Speedup of Wave-like Dark Matter Searches Using Entangled Qubits

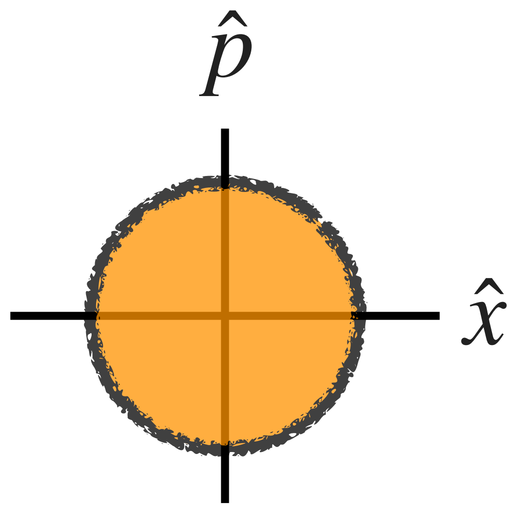
Arushi Bodas
University of Chicago & Fermilab

Based on: AB, S. Ghosh, R. Harnik, 2510.11795

High energy physics seminar, University of Minnesota
April 17, 2026

Motivation

- Dark matter (DM) signal is expected to be weak → reduce noise/enhance signal.

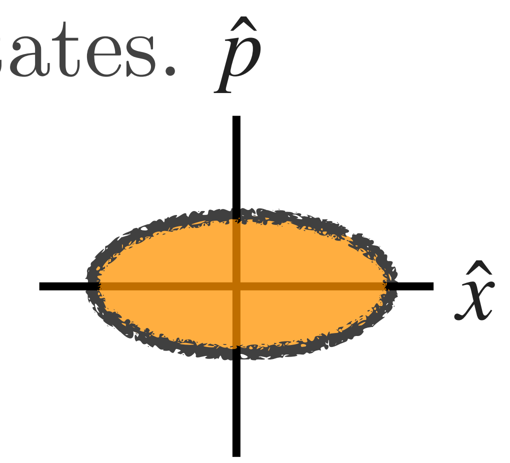


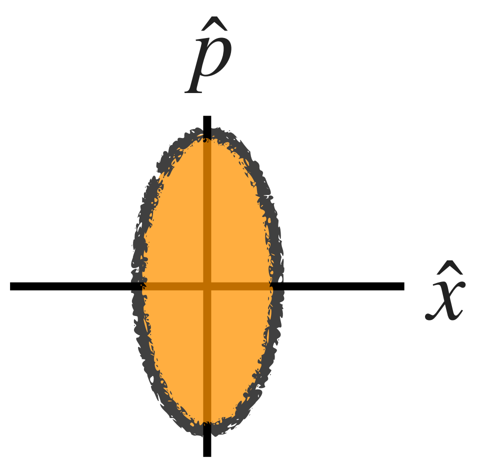
- Minimum noise is set by the uncertainty principle, or the standard quantum limit (SQL).

  Linear scaling: making larger experiments is logistically difficult.

- Going beyond requires non-classical states.

  Squeezing: redistribute uncertainty.



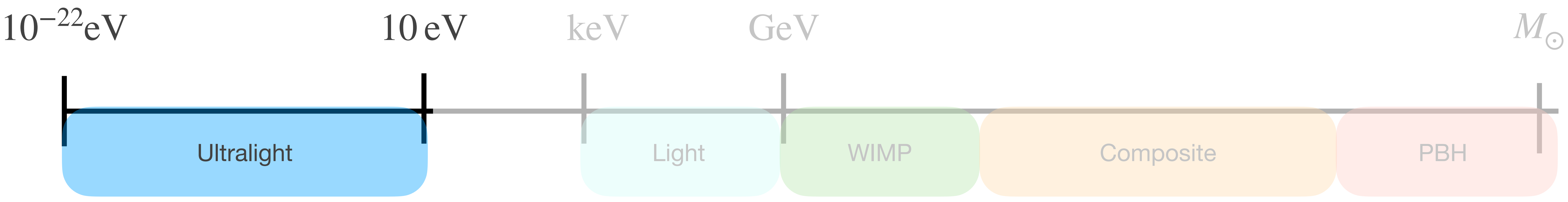


  Entangled states: add signal coherently at the level of amplitude.

- Entangled states of qubits are crucial to realize the advantage of quantum computers. Therefore, we can expect the prospects of generating entangled qubit states to improve in the future.

- We wanted to understand the qualitative and quantitative advantages that could be gained from using entangled qubits for DM detection.

DM landscape



For $m_{\rm DM} \lesssim 10\,$eV, DM field can be modeled as a classical wave.

$$f_{\rm DM} = \frac{\sqrt{2\rho_{\rm DM}}}{m_{\rm DM}} \cos(\omega_{\rm DM} t - \vec{k}\cdot\vec{x} - \varphi)\,\hat{n}$$

$\omega_{\rm DM} = m_{\rm DM}\,(1 + v^2/2)$

φ: random phase

Coherence time $\tau_{\rm DM} \sim \frac{2\pi}{m_{\rm DM} v^2} \sim \frac{10^6}{m_{\rm DM}}$

$(m_{\rm DM} \sim \mu{\rm eV} \rightarrow \tau_{\rm DM} \sim {\rm ms})$

Coherence length $\lambda_{\rm coh} \sim \frac{2\pi}{m_{\rm DM} v}$

$(m_{\rm DM} \sim \mu{\rm eV} \rightarrow \lambda_{\rm coh} \sim {\rm km})$

Light DM candidates

Axion-like particles (ALPs)

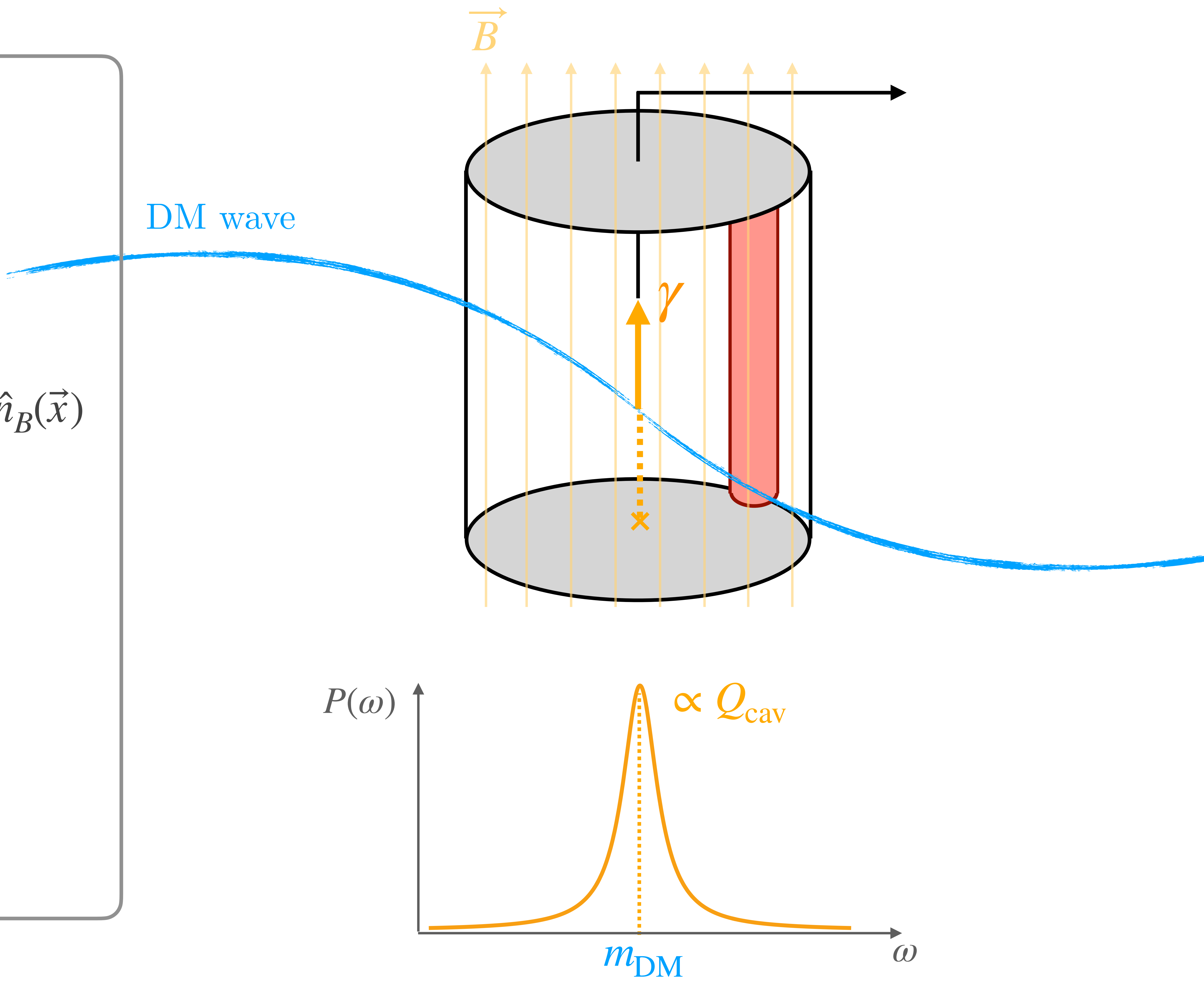
$$\mathscr{L}_{\rm int} \supset \frac{g_{a\gamma\gamma}}{4} aF\tilde{F} = g_{a\gamma\gamma} aE\,.\,B$$

$$\vec{E}_{\rm DM}(\vec{x},t) = g_{a\gamma\gamma}\frac{|\vec{B}\,|}{m_{\rm DM}} f(m_{\rm DM}R)\sqrt{2\rho_{\rm DM}}\,\cos(m_{\rm DM}t-\varphi)\,\hat{n}_B(\vec{x})$$

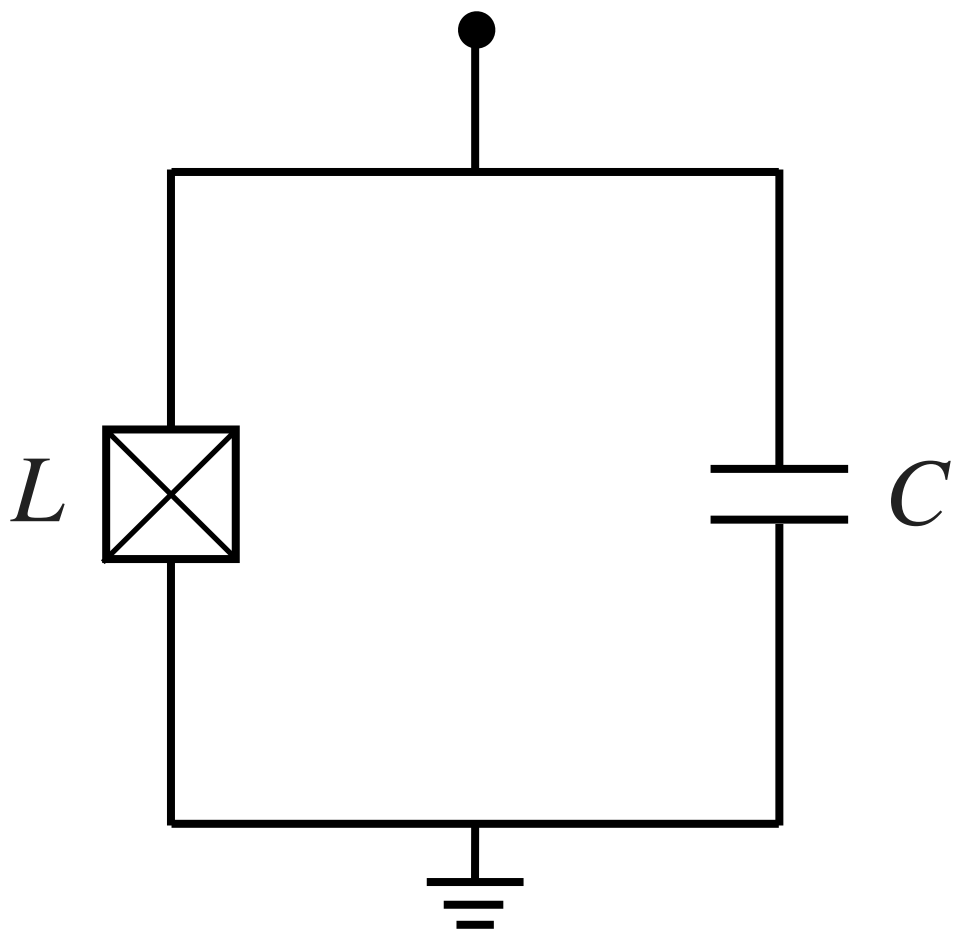
Dark photons

$$\mathscr{L}_{\rm int} \supset \epsilon F F' = \epsilon\,(E\,.\,E' + B\,.\,B')$$

$$\vec{E}_{\rm DM}(\vec{x},t) = \epsilon\sqrt{2\rho_{\rm DM}}\,\cos(m_{\rm DM}t-\varphi)\,\hat{n}_{A'}(\vec{x})$$



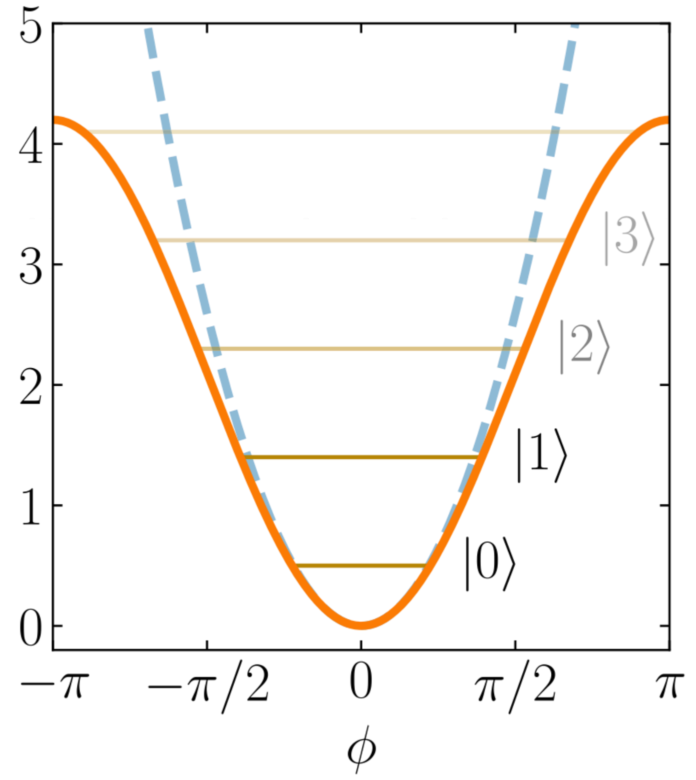
Interaction of E_{DM} with qubits



Transmon qubits

$$H_0 = 4E_C n^2 - E_L \cos\phi \equiv 4E_C n^2 + \frac{E_L}{2}\phi^2 + \mathcal{O}(\phi^3)$$

$$[\phi, n] = i\hbar \; ; \quad \phi = \left(\frac{2\hbar^2 E_C}{E_L}\right)^{1/4} i\,(a - a^\dagger)\,, \quad n = \left(\frac{\hbar^2 E_L}{32 E_C}\right)^{1/4} (a + a^\dagger)$$



$$H_0 = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) \qquad \text{with } \omega = \sqrt{8E_C E_L}/\hbar$$

$$H_{\text{int}} = (2e)\, n\, \vec{d}\,.\,\vec{E}_{\text{DM}} \longrightarrow H_{\text{int}} = (2e)\left(\frac{\hbar^2 E_L}{32 E_C}\right)^{1/4} \vec{d}\cdot\vec{E}_{\text{DM}}\,(a + a^\dagger)$$

$$2\eta\cos(m_{\text{DM}} t - \varphi)$$

For tunability, replace Josephson junction with SQUID. By changing external flux through the SQUID loop, E_L (and therefore ω) can be changed.

Qubits are much easier and faster to tune with tuning range of ~ 1-3 GHz. (Cavity tuning range is ~ 100 MHz)

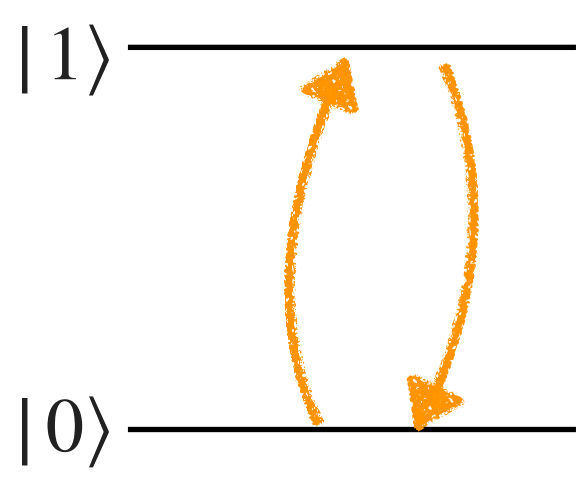
DM-qubit interaction

Qubit: two level system, $|0\rangle$ and $|1\rangle$

$$H_{\text{qubit}} = \omega a^\dagger a + 2\eta \cos(m_{\text{DM}} t - \varphi)\,(a + a^\dagger)$$

This leads to Rabi oscillations: $|\psi\rangle = \psi_0(t)\,|0\rangle + \psi_1(t) e^{-i\omega t}\,|1\rangle$

$$\begin{pmatrix} \psi_0(t) \\ \psi_1(t) \end{pmatrix}_{\text{res}} = \underbrace{\begin{pmatrix} \cos(\eta t) & i e^{-i\varphi} \sin(\eta t) \\ i e^{i\varphi} \sin(\eta t) & \cos(\eta t) \end{pmatrix}}_{U_{\text{DM}}} \begin{pmatrix} \psi_0(0) \\ \psi_1(0) \end{pmatrix}$$



Fastest on resonance

$$\omega_q = m_{\text{DM}}$$

Starting with ground state $|0\rangle$, probability of seeing an excitation due to DM: $p_{\text{sig}} = |\psi_1(t_{\text{exp}})|^2 \approx \sin^2(\eta\, t_{\text{exp}})$

If we had n_q qubits, probability of seeing at least one excitation would grow linearly: $p_{\text{sig}} \approx n_q \sin^2(\eta\, t_{\text{exp}})$

Entangled state protocol

S. Chen, H. Fukuda, T. Inada, T. Moroi, T. Nitta, and T. Sichanugrist, arXiv:2311.10413
A. Ito, R. Kitano, W. Nakano, and R. Takai, arXiv:2311.11632

$$\Delta_\omega = \omega - m_{\rm DM} = 0,\ \varphi = 0$$

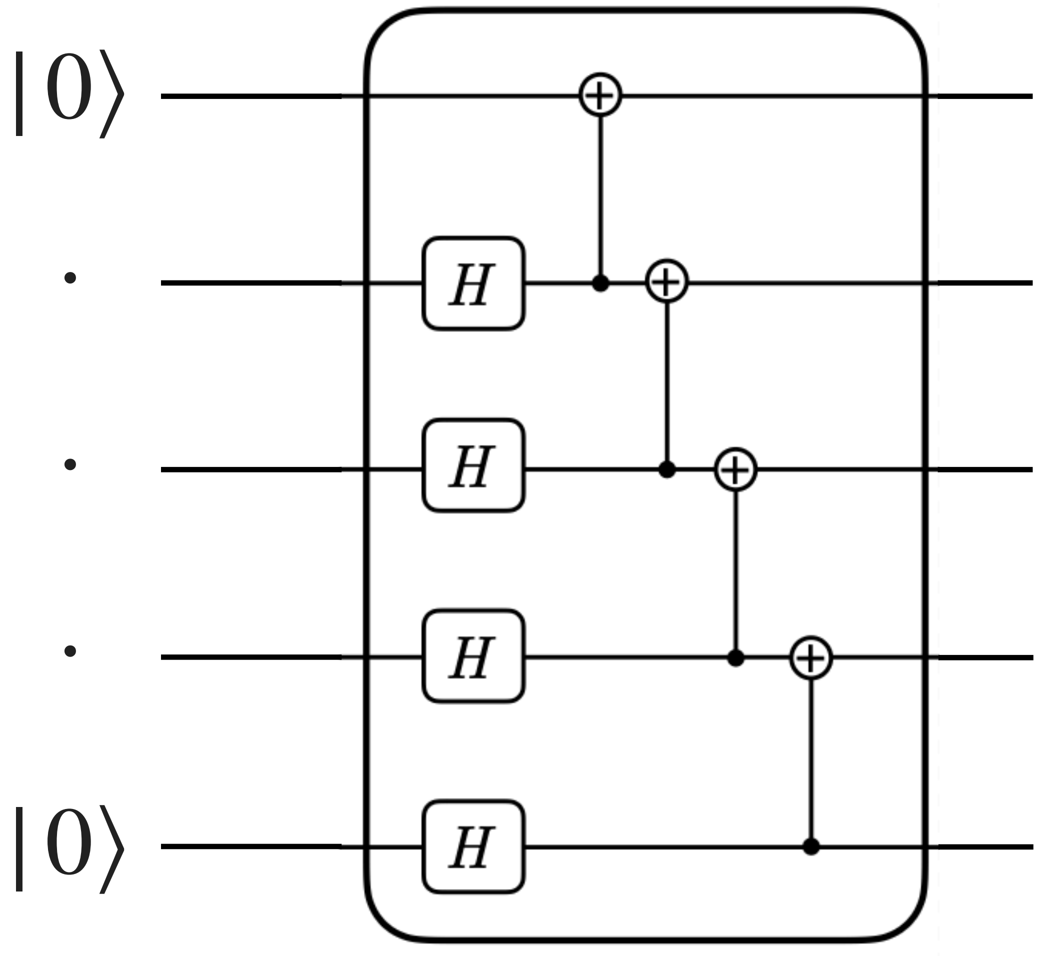
$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} \longrightarrow U_{\rm DM}|\pm\rangle = e^{\pm i\eta t}|\pm\rangle$$

$$\frac{1}{\sqrt{2}}\left(\underbrace{|+\rangle^{\otimes n_q} + |-\rangle^{\otimes n_q}}_{\text{GHZ}}\right) \xrightarrow{U_{\rm DM}^{\otimes n_q}} \frac{1}{\sqrt{2}}\left(e^{i n_q \eta t}|+\rangle^{\otimes n_q} + e^{-i n_q \eta t}|-\rangle^{\otimes n_q}\right)$$

Coherent addition of amplitude

State preparation and readout

CNOT
$0_c \to$ nothing
$1_c \to$ flip q_t

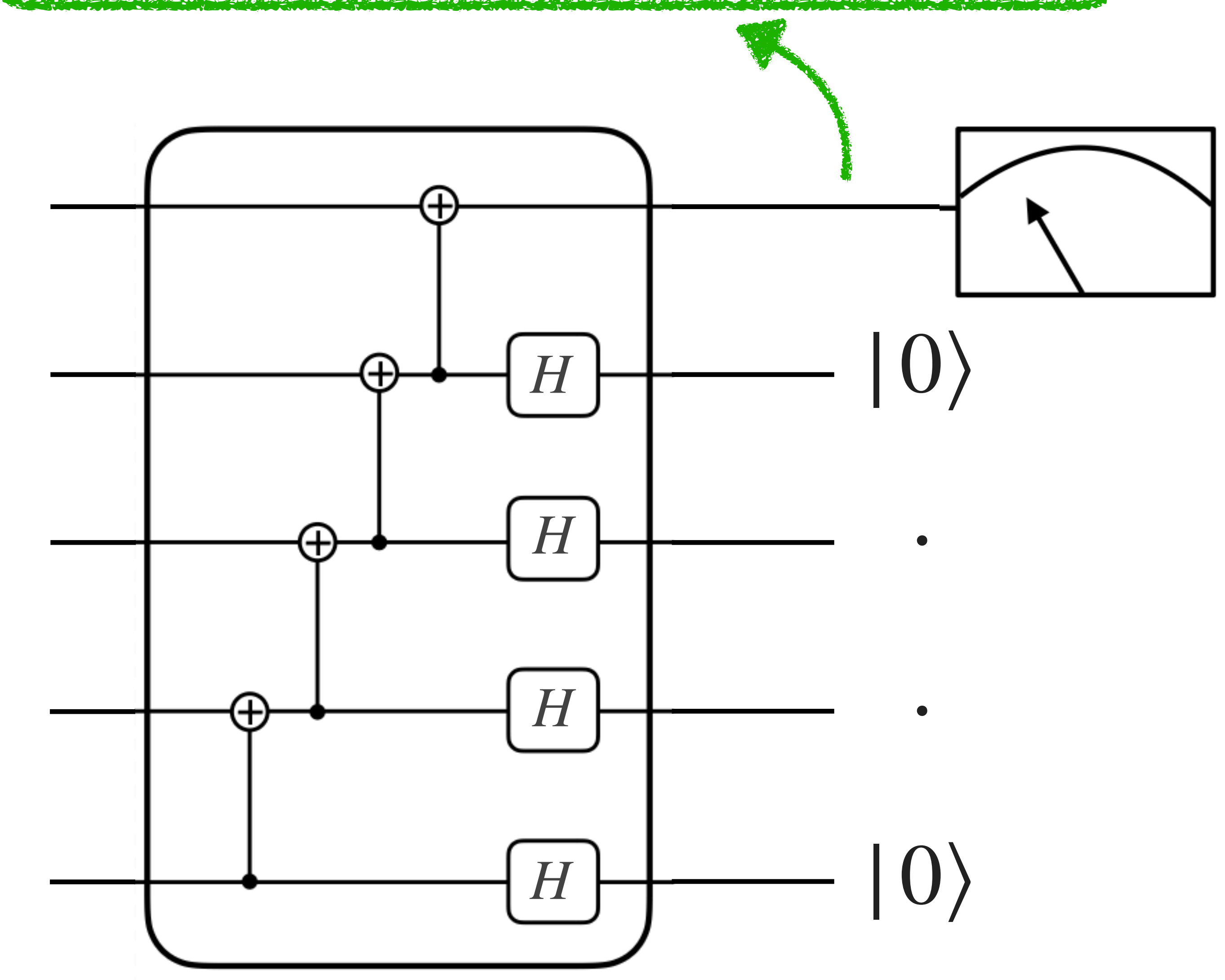


$$\frac{|+\rangle^{\otimes n_q} + |-\rangle^{\otimes n_q}}{\sqrt{2}}$$

DM →

$$\frac{e^{i n_q \eta t}|+\rangle^{\otimes n_q} + e^{-i n_q \eta t}|-\rangle^{\otimes n_q}}{\sqrt{2}}$$

$$\frac{\cos(n_q \eta t)|0\rangle + i\sin(n_q \eta t)|1\rangle}{\sqrt{2}}$$



Probability of seeing an excitation on the 1st qubit:

$$p_{\text{sig}} = \sin^2(n_q \eta t) \approx n_q^2 \eta^2 t^2 \cos^2\varphi$$

1. Are qubits necessary: is there an equivalent setup in conventional power-readout schemes?

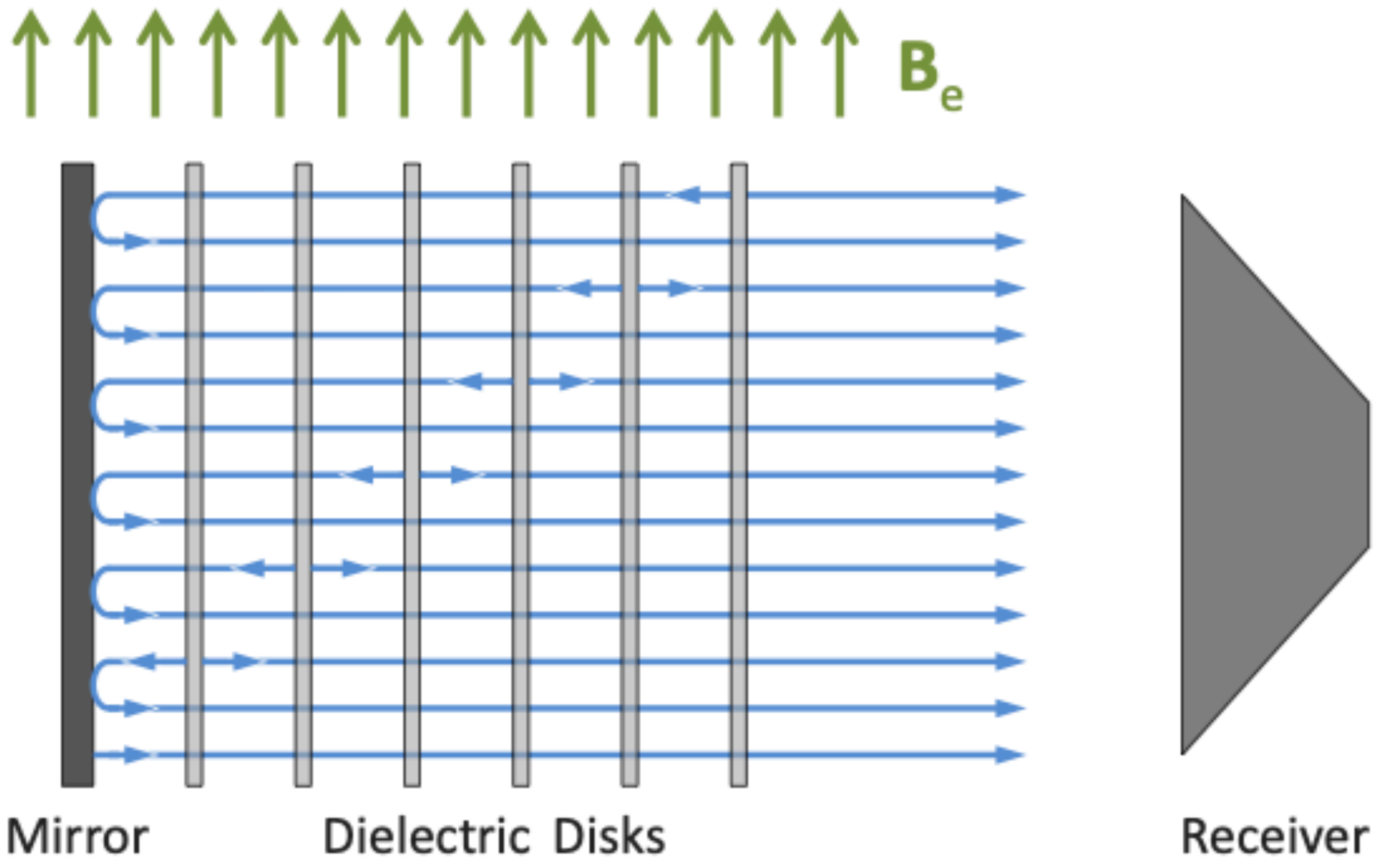
2. How much of the advantage is left after incorporating noise?

3. How many qubits would be needed to compete with a single cavity haloscope?

Are qubits necessary?

Similar amplitude addition can also occur in classical waves, where the energy/power $\propto n^2$.

Say we design a multi(n_{cav})cavity experiment where we manage to add electric fields from all cavities in phase. Then we would expect the power read out to scale as n_{cav}^2.



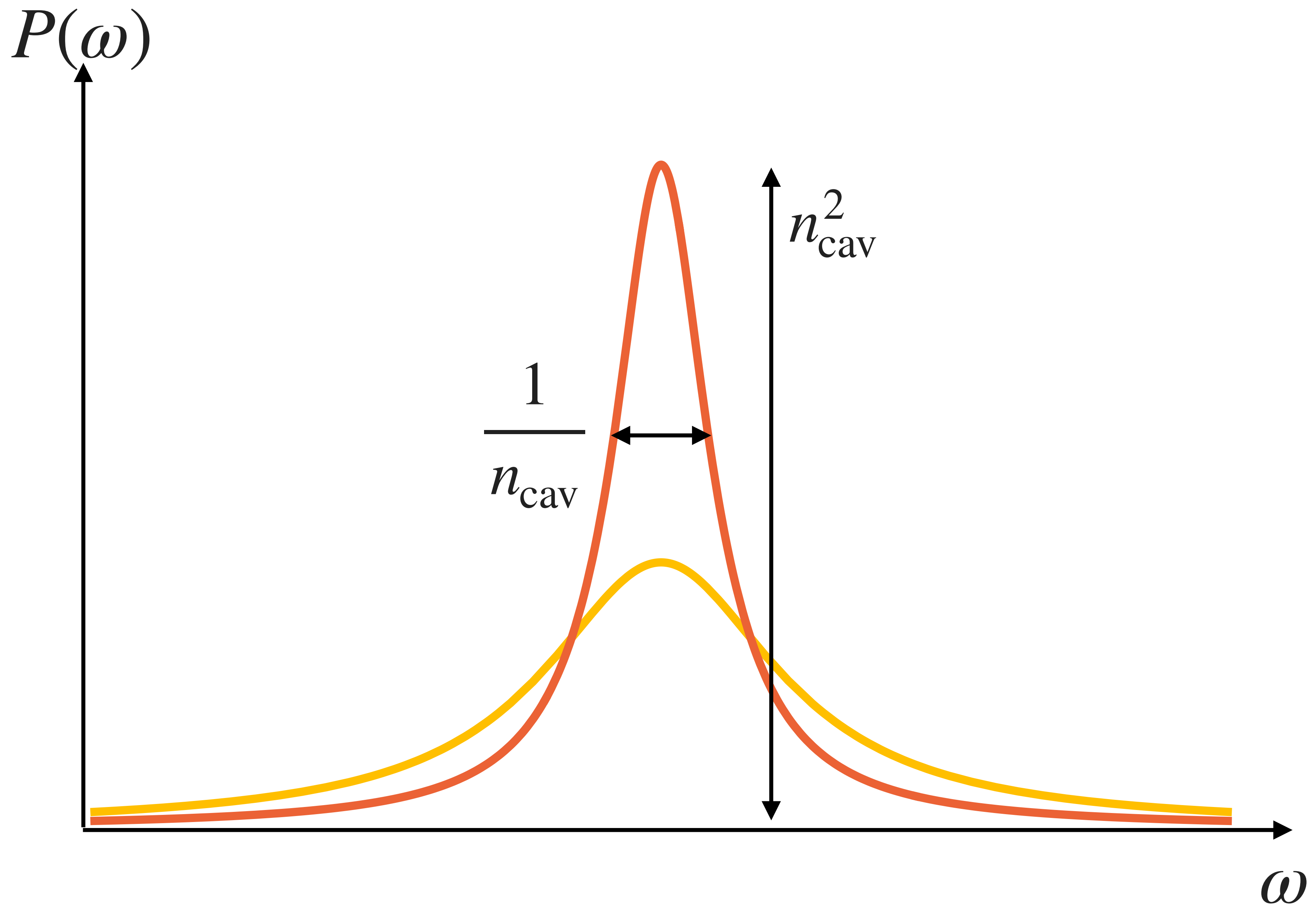
$$P_{\text{peak}} \propto n_{\text{layers}}^2$$

Image from: A. J. Millar et al, arXiv:1612.07057

Are qubits necessary?

Turns out, there is a power-bandwidth tradeoff.

$$\int P(\omega)\, d\omega \propto V_{\text{tot}} \propto n_{\text{cav}} V_{\text{cav}}$$



R. Lasenby, arXiv:1912.11467

N. Blinov, C. Gao, R. Harnik, R. Janish, and N. Sinclair, arXiv:2401.17260

A. J. Millar, G. G. Raffelt, J. Redondo, and F. D. Steffen, arXiv:1612.07057

M. Baryakhtar, J. Huang, and R. Lasenby, arXiv:1803.11455

Thomas-Reiche-Kuhn (TRK) sum rule

$$H = \frac{p^2}{2m} + V(x)\ , \quad [x, p] = i \qquad \text{and } \langle 0 | [x, [H, x]] | 0 \rangle = ?$$

1) $[H, x] = -\frac{i}{m} p$

$$\langle 0 | [x, [H, x]] | 0 \rangle = \frac{1}{m}$$

2) $\langle 0 | [x, [H, x]] | 0 \rangle = \langle 0 | x[H, x] | 0 \rangle - \langle 0 | [H, x] x | 0 \rangle$

Insert a complete basis of energy eigenstates $1 = \sum_n | n \rangle \langle n |$

$$\langle 0 | [x, [H, x]] | 0 \rangle = 2 \sum_n (E_n - E_0) | \langle n | x | 0 \rangle |^2$$

$$\sum_n (E_n - E_0) | \langle n | x | 0 \rangle |^2 = \frac{1}{2m} \xrightarrow{n \text{ particles}} \sum_{n,i} (E_n - E_0) | \langle n | x_i | 0 \rangle |^2 = \frac{n}{2m}$$

Thomas-Reiche-Kuhn (TRK) sum rule

If particles are charged and exposed to an external electric field E_{ext} ,

$$H = \frac{p^2}{2m} + V(x) - q\, E_{\text{ext}}\, x$$

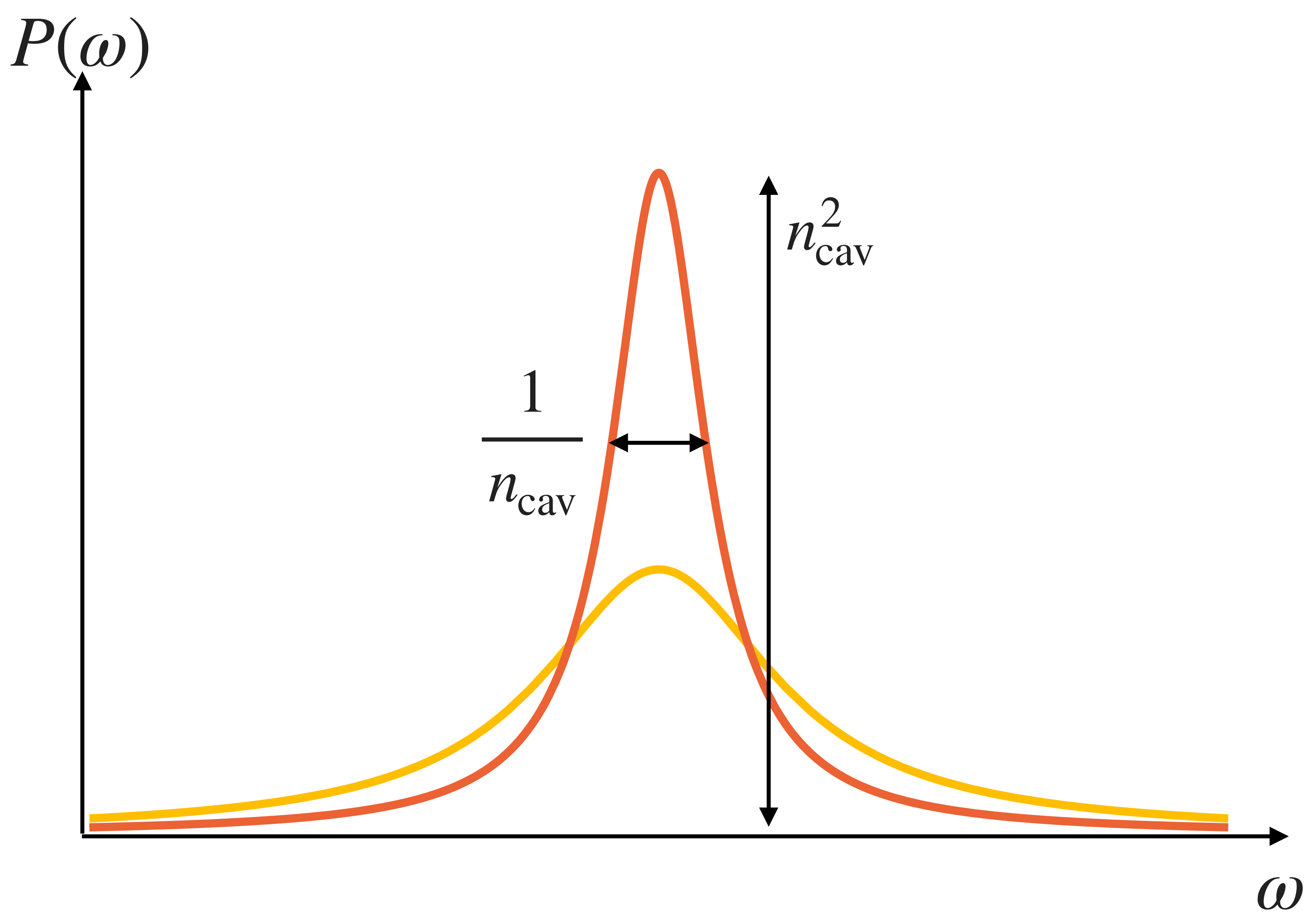
$$q^2 E_{\text{ext}}^2 \sum_{n,i} (E_n - E_0) |\langle n | x_i | 0 \rangle|^2 = q^2 E_{\text{ext}}^2 \frac{n}{2m}$$

Power absorbed

Sum rule for cavities

$$\sum_{n,i} (E_n - E_0) |\langle n | x_i | 0 \rangle|^2 = \frac{n}{2m} \longrightarrow \int d\omega\, \omega\, \mathrm{Im}\chi_{\mathrm{eff}}(\omega) = n_{\mathrm{cav}} \frac{\pi}{V}$$

$$\int_0^\infty dm\, P_m \propto V^2 \int d\omega\, \omega\, \mathrm{Im}\chi_{\mathrm{eff}}(\omega) = n_{\mathrm{cav}} V$$



For independent cavities, $\chi_{\mathrm{eff}} = \sum_i \chi \propto n_{\mathrm{cav}} \rightarrow \Delta\omega \propto n_{\mathrm{cav}}^0$

For coherent addition of E-fields, $\chi_{\mathrm{eff}} \propto n_{\mathrm{cav}}^2 \rightarrow \Delta\omega \propto n_{\mathrm{cav}}^{-1}$

All additional advantage from coherent addition is lost when considering scan rate!

Power transfer in qubit protocol

$$H_{\text{tot}} = \sum_{i=1}^{n_q} H_i \quad \text{where } H_i = \frac{1+\sigma_z}{2}\,\omega$$

Before DM action: $|\psi_i\rangle = \dfrac{|+\rangle^{\otimes n_q} + |-\rangle^{\otimes n_q}}{\sqrt{2}}, \qquad \langle\psi_i|H_{\text{tot}}|\psi_i\rangle = \dfrac{n_q\,\omega}{2}$

After DM action: $|\psi_f\rangle = \dfrac{e^{in_q\eta t}|+\rangle^{\otimes n_q} + e^{-in_q\eta t}|-\rangle^{\otimes n_q}}{\sqrt{2}}, \qquad \langle\psi_f|H_{\text{tot}}|\psi_f\rangle = \dfrac{n_q\,\omega}{2}$

The qubit protocol does not rely on power drawn from DM!

Bandwidth of GHZ protocol

$$U_{\rm DM} = U_1\, 1 + U_x X + U_y Y + U_z Z$$

$$U_{\rm DM}\,|\pm\rangle = C_{1\pm}\,|\pm\rangle + C_{2\pm}\,|\mp\rangle$$

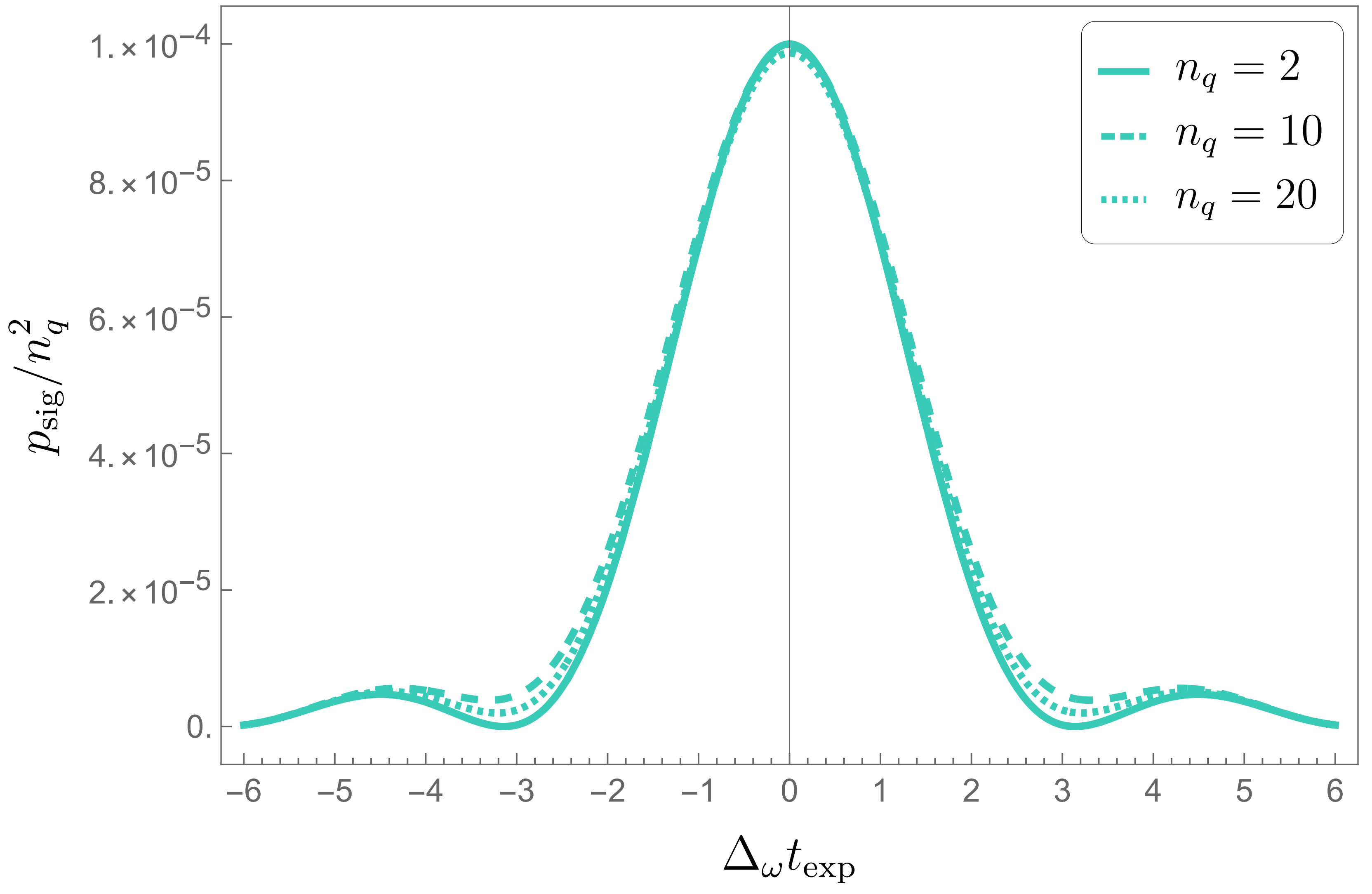
where $C_{1\pm} = U_1 \pm U_x,\ C_{2\pm} = U_z \mp iU_y$

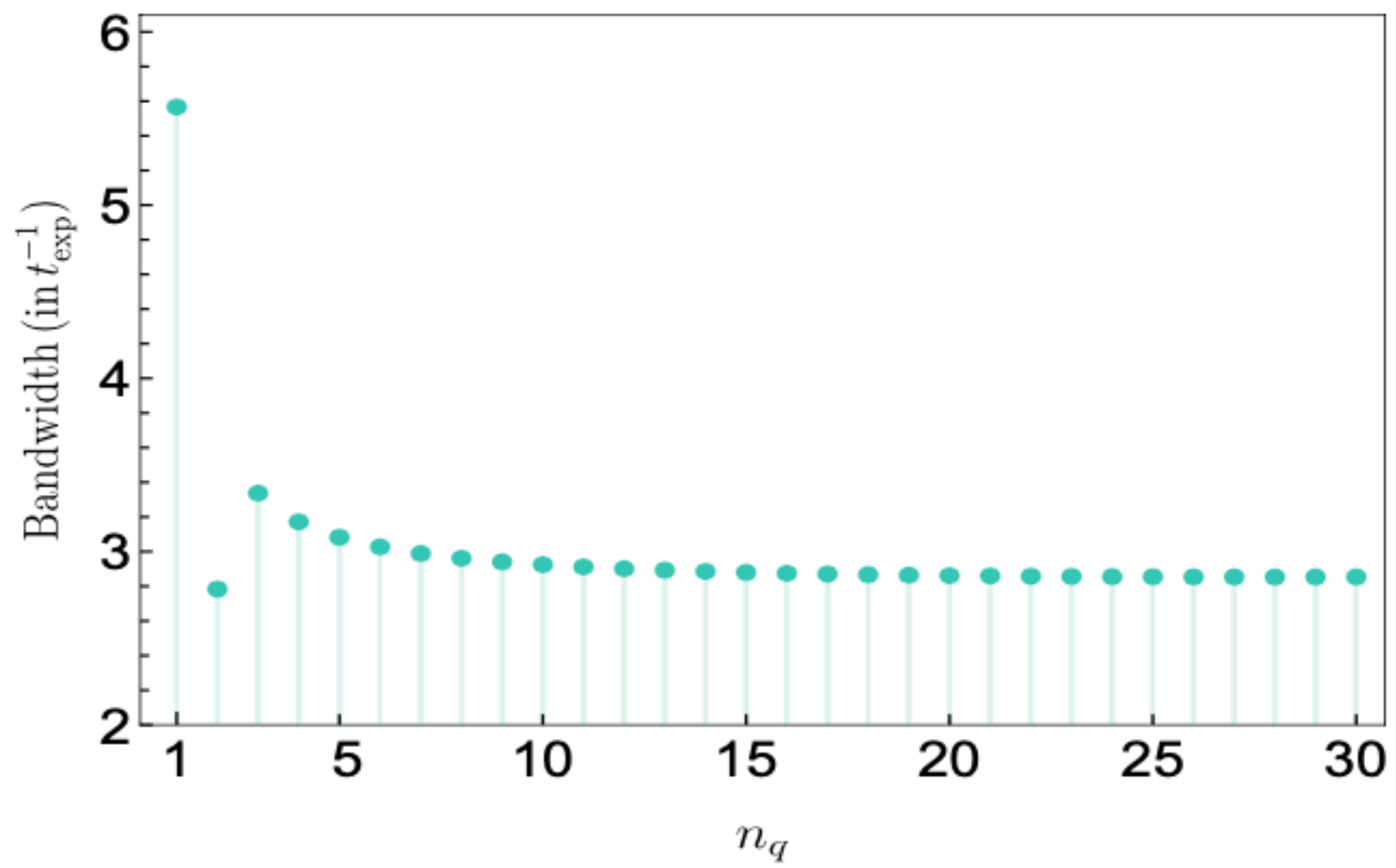
$$p_{\rm sig} = \frac{1}{8}\sum_{k=0}^{n_q}\binom{n_q}{k}\left|\left(C_{1+}^k C_{2+}^{n_q-k} + C_{2-}^k C_{1-}^{n_q-k}\right) - \left(C_{1+}^{n_q-k} C_{2+}^k + C_{2-}^{n_q-k} C_{1-}^k\right)\right|^2$$

In the limit $\Delta_\omega \gg \eta$:

$$p_{\rm sig} \xrightarrow{\text{off-res}} n_q^2\,\frac{4\eta^2}{\Delta_\omega^2}\sin^2\left(\frac{\Delta_\omega t_{\rm exp}}{2}\right)\cos^2\left(\varphi + \frac{\Delta_\omega t_{\rm exp}}{2}\right)$$

Bandwidth: $\Delta\omega \sim 1/t_{\rm exp}$ ← Independent of n_q!





1. Are qubits necessary: Yes!

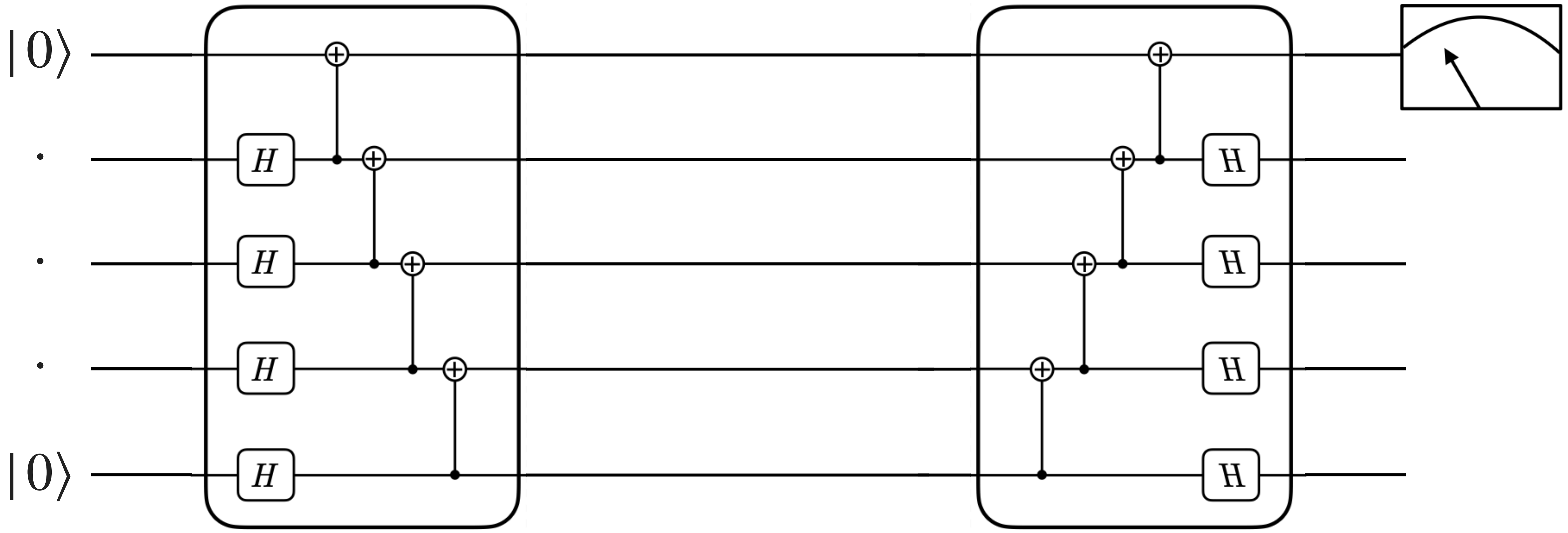
2. How much of the advantage is left after incorporating noise?

3. How many qubits would be needed to compete with a single cavity haloscope?

Noise sources

p_{g}: Gate error (CNOT gates)
(total error $\propto n_q$, independent of t_{exp})

p_{ro}: Readout error,
(independent of n_q, t_{exp})



p_{th}: thermal-like errors
(total error $\propto n_q$, t_{exp})

$$\tau_{\mathrm{GHZ}} = \frac{\tau_q}{n_q}$$

Scan rates

$$\mathrm{SR_{un}} \equiv \frac{\Delta\omega}{t_{\mathrm{exp}} \, . \, n_{\mathrm{rep}}} \xrightarrow{\Delta\omega \sim 1/t_{\mathrm{exp}}} \mathrm{SR_{un}} \equiv \frac{1}{t_{\mathrm{exp}}^2 \, . \, n_{\mathrm{rep}}}$$

Unentangled n_q qubits in parallel

$$S \approx (\eta^2 t_{\mathrm{exp}}^2)\, n_q\, n_{\mathrm{rep}}$$

$$N \approx \sqrt{p_{\mathrm{error}}\, n_q\, n_{\mathrm{rep}}}$$

$$\frac{S}{N} \approx \frac{(\eta^2 t_{\mathrm{exp}}^2)\sqrt{n_q\, n_{\mathrm{rep}}}}{\sqrt{p_{\mathrm{error}}}} \longrightarrow \text{gives } n_{\mathrm{rep}} \text{ needed}$$

For $t_{\mathrm{exp}} = \tau_{\mathrm{DM}}, \;\; p_{\mathrm{error}} \approx p_{\mathrm{ro}} + p_{\mathrm{th}}$

$$\mathrm{SR_{un}} \approx \frac{\boxed{n_q}\, \eta^4 \tau_{\mathrm{DM}}^2}{\mathrm{SNR}^2\, p_{\mathrm{error}}}$$

Entangled n_q qubits

$$S \approx (n_q^2\, \eta^2 t_{\mathrm{exp}}^2)\, n_{\mathrm{rep}}$$

$$N \approx \sqrt{p_{\mathrm{error}}\, n_{\mathrm{rep}}}$$

$$\mathrm{SR_{ent}} = \frac{\boxed{n_q^4}\, \eta^4 t_{\mathrm{exp}}^2}{\mathrm{SNR}^2\, p_{\mathrm{error}}}$$

$$p_{\mathrm{error}} = p_{\mathrm{ro}} + (p_g + p_{\mathrm{th}})\, n_q \;, \; t_{\mathrm{exp}} \neq \tau_{\mathrm{DM}}$$

Scan rates

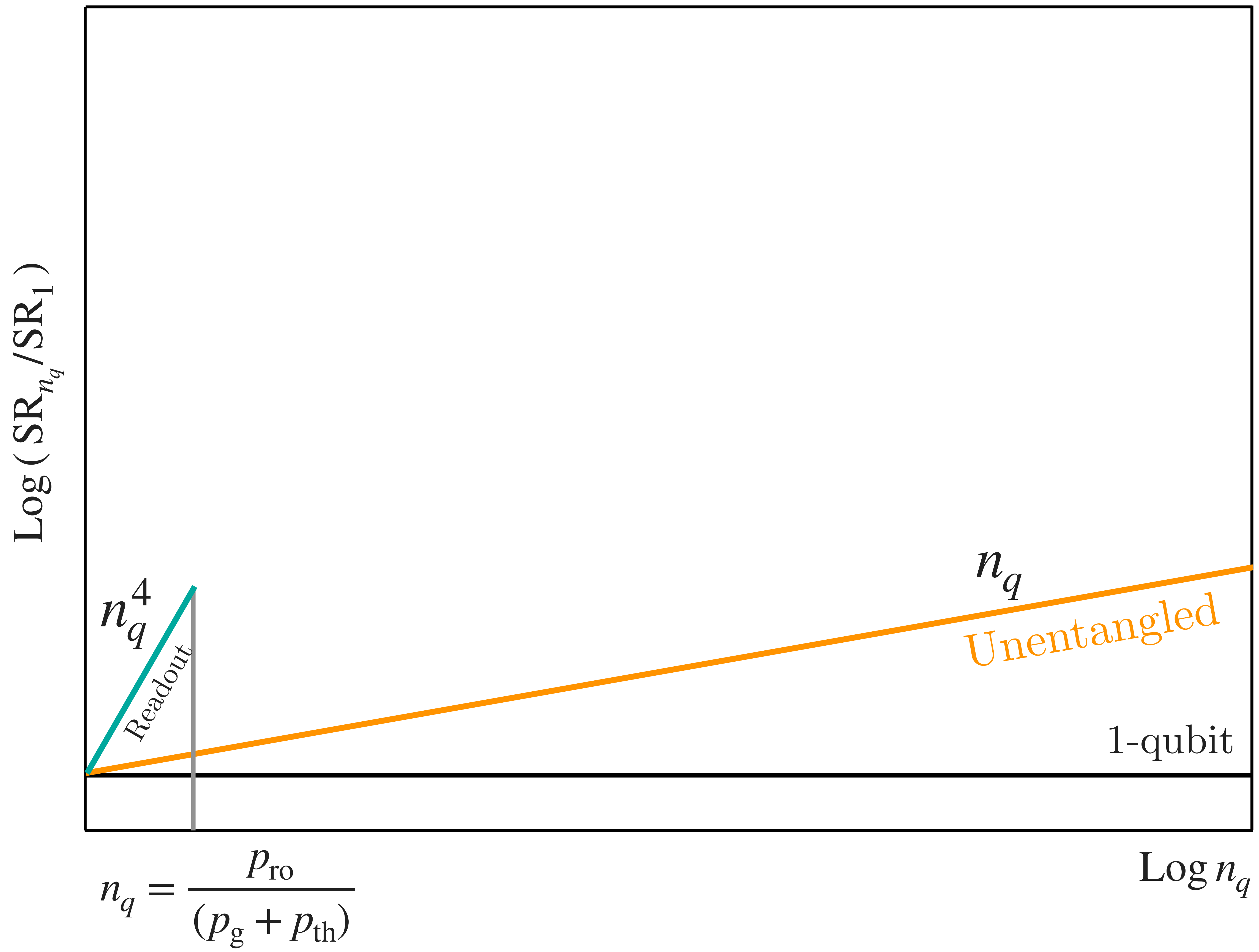
$$\mathrm{SR_{ent}} \propto \frac{n_q^4\, t_{\rm exp}^2}{p_{\rm ro} + \sout{(p_{\rm g} + p_{\rm th})\, n_q}}$$

$$\tau_{\rm GHZ} > \tau_{\rm DM} \implies t_{\rm exp} = \tau_{\rm DM}$$

If read-out error is dominant, then for

$$n_q \leq \frac{p_{\rm ro}}{p_g + p_{\rm th}},$$

$$\mathrm{SR_{ent}} \propto n_q^4$$

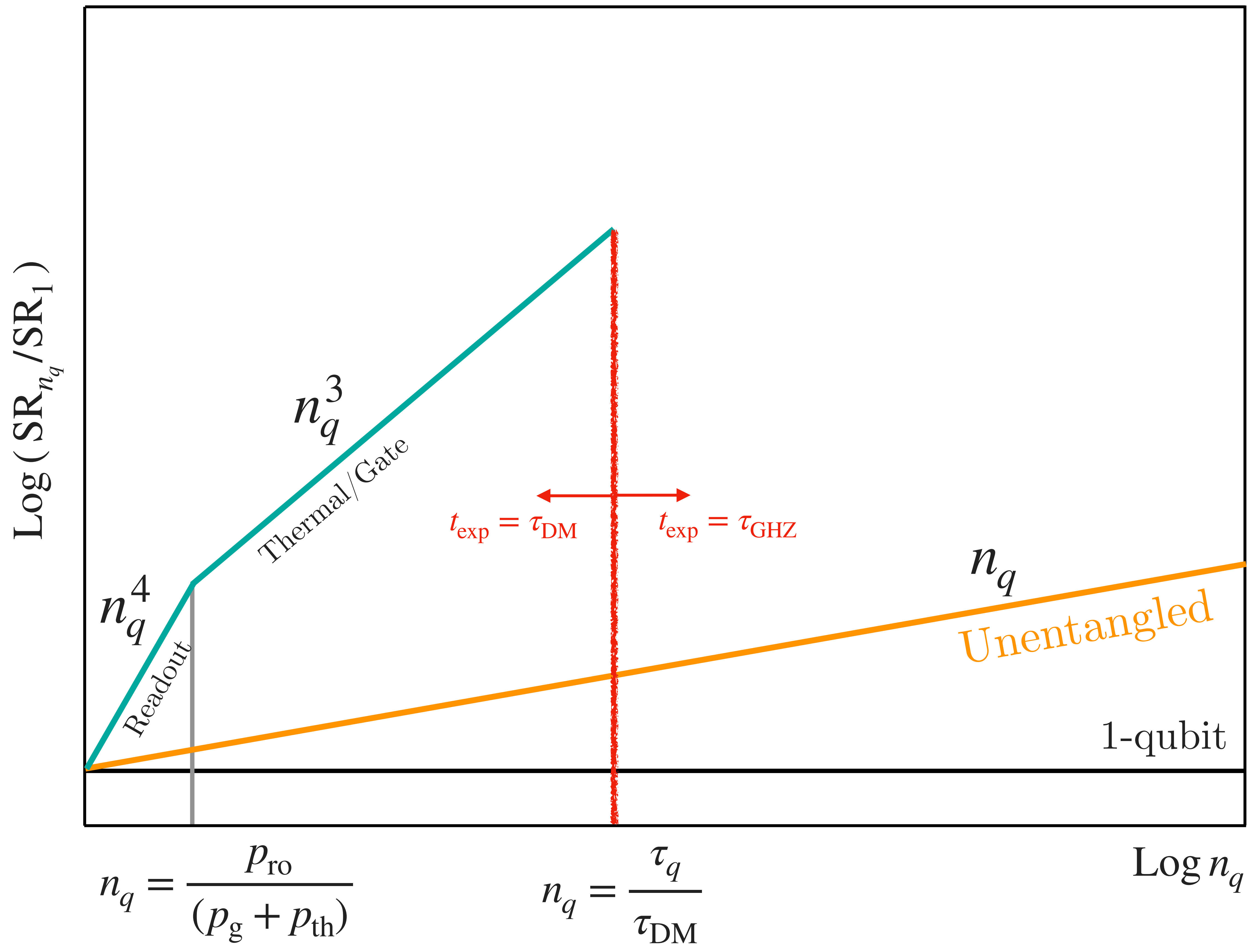


Scan rates

$$\mathrm{SR}_{\mathrm{ent}} \propto \frac{n_q^4\, t_{\mathrm{exp}}^2}{p_{\mathrm{ro}} + (p_{\mathrm{g}} + p_{\mathrm{th}})\, n_q}$$

$$\tau_{\mathrm{GHZ}} > \tau_{\mathrm{DM}} \implies t_{\mathrm{exp}} = \tau_{\mathrm{DM}}$$

$$\mathrm{SR}_{\mathrm{ent}} \propto n_q^3$$

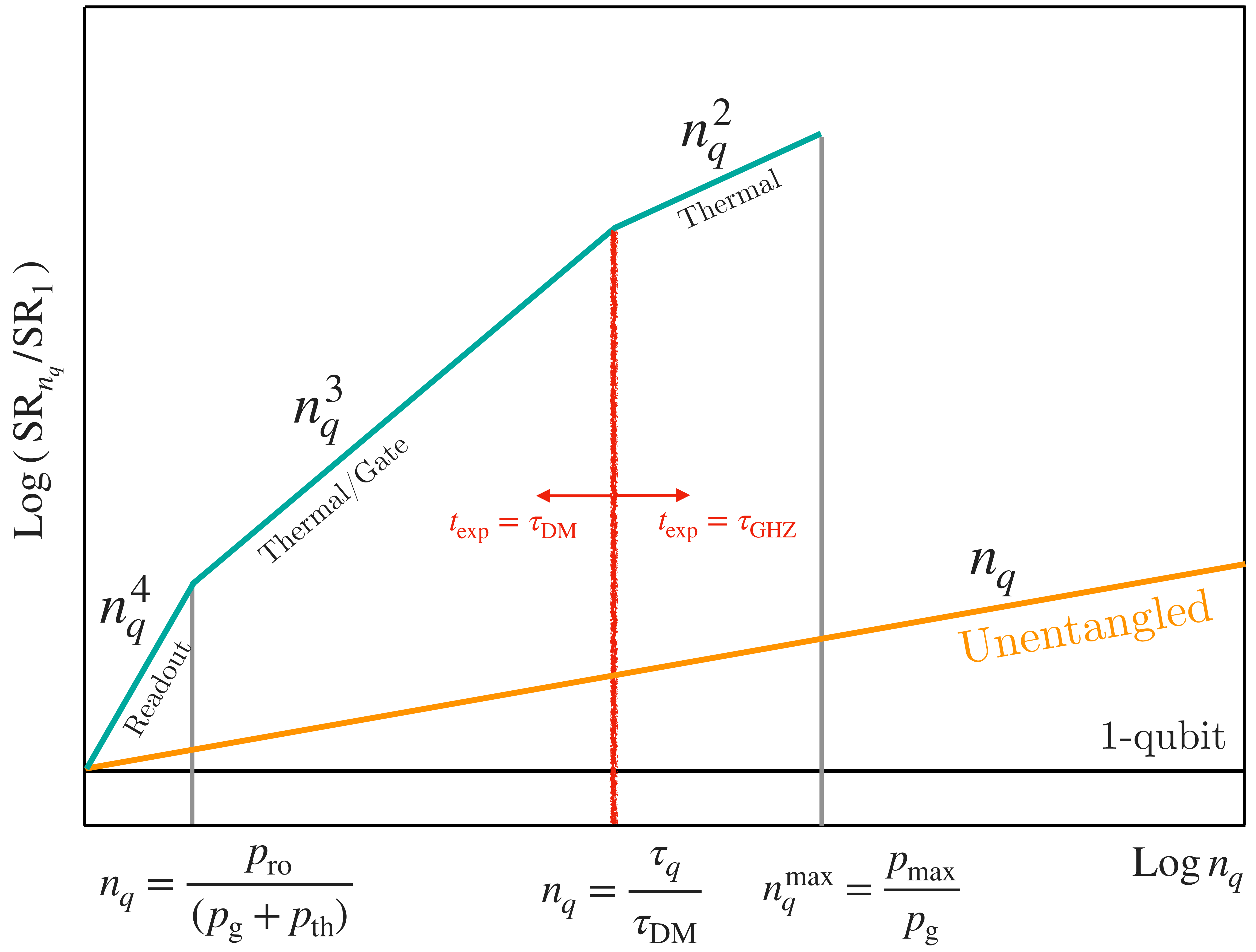


Scan rates

$$\mathrm{SR_{ent}} \propto \frac{n_q^4\, t_{\exp}^2}{p_{\mathrm{ro}} + (p_{\mathrm{g}} + p_{\mathrm{th}})\, n_q}$$

$$\tau_{\mathrm{GHZ}} < \tau_{\mathrm{DM}} \rightarrow t_{\exp} = \tau_{\mathrm{GHZ}} = \tau_q / n_q$$

$$\mathrm{SR_{ent}} \propto \frac{n_q^4 \left(\frac{\tau_q}{n_q}\right)^2}{p_{\max}} \propto n_q^2$$

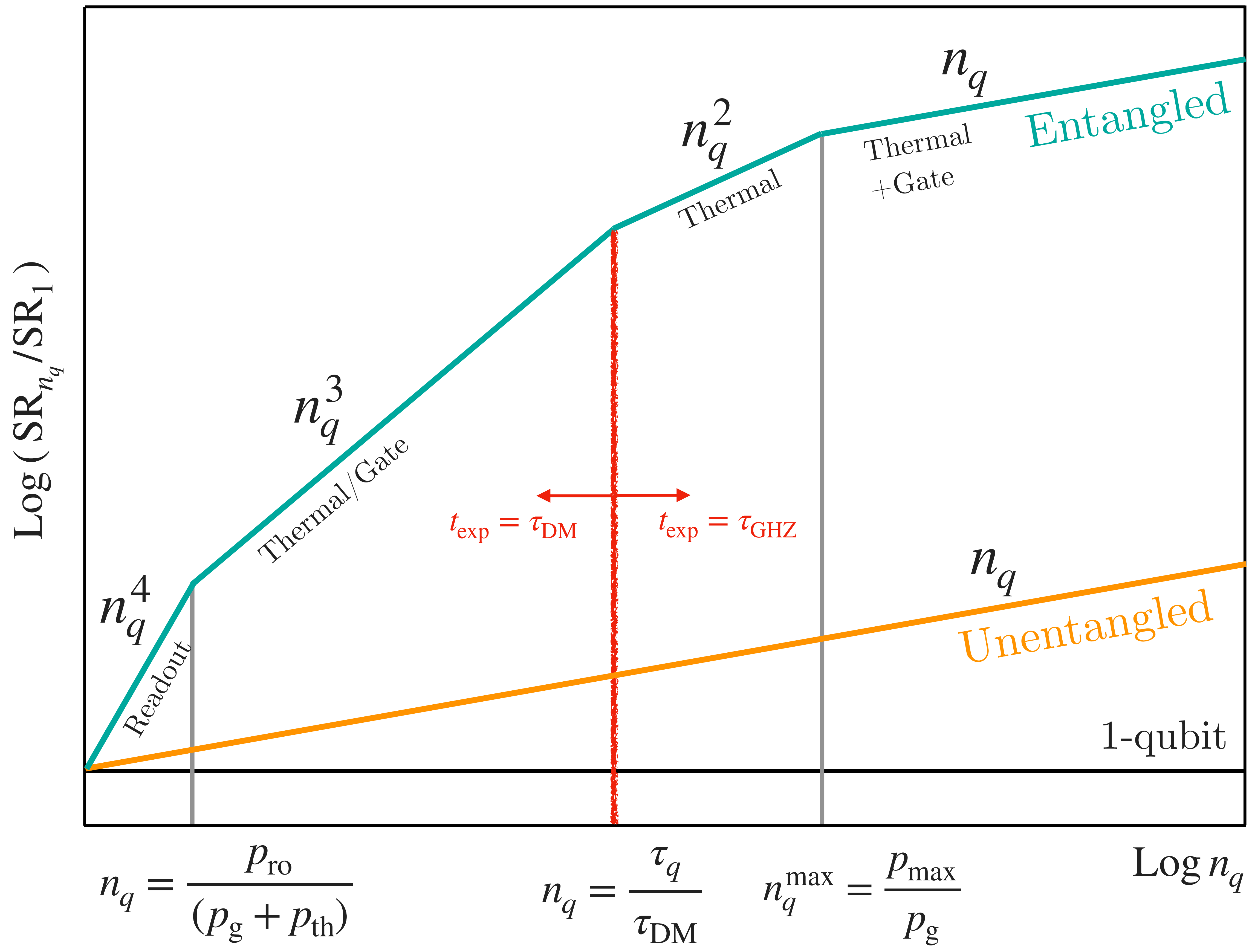


Scan rates

$$\mathrm{SR_{ent}} \propto \frac{n_q^4 \left(\frac{\tau_q}{n_q}\right)^2}{p_{\mathrm{ro}} + p_{\mathrm{g}} n_q + p_{\mathrm{max}}}$$

$$\mathrm{SR_{ent}} \propto n_q$$

Entanglement cannot be increased further. Additional qubits can be run in a parallel entangled setup.



1. Are qubits necessary: Yes!

2. How much of the advantage is left after incorporating noise?

3. How many qubits would be needed to compete with a single cavity haloscope?

Comparison with (single) cavity experiments

- In 1-10 GHz range ($m_{\rm DM} \in (5-50)\,\mu$eV), there are both cavity-based experiments and superconducting qubit technology available for dark photon searches.

- Cavity experiments have volume advantage. E.g. at GHz frequency, $V_{\rm cav} \sim 10^3\,{\rm cm}^3$, while effective volume for a superconducting transom qubit $V_{\rm qubit} \sim 1\,{\rm mm}^3$.

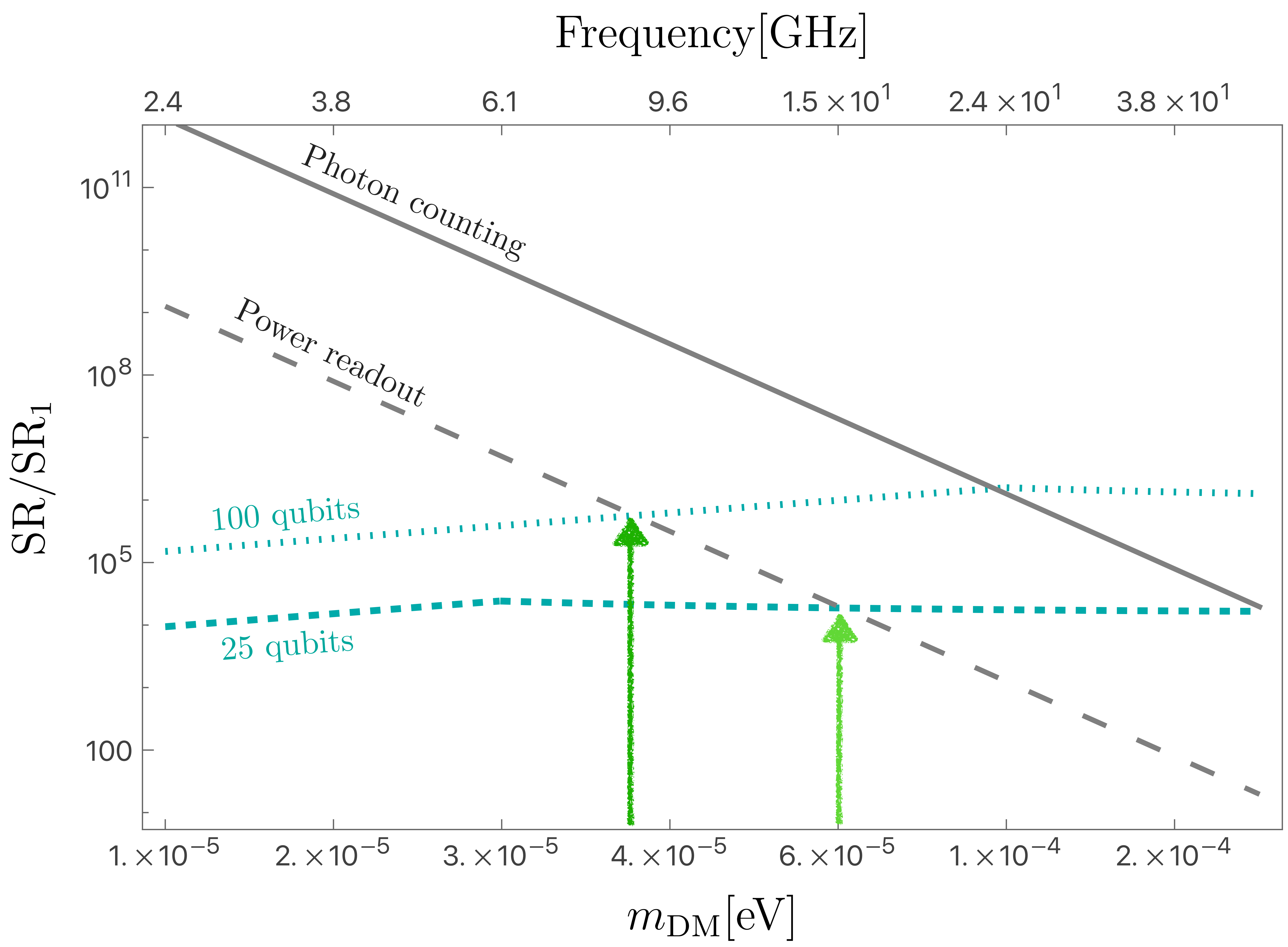
- Scaling advantage in scan rate for entangled states can make qubits competitive with single cavity experiments.

- Assuming thermal errors dominate, $\dfrac{\rm SR_{qubit}}{\rm SR_{cav}} \approx \left(\dfrac{V_{\rm qubit}^2}{V_{\rm cav}^2}\right)\left(\dfrac{p_{\rm error}^{\rm (cav)}}{p_{\rm error}^{\rm (qubit)}}\right) n_q^3$

- In cavity setups, we consider 1) Power-based readout: $p_{\rm error}^{\rm (cav)} \xrightarrow{\rm SQL} E_{\rm vac}/m_{\rm DM} \approx 1/2$

  2) Photon-counting readout: $p_{\rm error}^{\rm (cav)} \ll 1$

A. V. Dixit, S. Chakram, K. He, A. Agrawal, R. K. Naik, D. I. Schuster, and A. Chou, 2008.12231



$\tau_q = 1\,\text{ms}\,,$

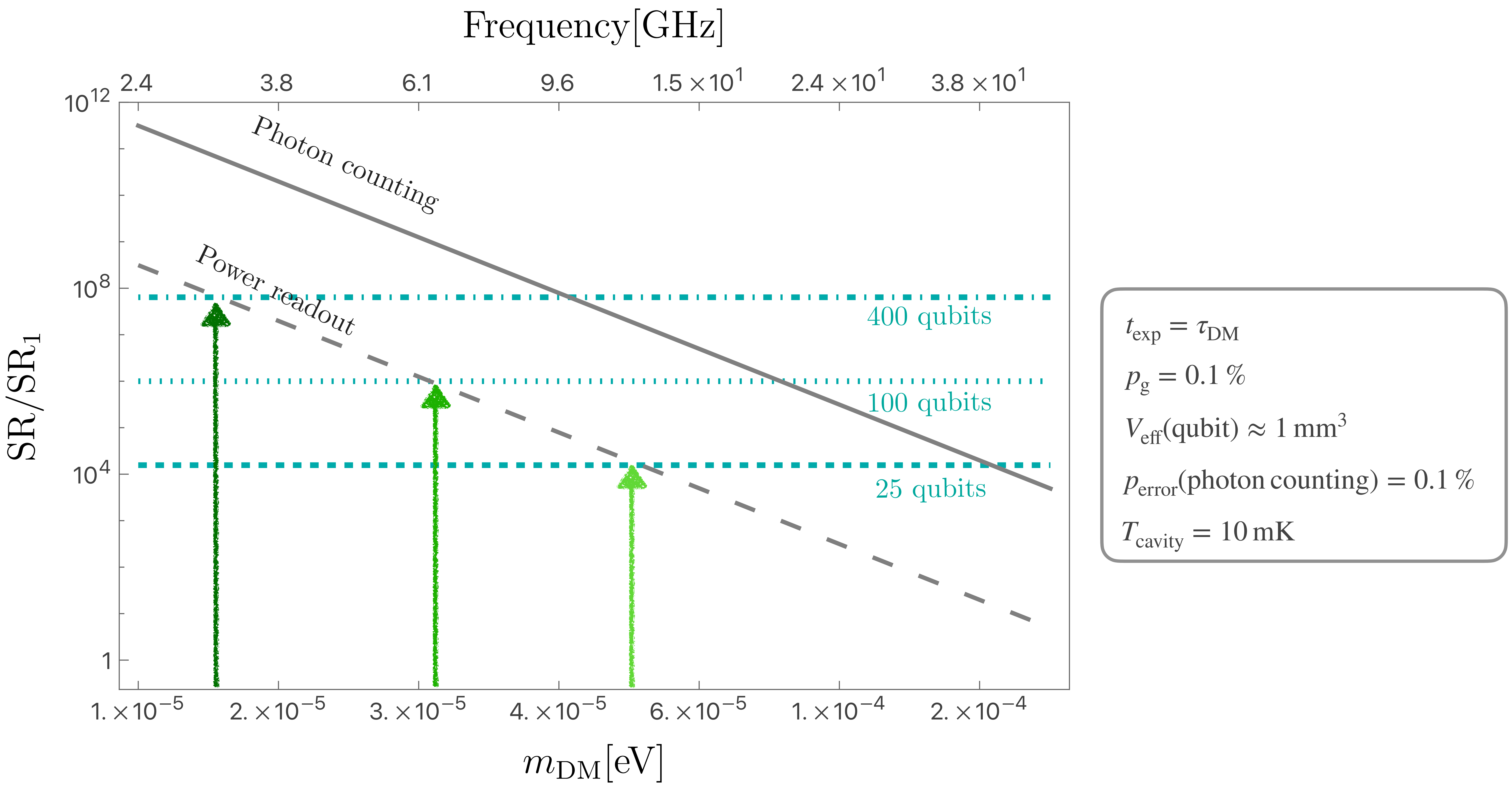
$t_{\text{exp}} = \min[\tau_{\text{DM}}, \tau_{\text{GHZ}}]$

$p_g = 0.5\,\%$

$V_{\text{eff}}(\text{qubit}) \approx 1\,\text{mm}^3$

$p_{\text{error}}(\text{photon counting}) = 0.1\,\%$

$T_{\text{cavity}} = 10\,\text{mK}$


Frequency[GHz]
2.4
3.8
6.1
9.6
1.5 × 10^1
2.4 × 10^1
3.8 × 10^1
10^12
10^8
10^4
1
SR/SR_1
Photon counting
Power readout
400 qubits
100 qubits
25 qubits
1. × 10^−5
2. × 10^−5
3. × 10^−5
4. × 10^−5
6. × 10^−5
1. × 10^−4
2. × 10^−4
m_DM[eV]
t_exp = τ_DM
p_g = 0.1 %
V_eff(qubit) ≈ 1 mm^3
p_error(photon counting) = 0.1 %
T_cavity = 10 mK

Summary and future directions

- Unlike cavity-based experiments, there is no loss of bandwidth for entangled qubit states $\rightarrow$ maintain advantage in scan-rate.

-

	Qubits	Cavity
Tunability	🥇	
Volume		🥇
Coherent addition	🥇	

- Can we incorporate error detection/correction to enhance sensitivity?

- Can we design a hybrid cavity-qubit setup that gets advantages of both?

Thank you!