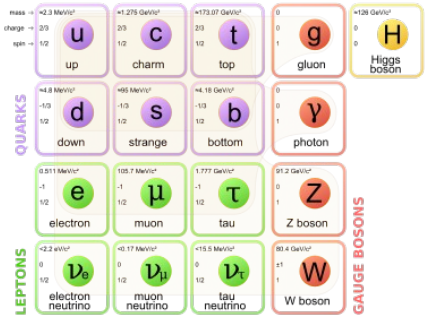
Modified Gravity before General Relativity

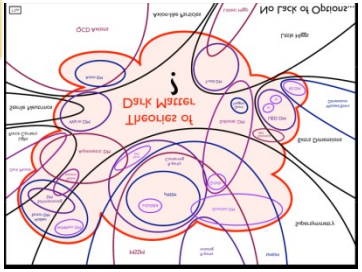
- Modifications to Newton's Law
- Inverse Cube Law.
- Extended Inverse-Square Law (Simon Newcomb -1880’s)
- Lord Kelvin - theory of everything (end of 19th century)
- Hendrik Lorentz: gravity on the basis of his ether theory and Maxwell's equations. (1900)
- Nordström's theory of gravitation (1912 and 1913)
- Einstein's scalar theory of gravity (1913)

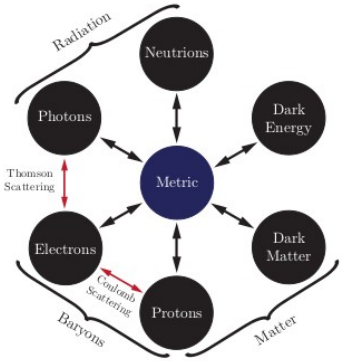
Summary of 20th century Observations

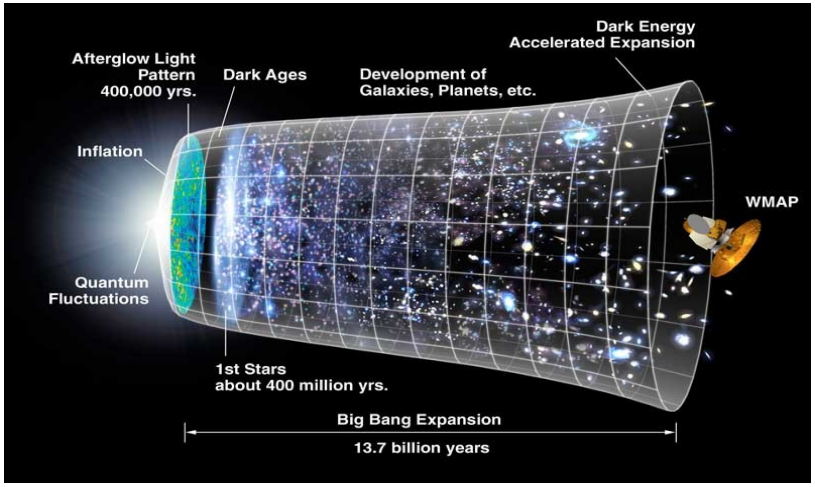
The Universe history:

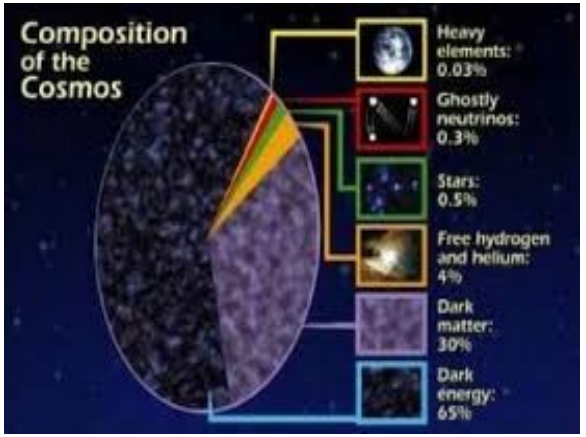












History of Major Shifts of Cosmological Models

Aristarchus of Samos (c. 310-230 BCE)

- **Geocentric (Ptolemy) to Heliocentric (Copernicus, 16th-17th c.):**
  - Retrograde motion (Copernicus)
  - Phases of Venus, Moons of Jupiter (Galileo, 1610)
- **Heliocentric to Infinite Universe (18th-19th c.):**
  - Improved telescopes, Uranus discovery (Herschel, 1781)
  - Stellar parallax (Bessel, 1838)
- **Infinite to Static Universe (Einstein, early 20th c.):**
  - Nebulae spectroscopy (Slipher)
  - Stellar distances (Leavitt, Hertzsprung)
- **Static to Expanding Universe (Lemaître, Hubble, 1920s-30s):**
  - Galactic redshift (Slipher, 1912-14)
  - Hubble's law (Hubble, 1929)
- **Expanding Universe to Inflationary Big Bang (1960s-80s):**
  - CMB (Penzias & Wilson, 1964)
  - Light element abundance (Alpher, Herman)
  - Inflation theory (Guth, Linde, 1980s)
- **Introduction of Dark Matter (1970s-80s):**
  - Galaxy rotation curves (Rubin, 1970s)
  - Gravitational lensing (Walsh et al., 1979)
  - Galaxy clusters (Zwicky, 1930s)
- **Lambda-CDM (late 1990s-present):**
  - Supernova observations (Perlmutter, Schmidt, Riess, 1998-99)
  - CMB (WMAP, Planck), BAO (SDSS, 2005)
- **Potential Future Shift (2020s-?):**
  - Hubble tension (Riess et al. vs Planck Collaboration)
  - S8 (growth rate) tension (KiDS, DES, Planck collaborations)
  - Cosmic dipoles tension (Various teams)
  - CMB anomalies (Planck Collaboration)
  - ISW (Integrated Sachs-Wolfe) tension
  - Lithium problem (Primordial Nucleosynthesis)



New Astronomy Reviews
Volume 95, December 2022, 101659



Challenges for ΛCDM: An update

L. Perivolaropoulos, F. Skara

Standard Model of Cosmology

ΛCDM Paradigm + Inflation

$$H(t)^2 + \frac{k}{a(t)^2} = \frac{8\pi G}{3}\left[\rho_{dm}(t) + \rho_b(t) + \rho_r(t)\right] + \frac{\Lambda}{3} \qquad w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} = -1$$

$$\dot{H}(t) - \frac{k}{a(t)^2} = -4\pi G\left[\rho_{dm}(t) + p_{dm}(t) + \rho_b(t) + p_b(t) + \rho_r(t) + p_r(t)\right]$$

ΛCDM concordance model is almost perfect!

- Describes the thermal history of the Universe at the background level
- Epochs of inflation, radiation, matter, late-time acceleration

Cosmology-background

- Homogeneity and isotropy: $$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

- Background evolution (Friedmann equations) in flat space

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_{DE})$$
$$\dot{H} = -4\pi G\,(\rho_m + p_m + \rho_{DE} + p_{DE})\,,$$

(the effective DE sector can be either Λ or any possible modification)

- One must obtain a H(z) and Ωm(z) and wDE(z) in agreement with observations (SNIa, BAO, CMB shift parameter, H(z) etc)

Cosmology-perturbations

- Perturbation evolutior $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\,\rho\,\delta \approx 0$ where $\delta \equiv \delta\rho/\rho$

  where $G_{\rm eff}(z,k)$ s the effective Newton's constant, given by

$$\nabla^2\phi \approx 4\pi G_{\rm eff}\rho\,\delta,$$

under the scalar metric perturbatio $ds^2 = -(1+2\phi)dt^2 + a^2(1-2\psi)d\vec{x}^{\,2}$

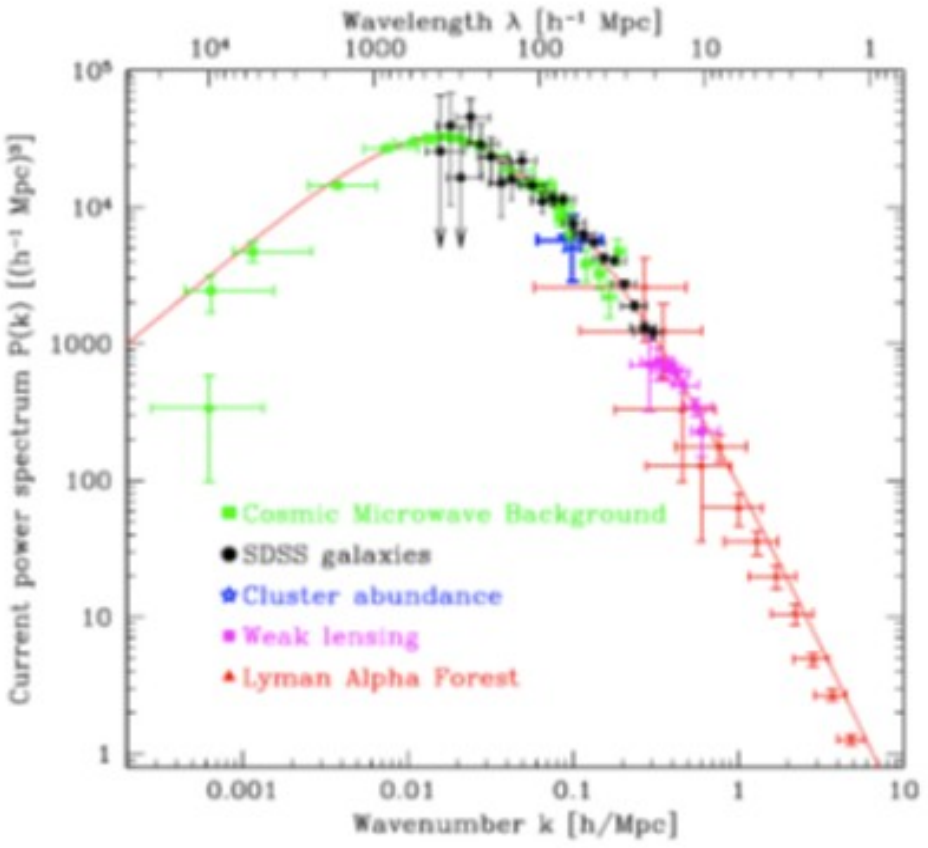
- Hence: $\delta'' + \left(\frac{(H^2)'}{2\,H^2} - \frac{1}{1+z}\right)\delta' \approx \frac{3}{2}(1+z)\frac{H_0^2}{H^2}\frac{G_{\rm eff}(z,k)}{G_N}\,\Omega_{0m}\delta$

  with $f(a) = \frac{dln\delta}{dlna}$ e growth rate, with $f(a) = \Omega_{\rm m}(a)^{\gamma(a)}$ $\Omega_{\rm m}(a) \equiv \frac{\Omega_{0m}\,a^{-3}}{H(a)^2/H_0^2}$

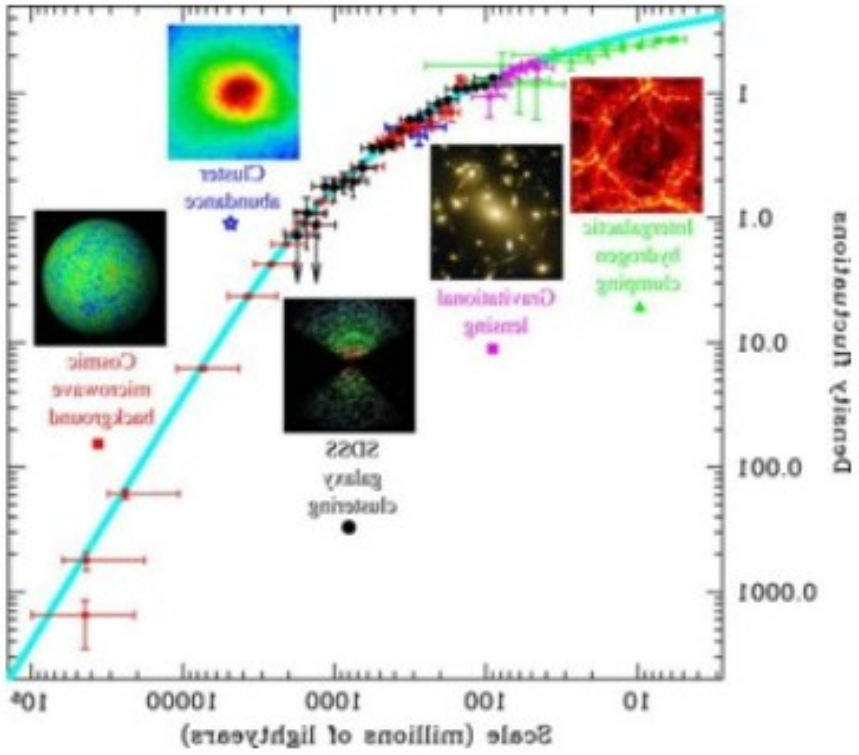
- One can define the observable: $f\sigma_8(a) \equiv f(a)\cdot\sigma(a) = \frac{\sigma_8}{\delta(1)}\,a\,\delta'(a)$

with $\sigma(a) = \sigma_8\frac{\delta(a)}{\delta 1}$ z-dependent rms fluctuations of the linear density field within spheres of radius $R = 8h^{-1}{\rm Mpc}$ d σ8 its value today.

Matter Density Fluctuation Power Spectrum



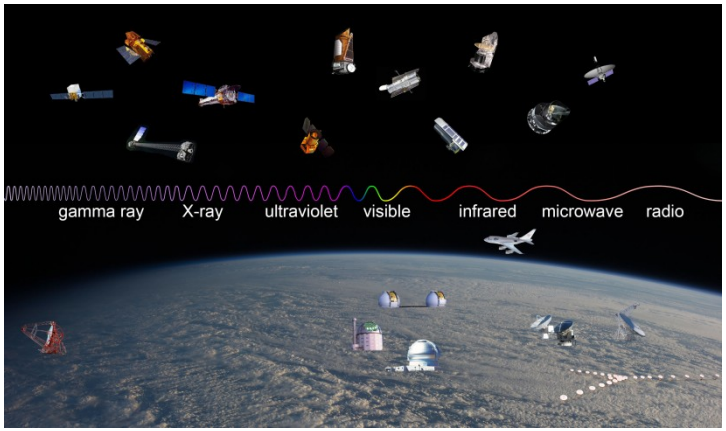
A different convention:
plot $P(k)k^3$



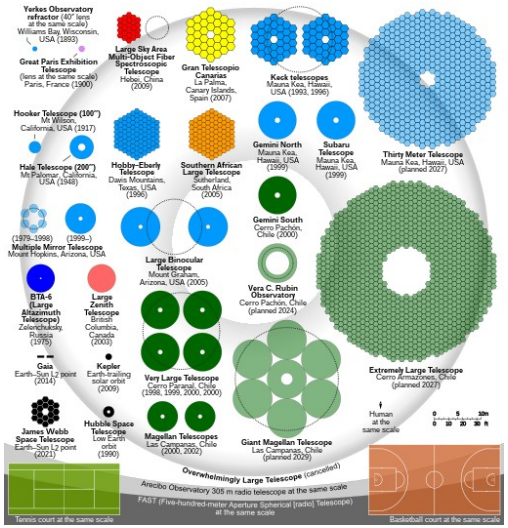
Cosmology in the 21st century

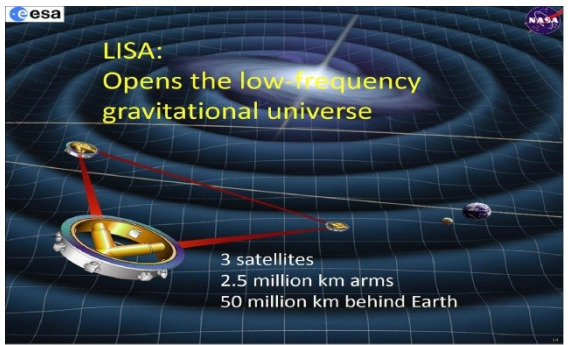


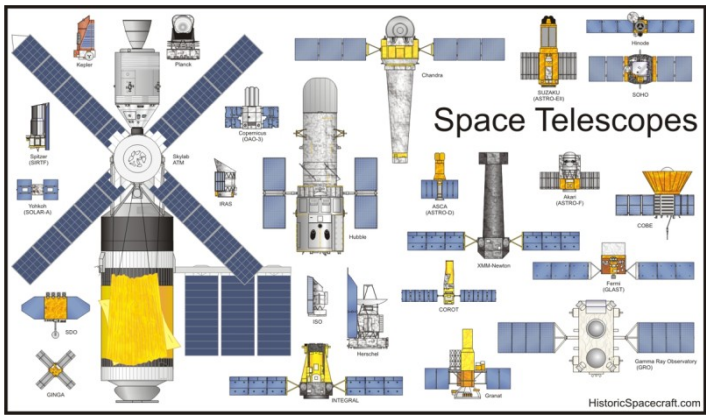








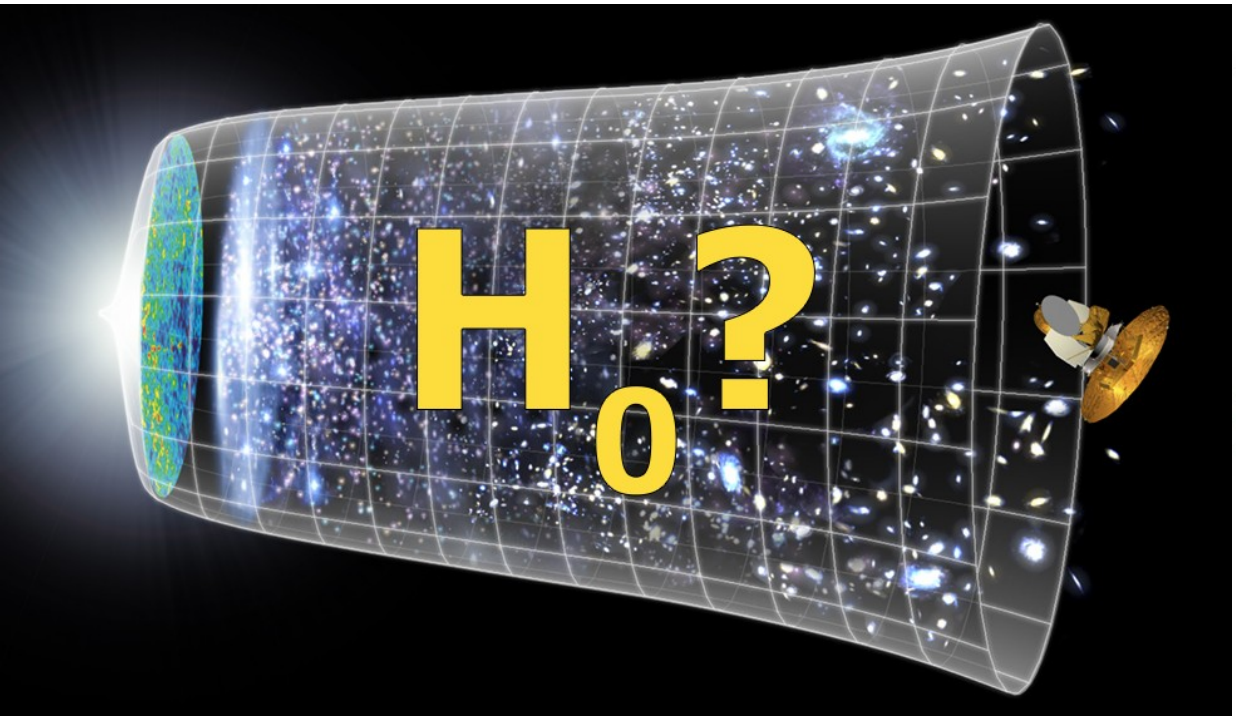


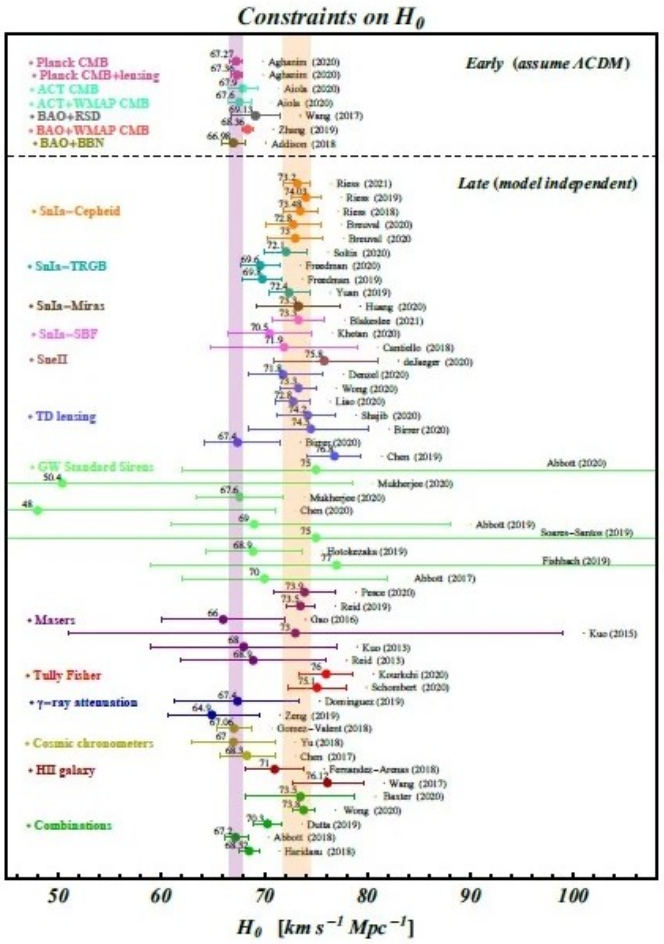


So what do these new observations tell us?

H0 tension

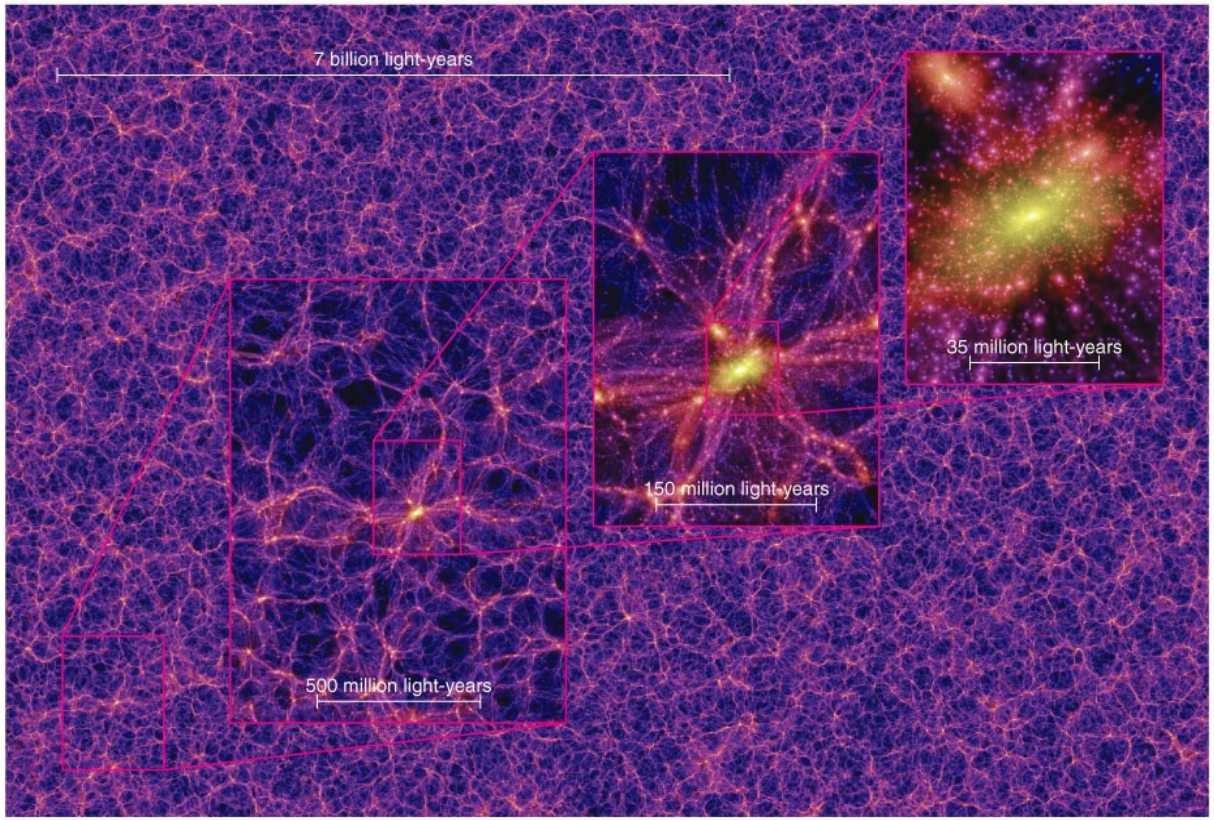
- The Universe expands faster than expected!



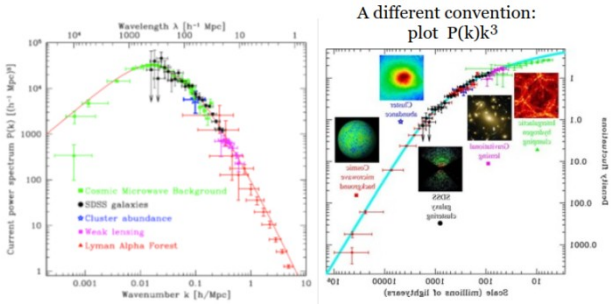


S8 Tension

- “Less” Matter clustering than expected!

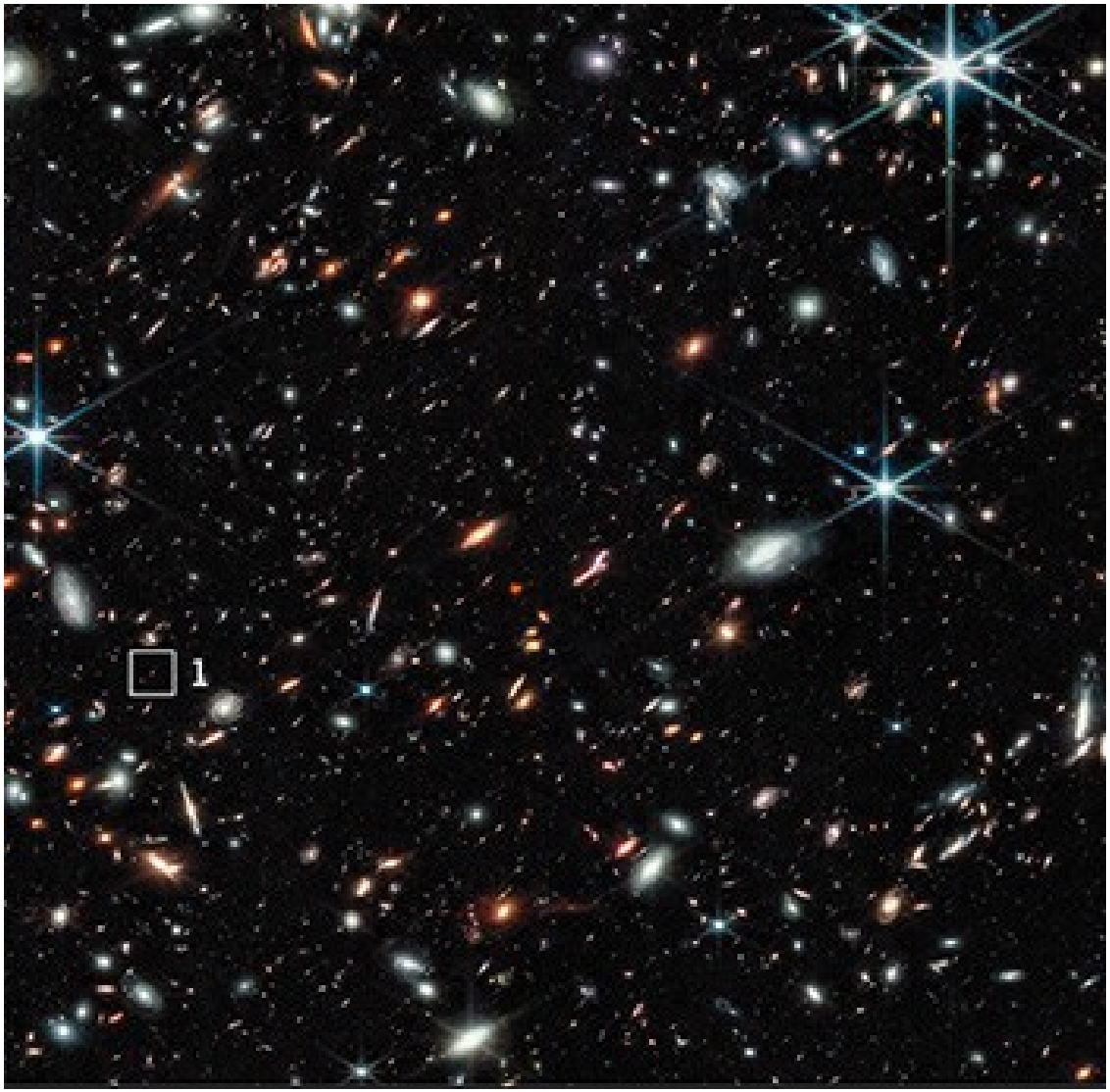


Matter Density Fluctuation Power Spectrum



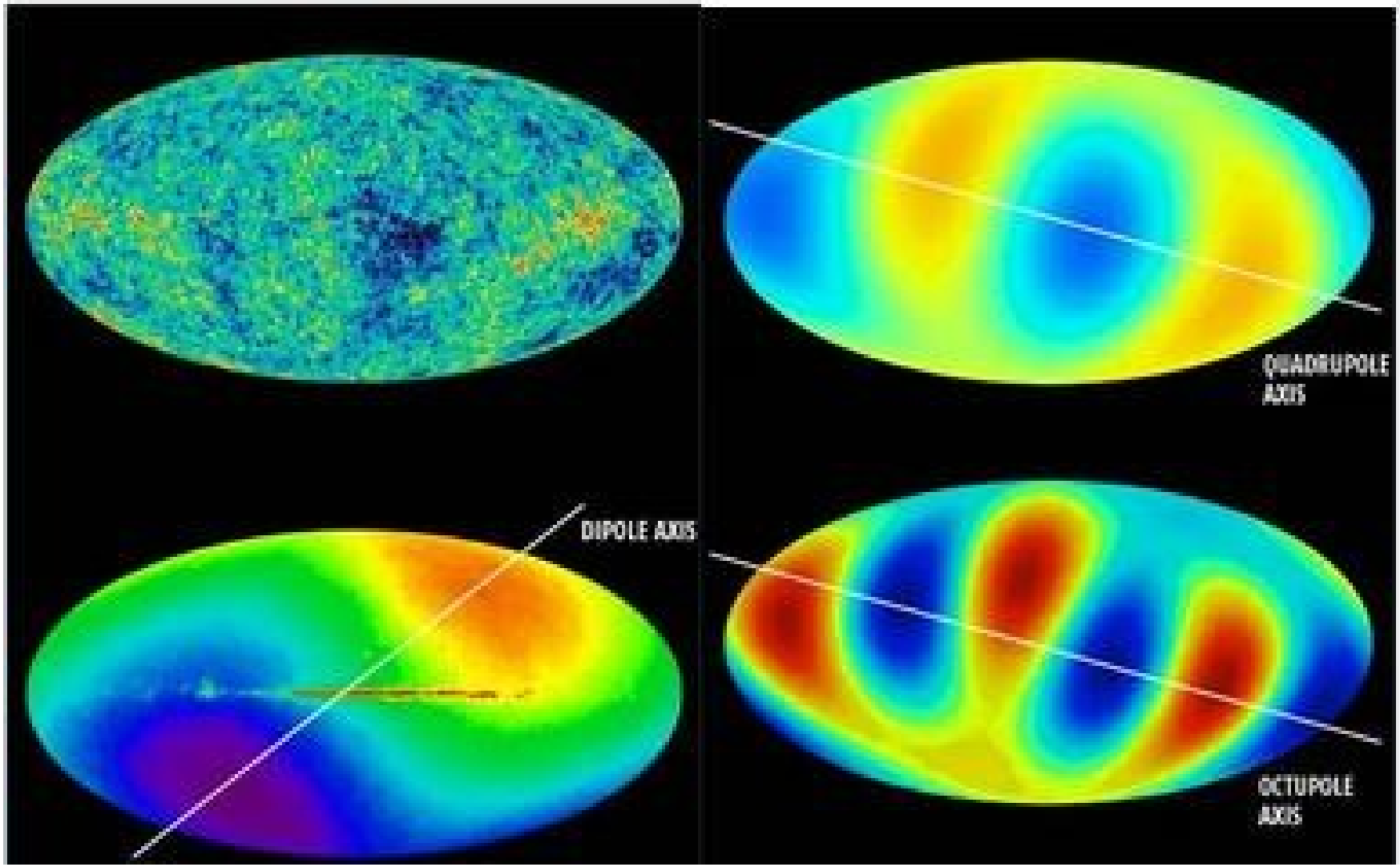
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Too many galaxies too early!

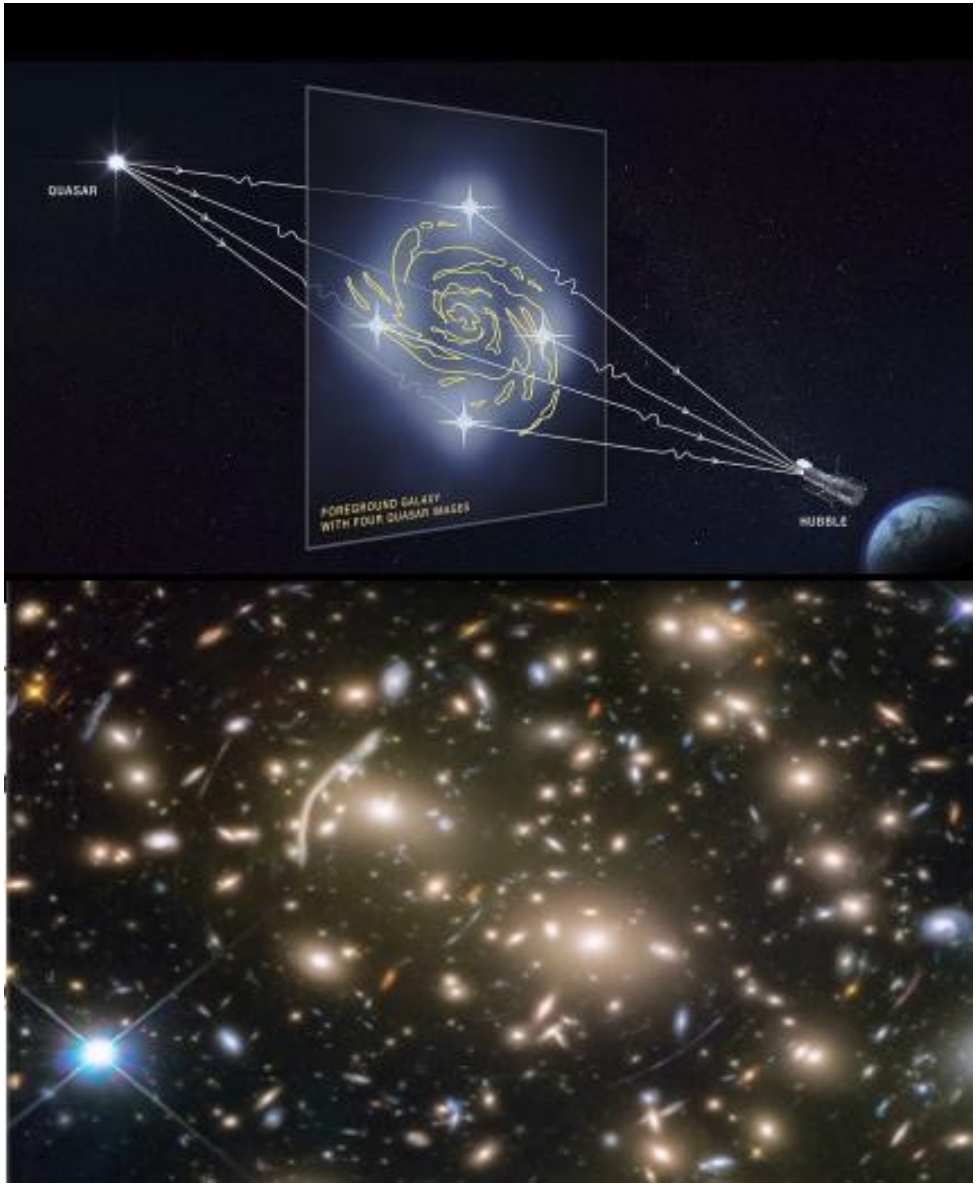


James Webb space telescope

Cosmic dipole tension!



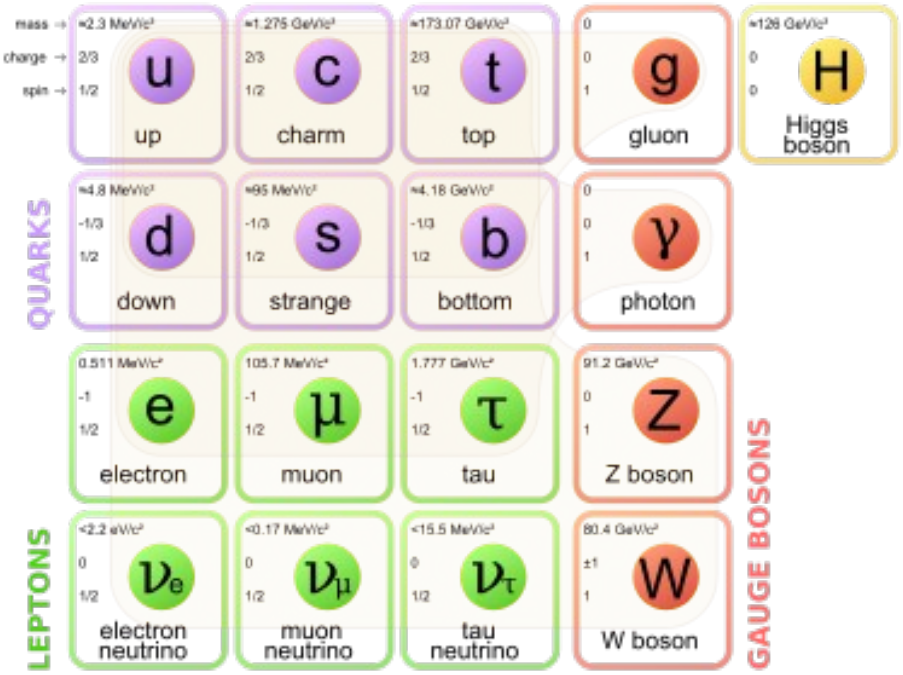
The lensing anomaly

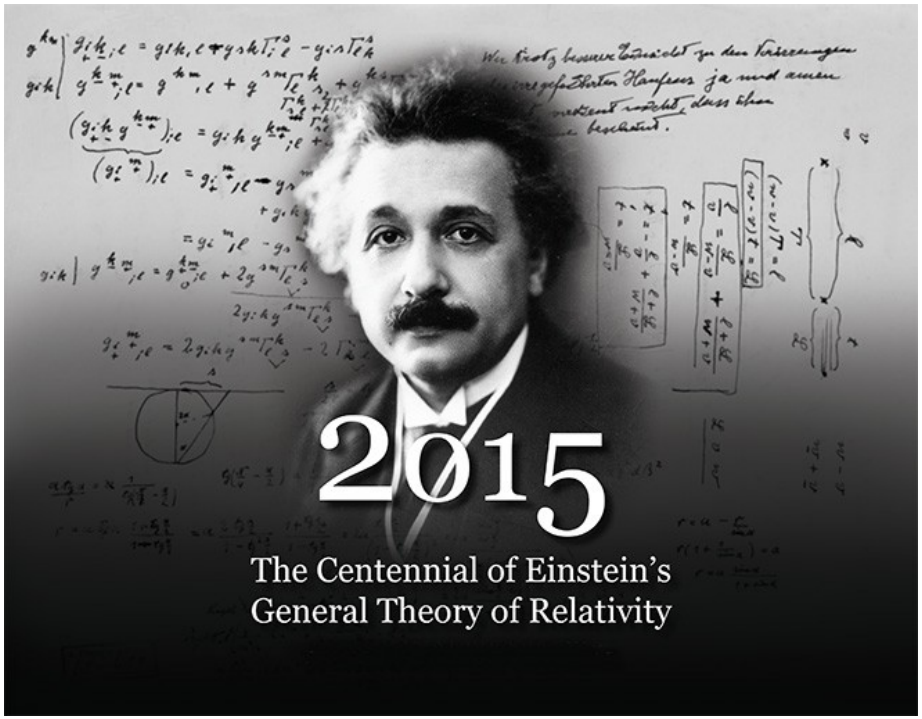



PBS
THE NEW CRISIS
IN COSMOLOGY

Are we approaching a Revolution in Physics?

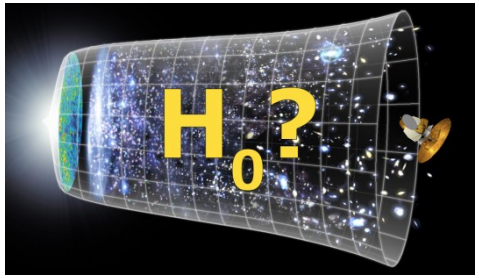
Knowledge of Physics: Standard Model + General Relativity

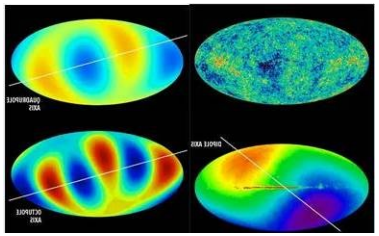


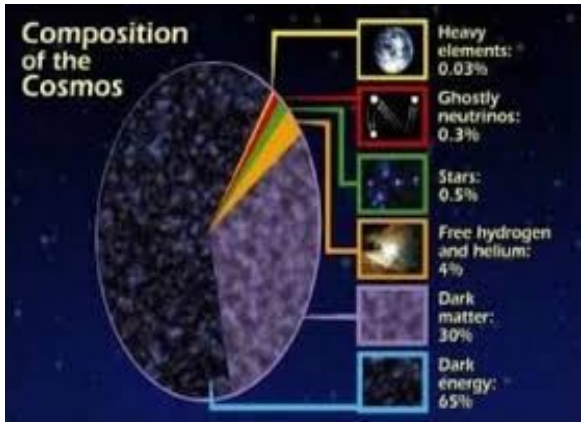


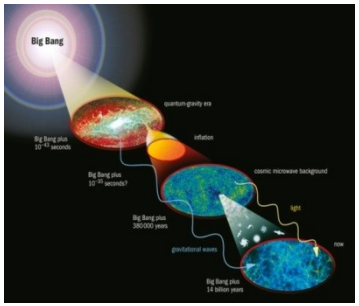
Are we approaching a Revolution in Physics?

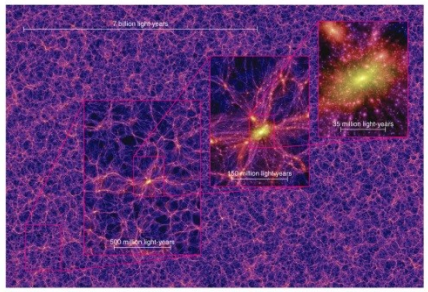
So can our knowledge of Physics describes all these?











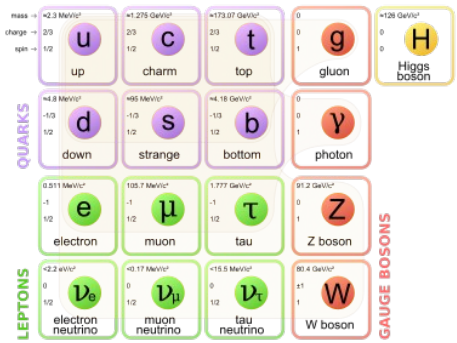


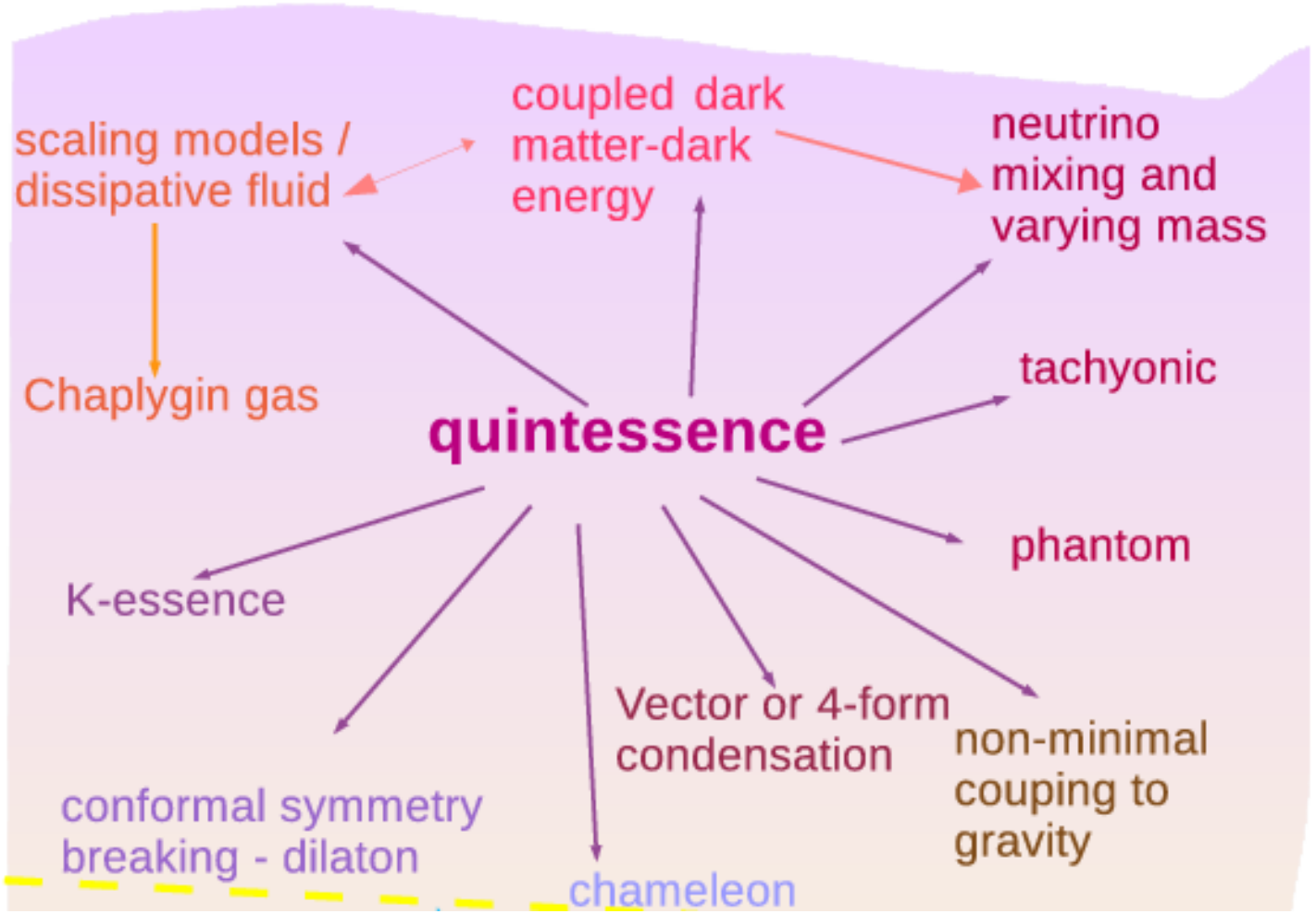


Most probably, no!

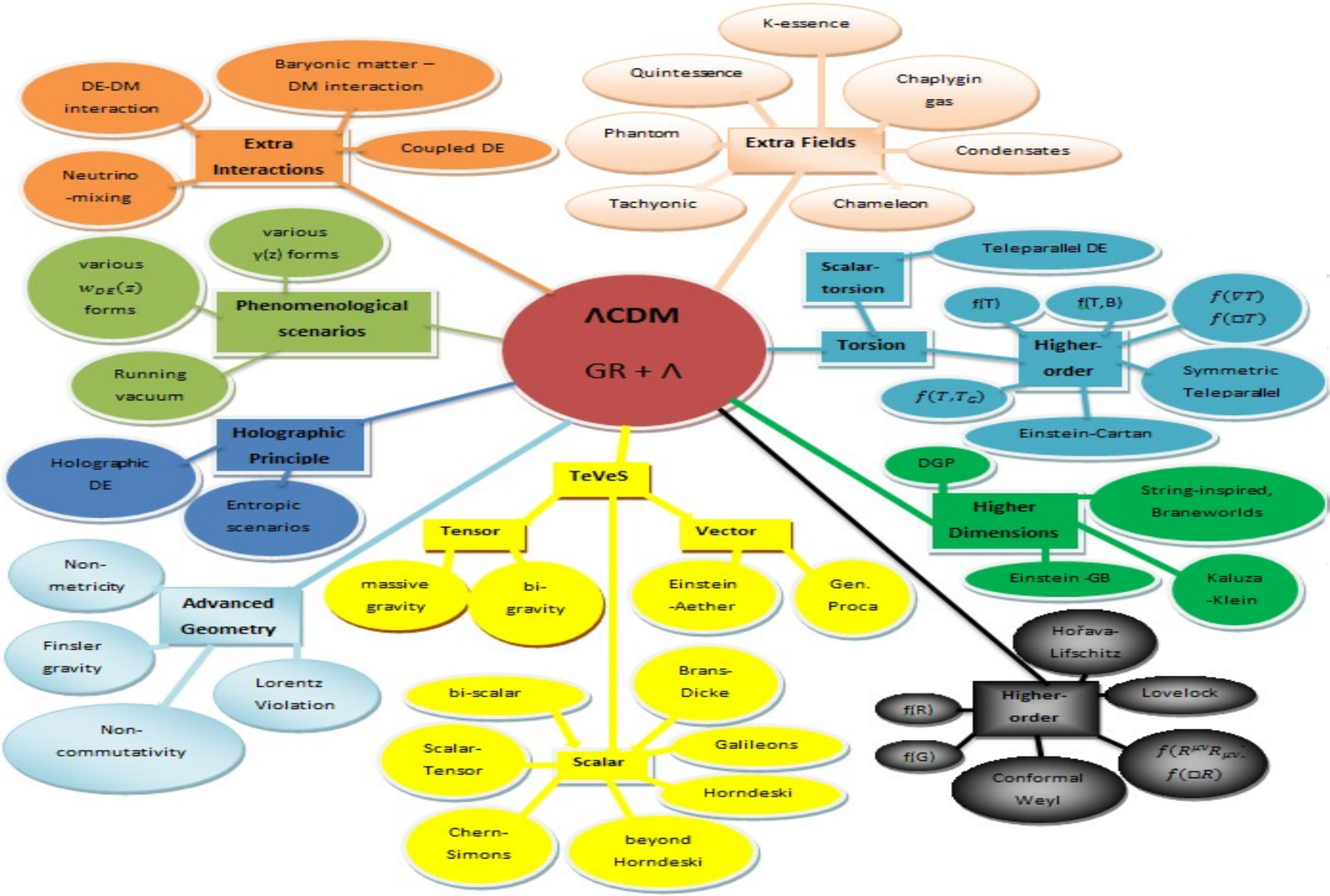
We definitely need new physics for Inflation and Dark matter. Maybe for dark energy.

New particles?





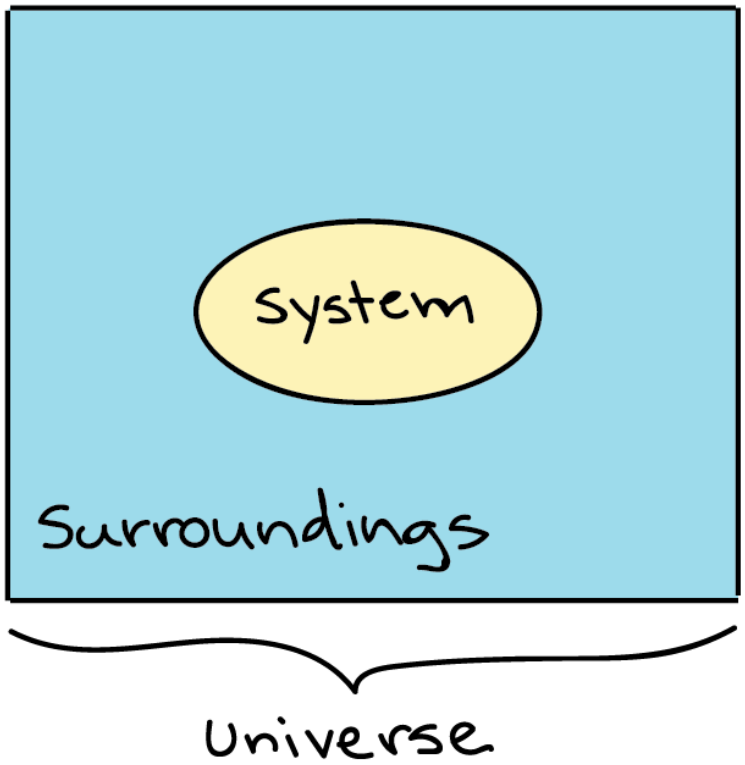
Modified Gravity?

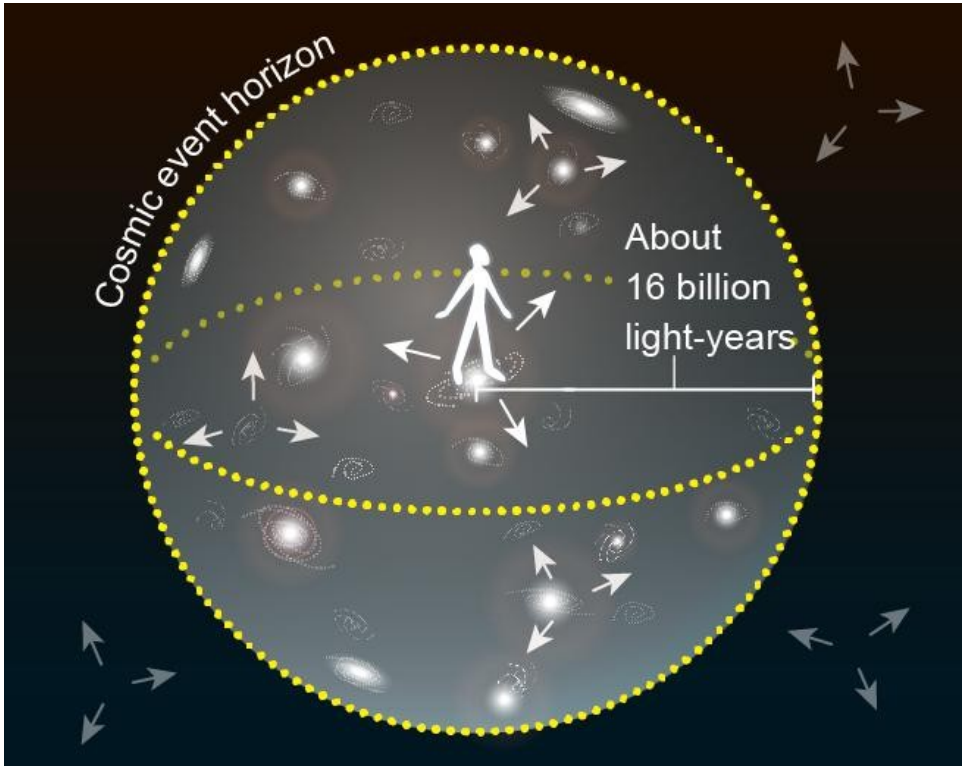


3rd way:

Change the Universe Thermodynamics!

Universe Thermodynamics?



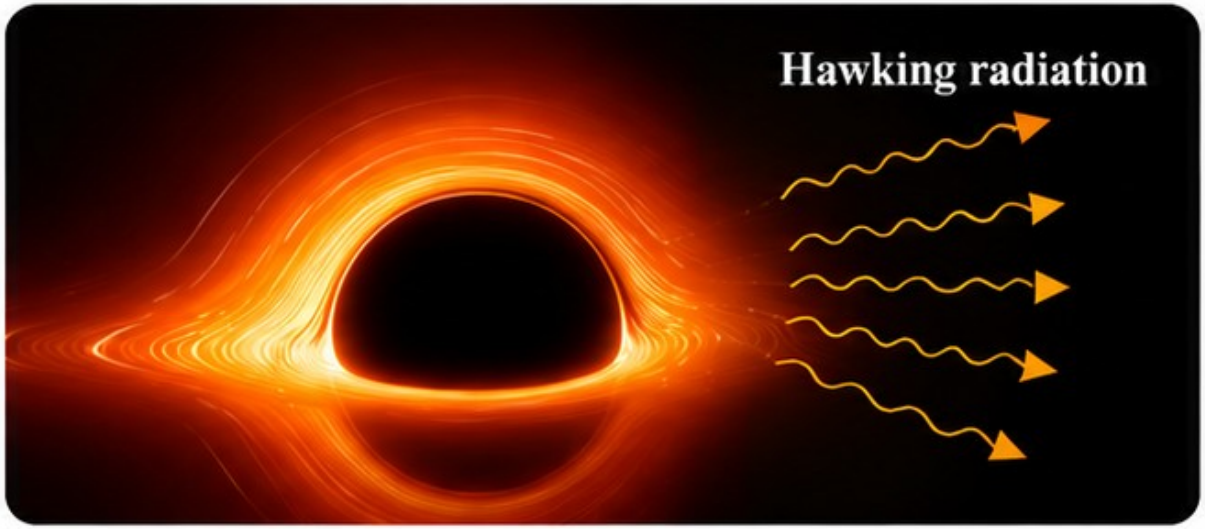


Black Hole Thermodynamics

In the 1970s, **Stephen Hawking** showed that black holes emit radiation with a **thermal spectrum**.



Stephen Hawking
(1942 – 2018)



Hawking temperature

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G M k_{\mathrm{B}}}$$

Black hole entropy

$$S_{\mathrm{BH}} = \frac{k_{\mathrm{B}} c^3}{4G\hbar} A \propto A$$

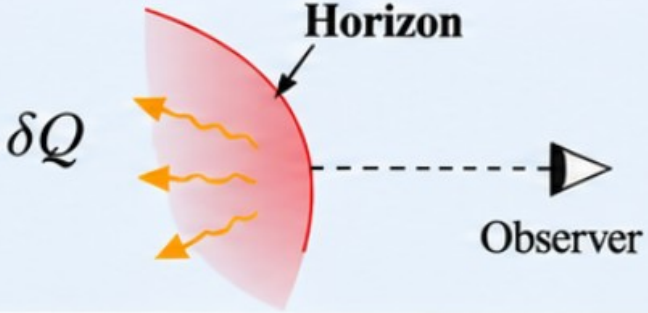
Black holes have temperature and entropy:
they are **thermodynamical systems**, and their properties are encoded in their **horizons**.

Why a Gravity–Thermodynamics Connection?

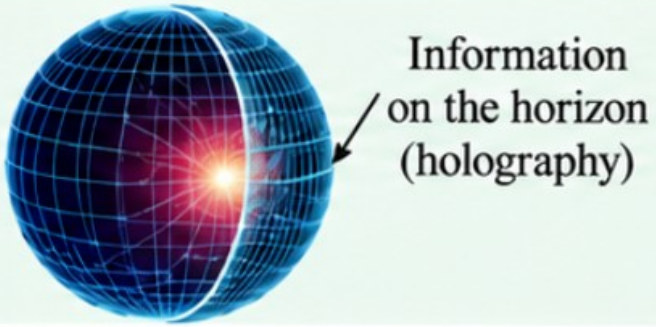
1. **Black holes are thermodynamic systems.**



2. **Gravitational dynamics can be derived from thermodynamics.**



3. **Gravity encodes information on boundaries** (holography).



➡ Gravity–thermodynamics provides a natural and powerful framework to explore new cosmological physics.

Thermodynamics of the Universe

Why can we apply thermodynamics to the whole Universe?



- ✓ Cosmological horizons behave like thermodynamic systems.
- ✓ The apparent horizon is the most convenient choice.

Possible horizons:

- Particle horizon
- Event horizon
- Hubble horizon
- Apparent horizon (most useful)

➡ The Universe can be treated as a thermodynamic system bounded by its apparent horizon.

Applying the first law of thermodynamics

$$\delta Q = -dE = T dS$$

Horizon radius

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}$$



+

Energy flux

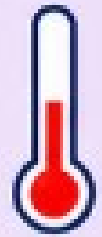
$$-dE = 4\pi\tilde{r}_A^2(\rho_m + p_m)H\,dt$$



+

Entropy & Temperature

$$S = \frac{A}{4G}, \quad T = \frac{1}{2\pi\tilde{r}_A}$$



Clausius relation at the horizon: $\delta Q = T dS$

Universe Thermodynamics

- Universe boundary: Apparent Horizon

$$\tilde{r}_a = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}$$

- First Law of Thermodynamics

$$\delta Q = -dE = A(\rho_m + p_m) H \tilde{r}_a dt$$

$$-dE = T dS.$$

$$dS = 2\pi \dot{\tilde{r}}_a dt / G.$$

[T. Jacobson, PRL 75]

Universe Thermodynamics

- 2nd Friedmann equation

$$-4\pi G(\rho_m + p_m) = \dot{H} - \frac{k}{a^2}$$

- 1st Friedmann equation

$$\frac{8\pi G}{3}\rho_m = H^2 + \frac{k}{a^2} - \frac{\Lambda}{3},$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0.$$

[R-G. Cai, S.P Kim, JHEP 2005]

Questions:

- Equilibrium?

- Which expression for Entropy?

Standard thermodynamics: What do we assume?

Boltzmann–Gibbs statistics

Entropy counts microscopic states:

$$S_{BG} = -k_B \sum p_i \ln p_i$$

1. **Additivity**
   Independent subsystems: S(A+B) = S(A)+S(B)
2. **Standard composition of probabilities**
   Ordinary logarithm and statistical weighting
3. **Conventional statistical framework**
   No relativistic deformation or altered microstate counting

Bekenstein–Hawking horizon

A horizon carries entropy:

$$S_{BH} = k_B A / (4 \ell_p^2)$$

1. **Smooth semiclassical horizon**
   Einstein gravity and the leading-order area law
2. **Area scaling**
   Entropy proportional to horizon area, not volume
3. **Equilibrium Clausius relation**
   Use $\delta Q = T\, dS$ at the cosmological horizon

Statistical assumptions and horizon-area assumptions are distinct.

Relax one assumption → Generalized entropy

1 **ADDITIVITY**

Allow non-additive composition

→ **TSALLIS**

Non-extensive statistical entropy

2 **NON-RELATIVISTIC DESCRIPTION**

Use Lorentz-compatible κ-statistics

→ **KANIADAKIS**

Relativistic deformation of BG statistics

3 **SMOOTH HORIZON**

Include quantum-gravity-inspired fractal deformation

→ **BARROW**

Entropy scales as $A^{(1+\Delta/2)}$

4 **SEPARABILITY AXIOM**

Generalize microscopic state counting / multi-scaling

→ **LUCIANO–SARIDAKIS**

Two-power horizon entropy: $\gamma\delta A^{\delta} + \gamma\varepsilon A^{\varepsilon}$

New entropy → modified Friedmann equations → new cosmological physics

Tsallis non-additive Entropy

$$S_T = \frac{\tilde{\alpha}}{4G} A^{\delta}$$

[C. Tsallis, J. Statist. Phys. 52]

$$dS = (4\pi)^{\delta} \frac{\delta \tilde{\alpha}}{2G} \tilde{r_a}^{2\delta-1} \dot{\tilde{r}}_a dt.$$

$$-\frac{(4\pi)^{2-\delta} G}{\tilde{\alpha}} (\rho_m + p_m) = \delta \frac{\dot{H} - \frac{k}{a^2}}{\left(H^2 + \frac{k}{a^2}\right)^{\delta-1}}.$$

$$\frac{2(4\pi)^{2-\delta} G}{3\tilde{\alpha}} \rho_m = \frac{\delta}{2-\delta} \left(H^2 + \frac{k}{a^2}\right)^{2-\delta} - \frac{\tilde{\Lambda}}{3\tilde{\alpha}}$$

[A. Lymperis. E.N. Saridakis, EPJC 78]

Tsallis Cosmology

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{DE}\right)$$

$$\dot{H} = -4\pi G\left(\rho_m + p_m + \rho_{DE} + p_{DE}\right)$$

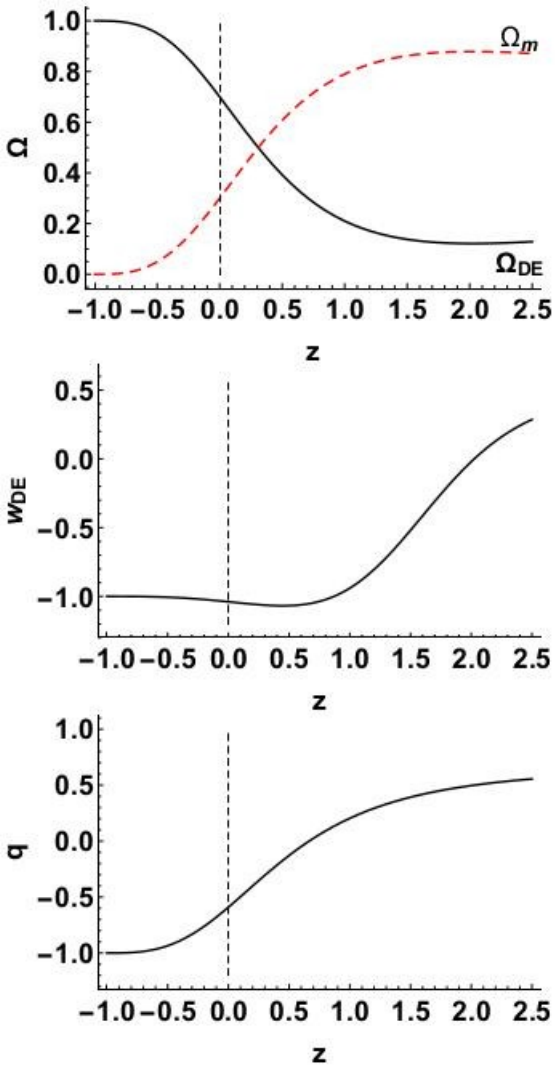
$$\rho_{DE} = \frac{3}{8\pi G}\left\{\frac{\Lambda}{3} + H^2\left[1 - \alpha\frac{\delta}{2-\delta}H^{2(1-\delta)}\right]\right\}$$

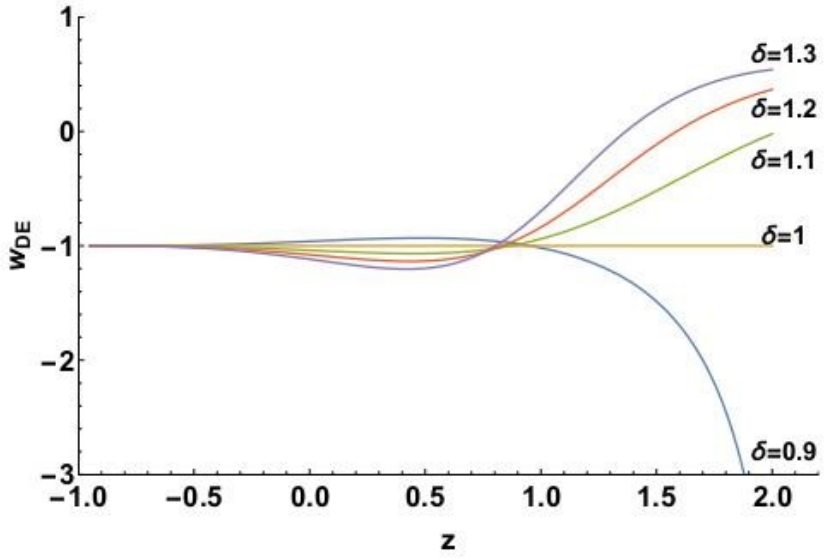
$$p_{DE} = -\frac{1}{8\pi G}\left\{\Lambda + 2\dot{H}\left[1 - \alpha\delta H^{2(1-\delta)}\right] + 3H^2\left[1 - \alpha\frac{\delta}{2-\delta}H^{2(1-\delta)}\right]\right\}$$

$$w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} = -1 - \frac{2\dot{H}\left[1 - \alpha\delta H^{2(1-\delta)}\right]}{\Lambda + 3H^2\left[1 - \frac{\alpha\delta}{2-\delta}H^{2(1-\delta)}\right]}$$

[A. Lymperis. E.N. Saridakis, EPJC 78]

Tsallis Cosmology





[A. Lymperis. E.N. Saridakis, EPJC 78]

Barrow quantum-corrected Entropy

$$S_B = \left(\frac{A}{A_0}\right)^{1+\Delta/2}$$

[J. Barrow, PLB 808]

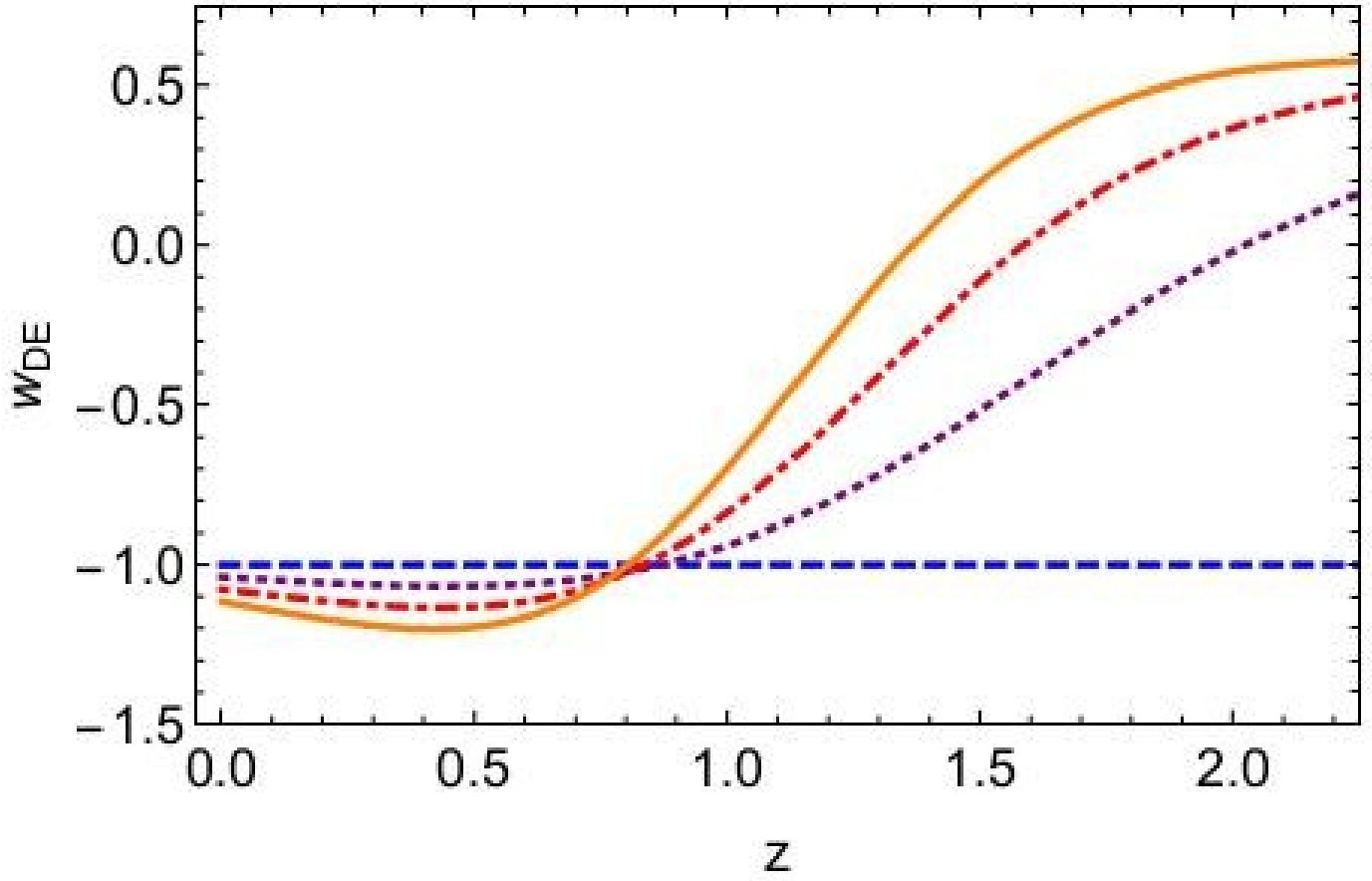
$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{DE}\right)$$
$$\dot{H} = -4\pi G\left(\rho_m + p_m + \rho_{DE} + p_{DE}\right),$$

$$\rho_{DE} = \frac{3}{8\pi G}\left\{\frac{\Lambda}{3} + H^2\left[1 - \frac{\beta(\Delta+2)}{2-\Delta}H^{-\Delta}\right]\right\},$$

$$p_{DE} = -\frac{1}{8\pi G}\left\{\Lambda + 2\dot{H}\left[1 - \beta\left(1+\frac{\Delta}{2}\right)H^{-\Delta}\right] + 3H^2\left[1 - \frac{\beta(2+\Delta)}{2-\Delta}H^{-\Delta}\right]\right\}$$

[E.N. Saridakis, JCAP 07]

Barrow Cosmology



[E.N. Saridakis, JCAP 07]

Barrow Cosmology with varying exponent

$$\Delta \equiv \Delta(x).$$

$$\frac{\Omega'_{DE}}{\Omega_{DE}\left(1-\Omega_{DE}\right)} = \sqrt{\Omega_{DE}}\left(\frac{C}{3M_p^2}\right)^{-\frac{1}{2}}\left[\frac{Q(1-\Omega_{DE})}{\Omega_{DE}}\right]^{\frac{\Delta}{2(\Delta-2)}}(2-\Delta)+\Delta+1$$
$$+\log\left[\frac{Q(1-\Omega_{DE})}{\Omega_{DE}}\right]^{\frac{\Delta'}{\Delta-2}},$$

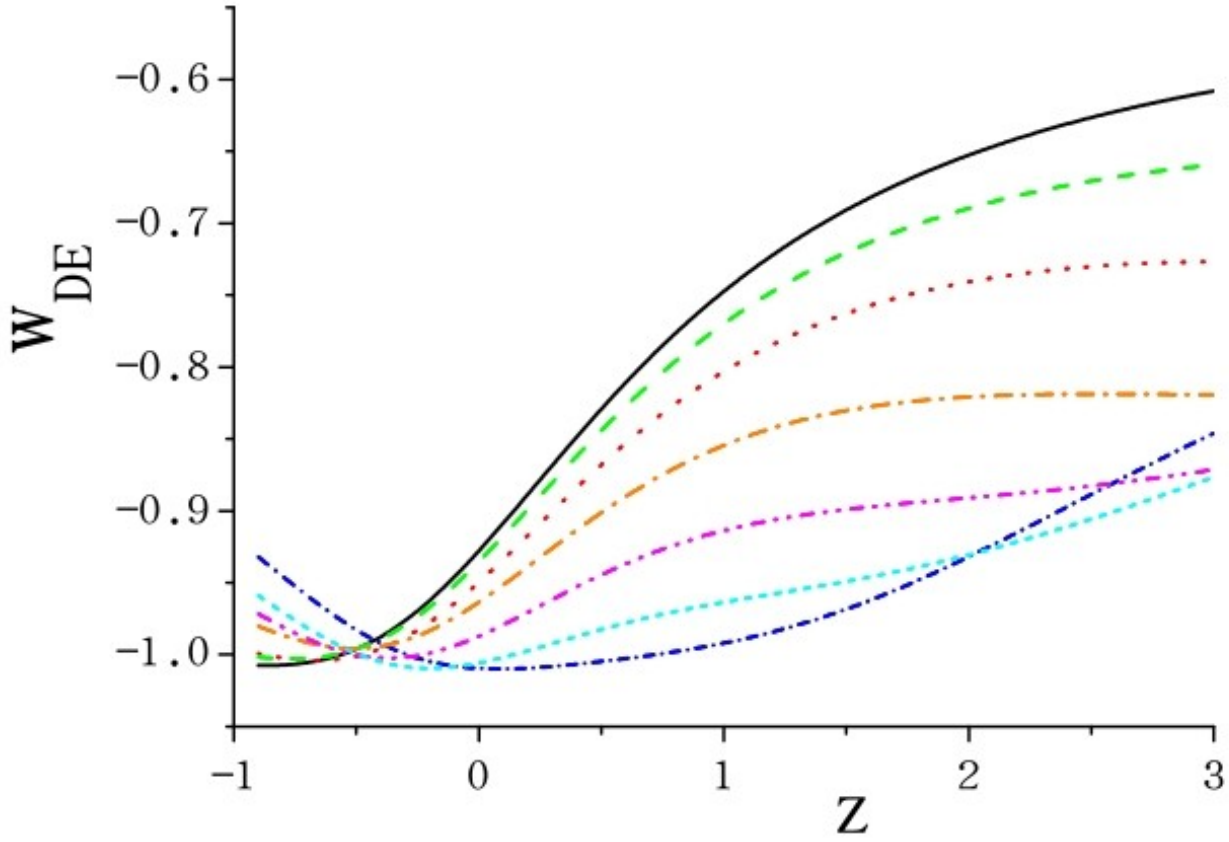
with

$$Q = Q(x) \equiv \frac{Ce^{3x}}{3M_p^2H_0^2\Omega_{m0}}.$$

$$w_{DE} = -\frac{\Delta+1}{3}+\frac{(\Delta-2)\sqrt{\Omega_{DE}}}{3}\left(\frac{C}{3M_p^2}\right)^{-\frac{1}{2}}\left[\frac{Q(1-\Omega_{DE})}{\Omega_{DE}}\right]^{\frac{\Delta}{2(\Delta-2)}}-\frac{\Delta'}{3}\log\left[\frac{Q(1-\Omega_{DE})}{\Omega_{DE}}\right]^{\frac{1}{2-\Delta}}$$

[S. Basilakos, A. Lymperis, M. Petronikolou, E.N. Saridakis, EPJC 80]

Barrow Cosmology with varying exponent

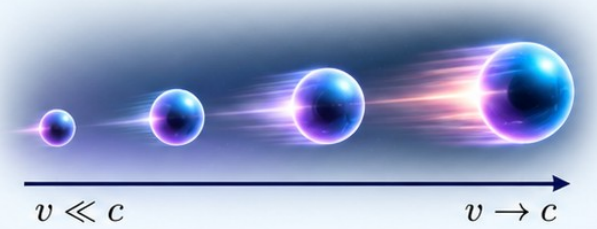


[S. Basilakos, A. Lymperis, M. Petronikolou, E.N. Saridakis, EPJC 80]

Kaniadakis relativistic Entropy

Motivation

- Standard Boltzmann–Gibbs statistics is **non-relativistic**.
- In high-energy / relativistic regimes, one should use a **relativistic distribution function**.
- Kaniadakis statistics is a consistent generalization of Boltzmann–Gibbs based on **special relativity** (deformed Lorentz symmetry).



1 Entropy definition

$$S_K = -k_B \sum_i n_i \ln_{\{K\}} n_i$$

with the κ-logarithm

$$\ln_{\{K\}} x = \frac{x^\kappa - x^{-\kappa}}{2\kappa},$$

κ is the relativistic deformation parameter ($\kappa \to 0$ recovers Boltzmann–Gibbs).

2 Relativistic distribution

$$n_i = \alpha \exp_{\{K\}} \left[-\beta(E_i - \mu)\right]$$

with the κ-exponential

$$\exp_{\{K\}} x = \left(\sqrt{1+\kappa^2 x^2} + \kappa x\right)^{\frac{1}{\kappa}}.$$

3 Equivalent entropy form

$$S_K = -k_B \sum_{i=1}^{W} \frac{P_i^{1+\kappa} - P_i^{1-\kappa}}{2\kappa}$$

Alternative expression:

$$S_K = \frac{1}{\kappa} \sinh\left(\kappa S_{BH}\right),$$

with $S_{BH} = -k_B \sum_i P_i \ln P_i$.

Kaniadakis entropy arises from using a **relativistic (κ-deformed) statistical mechanics,** providing a consistent extension of Boltzmann–Gibbs at high energies.

[G. Kaniadakis, PRE 66], [G. Kaniadakis, PRE 72]

Kaniadakis Cosmology

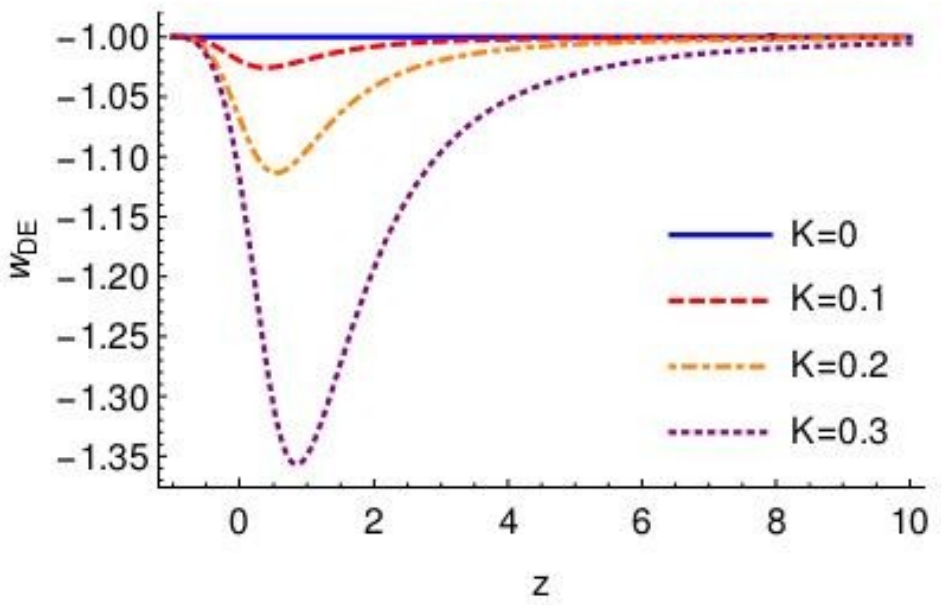
$$dS_K = \frac{8\pi}{4G}\cosh\left(K\frac{\pi r_a^2}{G}\right) r_a \dot{r}_a dt.$$

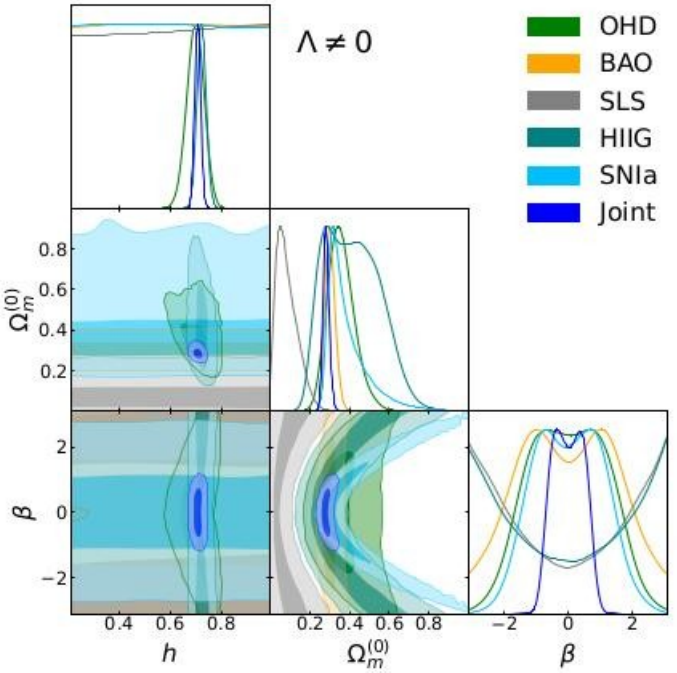
$$\rho_{DE} = \frac{3}{8\pi G}\left\{\frac{\Lambda}{3} + H^2\left[1-\cosh\left(K\frac{\pi}{GH^2}\right)\right] + \frac{K\pi}{G}\mathrm{shi}\left(K\frac{\pi}{GH^2}\right)\right\},$$

$$p_{DE} = -\frac{1}{8\pi G}\left\{\Lambda + (3H^2+2\dot{H})\left[1-\cosh\left(K\frac{\pi}{GH^2}\right)\right] + \frac{3K\pi}{G}\mathrm{shi}\left(K\frac{\pi}{GH^2}\right)\right\}.$$

[A. Lymperis, s. Basilakos, E.N. Saridakis, EPJC 81]

Kaniadakis Cosmology





[A. Lymperis, S. Basilakos, E.N. Saridakis, EPJC 81]

[A. Hernadez-Almada, G. Leon, J. Magana, M.A Garcia-Aseitia, V.Motta, E. N. Saridakis MNRAS 512]

Luciano-Saridakis entropy: Microscopic origin

1 Relax the separability axiom

Standard holographic counting: one characteristic scaling of microstates

$$W(L) \propto \exp\left[\xi L^2\right]$$

2 Introduce two independent microscopic scaling exponents

$$S_{\delta,\varepsilon} = \eta_\delta \sum_i p_i \left(\ln \tfrac{1}{p_i}\right)^\delta + \eta_\varepsilon \sum_i p_i \left(\ln \tfrac{1}{p_i}\right)^\varepsilon$$

Equiprobable states: $S_{\delta,\varepsilon} = \eta_\delta (\ln W)^\delta + \eta_\varepsilon (\ln W)^\varepsilon$

3 For a gravitational horizon

$$S_{\delta,\varepsilon}(A) = \gamma_\delta A^\delta + \gamma_\varepsilon A^\varepsilon$$

Two independent holographic scalings → generalized horizon entropy

[G. Luciano, E.N. Saridakis, PLB 879 (2026)]

Luciano-Saridakis cosmology: Modified Friedmann equations

1 First law at the apparent horizon

$$\delta Q = T_h \, dS_{\delta,\varepsilon}, \qquad A = 4\pi/H^2 \;\; (k=0)$$

2 Modified cosmic evolution

$$\alpha_\delta H^{4-2\delta} + \alpha_\varepsilon H^{4-2\varepsilon} = \frac{8\pi G}{3}\rho_m + C$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{\alpha_\delta(2-\delta)H^{2(1-\delta)} + \alpha_\varepsilon(2-\varepsilon)H^{2(1-\varepsilon)}}$$

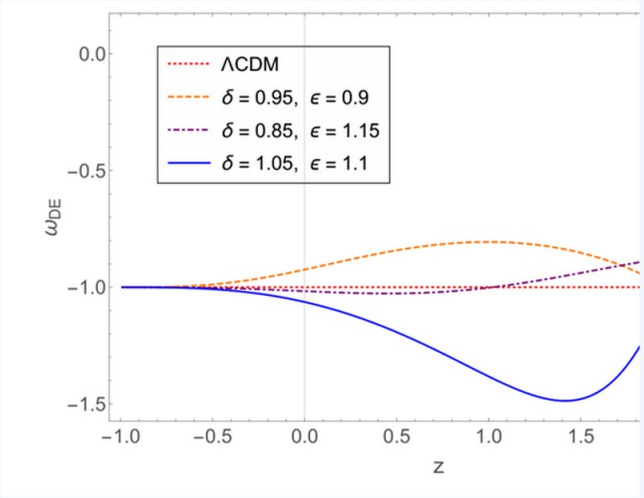
3 Effective dark energy formulation

$$H^2 = \tfrac{8\pi G}{3}(\rho_m + \rho_{\rm DE}), \qquad \dot{H} = -4\pi G(\rho_m + p_m + \rho_{\rm DE} + p_{\rm DE})$$

→ **Entropy modifies the expansion history without changing the gravitational action.**

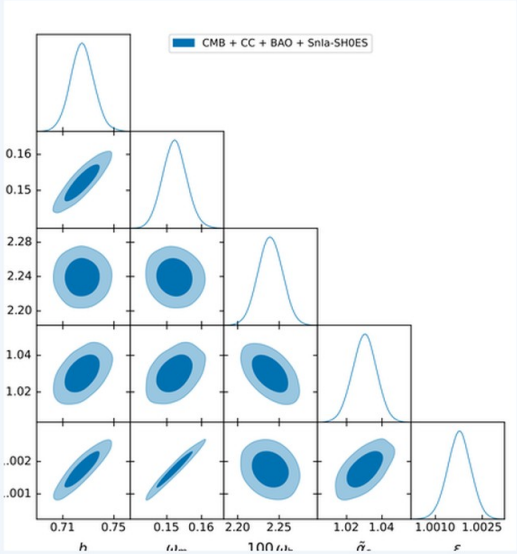
Luciano-Saridakis cosmology: Evolution and observations

Cosmological evolution



$w_{DE}(z)$: quintessence / phantom transitions

Combined observational constraints



CMB + CC + DESI DR2 BAO + Pantheon+/SH0ES

Joint posterior: restricted model (alpha_delta = 0)

Entropic dark energy: dynamical evolution and viable observational fits

Leizerovich et al., Phys. Dark Univ. 52 (2026) 102333, Fig. 5 (right)

Holographic Dark Energy

Holographic Dark Energy: From vacuum energy to a bound

1 Vacuum energy in a finite region

- Consider a region of size L filled with quantum vacuum energy (dark energy) with density ρ_{DE}.



$$E_{DE} = \rho_{DE} L^3 .$$

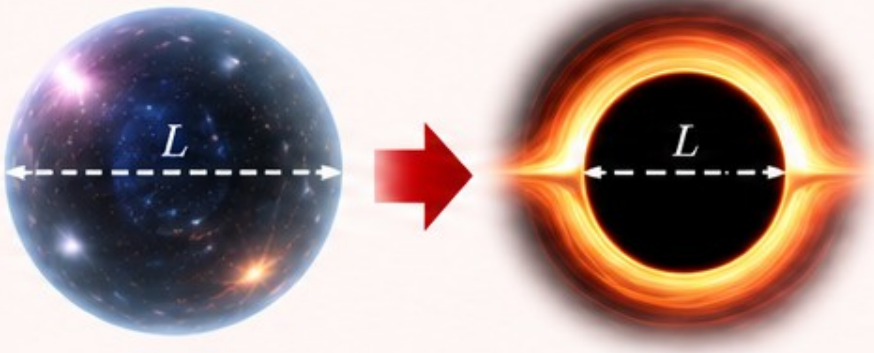
Total vacuum energy inside the region.

2 Avoiding gravitational collapse

- The energy inside the region must not be larger than the mass of a black hole with the same size L (Schwarzschild radius).

$$E_{DE} \leq M_{BH} \sim L M_p^2 ,$$

where $M_p^2 = \frac{1}{8\pi G}$ (reduced Planck mass).



If $E_{DE} > L M_p^2$,

the region would collapse into a black hole.

Holographic Dark Energy: Bound and resulting density

3 Holographic bound

From $E_{DE} \leq L M_p^2$ we obtain

$$\rho_{DE} L^4 \leq 3c^2 M_p^2 \,,$$

where $3c^2$ is a numerical constant of order unity.

- This is the **holographic bound**: the vacuum energy density cannot be arbitrarily large in a region of size L.
- It reflects the limit set by the formation of a black hole.

4 Saturation: holographic dark energy

Assuming that the bound is saturated, we obtain the holographic dark energy density

$$\rho_{DE} = \frac{3c^2 M_p^2}{L^2} \,.$$

- The infrared (IR) cutoff L sets the largest allowed size of the region without forming a black hole.
- It provides a dynamical dark energy density that can drive the accelerated expansion of the Universe.

5 Choice of the infrared cutoff

- The natural and consistent choice is the **future event horizon**
- It is the largest length scale in the Universe and accommodates the accelerated expansion.

$$L = R_h = a(t) \int_t^\infty \frac{dt'}{a(t')} \,,$$

Holographic Dark Energy

$$R_h \equiv a \int_t^{\infty} \frac{dt}{a} = a \int_a^{\infty} \frac{da}{Ha^2}.$$

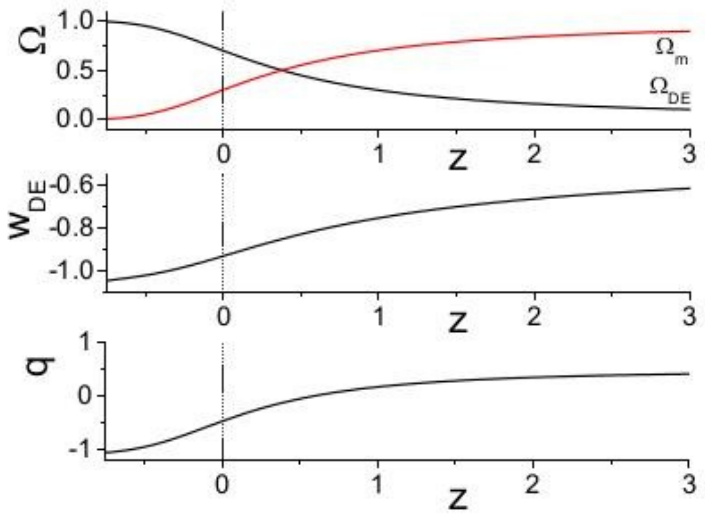
$$\frac{\Omega'_\Lambda}{\Omega_\Lambda^2} = (1-\Omega_\Lambda)(\frac{1}{\Omega_\Lambda} + \frac{2}{c\sqrt{\Omega_\Lambda}}).$$

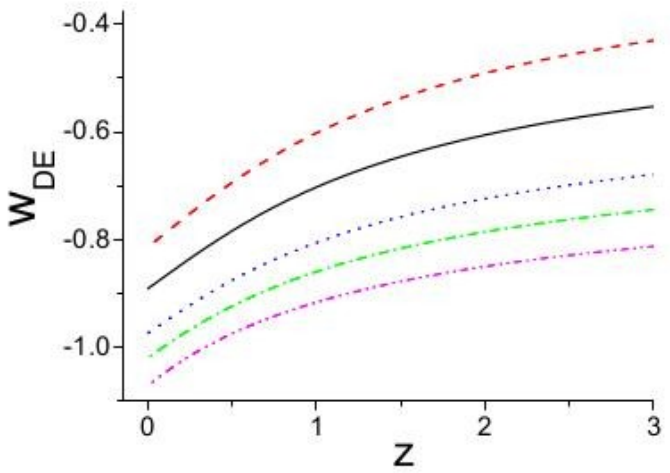
[M. Li, PLB 603]

Tsallis Holographic Dark Energy

$$\frac{\Omega'_{DE}}{\Omega_{DE}(1-\Omega_{DE})} = 2\delta - 1 + Q(1-\Omega_{DE})^{\frac{1-\delta}{2(2-\delta)}} \cdot(\Omega_{DE})^{\frac{1}{2(2-\delta)}} e^{\frac{3(1-\delta)}{2(2-\delta)}x}$$

$$Q \equiv 2(2-\delta)\left(\frac{B}{3M_p^2}\right)^{\frac{1}{2(\delta-2)}}\left(H_0\sqrt{\Omega_{m0}}\right)^{\frac{1-\delta}{\delta-2}}$$





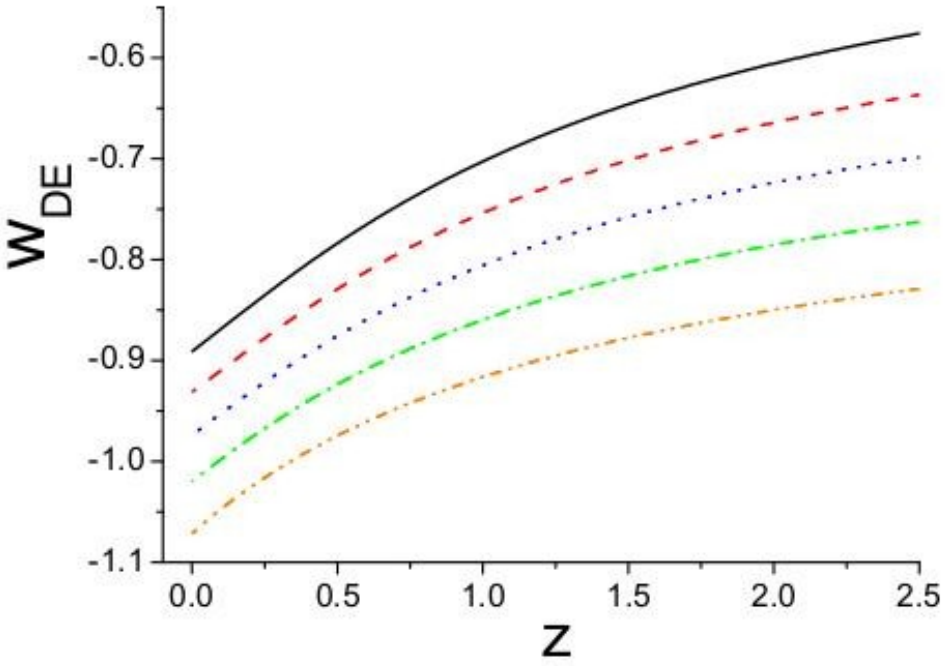
[E.N. Saridakis, K. Bamba, R. Myrzakulov, F. Anagnostopoulos, JCAP 2018]

Barrow Holographic Dark Energy

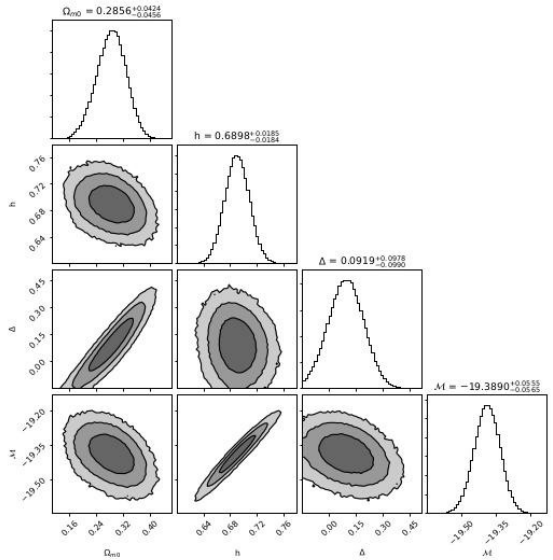
$$\rho_{DE} = CL^{\Delta-2},$$

$$\frac{\Omega'_{DE}}{\Omega_{DE}(1-\Omega_{DE})} = \Delta + 1 + Q(1-\Omega_{DE})^{\frac{\Delta}{2(\Delta-2)}} \cdot (\Omega_{DE})^{\frac{1}{2-\Delta}} e^{\frac{3\Delta}{2(\Delta-2)}x}$$

$$Q \equiv (2-\Delta)\left(\frac{C}{3M_p^2}\right)^{\frac{1}{\Delta-2}}\left(H_0\sqrt{\Omega_{m0}}\right)^{\frac{\Delta}{2-\Delta}}$$



[E.N. Saridakis, PRD 102]



[F. Anagnostopoulos , S. Basilakos, E.N. Saridakis, EPJC 80]

Kaniadakis Holographic Dark Energy

$$\rho_{DE} = 3c^2 M_p^2 R_h^{-2} + K^2 M_p^6 R_h^2.$$

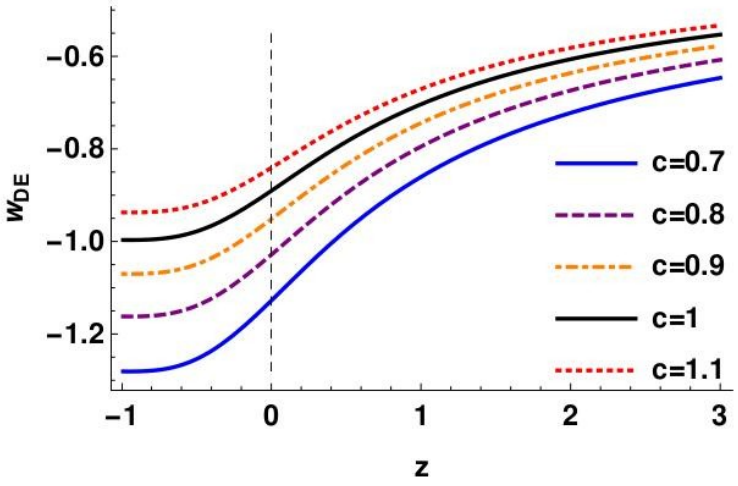
$$\Omega'_{DE} = \Omega_{DE}(1-\Omega_{DE})\left\{3 - \frac{2(\mathcal{A}-2K^2M_p^4\mathcal{B})}{\mathcal{A}}\left[1-\sqrt{3}\left(\frac{\Omega_{DE}}{\mathcal{A}\mathcal{B}}\right)^{\frac{1}{2}}\right]\right\}$$

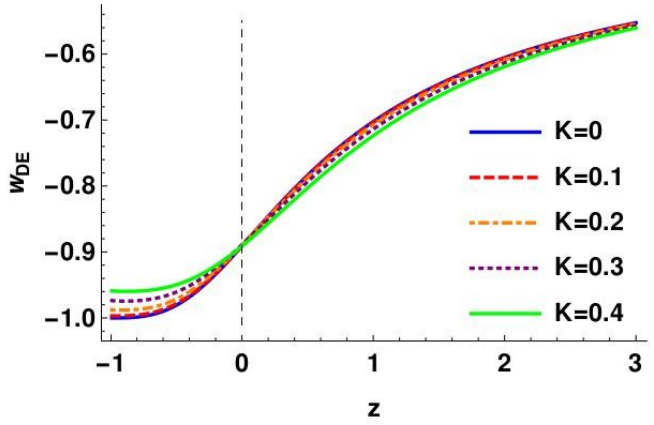
$$\mathcal{A} = \frac{3\mathrm{e}^{-3x}H_0^2\Omega_{m0}\Omega_{DE}}{1-\Omega_{DE}},$$

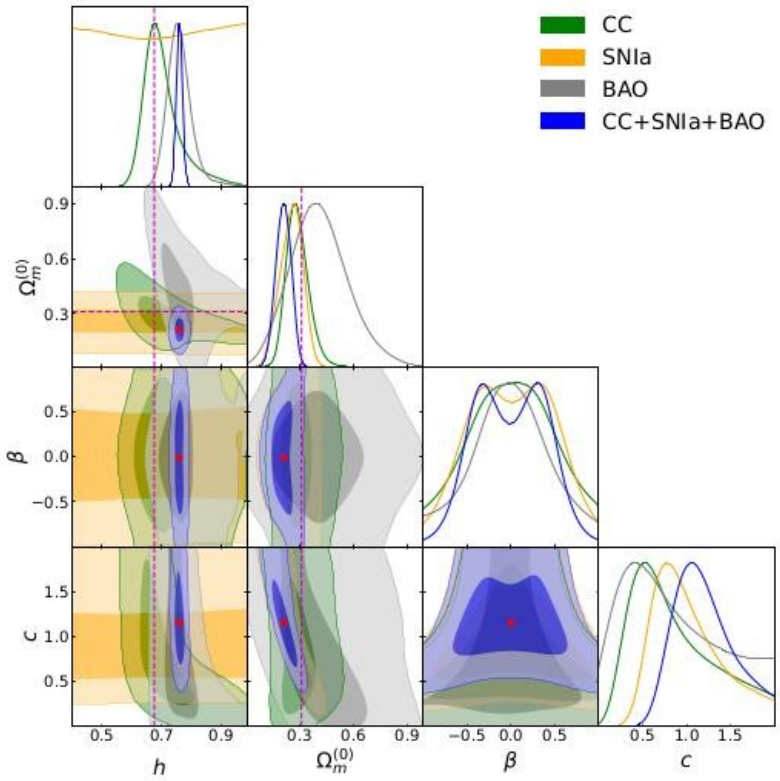
$$\mathcal{B} = \frac{\mathcal{A}-\sqrt{\mathcal{A}^2-12c^2K^2M_p^4}}{2K^2M_p^4}.$$

[N. Drepanou, A. Lymperis, E.N. Saridakis, EPJC 82]

Kaniadakis Holographic Dark Energy







[N. Drepanou, A. Lymperis, E.N. Saridakis, EPJC 82]

[A. Hernadez-Almada, G. Leon, J. Magana, M.A Garcia-Aseitia, V.Motta, E. N. Saridakis MNRAS 511]

Luciano-Saridakis holographic dark energy: The model

1 Generalized microscopic entropy → generalized area law

$$S_{\delta,\varepsilon} = \gamma_\delta A^\delta + \gamma_\varepsilon A^\varepsilon$$

Two independent entropy scalings, with standard Bekenstein-Hawking as a limiting case

2 Apply the holographic bound and saturate it

$$\rho_{\mathrm{DE}} L^4 \lesssim S_{\delta,\varepsilon}(L)$$

$$\rho_{\mathrm{DE}} = \gamma_\delta L^{2(\delta-2)} + \gamma_\varepsilon L^{2(\varepsilon-2)}$$

Hubble-horizon cutoff

$$L = H^{-1}$$

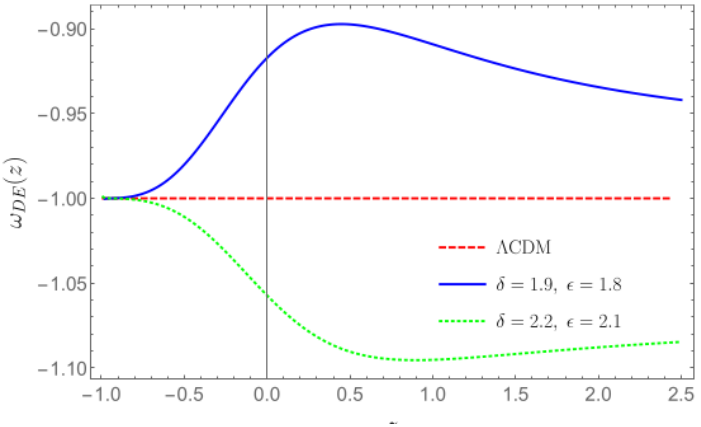
Future event horizon

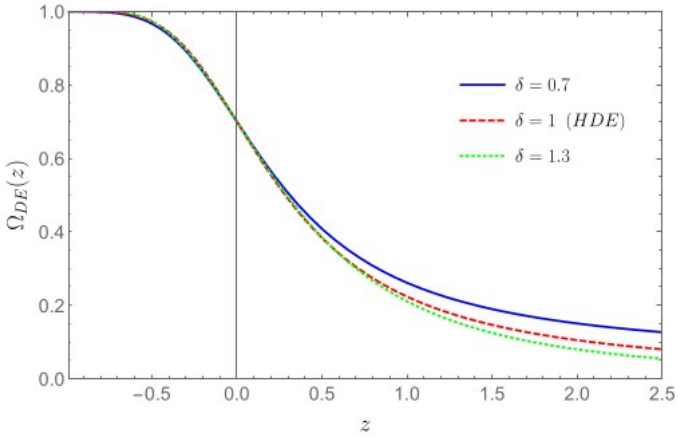
$$L = a(t)\int_t^\infty \frac{dt'}{a(t')}$$

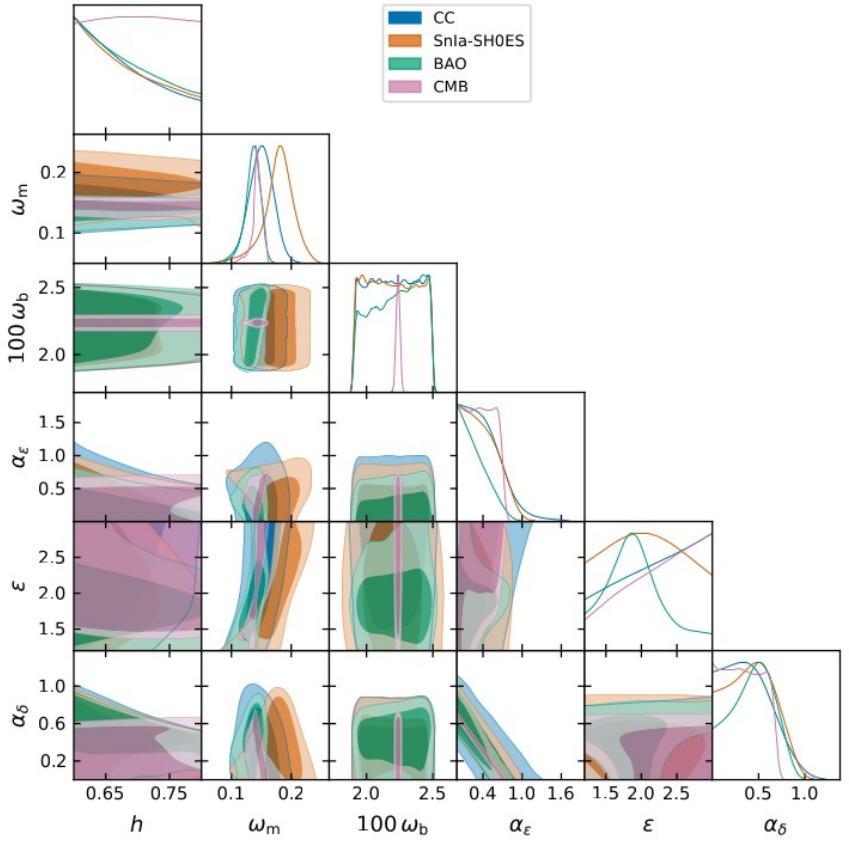
Limiting cases: standard HDE • cosmological constant / ΛCDM

G. G. Luciano & E. N. Saridakis, Phys. Lett. B 879 (2026) 140674

Luciano-Saridakis Holographic Dark Energy





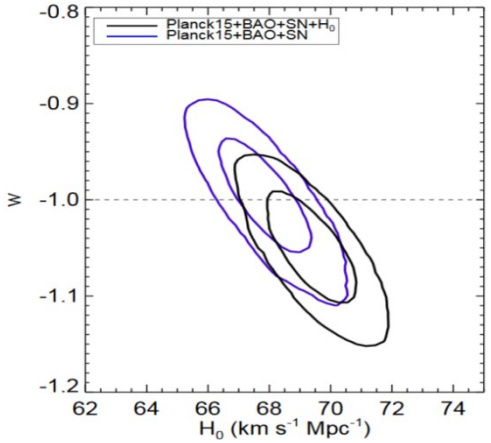


[M. Leizerovich, Susana J. Landau, G. G. Luciano, A. Papatriantafyllou , E. N. Saridakis PDU 54 (2026)]

Alleviating the tensions

H0 tension

- Tension (5σ!) between the data (direct measurements) and Planck/ΛCDM (indirect measurements). The data indicate a lack of "gravitational power".



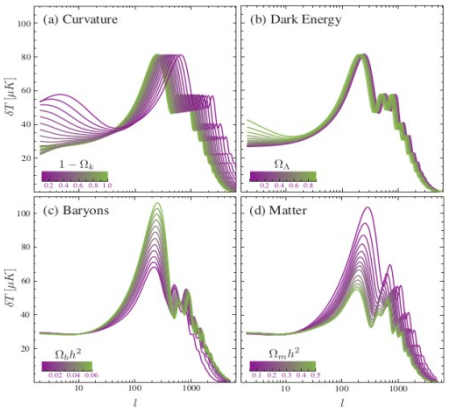
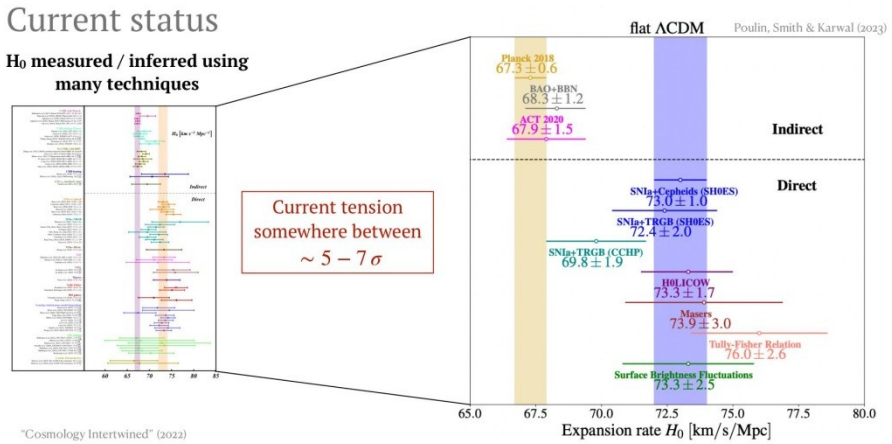
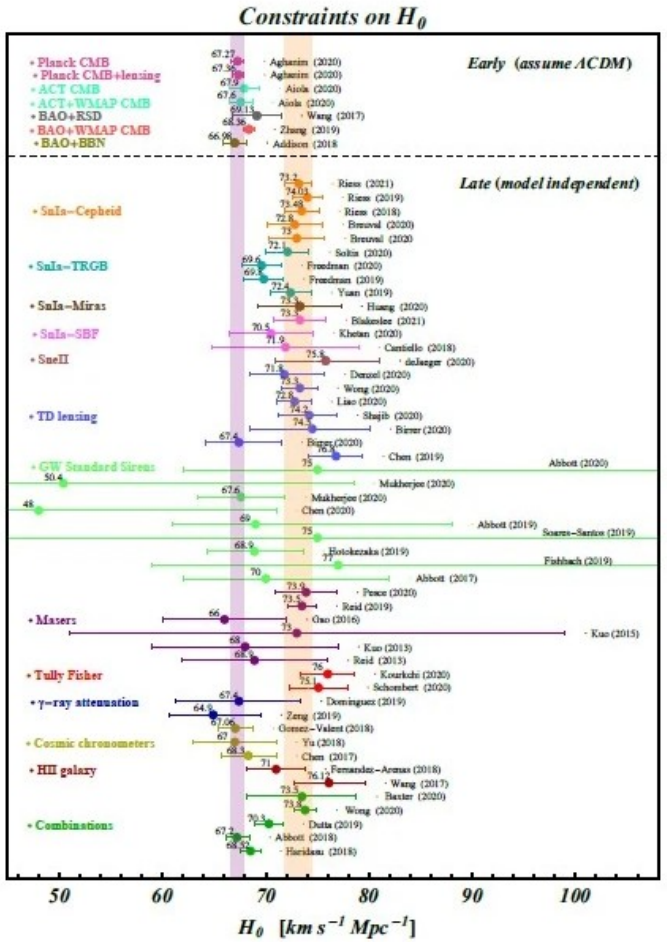


Figure 26. The CMB power spectrum as a function of cosmological parameters

[Riess et al, Astrophys.J 826]

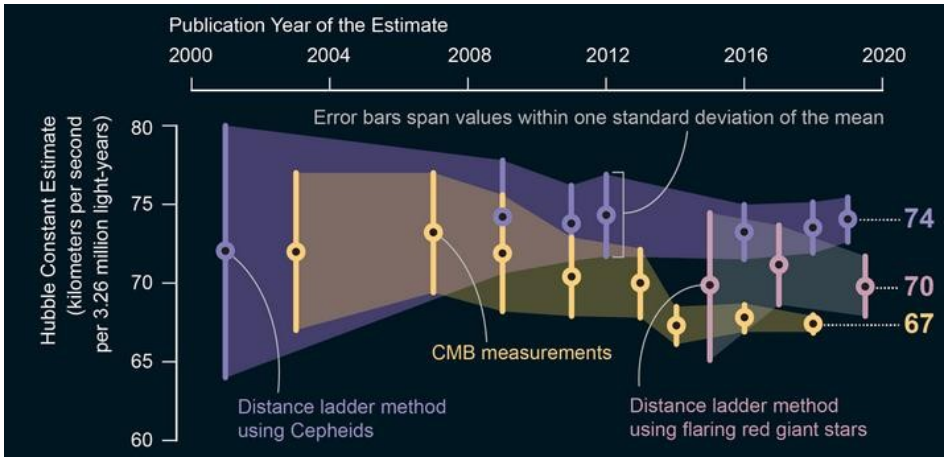




[Abdalla et al, JHEAp (2022)]

H0 tension

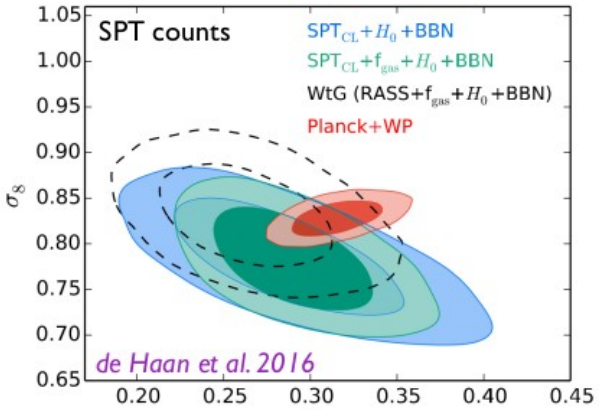
- Tension between the data (direct measurements) and Planck/ΛCDM (indirect measurements). This tension could be due to systematics.
- If not systematics then we may need changes in ΛCDM in early or late time behavior. 5σ seems to be very serious!

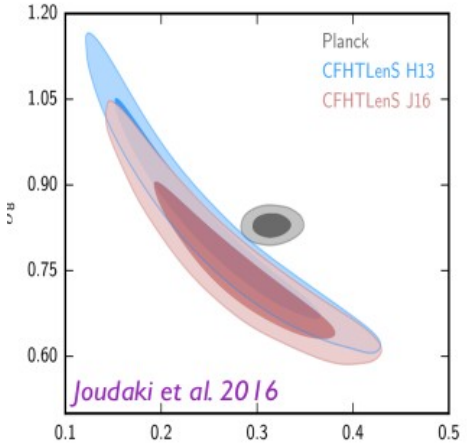


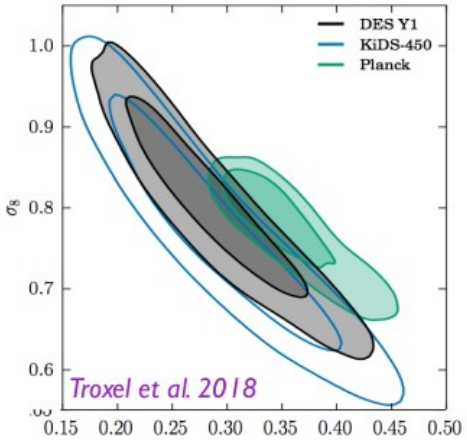
- Change early or late Universe physics. Higher number of effective relativistic species, dynamical dark energy, non-zero curvature, etc.
- The data indicate a lack of “gravitational power”. Modified Gravity.

S8 Tension

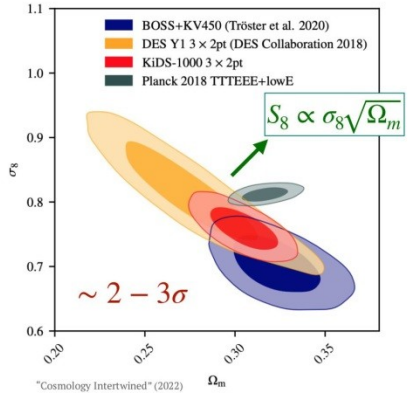
- Tension between direct data and Planck/ΛCDM estimation. The data indicate less matter clustering in structures at intermediate-small cosmological scales.

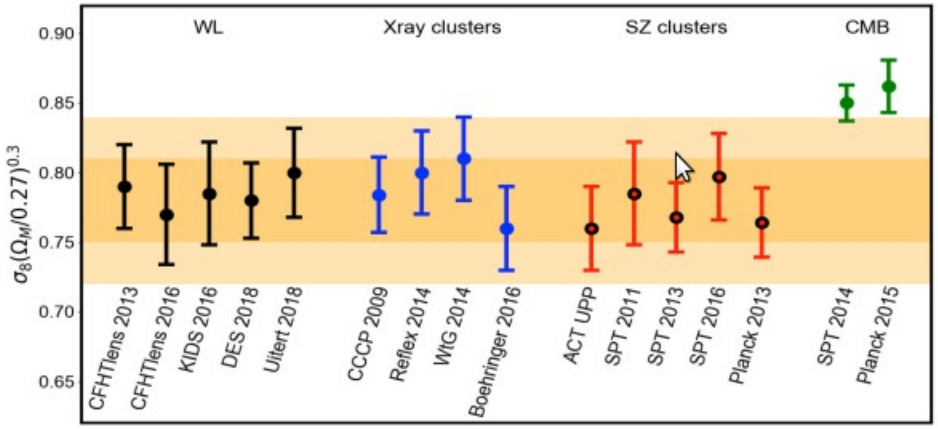






The S8 Tension





Tension2 – fσ8

- Tension between the data and Planck/ΛCDM.
- This tension could be due to systematics.

- If not systematics, the data less matter clustering in structures at intermediate-small cosmological scales (expressed as smaller Ωm at z<0.6, or smaller σ8, or wDE<-1).

- It could be reconciled by a mechanism that reduces the rate of clustering between recombination and today: Hot Dark Matter, Dark Matter that clusters differently at small scales, or Modified Gravity.

Possible Solutions of H0 and S8 tensions

tension $\lesssim 1\sigma$ *"Excellent models"*	tension $\lesssim 2\sigma$ *"Good models"*	tension $\lesssim 3\sigma$ *"Promising models"*
Dark energy in extended parameter spaces [289]	Early Dark Energy [235]	Early Dark Energy [229]
Dynamical Dark Energy [309]	Phantom Dark Energy [11]	Decaying Warm DM [474]
Metastable Dark Energy [314]	Dynamical Dark Energy [11, 281, 309]	Neutrino-DM Interaction [506]
PEDE [392, 394]	GEDE [397]	Interacting dark radiation [517]
Elaborated Vacuum Metamorphosis [400–402]	Vacuum Metamorphosis [402]	Self-Interacting Neutrinos [700, 701]
IDE [314, 636, 637, 639, 652, 657, 661–663]	IDE [314, 653, 656, 661, 663, 670]	IDE [656]
Self-interacting sterile neutrinos [711]	Critically Emergent Dark Energy [997]	Unified Cosmologies [747]
Generalized Chaplygin gas model [744]	$f(\mathcal{T})$ gravity [814]	Scalar-tensor gravity [856]
Galileon gravity [876, 882]	Über-gravity [59]	Modified recombination [986]
Power Law Inflation [966]	Reconstructed PPS [978]	Super ΛCDM [1007]
$f(\mathcal{T})$ [818]		Coupled Dark Energy [650]
Early Dark Energy [228, 235, 240, 250]	Early Dark Energy [212, 229, 236, 263]	DE in extended parameter spaces [289]
Exponential Acoustic Dark Energy [259]	Rock 'n' Roll [242]	Dynamical Dark Energy [281, 309]
Phantom Crossing [315]	New Early Dark Energy [247]	Holographic Dark Energy [350]
Late Dark Energy Transition [317]	Acoustic Dark Energy [257]	Swampland Conjectures [370]
Metastable Dark Energy [314]	Dynamical Dark Energy [309]	MEDE [399]
PEDE [394]	Running vacuum model [332]	Coupled DM - Dark radiation [534]
Vacuum Metamorphosis [402]	Bulk viscous models [340, 341]	Decaying Ultralight Scalar [538]
Elaborated Vacuum Metamorphosis [401, 402]	Holographic Dark Energy [350]	BD-ΛCDM [852]
Sterile Neutrinos [433]	Phantom Braneworld DE [378]	Metastable Dark Energy [314]
Decaying Dark Matter [481]	PEDE [391, 392]	Self-Interacting Neutrinos [700]
Neutrino-Majoron Interactions [509]	Elaborated Vacuum Metamorphosis [401]	Dark Neutrino Interactions [716]
IDE [637, 639, 657, 661]	IDE [659, 670]	IDE [634–636, 653, 656, 663, 669]
DM - Photon Coupling [685]	Interacting Dark Radiation [517]	Scalar-tensor gravity [855, 856]
$f(\mathcal{T})$ gravity theory [812]	Decaying Dark Matter [471, 474]	Galileon gravity [877, 881]
BD-ΛCDM [851]	DM - Photon Coupling [686]	Nonlocal gravity [886]
Über-Gravity [59]	Self-interacting sterile neutrinos [711]	Modified recombination [986]
Galileon Gravity [875]	$f(\mathcal{T})$ gravity theory [817]	Effective Electron Rest Mass [989]
Unimodular Gravity [890]	Über-Gravity [871]	Super ΛCDM [1007]
Time Varying Electron Mass [990]	VCDM [893]	Axi-Higgs [991]
ΛCDM [995]	Primordial magnetic fields [992]	Self-Interacting Dark Matter [479]
Ginzburg-Landau theory [996]	Early modified gravity [859]	Primordial Black Holes [545]
Lorentzian Quintessential Inflation [979]	Bianchi type I spacetime [999]	
Holographic Dark Energy [351]	$f(\mathcal{T})$ [818]	

Possible Solutions of H0 and S8 tensions

Early-Time Alternative Proposed Models
1. Axion Monodromy
2. Early Dark Energy
3. Extra Relativistic Degrees of Freedom
4. Modified Recombination History
5. New Early Dark Energy

Late-Time Alternative Proposed Models
1. Bulk Viscous Models
2. Chameleon Dark Energy
3. Clustering Dark Energy
4. Diffusion Models
5. Dynamical Dark Energy
6. Emergent Dark Energy
7. Graduated Dark Energy - AdS to dS Transition in the Late Universe
8. Holographic Dark Energy
9. Interacting Dark Energy
10. Quintessence Models and their Various Extensions
11. Running Vacuum Models
12. Time-Varying Gravitational Constant
13. Vacuum Metamorphosis

Modified Gravity Models
1. Effective Field Theory Approach to Dark Energy and Modified Gravity
2. $f(T)$ Gravity
3. Horndeski Theory
4. Quantum Conformal Anomaly Effective Theory and Dynamical Vacuum Energy
5. Ultra-Late Time Gravitational Transitions

Beyond the FLRW Framework
1. Cosmological Fitting and Averaging Problems
2. Data Analysis in an Universe with Structure: Accounting for Regional Inhomogeneity and Anisotropy
3. Local Void Scenario

Specific Solutions Assuming FLRW
1. Active and Sterile Neutrinos
2. Cannibal Dark Matter
3. Decaying Dark Matter
4. Dynamical Dark Matter
5. Extended Parameter Spaces Involving $A_{\rm lens}$
6. Cosmological Scenario with Features in the Primordial Power Spectrum
7. Interacting Dark Matter
8. Quantum Landscape Multiverse
9. Quantum Fisher Cosmology
10. Quartessence
11. Scaling Symmetry and a Mirror Sector
12. Self-Interacting Neutrinos
13. Self-Interacting Sterile Neutrinos
14. Soft Cosmology
15. Two-Body Decaying Cold Dark Matter into Dark Radiation and Warm Dark Matter

10 commandments for Hubble hunters

1. I am $H_0 \approx 74$ thy Goal
2. Thou shalt not fail to fit key data (BAO, SNela, polarization)...
3. ...or include a local H_0 prior in vain
4. Remember to not just blow up the uncertainty on H_0...
5. ...honour its central value, and keep an eye on your $\Delta\chi^2$/Bayesian evidence
6. Thou shalt not murder σ_8/S_8...
7. ...but aim to solve this and other tensions/puzzles at the same time
8. Thy solution shall come from a compelling particle/gravity model...
9. ...which makes verifiable predictions...
10. ...which later better be verified!



Credits: Gustave Doré

Efficient model independent requirements to solve the tensions

- In general, to avoid the H_0 tension one needs a positive correction to the first Friedmann equation at late times that could yield an increase in H_0 compared to the ΛCDM scenario.

Efficient model independent requirements to solve the tensions

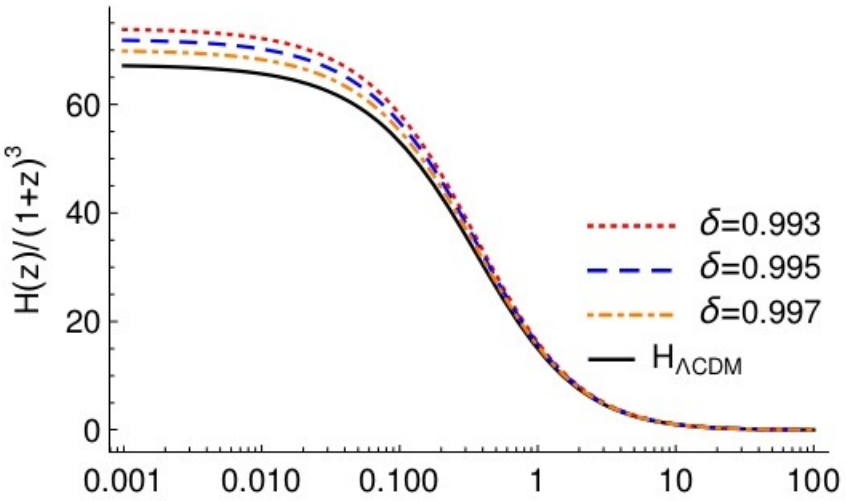
- For the σ_8 tension, we recall that in any cosmological model, at sub-Hubble scales and through matter epoch, the equation that governs the evolution of matter perturbations in the linear regime is

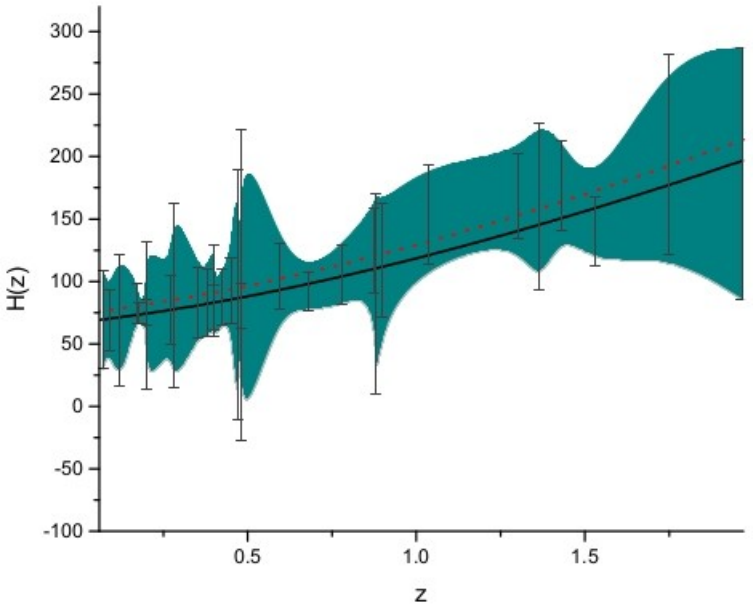
$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\rm eff}\rho_m\delta \ , \tag{1}$$

  where $G_{\rm eff}$ is the effective gravitational coupling given by a generalized Poisson equation.

- Solving for $\delta(a)$ provides the observable quantity $f\sigma_8(a)$, following the definitions $f(a) \equiv d\ln\delta(a)/d\ln a$ and $\sigma(a) = \sigma_8\delta(1)/\delta(a=1)$. Hence, alleviation of the σ_8 tension may be obtained if $G_{\rm eff}$ becomes smaller than G_N during the growth of matter perturbations and/or if the "friction" term in (1) increases.

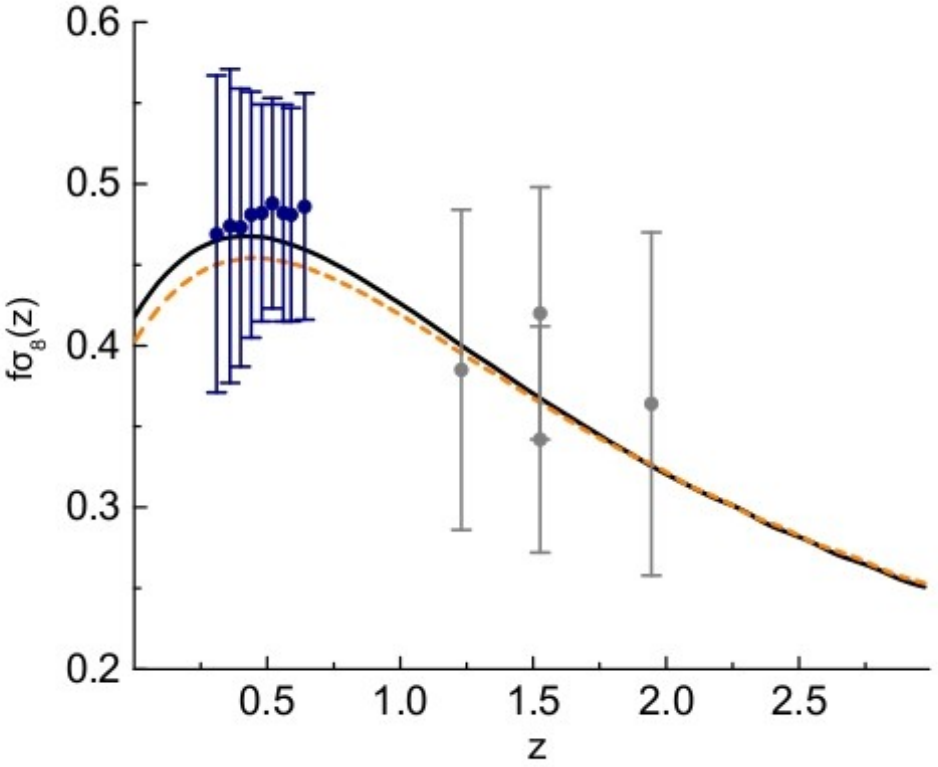
Tsallis Cosmology

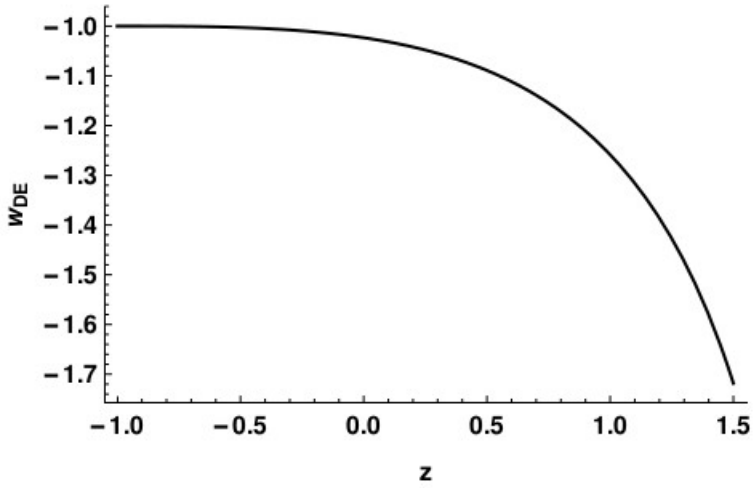




[S. Basilakos, A. Lymperis, M. Petronikolou, E.N. Saridakis, EPJC 84]

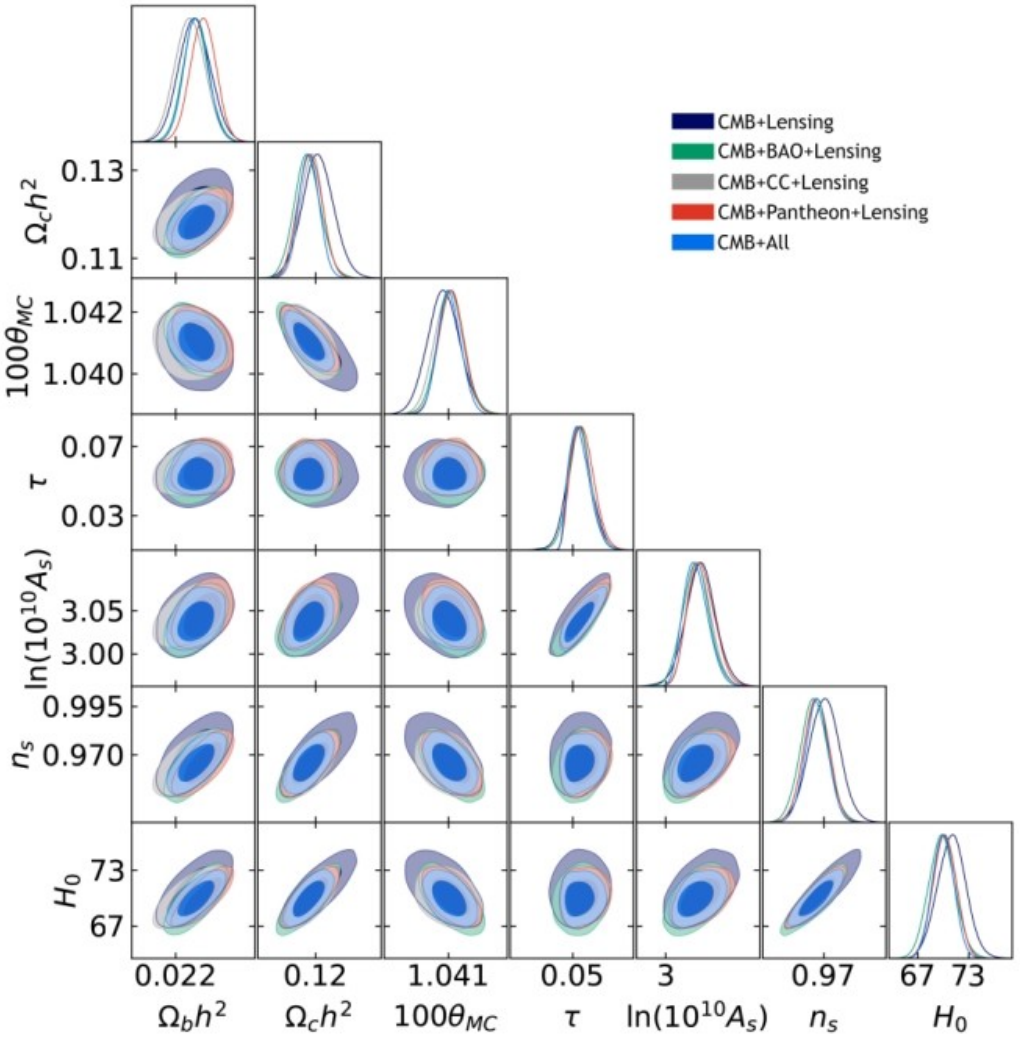
Tsallis Cosmology





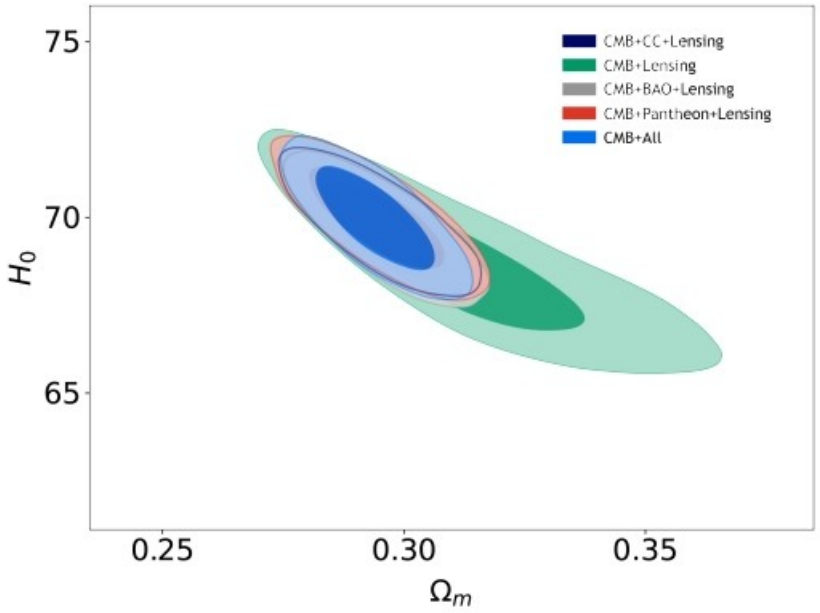
[S. Basilakos, A. Lymperis, M. Petronikolou, E.N. Saridakis, EPJC 84]

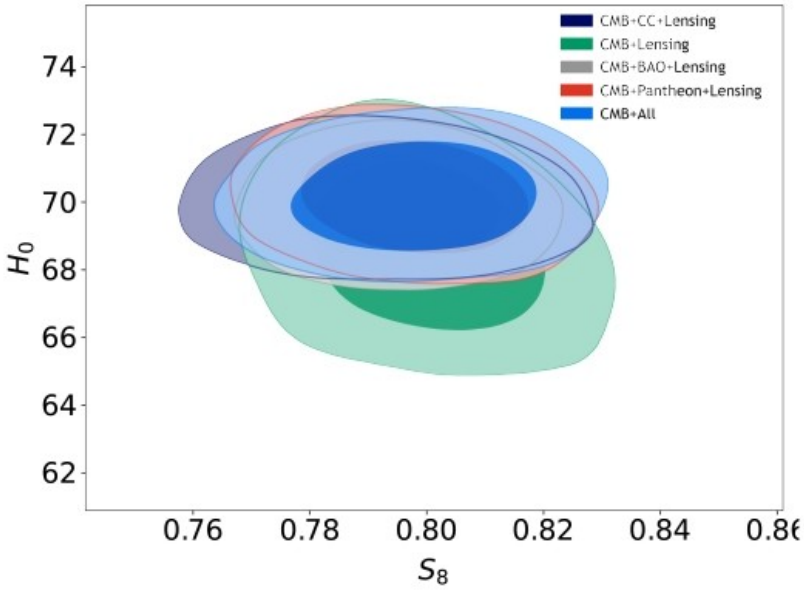
Barrow Holographic Dark energy



[M. Yarahmadi, A.Salehi MNRAS 534 (2024)] [M. Yarahmadi Phys. Dark. Univ. 47 (2025)]

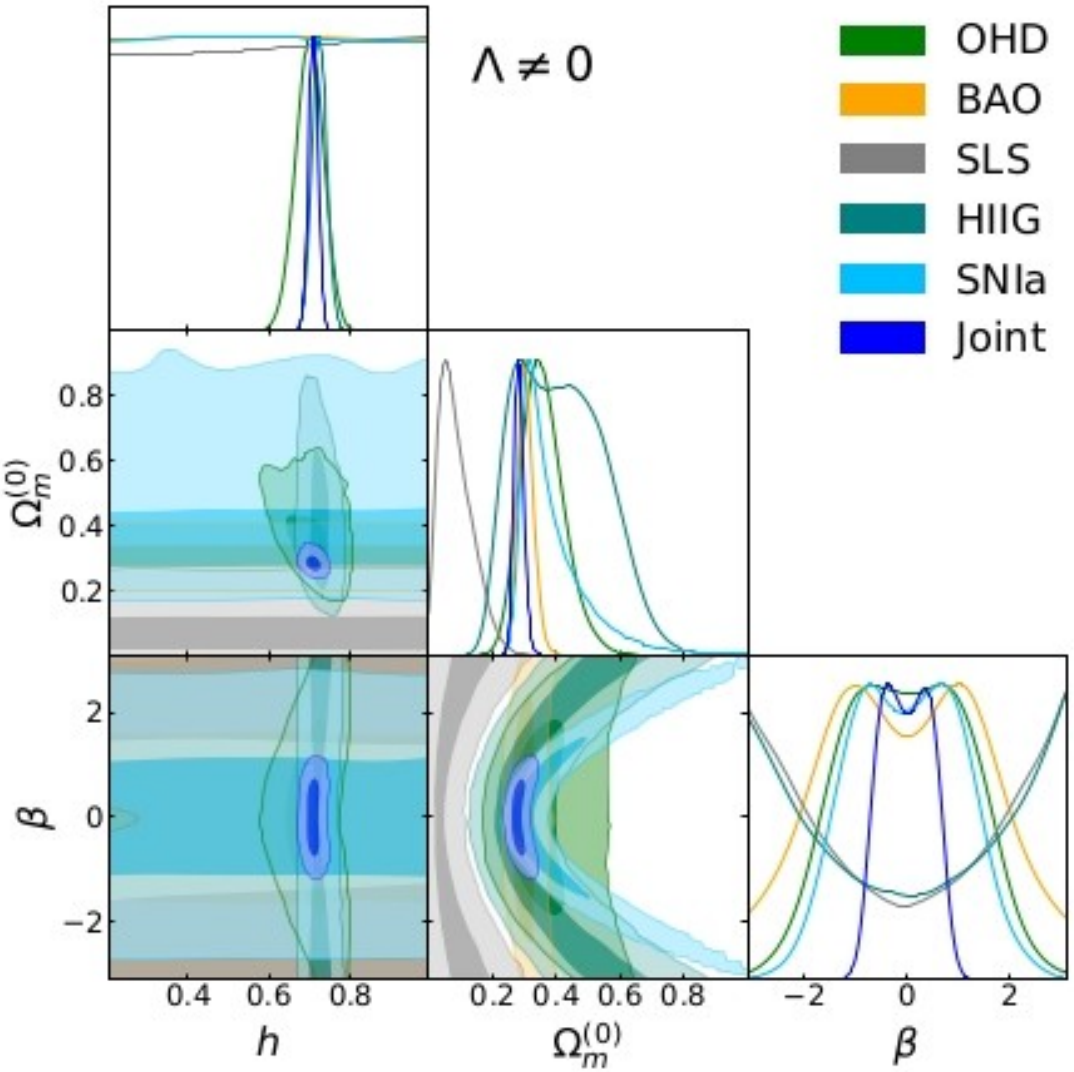
Barrow Holographic Dark energy





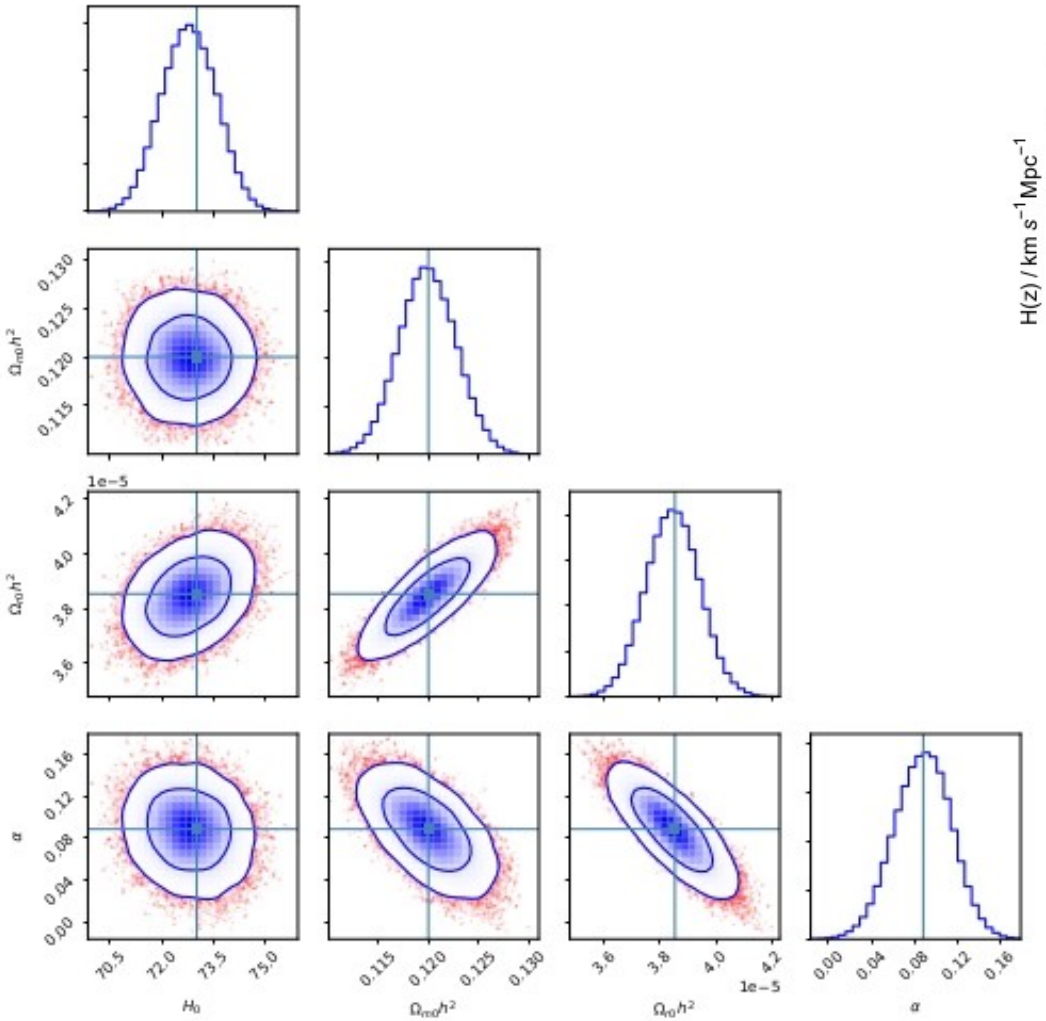
Dataset	Value	Tension with R22	Tension with Planck 2018
CMB + Lensing	$H_0 = 69.83 \pm 3.12$ km/s/Mpc	0.96σ	0.75σ
CMB + BAO + Lensing	$H_0 = 70.38 \pm 2.32$ km/s/Mpc	1.04σ	1.15σ
CMB + Lensing + CC	$H_0 = 70.27 \pm 2.38$ km/s/Mpc	0.99σ	1.14σ
CMB + Lensing + Pantheon	$H_0 = 70.58 \pm 2.54$ km/s/Mpc	0.89σ	1.18σ
CMB + All	$H_0 = 70.47 \pm 2.01$ km/s/Mpc	1.13σ	1.43σ

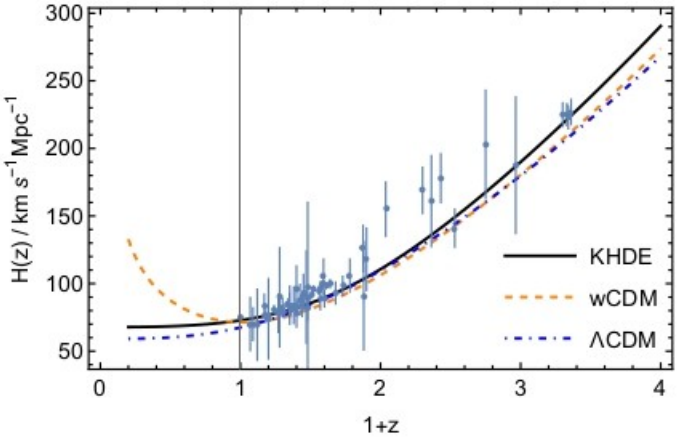
Kaniadakis Cosmology



[A. Hernadez-Almada, G. Leon, J. Magana, M.A Garcia-Aseitia, V.Motta, E. N. Saridakis MNRAS 512]

Kaniadakis Holographic Dark energy





[W. Fang, G. Chen,C-J Feng, W. Du, Chenganq Shu 2406.09209]

Conclusions

- Spacetime thermodynamics conjecture
- Holographic Dark Energy

- Modified Cosmology can arise from Modified Entropy

- Can provide alleviation to the cosmological tensions!

Conclusions

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Thank you!