Precision Theory for Precision Physics: The HL-LHC and Future Collider Challenge

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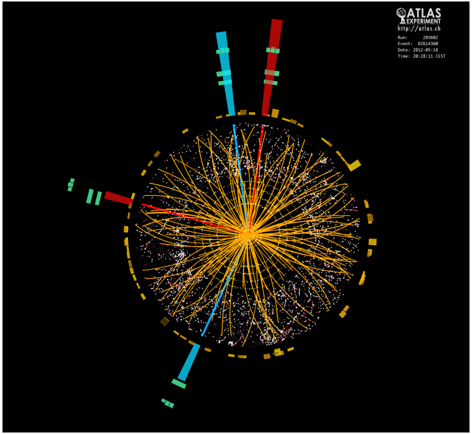
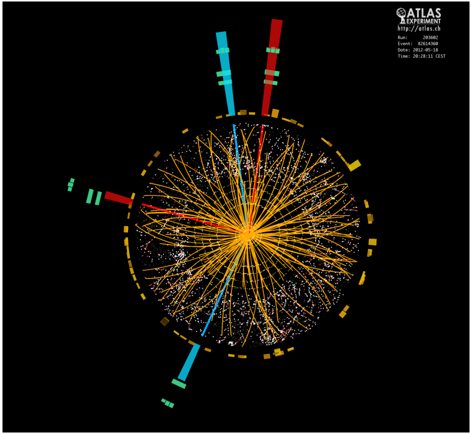
Outlook

- **Introduction**
  $\rightarrow$ Understand fundamental laws of nature with collisions
- **Precision at the LHC**
  $\rightarrow$ Illustration with W+c production
- **Precision at future colliders**
  $\rightarrow$ Illustration with Higgs and Electroweak physics

→ Common point between these pictures?



→ Display **known** and **unknown**!



→ Observing fragments of reality and looking for the truth

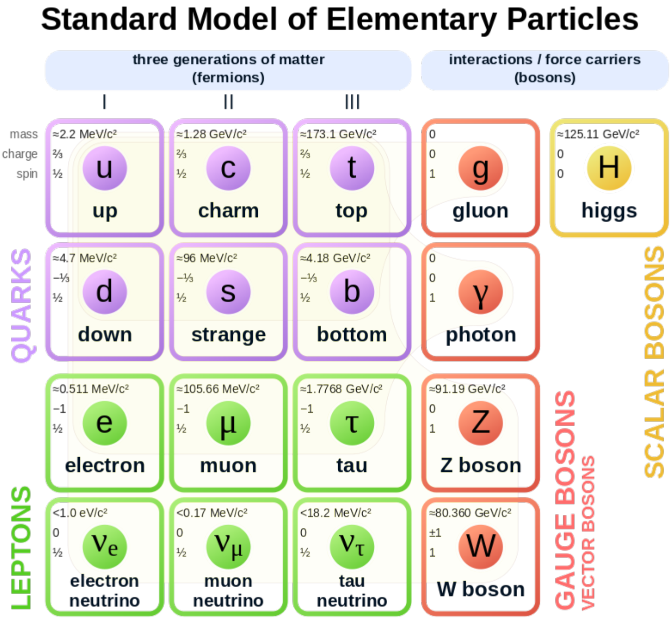
Allegory of the Cave - Plato

[source: MatiasEnElMundo/Getty Images]

→ Same in particles physics!
… but with some differences

Fragments of reality:

The particles of the
Standard Model

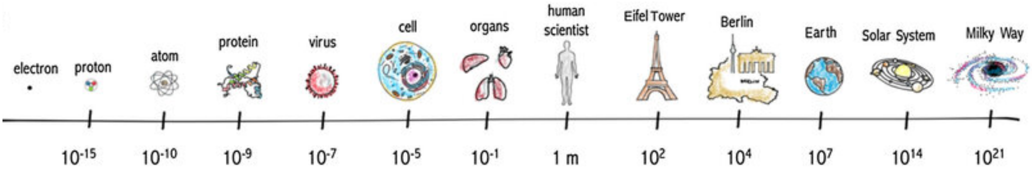


The truth about ...

- Dark Matter
- Matter-antimatter asymmetry
- Neutrino nature/masses
- Electroweak Symmetry Breaking
- How W and Z boson get their masses
- Higgs-boson properties
- ...

[source: bing image creator]

→ Same in particles physics! ... but with some differences...



[source: Researchgate]

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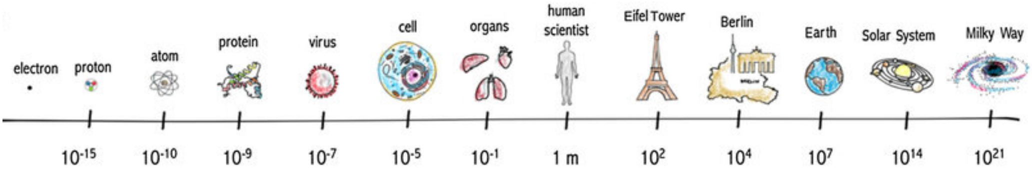
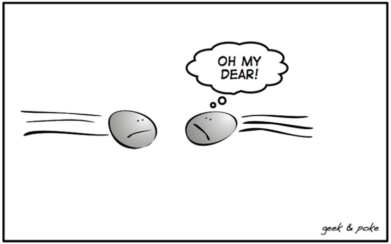


[source: Researchgate]

→ $L \sim c\hbar/E \;\Rightarrow$ need a lot of energy!

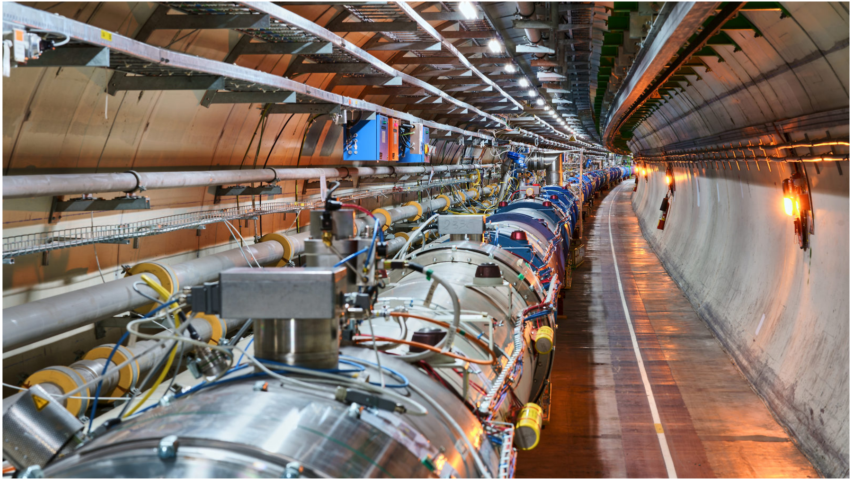


LATELY INSIDE THE LHC:
2 PROTONS 0.000000000000000001 SEC BEFORE THE COLLISION

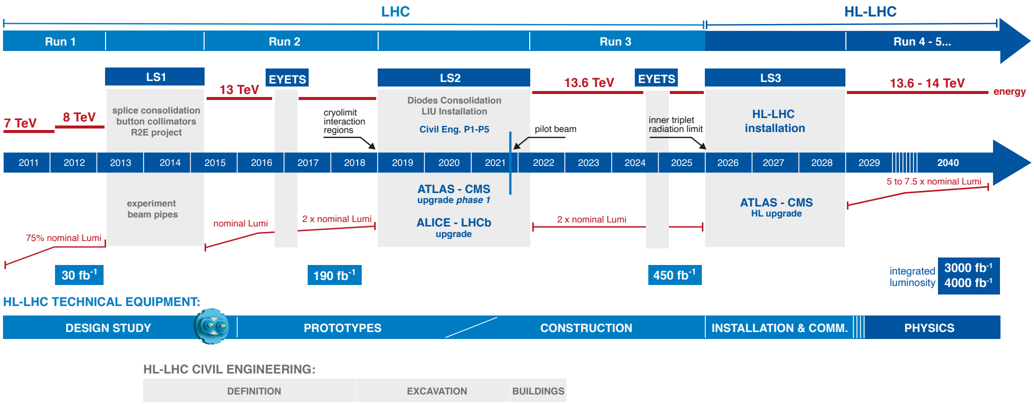
LHC @ CERN, Geneva (Switzerland)

→ 27km-long tunnel where protons are collided at high-energy (13.6 TeV currently)

→ Allow to probe fundamental interactions



LHC / HL-LHC Plan



⚠ A lot more data ($N_{\text{event}} = L \times \sigma$)

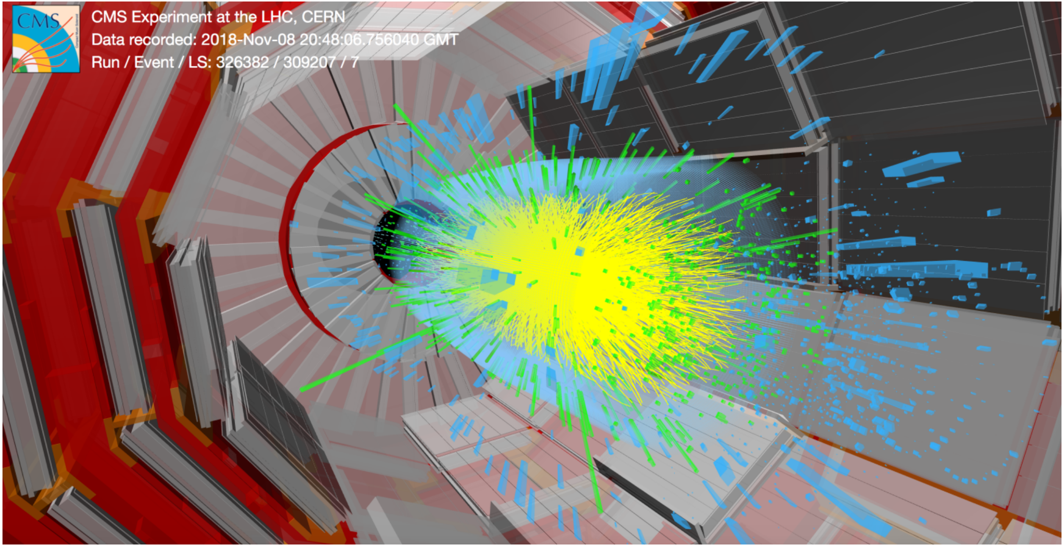
→ Very high-precision measurements

→ Access to new rare processes

→ Era of precision

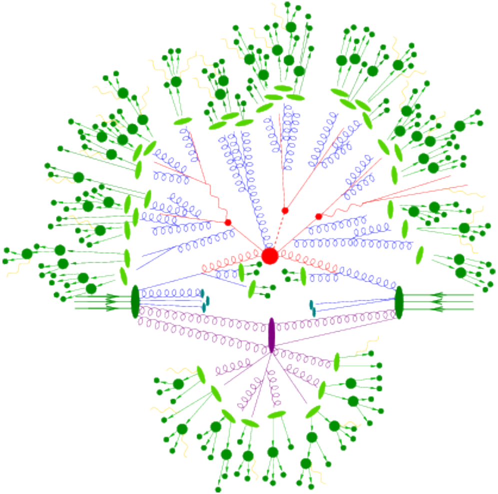
→ Challenges on theory and experiment sides!

Life at the LHC (in reality)



Life at the LHC (on the theory side)

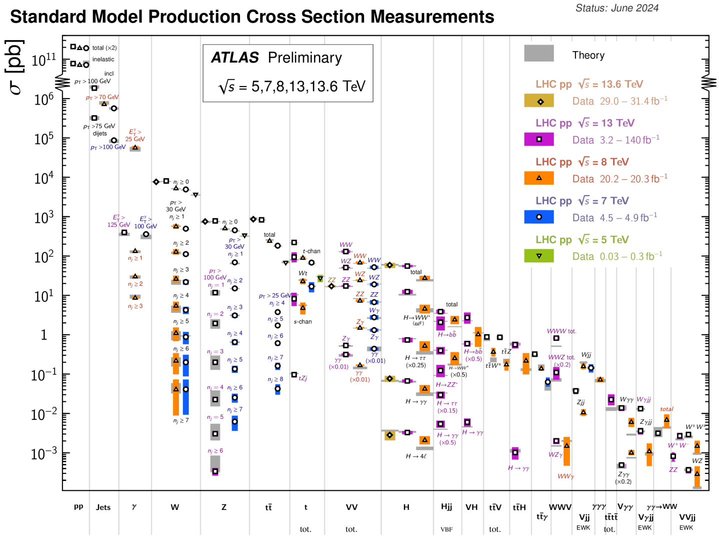
- Factorisation of mechanisms:
  - high-energy (perturbative)
    → Hard-scattering, parton-shower
  - low-energy (non-perturbative)
    → parton distribution function, hadronisation, underlying events, ...
- **Focus of the presentation: hard scattering part**
  → Calculated from first principle
  → Where new physics might hide



[source: Sherpa]


$\mathcal{L}_{SM}$
[source: bing image creator, Engel, Pellen]

Learning about fundamental physics through comparison
data vs. first-principle theoretical predictions



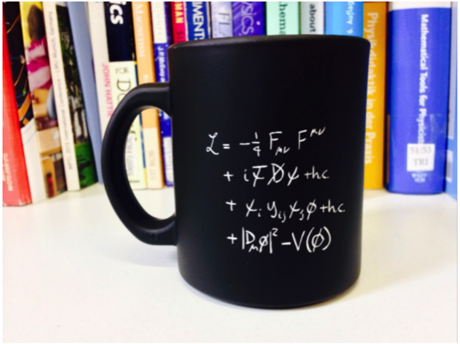
Learning about fundamental physics through comparison
data vs. **first-principle theoretical predictions**

process	known	desired
$pp \to V$	$\mathrm{N^3LO_{QCD}} + \mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $+$ Full $\mathrm{NLO_{QCD}}$ $(gg \to ZZ)$, approx. $\mathrm{NLO_{QCD}}$ $(gg \to WW)$	Full $\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V + j$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$	hadronic decays
$pp \to V + 2j$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$
$pp \to V + b\bar{b}$	$\mathrm{NLO_{QCD}}$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to W + b\bar{b}$	$\mathrm{NNLO_{QCD}}$	
$pp \to VV' + 1j$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$

[Huss, Huston, Jones, MP, Röntsch; 2504.06689]

→ Les Houches wishlist (updated every two years)
→ Situation very much process dependent

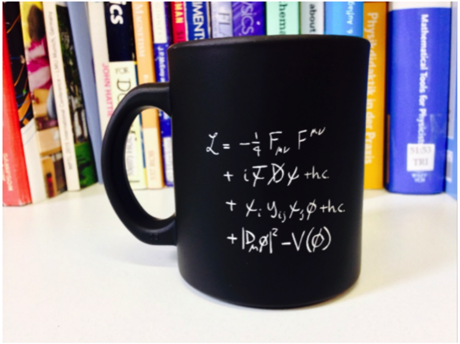
- **Precision at the LHC**
  $\rightarrow$ Illustration with W+c production



[source: CERN]

The Standard Model of particle physics

- Particles are represented by fields
- Lagrangian displays interactions of fields
- Based on gauge symmetry
  $\rightarrow SU_{\mathrm{QCD}}(3) \times SU_{\mathrm{L}}(2) \times U_{\mathrm{Y}}(1)$



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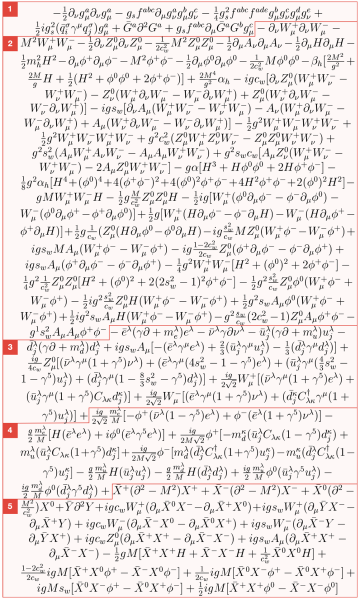
- Strong force
  $\rightarrow$ Quantum chromodynamics (QCD) mediated by gluon
- Electromagnetic force
  $\rightarrow$ Quantum electrodynamics (QED) mediated by photon
- Weak force
  $\rightarrow$ Weak interaction mediated by W and Z



[source: symmetrymagazine, Thomas Gutierrez]

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Perturbation theory and Feynman diagrams

Quark and gluon interaction

$$\mathcal{L}_\psi = \sum_q \overline{\psi}_q \left(\mathrm{i}\not{\partial} - g_\mathrm{s} \frac{\lambda_a}{2} \not{G}^a - m_q \right) \psi_q$$

After quantisation:
(e.g. with path integral formalism)
→ *Feynman rules* in momentum space

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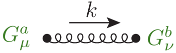
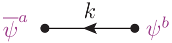
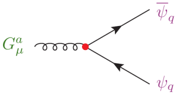


$$\frac{-\mathrm{i}\delta^{ab} g_{\mu\nu}}{k^2 + \mathrm{i}\epsilon}$$



$$\frac{\mathrm{i}(k\!\!\!/ + m_q)}{k^2 - m_q^2 + \mathrm{i}\epsilon}$$



$$-\mathrm{i}g_\mathrm{s} \frac{\lambda^a}{2} \gamma_\mu$$

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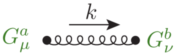

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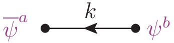

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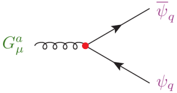
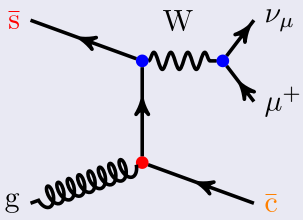


$$-\mathrm{i}g_\mathrm{s}\frac{\lambda^a}{2}\gamma_\mu$$

→ Building blocks for transition **amplitudes**:

$$\langle f|\, S\, |i\rangle = (2\pi)^4 \delta \left(\sum_j p_j - \sum_j k_j \right) \mathrm{i}\mathcal{M}_{\mathbf{fi}}, \quad |i\rangle \neq |f\rangle$$

1) Write all possible diagrams at the lowest order:



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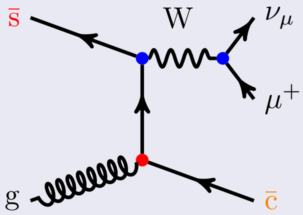

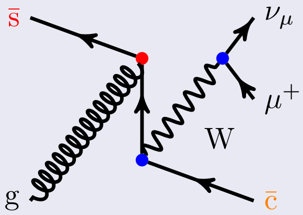

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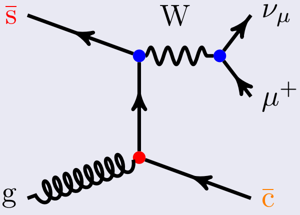

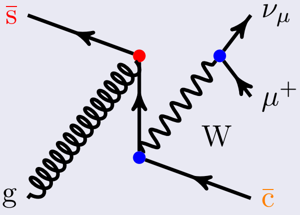

→ Complicated example: $gg \to \mu^-\bar{\nu}_\mu e^+\nu_e \bar{b}b\bar{b}b$: 3904 diagrams [Denner,MP,Lang; 2008.00918]

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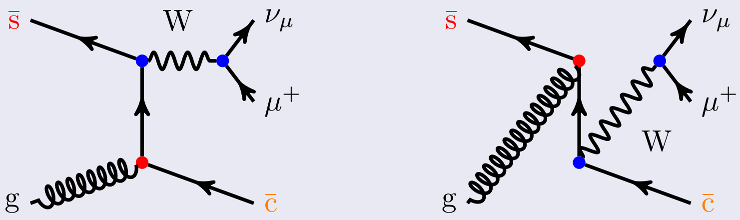

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$$\sigma \propto \int \mathrm{d}\Phi |\mathcal{M}|^2 \quad \text{with} \quad \mathrm{d}\Phi : \text{ phase space/momentum integration}$$

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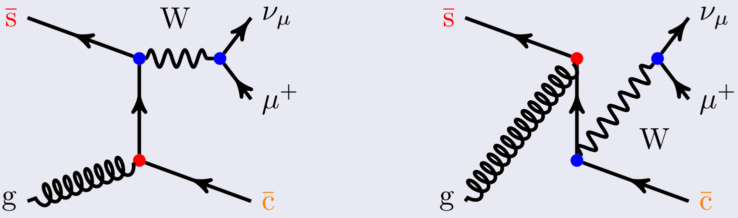

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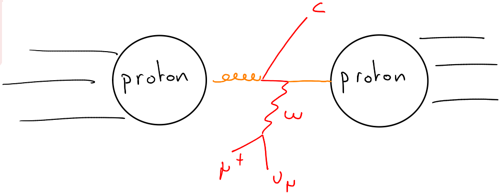
Integrating over the full phase space provides *inclusive/total* cross section!
→ Probability for the process $g\bar{s} \to \mu^+ \nu_\mu \bar{c}$ to happen!

⚠ LHC is not a $\bar{s}$-g collider...
It is a proton-proton collider!

↪ parton model: gluon and all possible quarks available in proton at high energy



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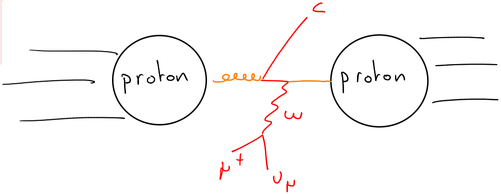
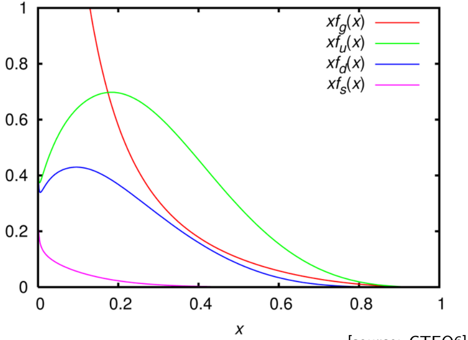


$$\sigma_{\text{hadron}} \propto \sum_{i,j} \int \mathrm{d}x_a \mathrm{d}x_b f_i(x_a) f_j(x_b) \int \mathrm{d}\Phi |\mathcal{M}|^2$$

- f_i **Parton Distribution Function** (PDF)
- f_i universal
- f_i used at LHC, fitted to data
- QCD constrains relations between f_i

→ Lowest-order approximation:
leading order (LO)



[source: CTEQ6]

- Experimental measurements performed in parts of the phase space
  $\rightarrow$ where detectors are placed
- Experiments also measure observables
  $\rightarrow$ transverse mom. $\left[p_{\mathrm{T}}(p) = \sqrt{p_x^2 + p_y^2}\right]$, rapidity $\left[y(p) = \frac{1}{2}\log\left(\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}\right)\right]$ ...
- Analytical integration not an option!

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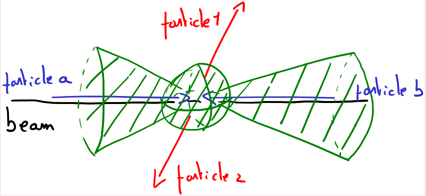
$\Rightarrow$ Solution: **Monte Carlo integration!**

$\rightarrow$ Trade integration variables (angles, energies) vs. random numbers (x_i)

Examples: $\phi = 2\pi \cdot x_1,\ E = (E_{\max} - E_{\min})x_2 - E_{\min}$

$$\Rightarrow\ \sigma \propto \int_0^1 \mathrm{d}x_1 \cdots \int_0^1 \mathrm{d}x_n\, g(x_1, \cdots, x_n)\, \Theta(t(x_1, \cdots, x_n))$$



- $\Theta(t)$ encodes experimental selection
- g highly complex function with singularities

Multidimensional integration of complex integrand

- Error scaling as $1/\sqrt{N_{\mathrm{points}}}$
- Peaked structure due to propagators $\frac{1}{(s-m^2)^2+m^2\Gamma^2}$ (Breit-Wigner distribution)

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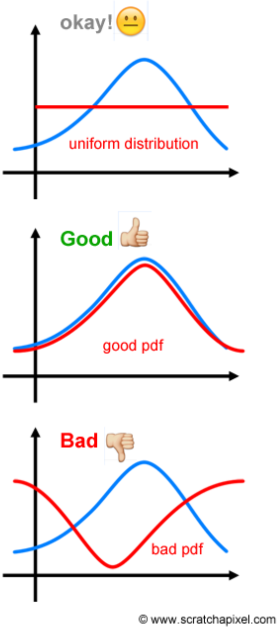
  $\rightarrow$ **importance sampling**:
  generates points with density close to integrand



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- Peaked structure due to propagators $\frac{1}{(s-m^2)^2+m^2\Gamma^2}$ (Breit-Wigner distribution)
  $\rightarrow$ **importance sampling**:
  generates points with density close to integrand
- Propagator structure known
  $\rightarrow$ important sampling for multiple resonance structures
  $\hookrightarrow$ Multi-channelling (dedicated programs)
  $\sim \mathcal{O}(10^3 - 10^4)$ for $gg \rightarrow \mu^-\bar{\nu}_\mu e^+\nu_e \bar{b}b\bar{b}b$ [Denner,MP,Lang; 2008.00918]
  $\rightarrow$ **automation is key!**

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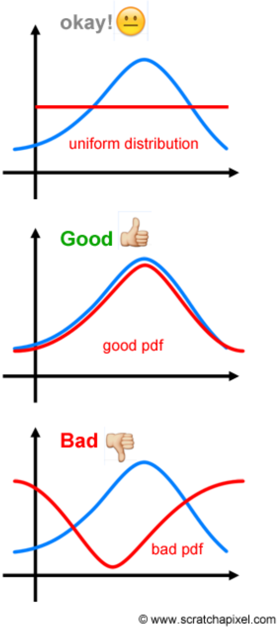

Perturbative theory appropriate for high energy!

$\rightarrow$ Expansion parameter: the couplings α_{s} (QCD) and α (EW)

$$\sigma = \sigma_{\mathrm{LO}} + \alpha_{\mathsf{s}}\, \sigma_{\mathrm{NLO\ QCD}} + \alpha\, \sigma_{\mathrm{NLO\ EW}} + {\alpha_{\mathsf{s}}}^2\, \sigma_{\mathrm{NNLO\ QCD}} + \dots$$

NB: NLO = next-to-leading order, NNLO = next-to-next-to-leading order, etc.

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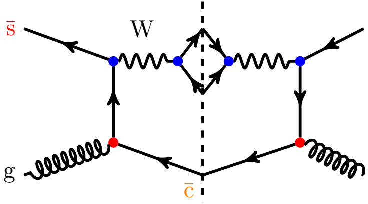
$$\left.\begin{array}{l} \alpha_{\mathsf{s}} \sim 0.1 \\ \alpha \sim 0.01 \end{array}\right\} \quad {\alpha_{\mathsf{s}}}^2 \sim \alpha \quad \text{(naive scaling)}$$

$\rightarrow$ Typical LHC processes requires NNLO QCD + NLO EW to match experimental accuracy

$\hookrightarrow$ Some processes requires $\mathcal{O}({\alpha_{\mathsf{s}}}^3)$ and/or $\mathcal{O}(\alpha_{\mathsf{s}}\alpha)$ corrections

$$\sigma_{\mathrm{LO}} \propto \sum_{i,j} \int \mathrm{d}x_a \mathrm{d}x_b f_i(x_a) f_j(x_b) \int \mathrm{d}\Phi |\mathcal{M}|^2 = \mathcal{O}(\alpha^2 \alpha_s) \quad \text{with} \quad \alpha_s = \frac{g_s^2}{4\pi}$$

$\mathcal{M}$



$\mathcal{M}^*$

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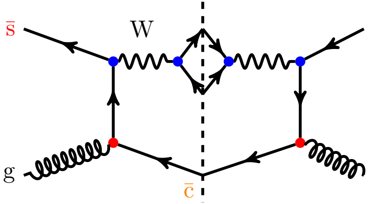
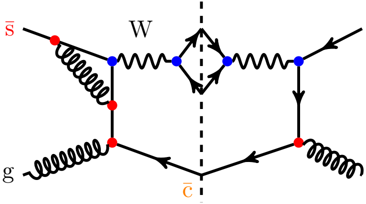
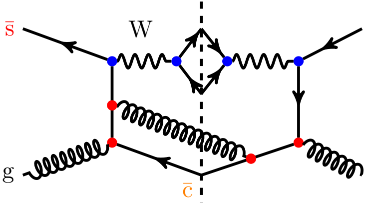


$\mathcal{M}^*$

$\rightarrow$ NLO QCD corrections ($\alpha_s\, \sigma_{\mathrm{NLO\ QCD}}$) contains all contributions $\mathcal{O}(\alpha^2 {\alpha_s}^2)$



Virtual



Real

Virtual corrections lead to loop integrals over the internal momenta of the type

$$\int \frac{\mathrm{d}^4\ell}{(2\pi)^4} \frac{1}{(\ell^2 - \Delta)^n} \cdots$$

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They feature

- Infrared (IR) divergences: $\ell \to 0$
- Ultraviolet (UV) divergences: $\ell \to \infty$
  $\to$ *Renormalisation*: absorption of div. in $\mathcal{L}$ parameters using physical conditions
- Regularisation (mathematically sound) to obtain finite expression

Virtual corrections lead to loop integrals over the internal momenta of the type

$$\int \frac{\mathrm{d}^4\ell}{(2\pi)^4} \frac{1}{(\ell^2 - \Delta)^n} \cdots$$

They feature

- Infrared (IR) divergences: $\ell \to 0$
- Ultraviolet (UV) divergences: $\ell \to \infty$
  $\to$ *Renormalisation*: absorption of div. in $\mathcal{L}$ parameters using physical conditions
- Regularisation (mathematically sound) to obtain finite expression

- Virtual corrections are obtained
  - Numerically
  - Analytically
- Nowadays one-loop automatised and available in public codes
  $\hookrightarrow$ More than 200 000 diagrams in $gg \to \mu^-\bar{\nu}_\mu e^+\nu_e b\bar{b}b\bar{b}$ [Denner,MP,Lang; 2008.00918]
- Current frontier for LHC phenomenology: 2-loop with 5/4 external legs in QCD/EW

Integration of **real corrections** also lead to IR divergences!

$$\int \mathrm{d}\Phi^{n}\, 2\mathrm{Re}\left\{\mathcal{M}_{\mathrm{tree}}\mathcal{M}^{*}_{\mathrm{virt}}\right\} + \int \mathrm{d}\Phi^{n+1}\, \left|\mathcal{M}_{\mathrm{real}}\right|^{2} = \mathrm{IR\ finite}$$

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- Reason behind: KLN (Kinoshita-Lee-Nauenberg) theorem

"IR divergences coming from loop integrals are canceled by IR divergences coming from phase-space integrals"

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$\rightarrow$ IR divergences arise when particles are soft ($E \rightarrow 0$) or collinear to each other
$\rightarrow$ Physical picture: detectors cannot resolve low energy and collinear particles

$\rightarrow$ How to compute them numerically with Monte Carlo integration?!

Subtraction procedure

$$
\alpha_s\, \sigma_{\text{NLO QCD}} = \int \mathrm{d}\Phi^{n} \quad 2\mathrm{Re}\left\{\mathcal{M}\mathcal{M}_{\text{virt}}\right\}
+ \int \mathrm{d}\Phi^{n+1} \left|\mathcal{M}_{\text{real}}\right|^2
$$

Subtraction procedure

$$\alpha_s\, \sigma_{\text{NLO QCD}} = \int \mathrm{d}\Phi^n \left[2\mathrm{Re}\left\{\mathcal{M}\mathcal{M}_{\text{virt}}\right\} + \tilde{D}_{\text{sub}} \right] + \int \mathrm{d}\Phi^{n+1} \left|\mathcal{M}_{\text{real}}\right|^2 - \int \mathrm{d}\Phi^n \underbrace{\mathrm{d}\Phi^1\, D_{\text{sub}}}_{=\tilde{D}_{\text{sub}}}$$

- D_{sub} has same singular behaviour as $|\mathcal{M}_{\text{real}}|^2$
- D_{sub} simple enough to be integrated analytically

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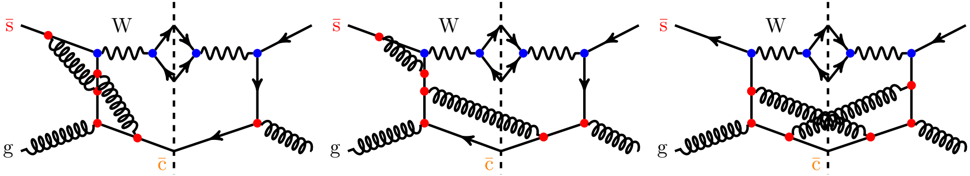
- Requires profound understanding of QFT amplitudes
- Requires high level of automation
  $\rightarrow$ 30 conterterms in $\mathrm{gg} \rightarrow \mu^-\bar{\nu}_\mu \mathrm{e}^+\nu_\mathrm{e}\bar{b}b\bar{b}b\mathrm{g}$ [Denner,MP,Lang; 2008.00918]
- Numerical stability is key
  $\rightarrow$ General algorithms for NNLO QCD calculations available
  $\rightarrow$ for N^{3}LO QCD: only working for specific cases

Moving to NNLO QCD

→ NNLO QCD corrections ($\alpha_s{}^2\, \sigma_{\text{NNLO QCD}}$) contains all contributions $\mathcal{O}(\alpha^2 \alpha_s{}^3)$



Double virtual **Real-virtual** **Double real**

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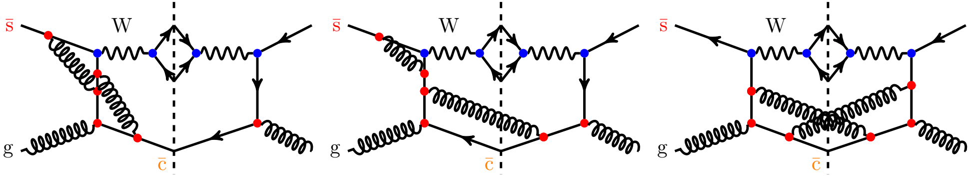

Double virtual **Real-virtual** **Double real**

→ Moving from NLO to NNLO is not incremental!
↪ Everything more complicated!

$pp \to W^+ j_c$

Contrib.	LO	NLO	NNLO
$\bar{s}g$	✓	✓	✓
sg	—	—	✓
$s\bar{s}$	—	✓	✓
$\bar{s}\bar{s}$	—	✓	✓
$\bar{s}q$	—	✓	✓
qq'	—	✓	✓
gq	—	—	✓
gg	—	✓	✓

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→ Bottleneck(s)

- two-loop virtual corrections
- numerically stable subtraction scheme
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gq	—	—	✓
gg	—	✓	✓

→ Bottleneck(s)

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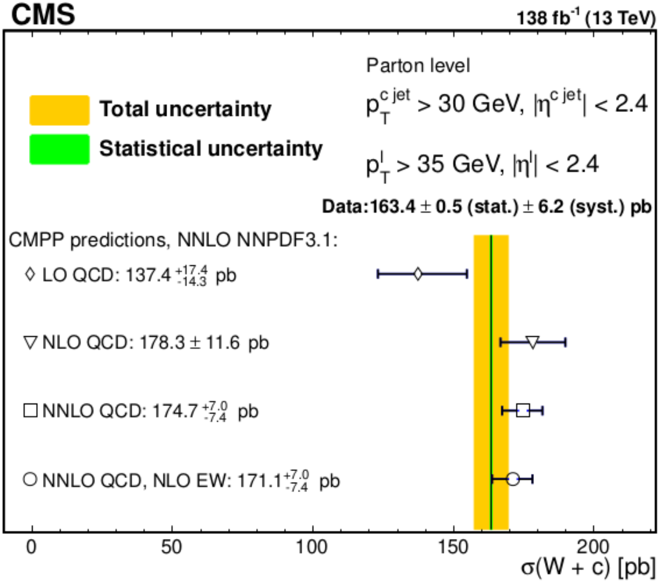
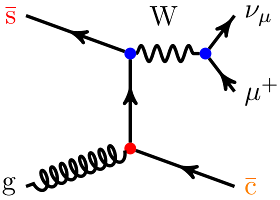
- numerically stable amplitudes

→ CPU intensive business

- State-of-the-art computation: $\mathcal{O}(0.3 - 2 \text{ Millions cpu.h})$
- Most complicated to date: $\mathcal{O}(10 \text{ Millions cpu.h})$
  $\Rightarrow$ No calculations on laptops
  $\hookrightarrow$ High Performance Computing clusters





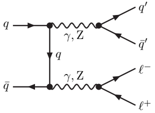
[CMS + Czakon, Mitov, MP, Poncelet; 2308.02285]

What could we do more?

Theory

- Subleading EW contributions $\sim 1\%$

  $\rightarrow$ [Denner, Dittmaier, MP, Schwan; 1907.02366]



- N3LO QCD $\sim 1\%$ (?)
- NNLO QCD matched to parton shower
  $\rightarrow$ inclusion of EW corrections (?)
- NNLO QCD corrections to W+D hadron [Generet, Poncelet, Muškinja; 2510.24525]
  $\rightarrow$ inclusion of EW corrections (?)

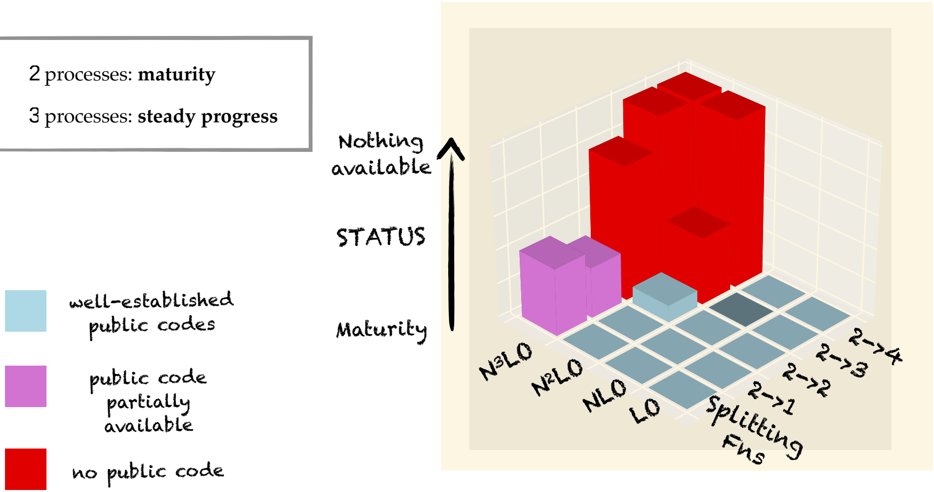
Theory-Experiment

- Improvement of PDF
- Use of common jet algorithm

[source: Luca Buonocore, DIS24, Grenoble, France]

Where Do We Come From? What Are We? Where Are We Going?

- 2   2 processes: **maturity**
- 2   3 processes: **steady progress**



5

Theoretical predictions at the LHC in very good state
Each further step become increasingly more complicated

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Each further step become increasingly more complicated

- N3LO QCD for colourless processes (Higgs, Drell-Yan, di-photon, etc.)
  $\rightarrow$ N3LO QCD PDFs
- Mixed NNLO QCD-EW corrections (Drell-Yan)
- 2-loop amplitudes for $2 \rightarrow 2/3$ with many scales
- NLO QCD+EW for $2 \rightarrow 8$ processes (ttbb, ttV, etc.)
- Matching of NNLO QCD to PS (colourless processes)
  $\rightarrow$ (N)NLO QCD+EW matching to PS (Drell-Yan, di-boson)
- Improvement of PS accuracy

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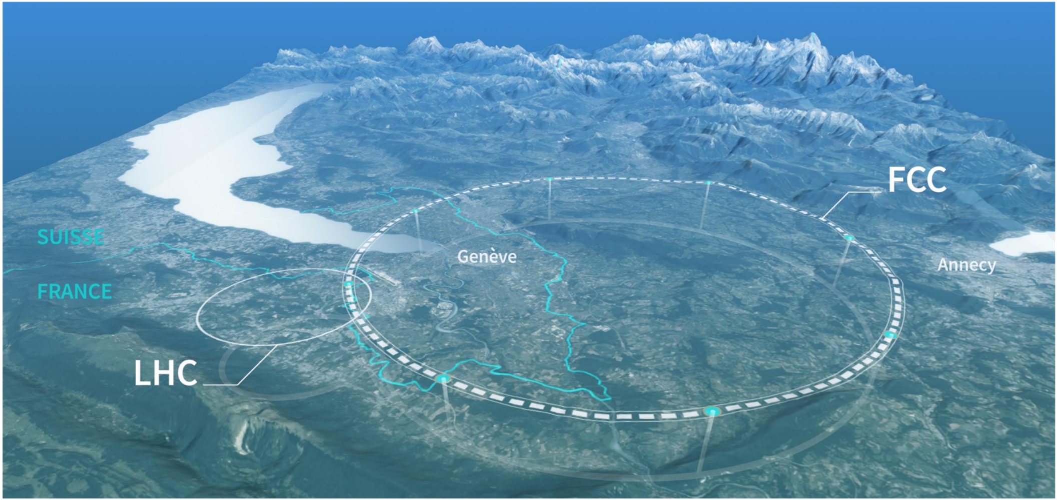
Limiting factors (?)

$\rightarrow$ Non-perturbative physics: PDF, Hadronisation, Multiple-parton interaction (MPI)

- **Precision at future colliders**
  $\rightarrow$ Illustration with Higgs and Electroweak physics

- **FCC**: 90km-long tunnel to collide electron/proton up to even higher energy (100 TeV)
  → choice number 1 for the future of European community



Why FCC?

→ Review of Christophe Grojean "Future Circular Collider: A Physics Manifesto"

https://indico.cern.ch/event/1582427/

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- Much cleaner initial state
- Known collision energy
- Huge luminosity at specific energies
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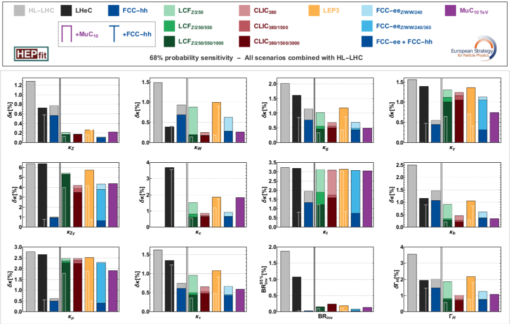
→ LHC → FCC-ee:
Dominant uncertainties shift from experimental modelling to theoretical predictions

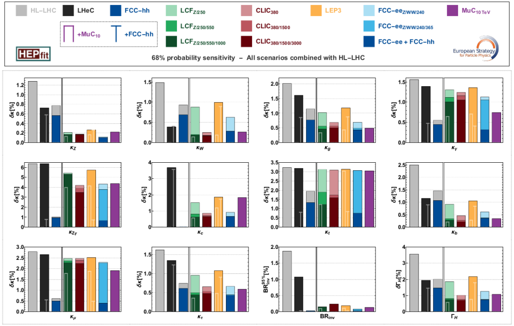
Illustration of precision physics



[de Blas et al.; 2511.03883]

Precision everywhere! Example for Higgs and EW physics

Illustration of precision physics

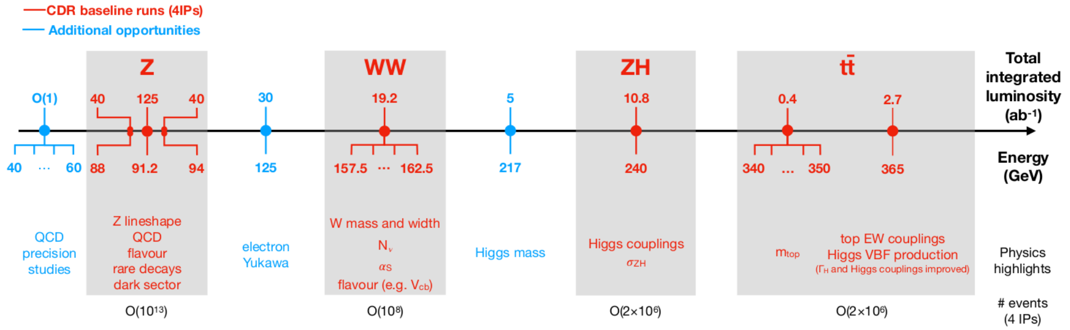


Observable	Current	FCC-ee	LCF	LEP3
Δm_Z (keV)	2000	4 (100)	200	7.5 (100)
$\Delta\Gamma_Z$ (keV)	2300	4 (12)	125	7.5 (23)
δR_μ $(\times 10^{-6})$ $R_\mu \equiv \frac{\Gamma_{\text{had}}}{\Gamma_\mu}$	1600	2.4 (2.3)	90 (90)	4.5 (2.3)
δR_b $(\times 10^{-6})$ $R_b \equiv \frac{\Gamma_b}{\Gamma_{\text{had}}}$	3300	1.2 (1.6)	70 (60)	2.2 (3.0)
$\Delta \sin^2\theta_W$ $(\times 10^6)$	130	0.4 (0.5)	2.7 (2.3)	0.75 (0.95)
$\Delta\alpha(m_Z)^{-1}$ $(\times 10^3)$	14	0.8, 3.8	–	1.4, 7.3
Δm_W (keV)	9900	180 (160)	500 (1600)	430 (700)
$\Delta\Gamma_W$ (keV)	42000	270 (200)	2000	650 (500)

[de Blas et al.; 2511.03883]

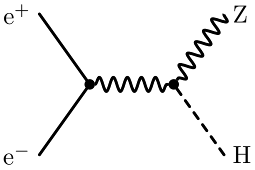
Precision everywhere! Example for Higgs and EW physics

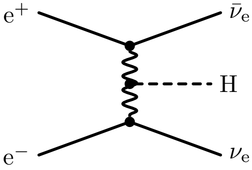
FCC-ee physics



- Higgs physics at FCC-ee through ZH production (and VBF)
- W physics at FCC-ee through WW production

Higgs physcis





Higgs decay widths and Higgs couplings at ILC and FCC-ee

LHC HXS WG; de Blas et al., 1905.03764; HL-LHC: Cepeda et al., 1902.00134; ILC: Bambade et al., 1903.01629 FCC-ee: Freitas et al., 1906.05379

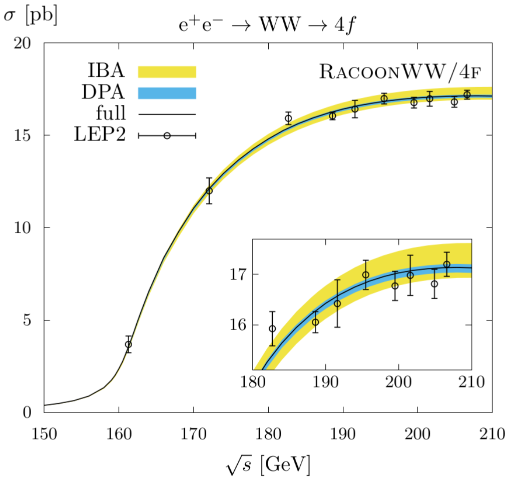
	experimental accuracy			theory uncertainty			param. unc.	
	HL-LHC	ILC250	FCC-ee	current	source	prospect	prospect	source
$H \to b\bar{b}$	4.4%	2%	0.8%	0.4%	α_s^5	0.2%	0.6%	m_b
$H \to \tau\tau$	2.9%	2.4%	1.1%	0.3%	α^2	0.1%	negligible	
$H \to \mu\mu$	8.2%	8%	12%	0.3%	α^2	0.1%	negligible	
$H \to gg$	1.6% (prod.)	3.2%	1.6%	3.2%	α_s^4	1%	0.5%	α_s
$H \to \gamma\gamma$	2.6%	2.2%	3.0%	1%	α^2	1%	negligible	
$H \to \gamma Z$	19%			5%	α	1%	0.1%	M_H
$H \to WW$	2.8%	1.1%	0.4%	0.5%	$\alpha_s^2, \alpha_s\alpha, \alpha^2$	0.3%	0.1%	M_H
$H \to ZZ$	2.9%	1.1%	0.3%	0.5%	$\alpha_s^2, \alpha_s\alpha, \alpha^2$	0.3%	0.1%	M_H

[Source: Stefan Dittmaier]

→ **Theory challenges**

- full NNLO EW for $e^+e^- \to ZH$
- leading NNLO EW for $e^+e^- \to \nu\nu H$ (VBF)
- Higher precision in Higgs decay

State of the art:



[Denner, Dittmaier, Roth, Wieders; hep-ph/0505042, hep-ph/0502063]

+[Schael et al.; 1302.3415]

→ NLO EW for $e^+e^- \to 4f$

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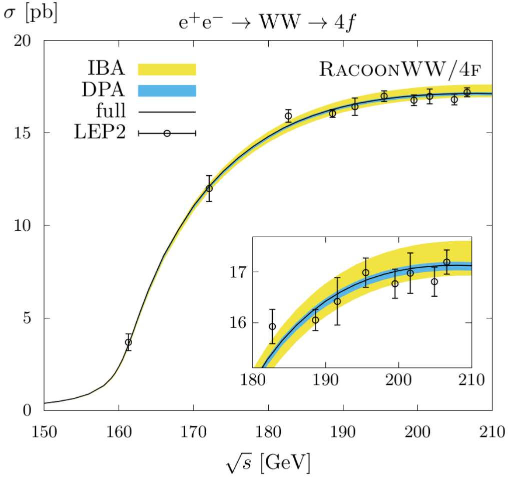

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→ NLO EW for $e^+e^- \to 4f$

→ ΔM_W[MeV]:
33 (LEP2), 12 (current), 10 (CMS), ...
... **0.5 (FCC-ee)**

→ **Theory challenges**

- Considering also hadronic decays
- NNLO EW for $e^+e^- \to 4f$
  → use for EFT at threshold
  → **resonance treatement!**
- mixed NNLO QCD-EW and NNLO QCD for hadronic channels

List of priorities (draft)

1. Lay the foundations for the conceptual design studies of four (or more) detectors
2. Consolidate IR layout, detector integration, and related background mitigation
3. Collaborate with IT to develop a computing architecture model for experiments
4. Complete the software & analysis toolkit to ease detector performance comparison
5. Confirm, with full analyses, the current uncertainty estimates on EWPOs (Z and W)
6. Gather the worldwide theory community to address the theoretical challenges
7. Streamline and optimise the procedure for centre-of-mass energy calibration
8. Develop an efficient PED Education/Communication/OutReach/InReach strategy
9. Ascertain the detector cost estimate
10. Articulate the physics case, feasibility, and schedule implications of other $\sqrt{s}$ stages
11. Anticipate FCC-PED structure and management in the project phase (2027-2033)

C. Grojean, `https://indico.cern.ch/event/1582427/`

$\rightarrow$ Theory predictions are a critical aspect for the success of the FCC!

Summary/Outlook

Exciting physics at the LHC! (... and future colliders)

$\rightarrow$ Explore fundamental aspect of particle physics
$\rightarrow$ Precision era at the LHC (and even more at future colliders)

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  - Searches for new phenomena

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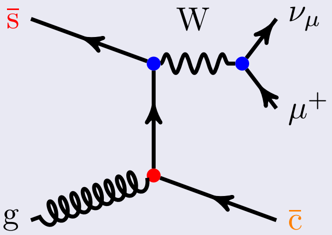
[source: bing image creator, Engel, Pellen]

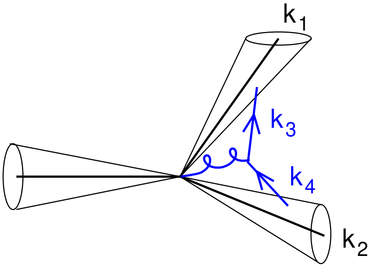
BACK-UP



- Direct link between W+c measurements and strange-quark PDF
  $\rightarrow$ Insight in proton content
- Critical for other measurements (e.g. W-boson mass)
- Test of (perturbative) QCD
  $\rightarrow$ s-s̄ asymmetry predicted at 3-loop in QCD [Catani, de Florian, Rodrigo, Vogelsang; hep-ph/0404240]
- Study of flavour jets
- Further test of the Standard Model

Jet algorithm

- Beyond NLO, flavour jet algorithm is required
  $\rightarrow$ Otherwise no IR-safe definition of flavour jets
  $\rightarrow$ Soft wide angle radiations are problematic
- flavour k_T algorithm with $R = 0.4$ [Banfi, Salam, Zanderighi; hep-ph/0601139]
  $\rightarrow$ Soft radiations are clustered first
  $\rightarrow$ rules:
  - $\mathrm{c} + \mathrm{c} = \mathrm{j}$ or $\mathrm{c} + \bar{\mathrm{c}} = \mathrm{j}$
  - $\mathrm{c} + \mathrm{c} + \bar{\mathrm{c}} = \mathrm{j}_\mathrm{c}$ or $\bar{\mathrm{c}} + \mathrm{c} + \bar{\mathrm{c}} = \mathrm{j}_\mathrm{c}$



Jet algorithms - definitions (1)

$\rightarrow$ Freedom in choosing whether cc and $\bar{c}\bar{c}$ are flavoured

Variation of flavour k_{T} algorithm [Banfi, Salam, Zanderighi; hep-ph/0601139]

- flavoured k_{T} algorithm, charge dependent (k_{T}CD)
- flavoured k_{T} algorithm, charge agnostic (k_{T}CA)
- flavoured k_{T} algorithm, charge dependent, with beam definition including W momenta (k_{T}CDB)

Best predictions @ 13 TeV - cross sections

Order	$\sigma_{\mathrm{W^+j_c}}$ [pb]	$\sigma_{\mathrm{W^-j_c}}$ [pb]	$R_{\mathrm{W^\pm j_c}} = \sigma_{\mathrm{W^+j_c}}/\sigma_{\mathrm{W^-j_c}}$
LO	$113.817(2)^{+12.4\%}_{-9.87\%}$	$119.711(2)^{+12.4\%}_{-9.88\%}$	$0.95076(2)^{+0.013\%}_{-0.021\%}$
NLO QCD	$162.4(1)^{+7.2\%}_{-6.6\%}$	$168.1(1)^{+6.9\%}_{-6.4\%}$	$0.9659(9)^{+0.29\%}_{-0.21\%}$
NNLO QCD	$168.6(8)^{+0.7\%\ +3.8\%(\mathrm{PDF})}_{-2.1\%\ -3.8\%(\mathrm{PDF})}$	$173.9(1.9)^{+0.6\%\ +3.7\%(\mathrm{PDF})}_{-1.8\%\ -3.7\%(\mathrm{PDF})}$	$0.96(1)^{+0.2\%\ +2.1\%(\mathrm{PDF})}_{-0.3\%\ -2.1\%(\mathrm{PDF})}$

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Theory uncertainty in QCD

- In perturbation theory, dependence on *renormalisation* and *factorisation* scales
- Variation of the scales: proxy for evaluating missing higher orders in QCD
  $\rightarrow$ Simple recipe to catch some of the theory errors

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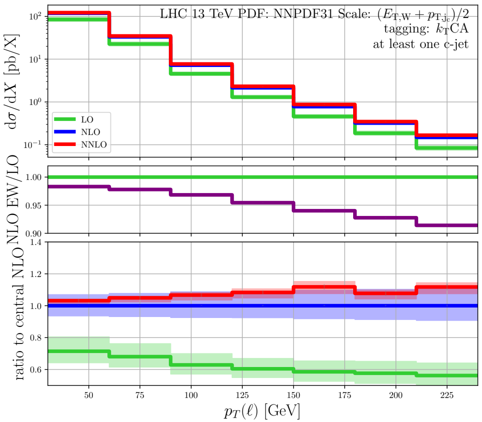
- PDF uncertainty dominant at NNLO QCD

Order	$\sigma_{\mathrm{W^+j_c}}$ [pb]	$\sigma_{\mathrm{W^-j_c}}$ [pb]	$R_{\mathrm{W^\pm j_c}} = \sigma_{\mathrm{W^+j_c}}/\sigma_{\mathrm{W^-j_c}}$
$\sigma_{\mathrm{NLO\ EW}}$	117.399(2)	111.627(2)	0.95084(2)
$\delta_{\mathrm{NLO\ EW}}$[%]	−1.93	−1.92	−0.01

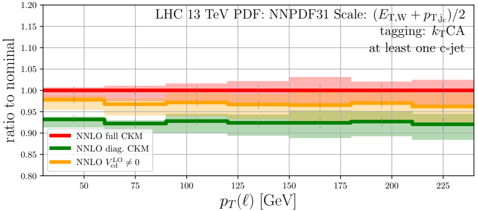
- EW corrections null in the ratio

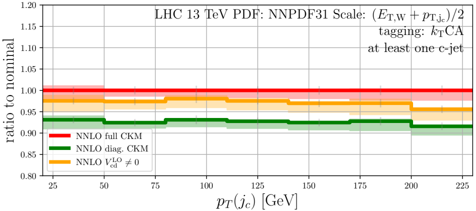
Best predictions @ 13 TeV - Differential distributions



- Good perturbative behaviour for QCD corrections
- Sudakov logarithm for EW corrections

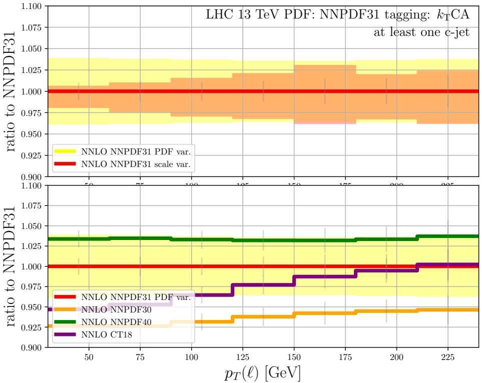
Effect of non-diagonal CKM @ NNLO QCD





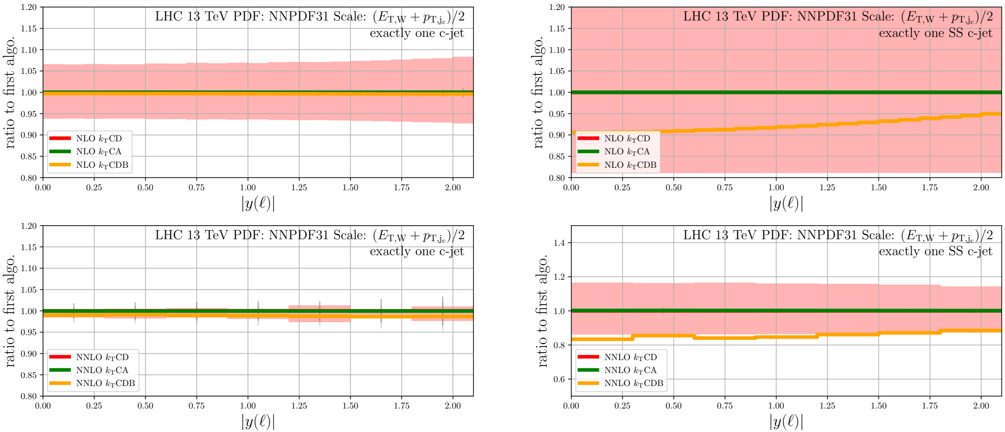
- full CKM / no CKM $\sim 7.5 - 11\%$
- full CKM / $V_{\mathrm{cd}}^{\mathrm{LO}} \neq 0 \sim 3\%$
  $\rightarrow$ Original approximation rather good
  $\rightarrow$ Full CKM dependence up to NNLO QCD for precise predictions

PDF uncertainty



- NNPDF3.1 variation $\sim \pm 4\%$
- Spread of various PDF sets $\sim 10\%$
  $\rightarrow$ PDF error is the largest theoretical uncertainty

Jet algorithm (1)



- No difference at NLO and NNLO for exactly one-jet
- Large differences for exactly one SS c-jet

Jet algorithms - definitions (2)

→ Flavoured anti-k_T algorithm

$$d_{ij}^{(flavored)} = d_{ij}^{(standard)} \times \begin{cases} \mathcal{S}_{ij}\,, & \text{if both } i \text{ and } j \text{ have non-zero flavor of opposite sign,} \\ 1\,, & \text{otherwise.} \end{cases}$$

where

$$\mathcal{S}_{ij} = 1 - \theta\left(1-\kappa_{ij}\right)\cos\left(\frac{\pi}{2}\kappa_{ij}\right) \quad \text{with} \quad \kappa_{ij} \equiv \frac{1}{a}\,\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}\,.$$

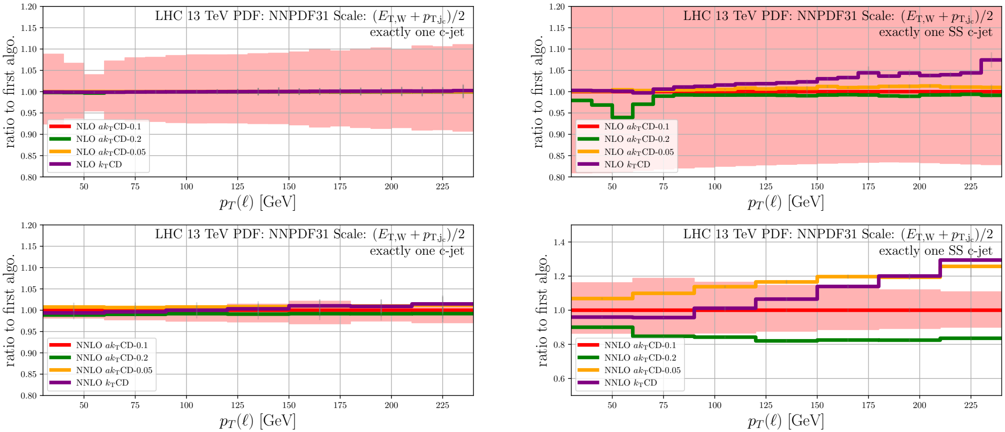
[Czakon, Poncelet, Mitov; 2205.11879]

Variation of anti-k_T algorithm

- flavoured anti-k_T algorithm, charge dependent, with $a = 0.2, 0.1, 0.05$ ($\mathrm{a}k_\mathrm{T}$CD-0.2, $\mathrm{a}k_\mathrm{T}$CD-0.1, $\mathrm{a}k_\mathrm{T}$CD-0.05)
- flavoured anti-k_T algorithm, charge agnostic, with a $= 0.1$ ($\mathrm{a}k_\mathrm{T}$CA-0.1).

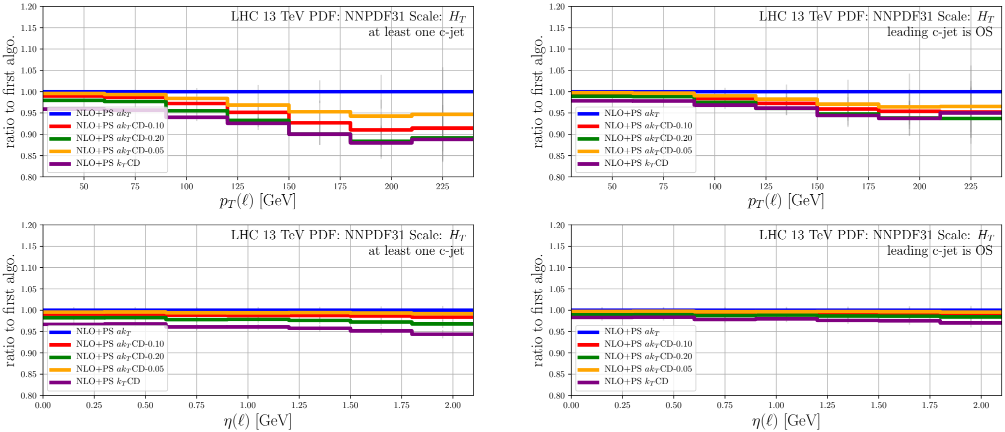
Alternatives: [Caletti, Larkoski, Marzani, Reichelt; 2205.01117, 2205.01109], [Gauld, Huss, Stagnitto; 2208.11138], [Caola et al.; 2306.07314]

Jet algorithm (2)



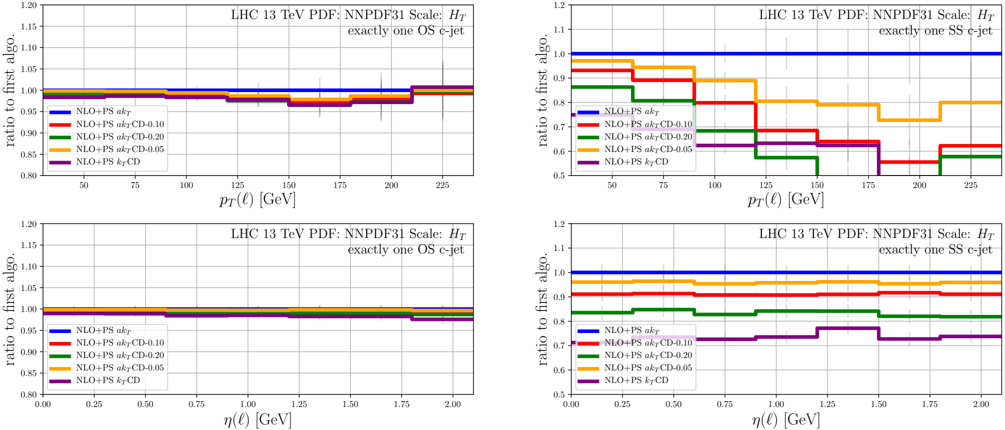
- No (small) difference at NLO and NNLO for exactly one-jet
- Large differences for exactly one SS c-jet

Jet algorithm (3) at NLO+PS



- 5–10% differences for at least one c-jet (inclusive)
- Below 5% differences for leading c-jet is OS

Jet algorithm (4) at **NLO+PS**



- $<3\%$ differences for exactly one OS c-jet
- Huge differences for exactly one SS c-jet
  $\rightarrow$ exactly one OS c-jet is preferred in this respect

D meson and charm quark

D mesons

D^+ : $c\bar{d}$, D^- : $d\bar{c}$, D^0 : $c\bar{u}$, $\bar{D}^0$: $\bar{c}u$, D_s^+ : $c\bar{s}$, D_s^- : $s\bar{c}$
$\rightarrow$ Masses $\sim$ 1.9 GeV

- Charm quarks/jets are not detected at colliders
  $\rightarrow$ instead D mesons
- Mesons identified through their decay products
- Use of Monte Carlo simulation to match W+D meson measurement to a W+c process
  $\rightarrow$ Transfer function