On the probability of inflation in quantum bounce cosmology[1]

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[1] based on PRD102, 103521 (2020), PRD106, 103535 (2022), PRD108, 023517 (2023), PRD110, 043507 (2024)

1. Bounce and Inflation
2. Bounce in LQC
3. Predicting N_{infl}
4. Analytical Results
5. Perturbations
6. Constraining γ
7. Summary

Bouncing or Inflation ?

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Alternative solution $\Rightarrow$ Bounce: Early phase of contraction ($\dot{a} < 0$)

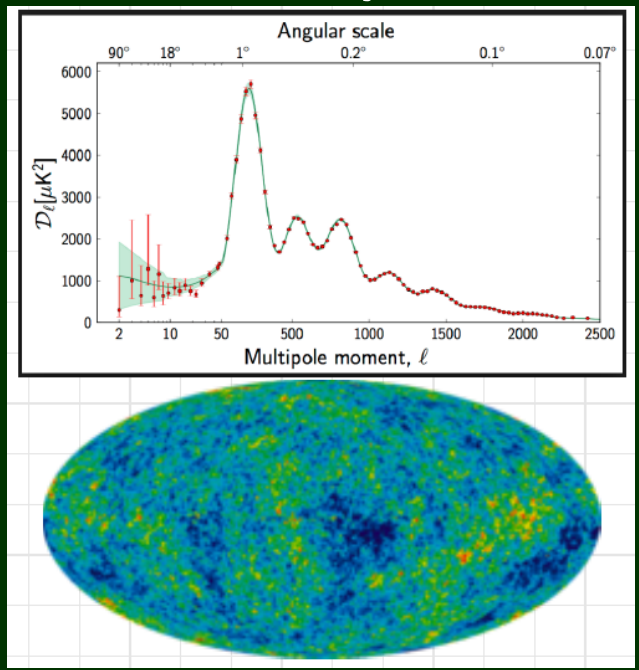
$$\frac{dR_H}{da} = -\frac{\ddot{a}}{\dot{a}^3}, \qquad \frac{d|\Omega(t)-1|}{dt} = -2|k|\frac{\ddot{a}}{\dot{a}^3}$$

$$\text{both } < 0 \text{ for : } \begin{cases} \ddot{a} > 0 \text{ and } \dot{a} > 0 & \text{(inflation), or} \\ \ddot{a} < 0 \text{ and } \dot{a} < 0 & \text{(contraction)} \end{cases}$$

Inflation gives a sound description for the origin of cosmological perturbations, the properties of their power spectra (almost scale invariant cosmological perturbations at large scales and small non-gaussianities)

All observations carried so far support Inflation (or do not contradict it)

CMB measured by Planck 2018



Problems that inflation is assumed to face:

- Initial Singularity Problem: General relativity (GR) inevitably leads inflation to an initial singularity[2], with which in principle it is not clear how to impose the initial conditions.

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- Trans-Planckian problem: the wavelengths corresponding to present observations can be smaller than the Planck length at the beginning of the inflation $\Rightarrow$ quantum gravity becomes important

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- Planck-scale problem: Inflation is very sensitive to Planck-scale physics, and effects of quantum gravity in the early universe are important and need to be taken into account[3]

[3] D. Baumann, TASI Lectures on Inflation, arXiv:0907.5424; C.P. Burgess, M. Cicoli, F. Quevedo, JCAP 1311 (2013) 003; D. Baumann and L. McAllister, Inflation and String Theory (Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2015); E. Silverstein, TASI lectures on cosmological observables and string theory, arXiv:1606.03640

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- Other conceptual problems (eternal inflation, multiverse, etc)

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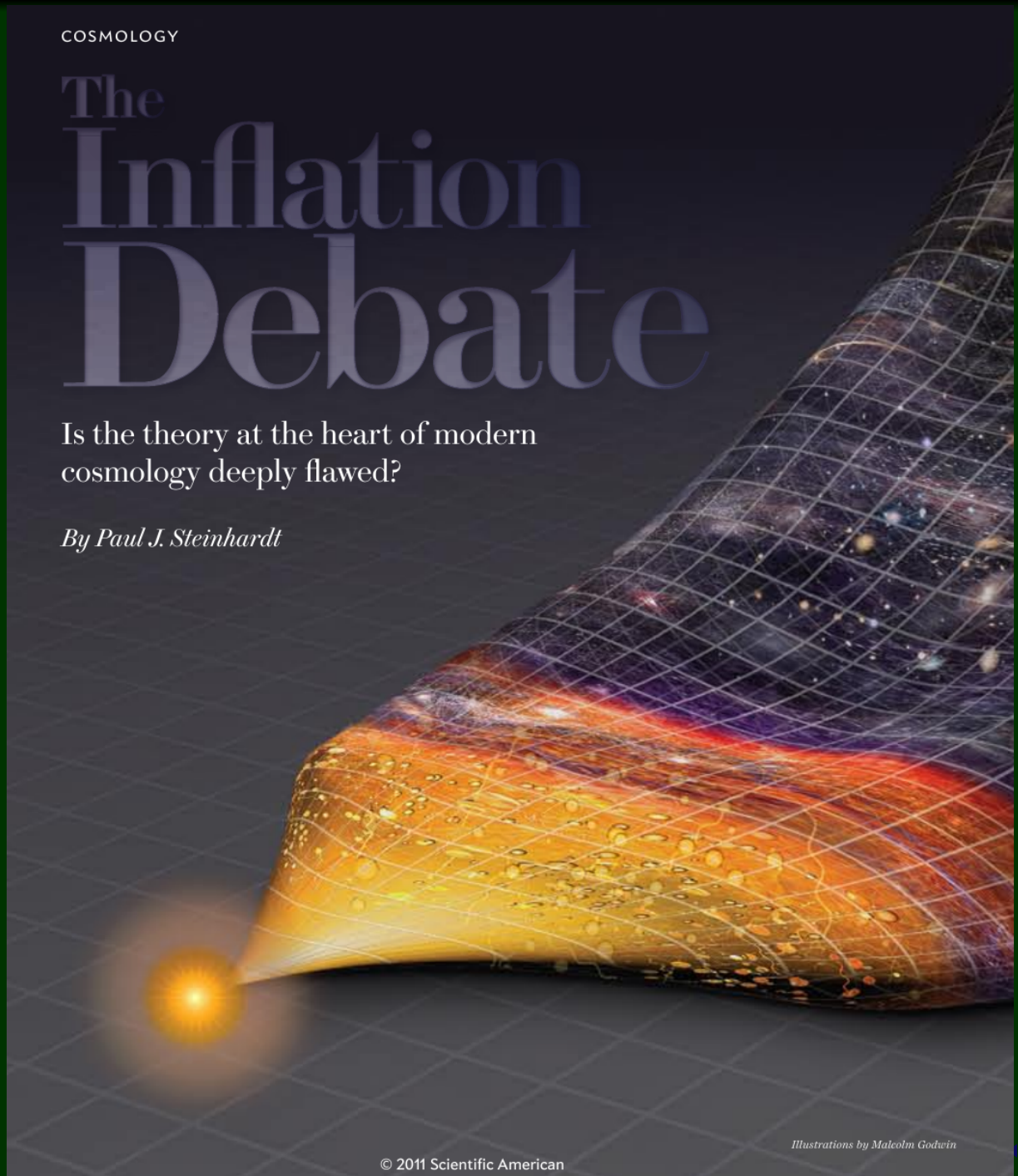
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- Bouncing models are assumed to be free of the aforementioned problems of inflation
- Bounces through quantum effects (LQC, de Broglie-Bohm, string theory, etc)
- Bounce models have shown to give predictions concerning structure formation and to produce equivalent perturbations results like those from inflation models[4]

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COSMOLOGY
The Inflation Debate
Is the theory at the heart of modern cosmology deeply flawed?
By Paul J. Steinhardt
Illustrations by Malcolm Godwin
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Yet, nothing prevents having both bouncing and inflation occurring concomitantly (best of the two worlds !)

Early bounce phase followed by inflation $\Rightarrow$ approach considered here !

Inflation in Loop Quantum Cosmology

- LQC is the symmetry reduced quantization of cosmology by mimicking the constructions used in LQG.
- One defines operators representing geometric observables, e.g., the area operator:

$$\hat{A}|0\rangle = \Delta|0\rangle,$$

  $\Delta \equiv 4\sqrt{3}\pi\gamma$ is the smallest nonzero eigenvalue, γ is the Barbero-Immirzi parameter.
- Δ represents the fundamental area gap and sets an energy scale,

$$\rho_c \equiv 18\pi \left(\frac{m_{\rm Pl}}{\Delta}\right)^3 m_{\rm Pl}.$$

- In LQC, the matter density is bounded from above[5],

$$\rho \leq \rho_c,$$

and the Friedmann equation is modified to

$$H^2 = \frac{8\pi}{3m_{\rm Pl}^2}\rho\left(1-\frac{\rho}{\rho_{\rm c}}\right).$$

Considering for instance $\gamma \simeq 0.2375$, as suggested from black hole thermodynamics: $\rho_{\rm c} \simeq 0.41 m_{\rm Pl}^4$.

- In LQC, the modifications are purely due to quantum geometry, and matter satisfies the same field equations as usual. For a scalar field,

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0.$$

[5] A. Ashtekar, T. Pawlowski and P. Singh, PRL96 (2006) 141301

$\Rightarrow$ The big bang singularity is replaced by a quantum bounce, which naturally happens at $\rho = \rho_{\rm c}$

$\Rightarrow$ In all cases studied so far, the singularity is resolved ($f(R)$ universe, Brans-Dicke models, the closed FLRW model, FLRW models with Λ with any signs, the Bianchi models, the Gowdy model, which incorporates the simplest types of inhomogeneities in full GR, ...)

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Does a slow-roll phase compatible with observations arise naturally from the quantum bounce ? or is an enormous fine tuning needed ?

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$\phi =$ scalar field – the inflaton

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How easily, or better, how probable is inflation ?

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But it can be well defined when the singularity is resolved by a (quantum) bounce Ashtekar & Sloan, GRG43, 3619 (2011).

It was found that the probability for the desired – i.e., in agreement with CMB measurements – slow roll inflation NOT TO OCCUR in an LQC solution is less than about one part in a million (Ashtekar and Sloan, GRG43 (2011) 3619)

$$P(\text{not realized}) \lesssim 2.74 \times 10^{-6}, \text{ for } N_e \simeq 60, \quad V = \frac{m^2}{2}\phi^2$$

$\Rightarrow$ the same conclusion applies to other common inflaton potentials, as also in WI (L. L. Graef & ROR, PRD98 (2018) 023531)

$$\Rightarrow \boxed{\text{Slow} - \text{roll inflation is an attractor in LQC !}}$$

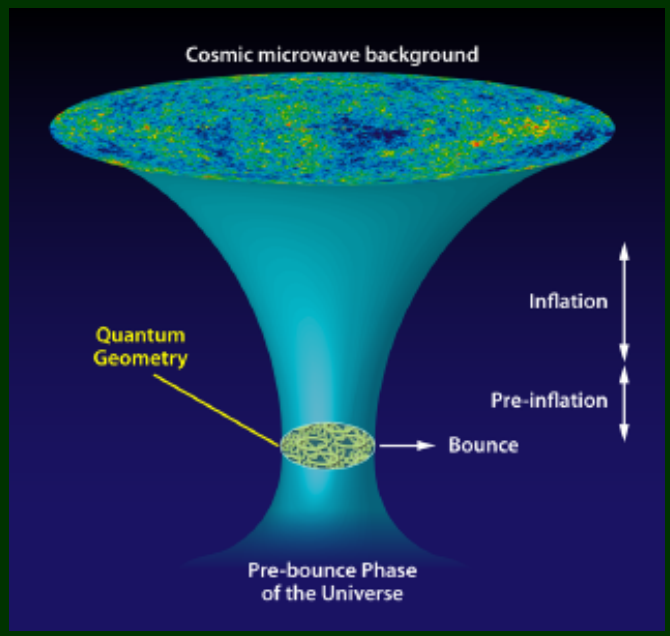
In the framework of LQC, the background evolution can be divided into two classes:

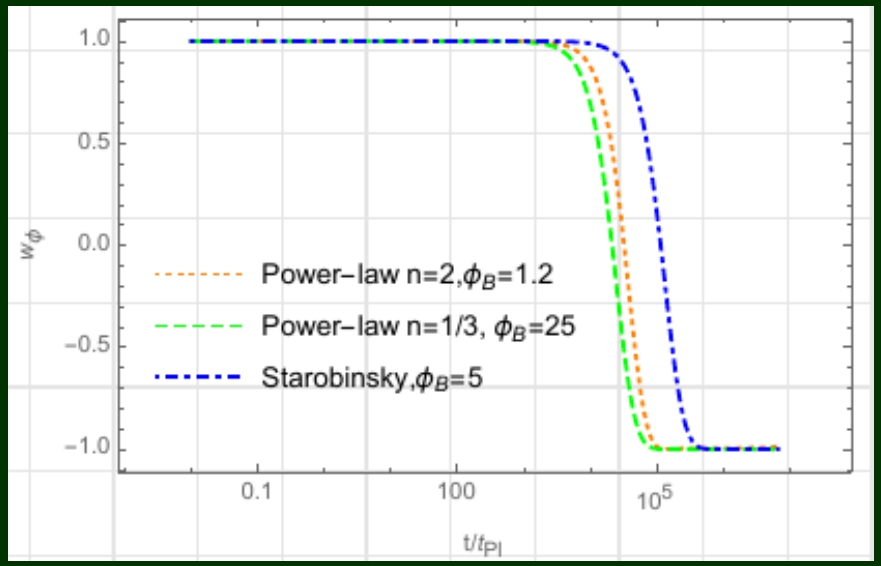
$$p_\phi = \frac{1}{2}\dot{\phi}^2(t_B) - V(\phi(t_B)) = \begin{cases} > 0, & \text{KE dominated} \\ < 0, & \text{PE dominated} \end{cases}$$

A potential dominated bounce is either not able to produce the desired slow-roll inflation or leads to a large amount of e-folds of expansion (Ashtekar and Barrau, CQG32 (2015) 234001). In the latter, any new physics due to the bounce is washed out (we will see this later).

In the kinetic energy initially (well) dominated case, $\frac{1}{2}\dot{\phi}(t_B)^2 \gg V(\phi(t_B))$, the evolution can always be divided into three phases (which are rather universal (Zhu, Wang, Cleaver, Kirsten, Sheng, PLB773 (2017) 196; PRD96 (2017) 083520))

$$w(\phi) \equiv \frac{p_\phi}{\rho_\phi} = \begin{cases} +1, & \text{bouncing} \\ \in (-1,1), & \text{transition} \\ -1, & \text{inflation} \end{cases}$$





$\Rightarrow$ This universal behavior appears both in CI and in WI (L. L. Graef & ROR, PRD98, 023531 (2018))

Predicting N_{infl}

Initial conditions set at the bounce predict a high probability of inflation (bounce where the kinetic energy dominates — stiff matter dominated).

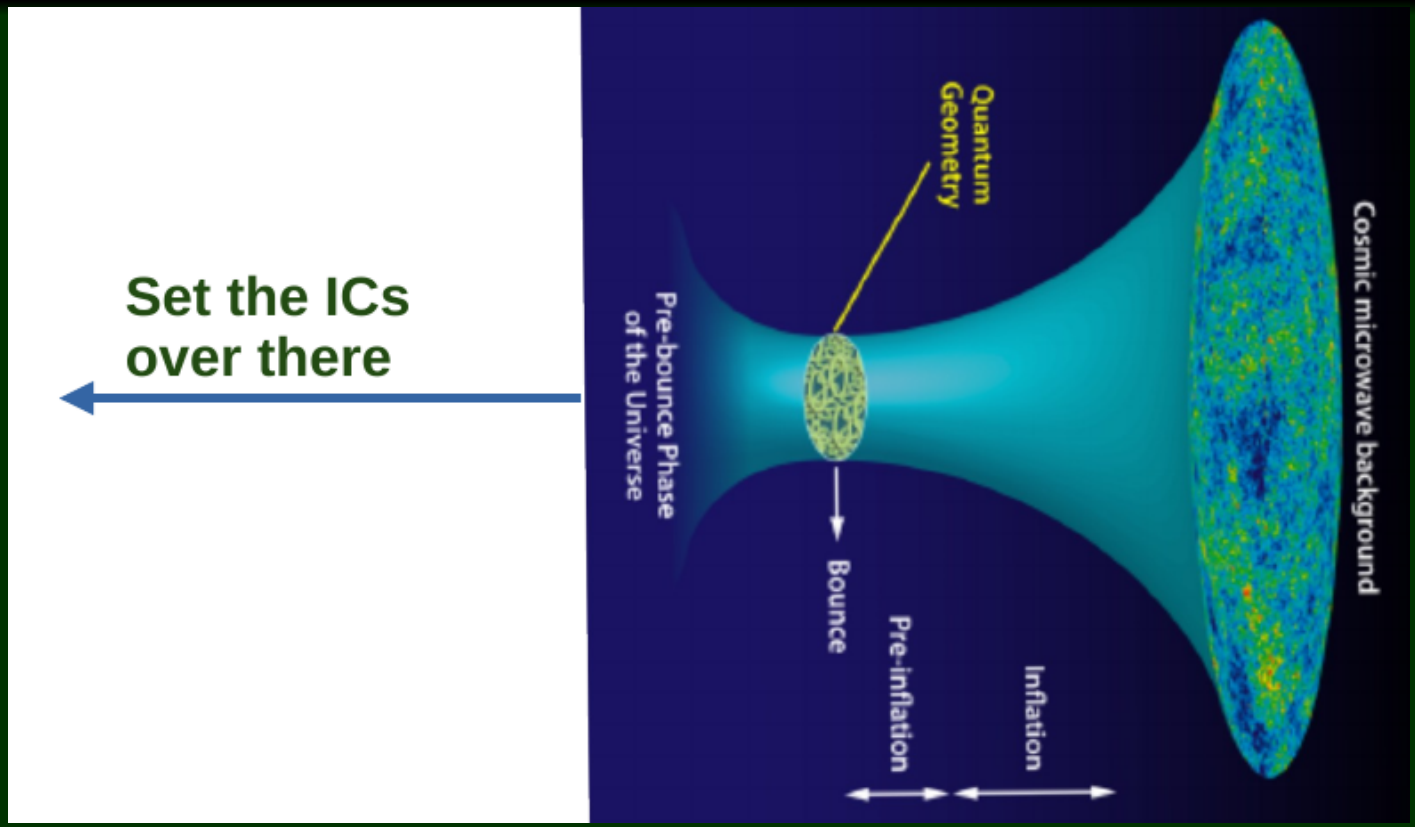
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Predicting $N_{\rm infl}$

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But can we make predictions about the total number of efolds given some $V(\phi)$?

YES, if instead of setting ICs at the bounce we choose them deep in the contracting phase (Linsefors and Barrau, PRD 87, 123509 (2013))

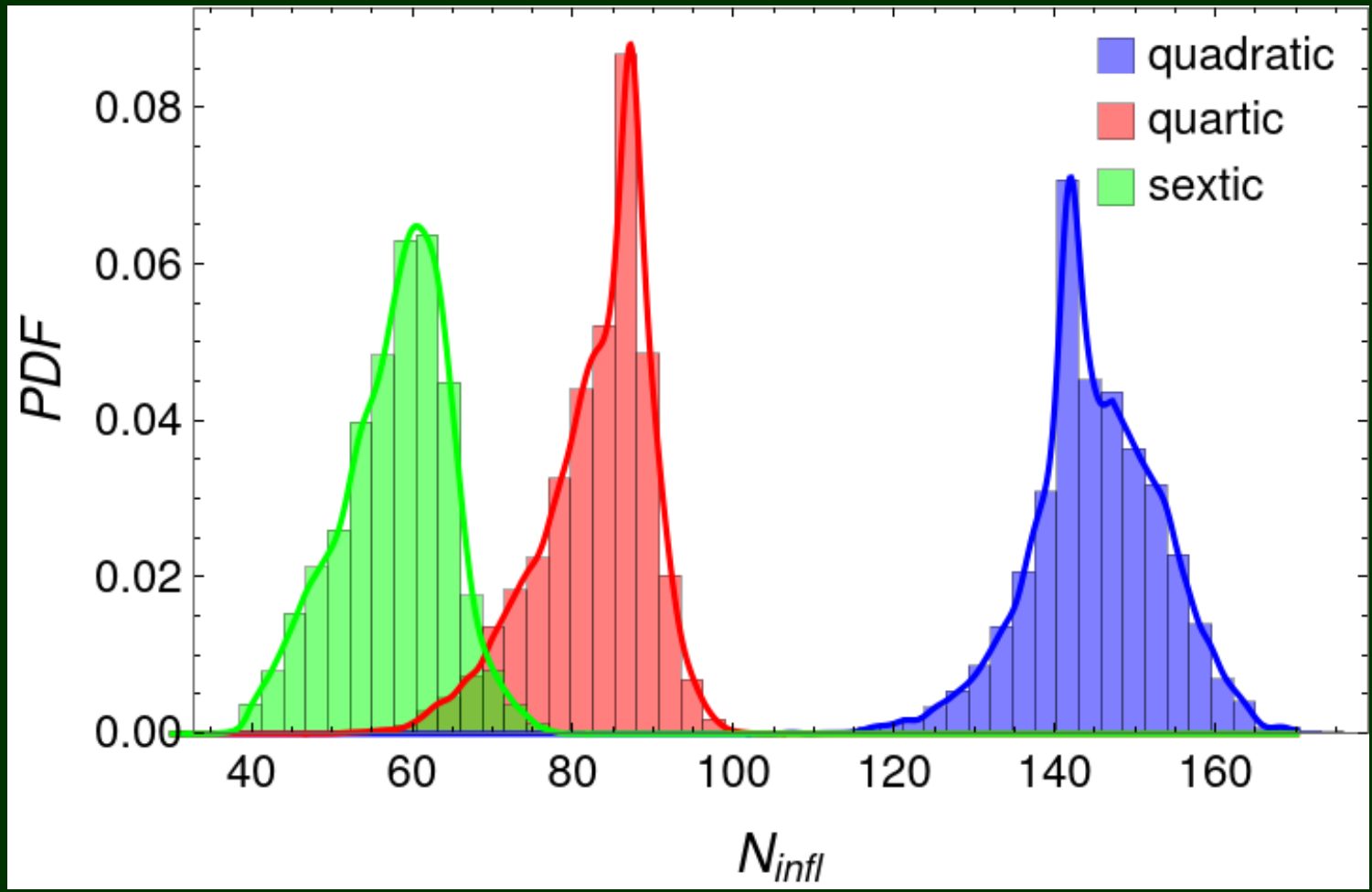


ICs taken far back in the contracting phase, where quantum effects are negligible, $\rho \ll \rho_{\rm cr} \Rightarrow$ classical (GR) evolution

ϕ and $\dot{\phi}$ randomly chosen (flat distribution) with a fixed $\rho(t_i) \ll \rho_{\rm cr}$, $t_i \ll t_B$. System evolved from the contracting phase up to the end of inflation.

Monomial (chaotic) potential:

$$V = \frac{V_0}{2n}\left(\frac{\phi}{M_{\rm Pl}}\right)^{2n}$$



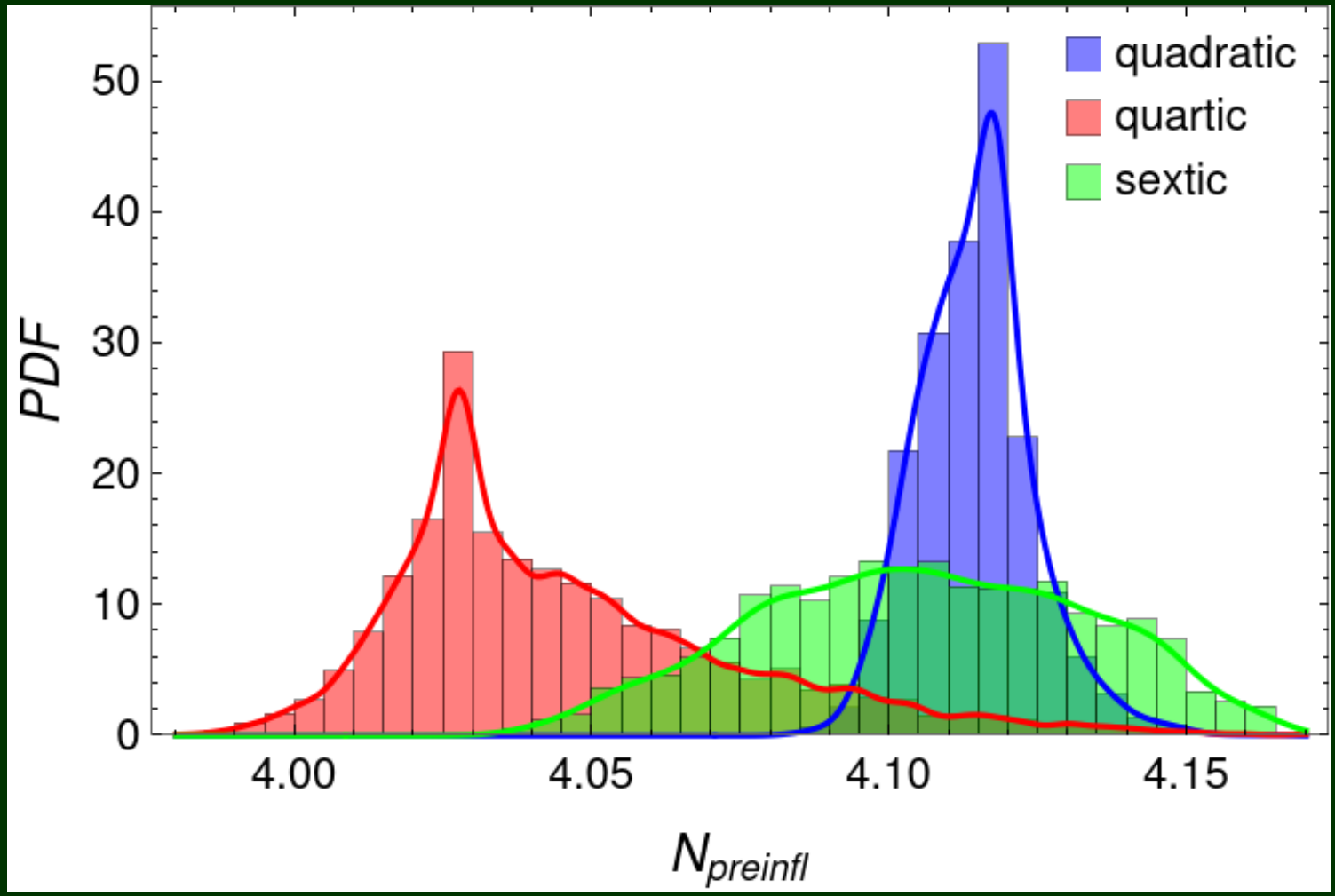
L. N. Barboza, L. L. Graef, and R.O.R., PRD 102, 103521 (2020)

The number of e-folds during the preinflationary phase $N_{\rm preinfl}$, from the bounce to the start of slow roll: around the start of slow roll, $t = t_{\rm sr}$, $\rho_{\rm kin}(t_{\rm sr}) \equiv \dot{\phi}^2(t_{\rm sr})/2 \sim V(\phi((t_{\rm sr}))$. At the bounce, $\rho_{\rm kin}(t_{\rm bounce}) \simeq \rho_{\rm cr}$, then, we have that

$$\rho_{\rm kinetic}(t_{\rm sr}) \simeq \frac{\rho_{\rm cr}}{a^6(t_{\rm sr})} \sim \rho_V(t_{\rm sr}).$$

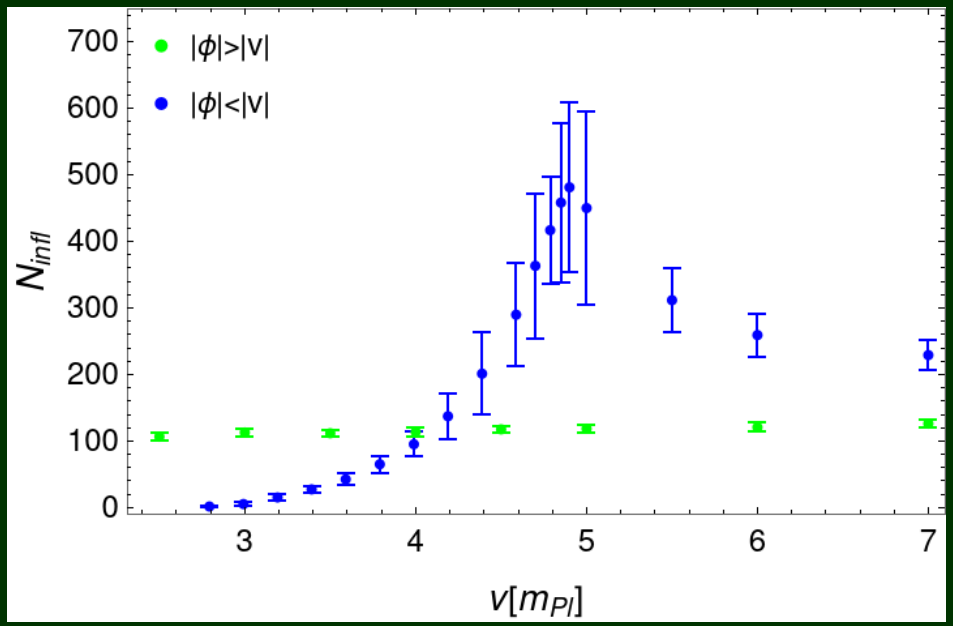
$$N_{\rm preinfl} = \ln\left[\frac{a(t_{\rm sr})}{a(t_{\rm bounce})}\right] \sim \frac{1}{6}\ln\left(\frac{\rho_{\rm cr}}{\rm V_*}\right) \sim 4.3.$$

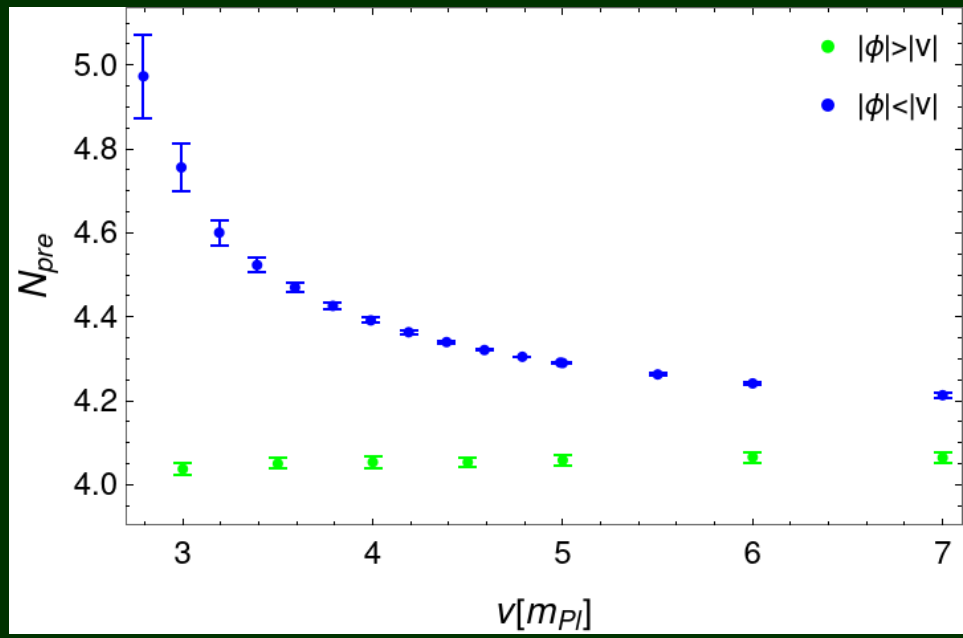
using $\rho_V(t_{\rm sr}) \sim V_* < (1.6 \times 10^{16}{\rm GeV})^4$ (Planck 2018)


quadratic
quartic
sextic
PDF
50
40
30
20
10
0
4.00
4.05
4.10
4.15
$N_{preinfl}$

Double well (Higgs-like) potential:

$$V = V_0 \left[1 - \left(\frac{\phi}{v}\right)^2\right]^2,$$





Analytical Results[6]

Dynamical regimes:

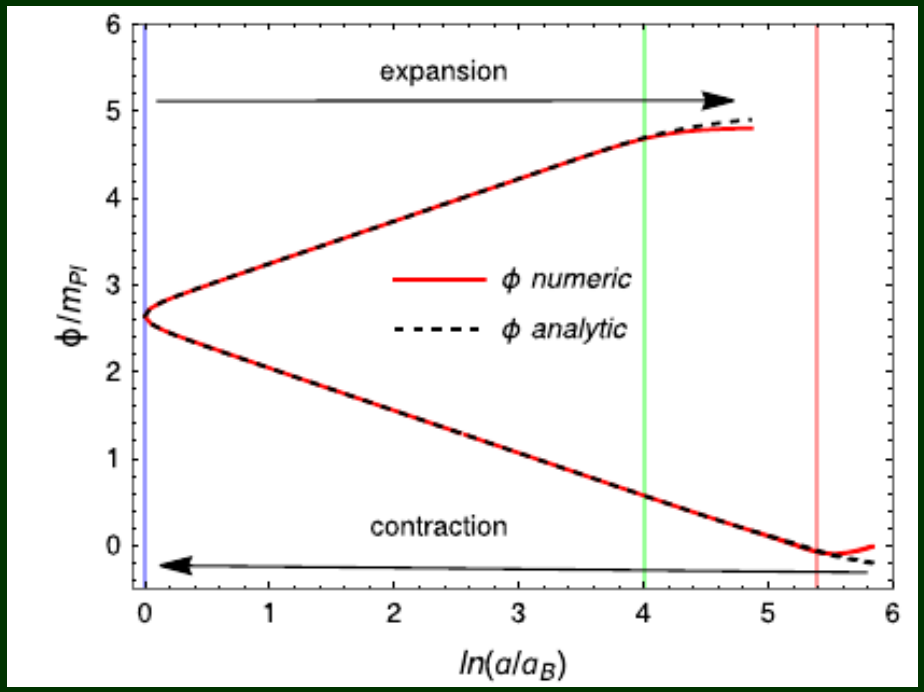
- Kinetic dominated regime $\dot{\phi}^2/2 \gg V(\phi)$:

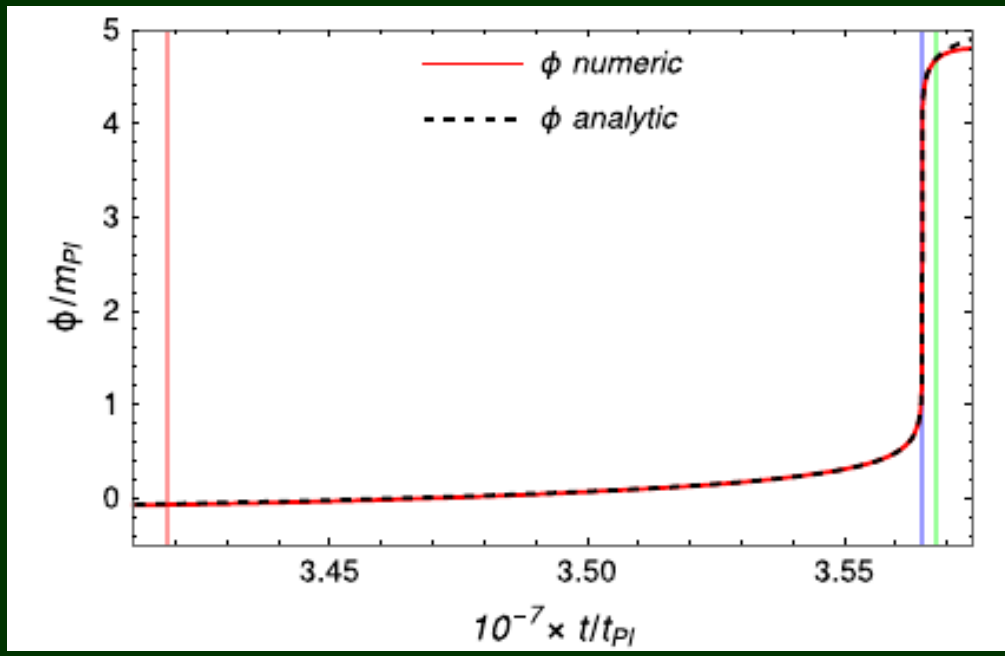
$$\ddot{\phi} + 3H\dot{\phi} \approx 0 \;\Rightarrow\; \dot{\phi}(t) = \pm\sqrt{2\rho_{\rm cr}}\left[\frac{a_{\rm B}}{a(t)}\right]^3$$

Using $a(t) = a_{\rm B}\left[1 + \frac{24\pi\rho_{\rm cr}}{m_{\rm Pl}^4}\frac{(t-t_{\rm B})^2}{t_{\rm Pl}^2}\right]^{1/6}$,

$$\begin{aligned}
\dot{\phi}(t) &= \pm\frac{\sqrt{2\rho_{\rm cr}}}{\left[1 + \frac{24\pi\rho_{\rm cr}}{m_{\rm Pl}^4}(t-t_{\rm B})^2/t_{\rm Pl}^2\right]^{1/2}}, \\
\phi(t) &= \phi_{\rm B} \pm \frac{m_{\rm Pl}}{2\sqrt{3\pi}}\,\text{arcsinh}\left[\sqrt{\frac{24\pi\rho_{\rm cr}}{m_{\rm Pl}^4}}\frac{(t-t_{\rm B})}{t_{\rm Pl}}\right].
\end{aligned}$$

[6] L. N. Barboza, G. L. L. W. Levy, L. L. Graef, and R.O.R., PRD 106, 103535 (2022)





$\Rightarrow$ the approximate equations work quite well from the transition point, $\dot{\phi}^2(t_{tr})/2 = V(\phi(t_{tr}))$, in the contracting phase up to the transition point in the expanding phase !

- Classical contracting regime $\rho \ll \rho_{\rm cr}$:

$$H \simeq \frac{2}{3(1+w)(t-t_B)} = \frac{1+\alpha}{3(t-t_B)} = -\sqrt{\frac{8\pi}{3m_{\rm Pl}^2}\frac{(1+\alpha)}{\alpha}V}$$

where $\alpha \equiv V/(\dot{\phi}^2/2)$, $w = (1-\alpha)/(1+\alpha)$. Taking some instant t_α between the transition time, $\dot{\phi}^2(t_{tr}^-)/2 = V(\phi(t_{tr}^-))$, and the bounce time, $t_{tr}^- < t_\alpha \ll t_B$, from H we find

$$\frac{V(\phi_\alpha)}{V'(\phi_\alpha)} = \frac{\sqrt{1+\alpha}}{4\sqrt{3\pi}} m_{\rm Pl}.$$

From the expression for H:

$$t_\alpha - t_B \simeq -\frac{1+\bar{\alpha}}{3}\sqrt{\frac{3m_{\rm Pl}^2\bar{\alpha}}{8\pi(1+\bar{\alpha})V(\phi_\alpha)}}.$$

Given some "averaged" $\bar{\alpha} \Rightarrow$ solve for $\phi_\alpha \Rightarrow$ obtain $t_\alpha - t_B \Rightarrow$ we find ϕ_B

- The postbounce transition regime ($t_B \to t_i$):
  At time t_{tr}^+ in the postbounce regime, $\dot{\phi}^2(t_{tr}^+)/2 = V(\phi(t_{tr}^+))$,

$$\phi(t_{tr}^+) = \phi_B + \frac{m_{\rm Pl}}{2\sqrt{3\pi}}\,\text{arcsinh}\left(\sqrt{\frac{24\pi\rho_{\rm cr}}{m_{\rm Pl}^4}}\,\frac{t_{tr}^+ - t_B}{t_{\rm Pl}}\right)$$

  From $\phi(t_{tr}^+)$ and $\dot{\phi}(t_{tr}^+)$ (using ϕ_B from previous step) $\Rightarrow$ obtain $t_{tr}^+ - t_B$ and then the value for $\phi(t_{tr}^+)$.
- Start of inflation ($w = -1/3$), t_i:
  t_i is not far from t_{tr}^+: we can expand $\phi(t)$ close to t_i,

$$\begin{aligned}
\phi_{\rm i} &\simeq \phi_{\rm tr} + \dot{\phi}_{\rm tr}\, t_{tr}^+ \,\ln\frac{t_{\rm i}}{t_{tr}^+},\\
\dot{\phi}_{\rm i} &\simeq \frac{t_{tr}^+}{t_{\rm i}}\,\dot{\phi}_{\rm tr},\\
V(\phi_{\rm i}) &\simeq V(\phi_{\rm tr}) + V_{,\phi}\,(\phi_{\rm tr})\, t_{tr}^+\,\dot{\phi}_{\rm tr}\,\ln\frac{t_{\rm i}}{t_{tr}^+},
\end{aligned}$$

  and at $t = t_i \Rightarrow w = -1/3 \Rightarrow \dot{\phi}_{\rm i}^2 = V(\phi_{\rm i}) \Rightarrow$ can solve for t_i and then obtain $\phi(t_i)$

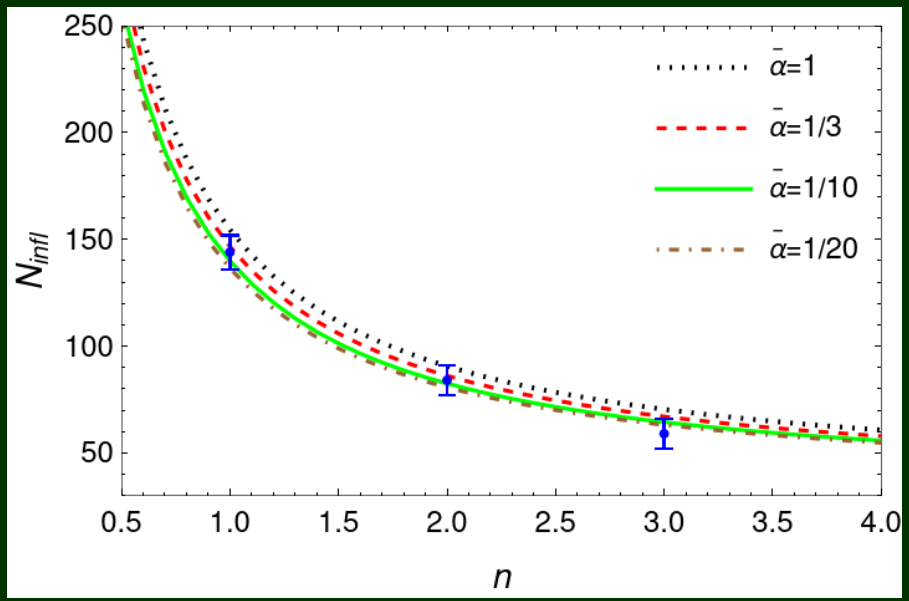
- Inflation regime ($t_i \to t_{\rm end}$):
  End of inflation:

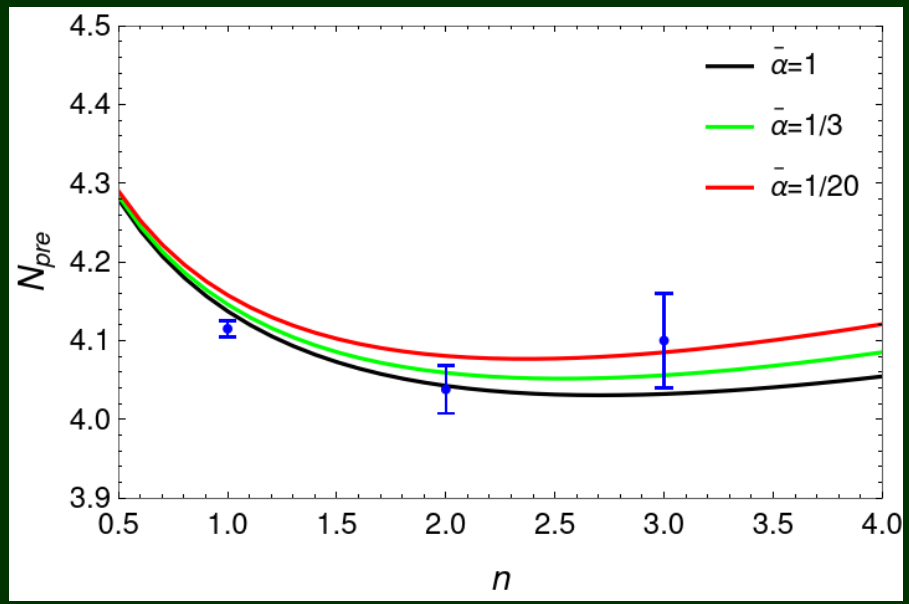
$$\epsilon = -\dot{H}/H^2 \simeq (m_{\rm Pl} V_{,\phi}/V)^2/(16\pi) = 1 \quad \Rightarrow \quad \phi_{\rm end}$$

  Number of efolds:

$$N_{\rm infl}(\phi) \equiv \ln\left(\frac{a_{\rm end}}{a_{\rm i}}\right) \approx \frac{8\pi}{m_{\rm Pl}^2}\int_{\phi_{\rm end}}^{\phi_{\rm i}}\left(\frac{V}{V'}\right) d\phi$$

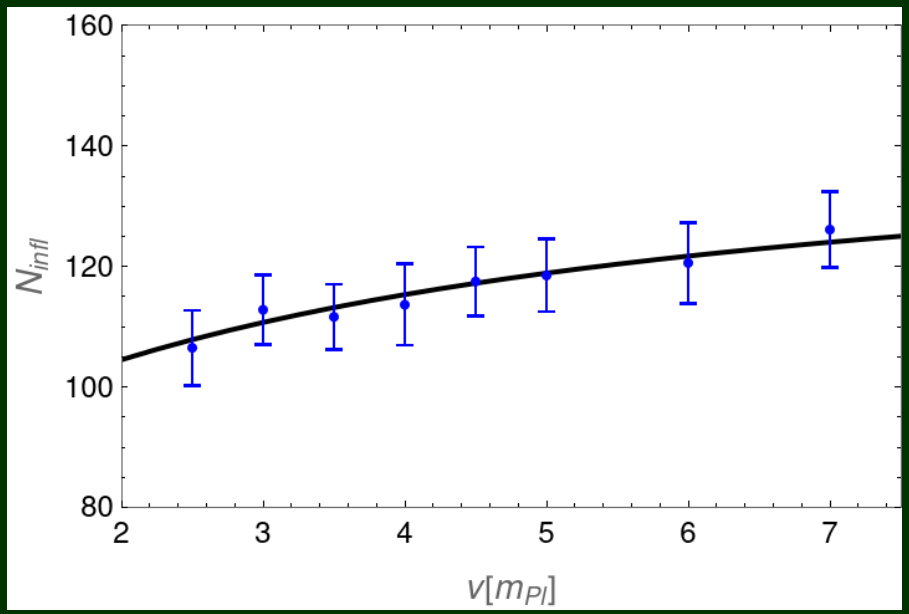
$$V = \frac{V_0}{2n}\left(\frac{\phi}{M_{\rm Pl}}\right)^{2n}$$



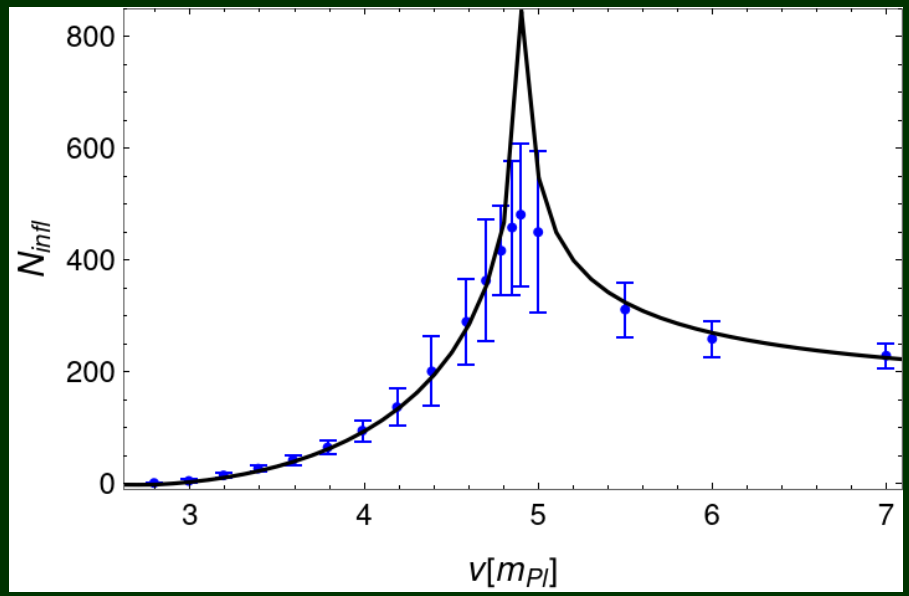


$\Rightarrow \bar{\alpha} = 1/3$

$$V = V_0 \left[1 - \left(\frac{\phi}{v}\right)^2\right]^2 ,$$



$|\phi| > |v|$



$|\phi| < |v|$

Perturbations in LQC

The effective scalar (and tensor) perturbations in LQC are given by

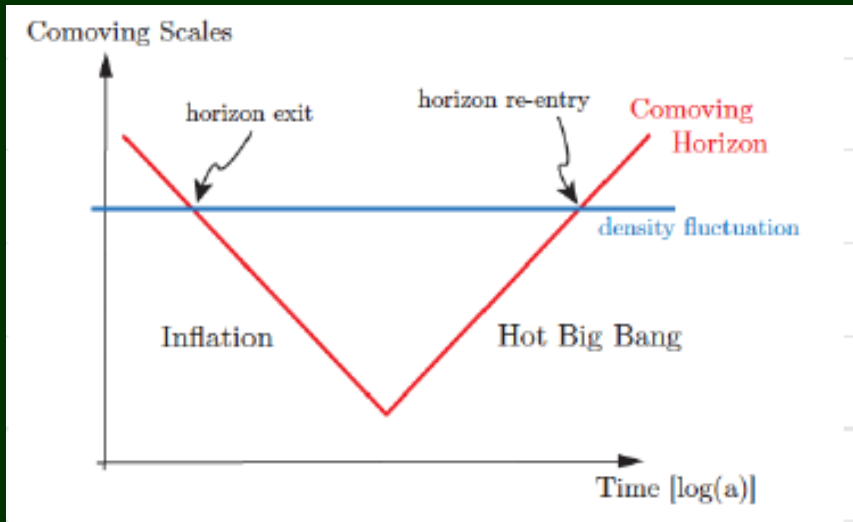
$$\mu_k'' + \Omega_k^2 \mu_k = 0, \qquad \Omega_k^2 \simeq k^2 - \frac{a''}{a},$$

where $a' \equiv da/d\eta, \;\; d\eta = dt/a(t)$.
For a kinetic energy dominated bounce,

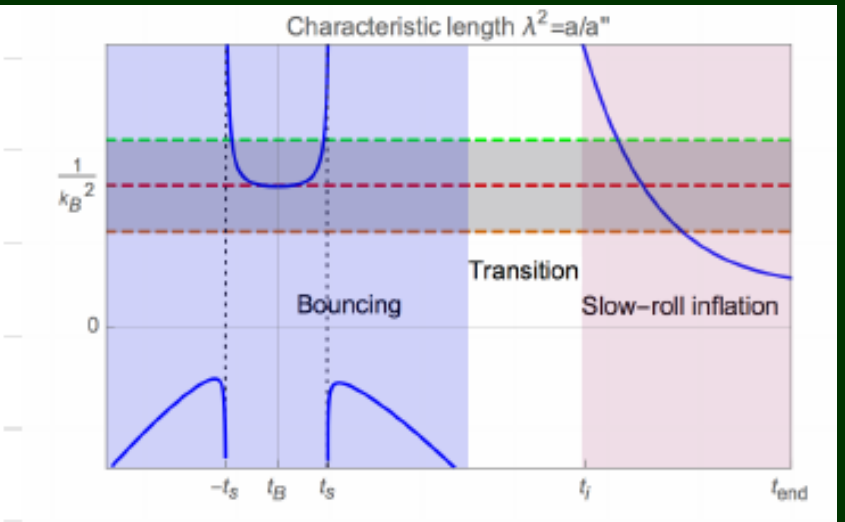
$$a(t) \simeq a_B \left[1 + \frac{24\pi\rho_c}{m_{\rm Pl}^4} \frac{(t-t_B)^2}{t_{\rm Pl}^2}\right]^{1/6}$$

The term a''/a defines a characteristic radius λ, $\lambda^2 = a/a''$ and a characteristic energy scale at the bounce ($\Omega_k(t_B) = 0$),

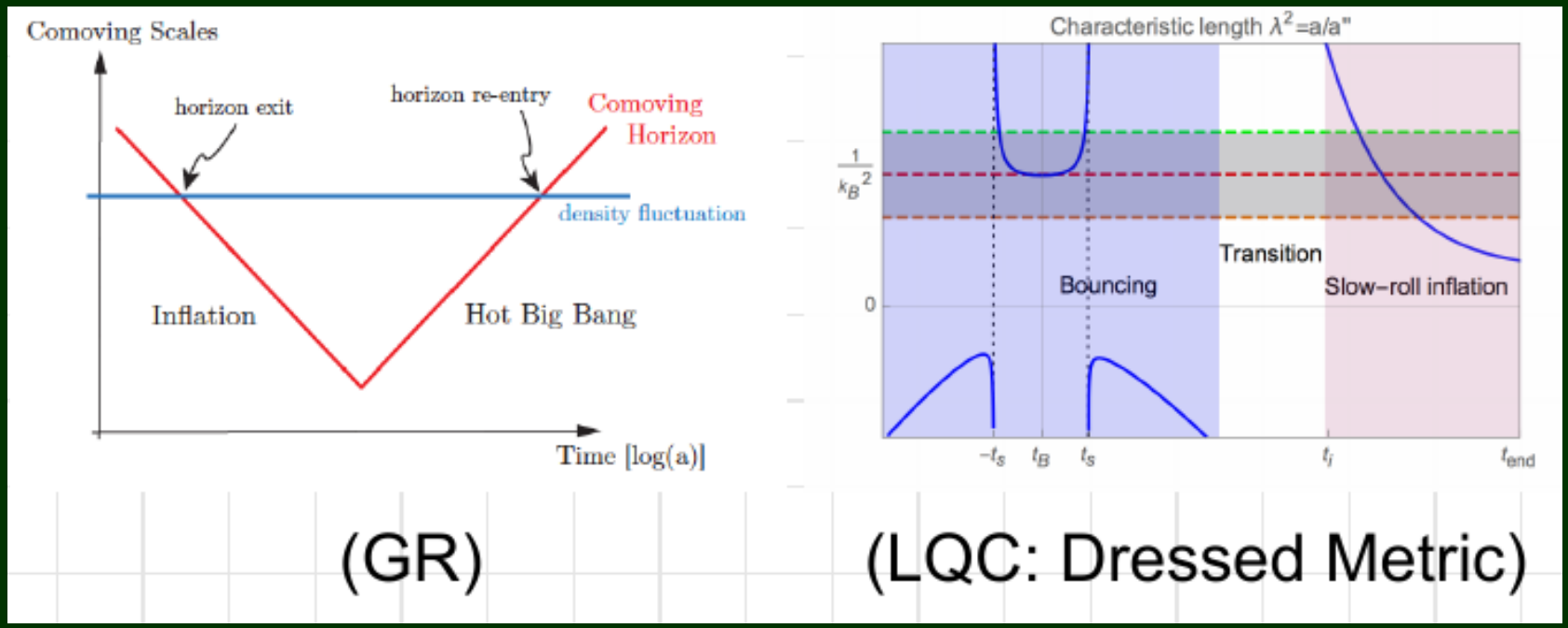
$$\boxed{k_B/a_B \equiv \sqrt{\left.\frac{a''}{a}\right|_{t=t_B}} = \frac{\sqrt{8\pi\rho_c}}{m_{\rm Pl}}}$$



(GR)



(LQC: Dressed Metric)



$\Downarrow$

Near the bounce, the wavelengths can be larger, equal, or smaller than the corresponding characteristic scale $\Rightarrow$ in general impossible to assume that the universe is in the Bunch-Davies vacuum at the bounce.

Modification of the scalar and tensor power spectrum due to particle production (Zhu, Wang, Cleaver, Kirsten, Sheng, PRD96 (2017) 083520)

$$\Delta_{\mathcal{R}}(k) = |\alpha_k + \beta_k|^2 \Delta^{GR}_{\mathcal{R}}(k), \quad \Delta_h(k) = |\alpha_k + \beta_k|^2 \Delta^{GR}_h(k)$$

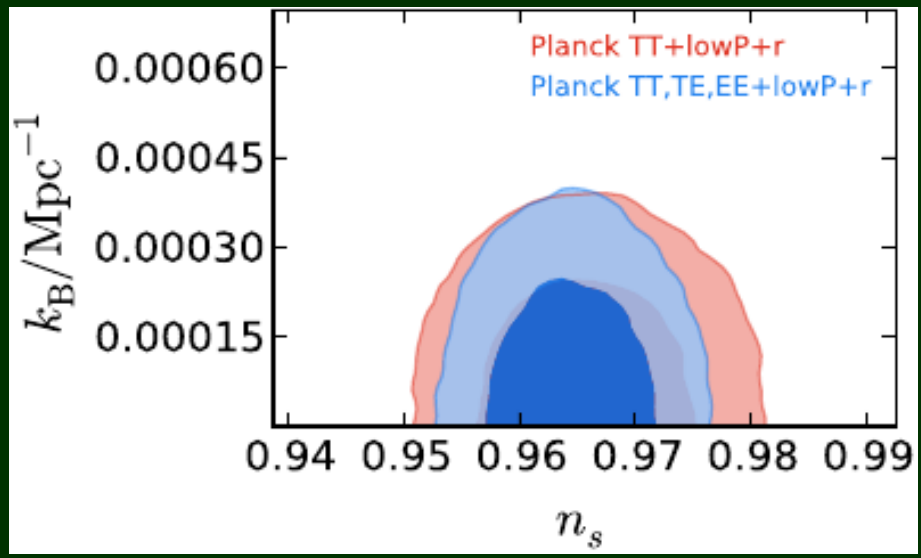
where

$$\begin{aligned} |\alpha_k + \beta_k|^2 &\simeq 1 + \left[1 + \cos\left(\frac{\pi}{\sqrt{3}}\right)\right] \mathrm{csch}^2\left(\frac{\pi k}{\sqrt{6} k_B}\right) \\ &+ \sqrt{2}\sqrt{\cosh\left(\frac{2\pi k}{\sqrt{6} k_B}\right) + \cos\left(\frac{\pi}{\sqrt{3}}\right)} \cos\left(\frac{\pi}{2\sqrt{3}}\right) \\ &\times \mathrm{csch}^2\left(\frac{\pi k}{\sqrt{6} k_B}\right) \cos(2k\eta_B + \varphi_k), \end{aligned}$$

where η_B is the conformal time at the bounce and $k_B = \sqrt{\rho_c} a_B / M_{\rm Pl}$ is a characteristic scale also at the bounce (the shortest scale, or largest wave number k, that feels the space-time curvature during the bounce)

$\Rightarrow$ there is now a k-dependence. To be consistent with the CMB observations (almost scale independent spectrum), it (the k-dependence) needs to be small

$\Rightarrow$ This gives a constraint on the characteristic scale k_B of the bounce:



$$\frac{k_B}{a_0} \lesssim \begin{cases} 3.1 \times 10^{-4}\mathrm{Mpc}^{-1}, & \text{in } 2\sigma \\ 1.9 \times 10^{-4}\mathrm{Mpc}^{-1}, & \text{in } 1\sigma \end{cases}$$

(M. Benetti, L. L. Graef and R.O.R., JCAP10(2019)066)

Likewise, this also gives a bound on the total number of e-folds from the bounce until now

$$\frac{k_B}{a_0} = \frac{\sqrt{8\pi\rho_c}}{m_{\rm Pl}}\frac{a_B}{a_0} = \frac{\sqrt{8\pi\rho_c}}{m_{\rm Pl}}e^{N_{\rm tot}}$$

$\Rightarrow N_{\rm tot} \gtrsim 142$ (using Barbero-Immirzi parameter $\gamma \simeq 0.2375$)

Since $N_{k_*} \sim 60 \Rightarrow$ from the bounce until the end of inflation: $N \gtrsim 82$

Model	$N_{\rm pre}$	$N_{\rm infl}$
Quadratic	4.15	146.55
Quartic	4.06	86.36
Sextic	4.06	67.30
Starobinsky	4.92	1.10×10^9

Model	$N_{\rm pre}$	$N_{\rm infl}$
Higgs-like ($\lvert\phi_{\rm B}\rvert > \lvert v = 3.5 m_{\rm Pl}\rvert$)	4.08	113.31
Higgs-like ($\lvert\phi_{\rm B}\rvert > \lvert v = 4.0 m_{\rm Pl}\rvert$)	4.08	115.46
Higgs-like ($\lvert\phi_{\rm B}\rvert > \lvert v = 4.5 m_{\rm Pl}\rvert$)	4.09	117.35
Higgs-like ($\lvert\phi_{\rm B}\rvert < \lvert v = 3.5 m_{\rm Pl}\rvert$)	4.53	32.64
Higgs-like ($\lvert\phi_{\rm B}\rvert < \lvert v = 4.0 m_{\rm Pl}\rvert$)	4.43	95.66
Higgs-like ($\lvert\phi_{\rm B}\rvert < \lvert v = 4.5 m_{\rm Pl}\rvert$)	4.37	235.92

Constraining the Barbero-Immirzi parameter

In general, the Barbero-Immirzi parameter parameter is an arbitrary parameter. Can we constrain it ?

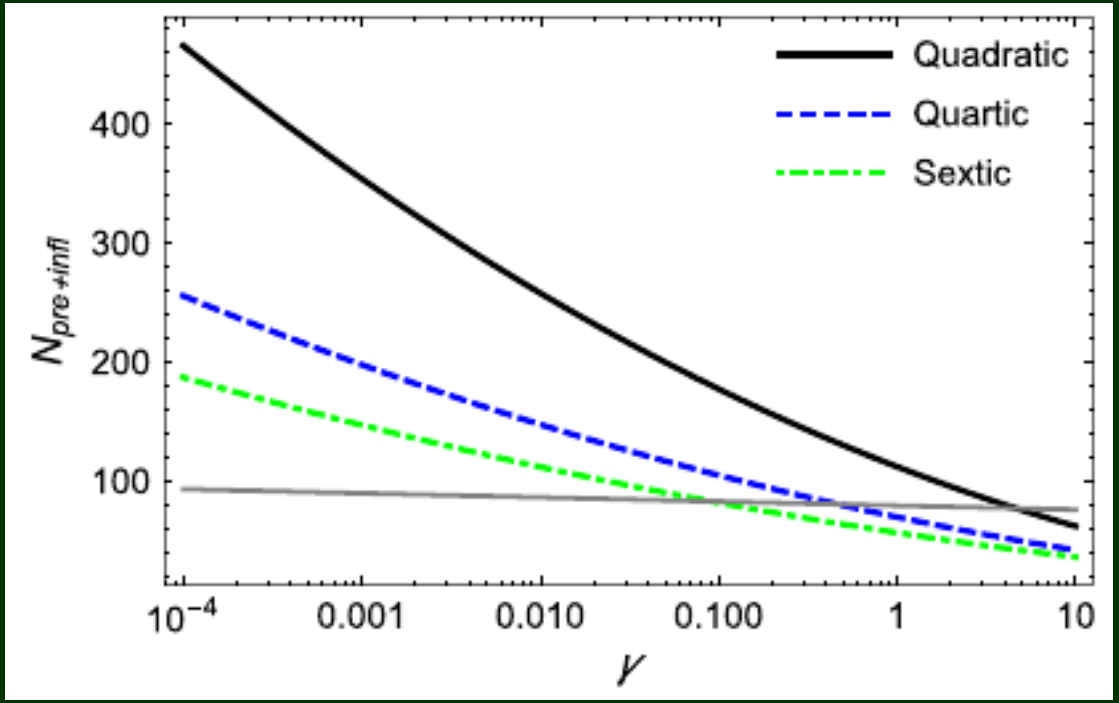
In LQC $\Rightarrow$ scale dependent corrections to the spectrum $\Rightarrow$ sets a constraint on the typical scale at the bounce, k_B:

$k_B \lesssim 1.9 \times 10^{-4} \mathrm{Mpc}^{-1}$ (at 1σ)

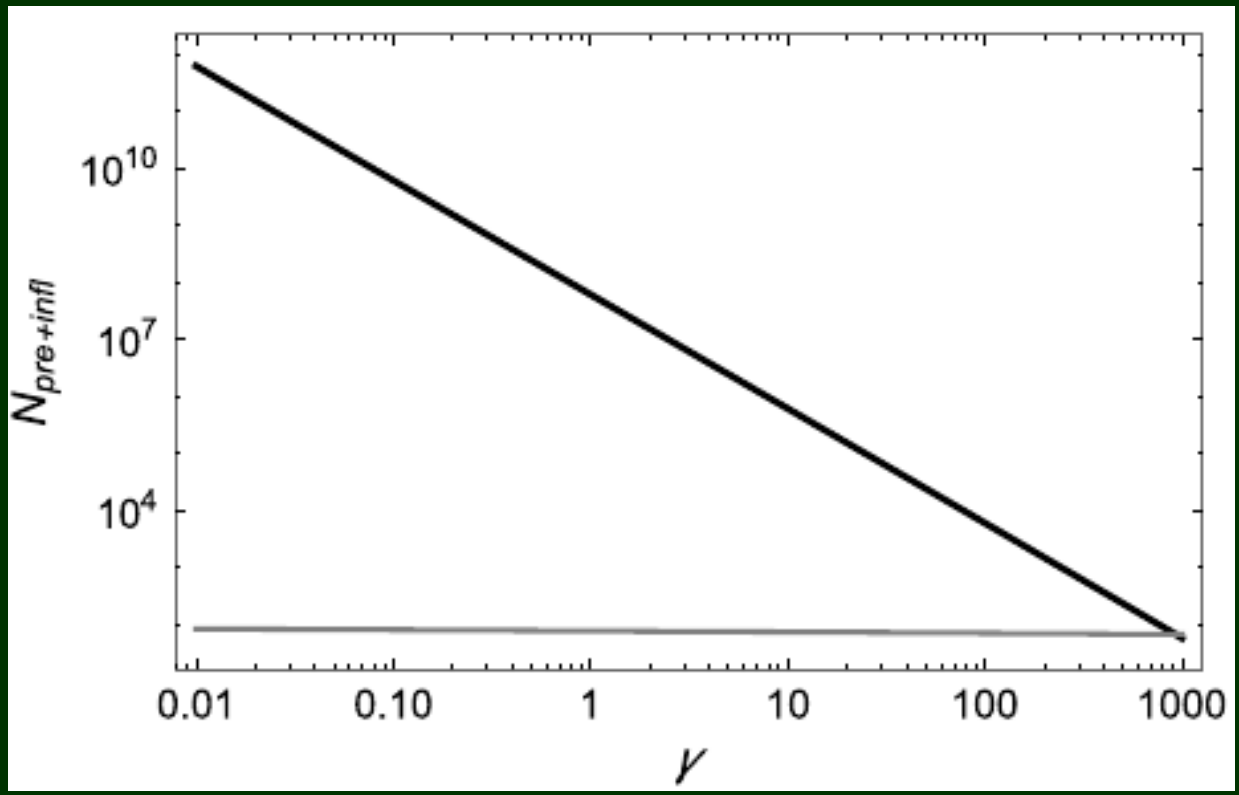
$$k_{\rm B} \equiv \frac{\sqrt{8\pi\rho_{\rm cr}}a_{\rm B}}{m_{\rm Pl}} = m_{\rm Pl}\left(\frac{\sqrt{3}}{4\pi\gamma^3}\right)^{1/2} e^{-N_{\rm T}} \;\Rightarrow\; N_{\rm T} \gtrsim 139 - \frac{3}{2}\ln(\gamma)$$

$$\begin{aligned} N_{\rm T} &= \ln\left(\frac{a_0}{a_{\rm B}}\right) = \ln\left(\frac{a_{\rm i}}{a_B}\frac{a_{\rm end}}{a_{\rm i}}\frac{a_{\rm reh}}{a_{\rm end}}\frac{a_0}{a_{\rm reh}}\right) \\ &= N_{\rm pre} + N_{\rm infl} + N_{\rm reh} + \ln\left(\frac{a_0}{a_{\rm reh}}\right), \\ \Rightarrow &\quad \boxed{N_{\rm pre} + N_{\rm infl} \gtrsim 79 - \frac{3}{2}\ln(\gamma)} \end{aligned}$$

$$V = \frac{V_0}{2n}\left(\frac{\phi}{M_{\rm Pl}}\right)^{2n}$$



$$V = V_0 \left(1 - e^{-\sqrt{\frac{16\pi}{3}}\frac{\phi}{m_{\rm Pl}}}\right)^2 .$$



For the α-attractor models[7]:

$$V(\phi) = V_0 \frac{\left[\tanh\left(\frac{\phi}{\sqrt{6\alpha}\, M_{\rm Pl}}\right)\right]^2}{\left[1+\tanh\left(\frac{\phi}{\sqrt{6\alpha}\, M_{\rm Pl}}\right)\right]^{2n}}$$

$n = 0 \Rightarrow$ T model
$n = 1 \Rightarrow$ E model (for $\alpha = 1 \Rightarrow$ Starobinsky model)

For $\alpha \ll 1$,

$$n_s \simeq 1 - \frac{2}{N_*}, \qquad r \simeq \alpha \frac{12}{N_*^2},$$

$N_* = 50 - 60$

[7] G. Levy and R.O.R., PRD110, 043507 (2024)

From:

$$n_s = 0.9649 \pm 0.0042, \qquad r < 0.036, \quad \text{at } 95\% CL$$

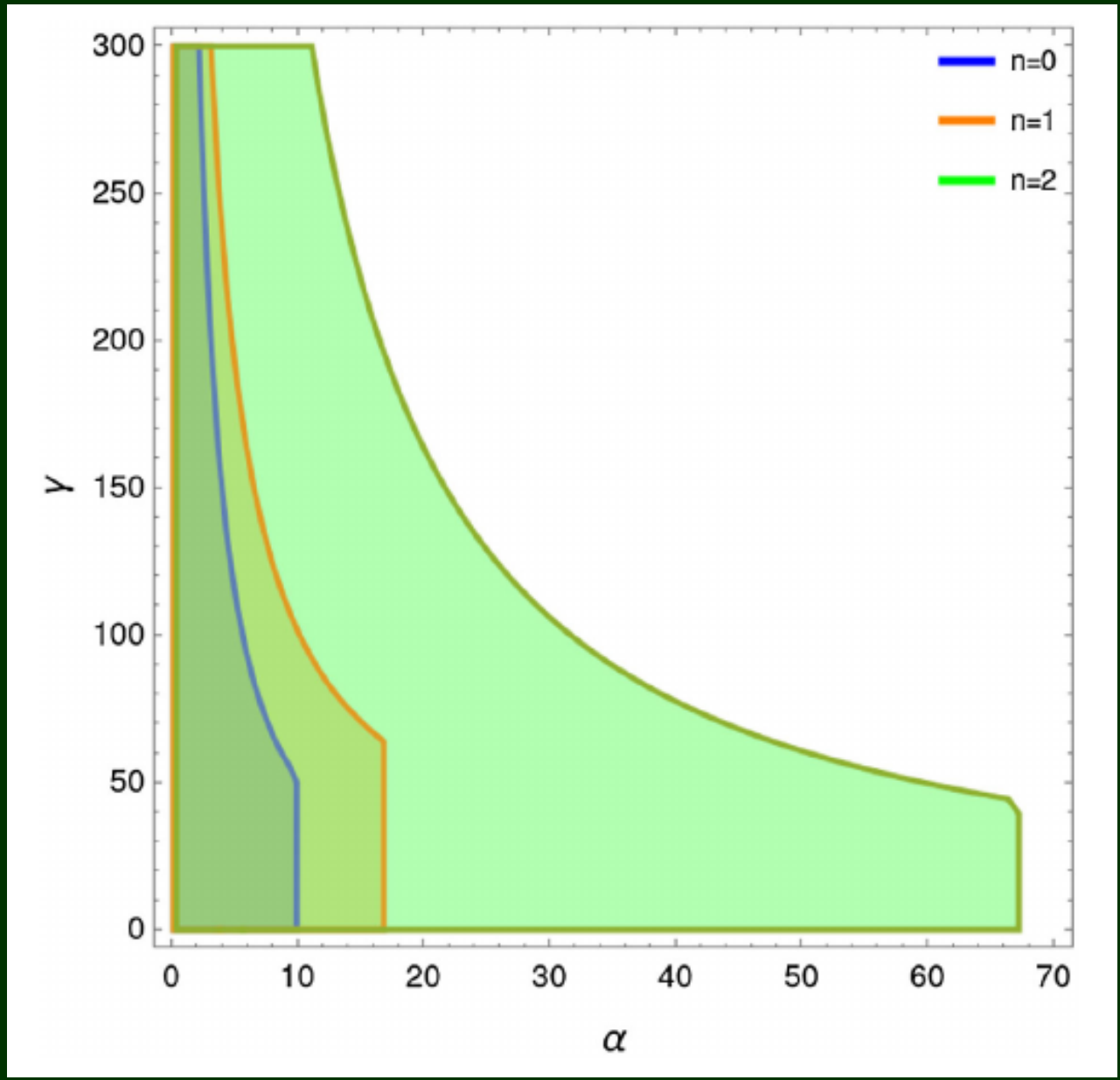
$$\alpha \lesssim \begin{cases} 10, & \text{for } n = 0, \\ 17, & \text{for } n = 1, \\ 67, & \text{for } n = 2. \end{cases}$$

From the constraint shown before:

$$N_{\rm pre+infl} \gtrsim 79 - \frac{3}{2}\ln(\gamma).$$

$$\Downarrow$$

$$\gamma \lesssim \begin{cases} 51.2, & \text{for } n = 0 \text{ and } \alpha = 10, \\ 63.4, & \text{for } n = 1 \text{ and } \alpha = 17, \\ 64.2, & \text{for } n = 2 \text{ and } \alpha = 67, \end{cases}$$


n=0
n=1
n=2
300
250
200
150
100
50
0
γ
0
10
20
30
40
50
60
70
α

The results are generic for any bounce model that is kinetic energy dominated (stiff matter) at the bounce.

Results extended also for the quantum bounce in the context of the de Broglie–Bohm interpretation (G. S. Vicente, R.O.R. and V. N. Magalhães, PRD **108** (2023) 023517)

Summary

- Inflation is the standard paradigm to solve the outstanding problems of the Hot Big Bang model;
- Bouncing models have also shown to lead to similar results and to be free of issues that inflation is claimed to have;
- Yet, nothing prevents inflation to emerge even in bouncing models $\Rightarrow$ and in fact inflation is highly probable in bouncing models !
- Initial conditions set at the bounce $\Rightarrow$ predicts a high probability for inflation to happen;
- Initial conditions set at the deep contracting phase $\Rightarrow$ precise predictions can be made about the duration of inflation (impossible in GR !);
- Adds new level of constraints that can then be put on different inflation models and also on free parameters of the bounce model when contrasting the results with the observations.