

Freeze-in Dark Matter in the Presence of an additional Scalar

Vassilis C. Spanos



**National and Kapodistrian
University of Athens**

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Outline

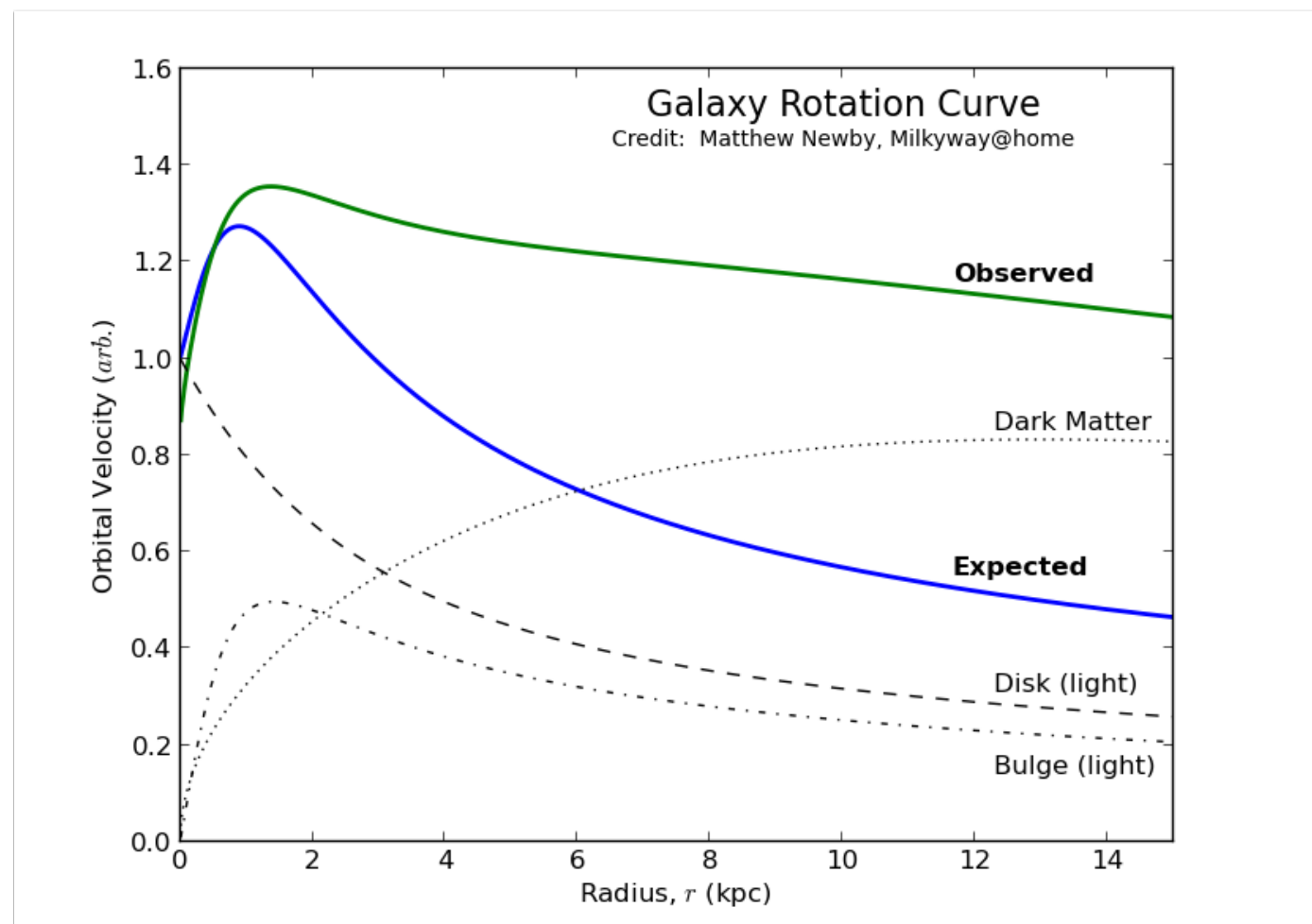
1. Evidence for Dark Matter
2. From the WIMP Miracle to Freeze-In
3. Gravitino Freeze-In Dark Matter
4. The Reheating Temperature Puzzle
5. A New Scalar Component
6. Numerical Results
7. Conclusions and Outlook

Eberl, Gialamas, VCS (2021), (2024)

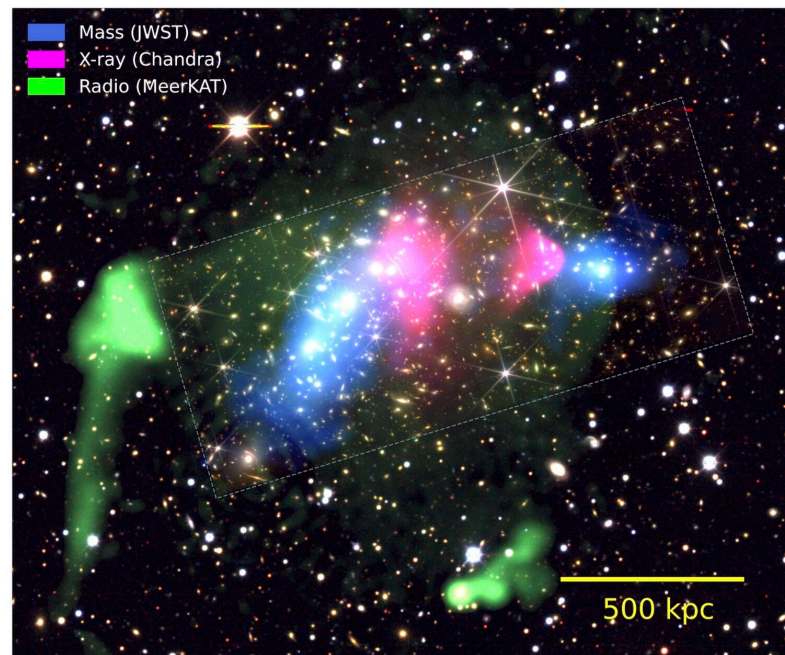
Georgilas, VCS (2026)

Evidence for the Existence of Dark Matter

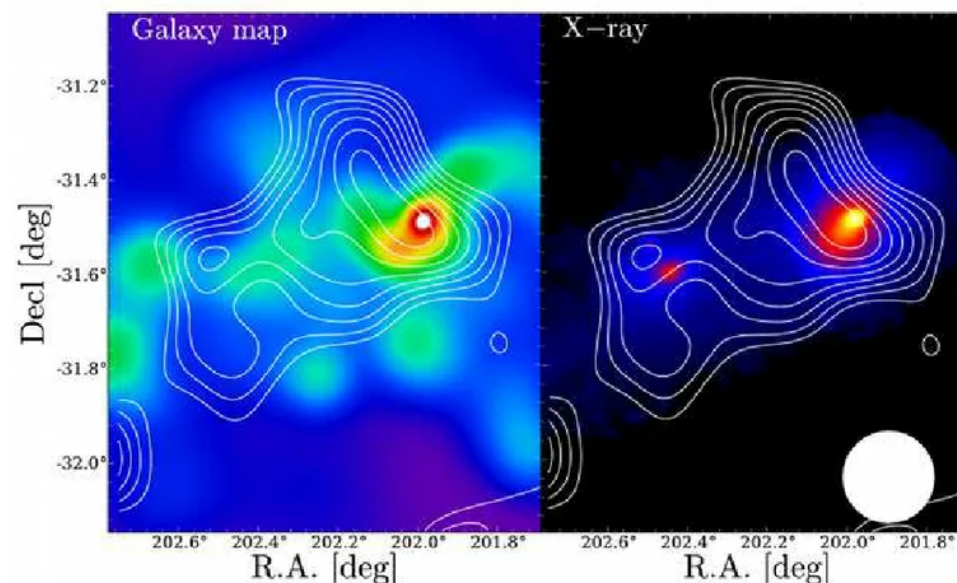
- **Galaxy rotation curves:** Stars in spiral galaxies rotate with nearly constant velocity well beyond the visible disk, implying the existence of a massive dark halo.



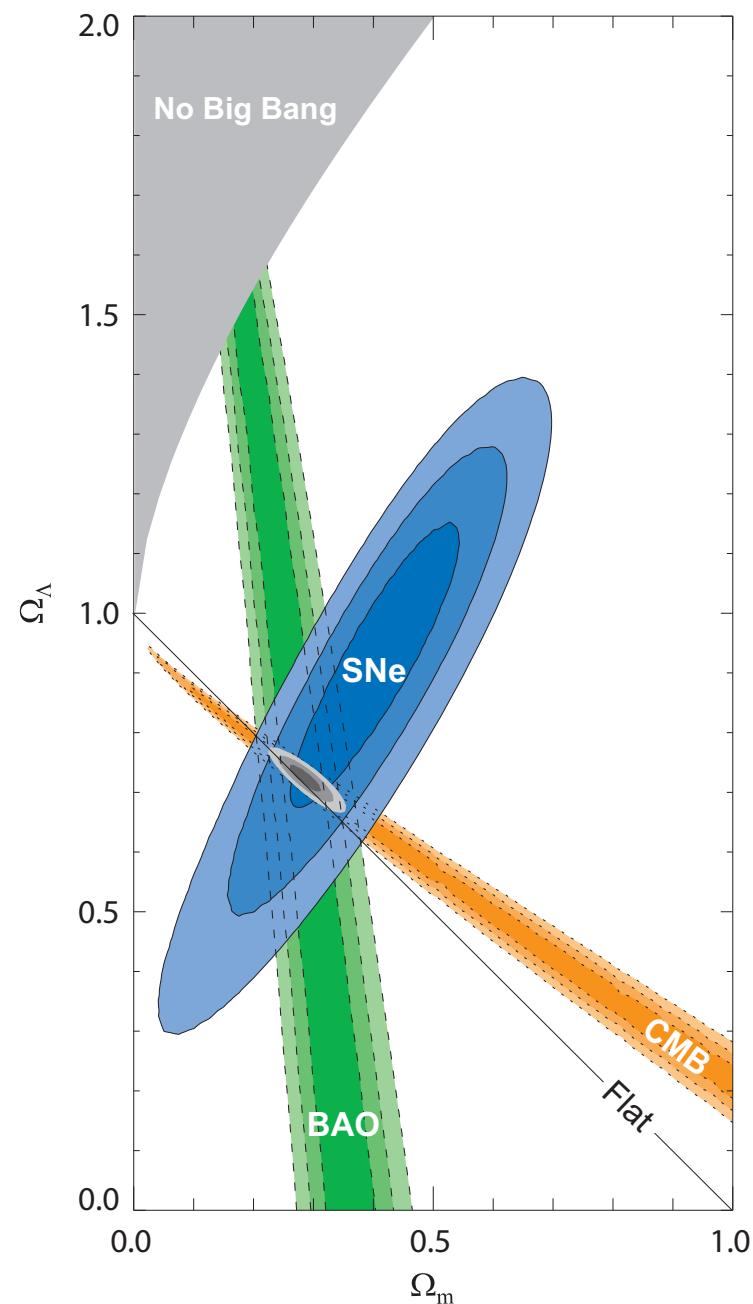
- **Galaxy clusters:** The dynamics of galaxies and the X-ray emitting intra-cluster gas require substantially more gravitating matter than is directly observed.



- **Gravitational lensing:** Weak and strong lensing measurements reveal a matter distribution extending far beyond the luminous component. The Bullet Cluster provides particularly strong evidence that most of the gravitating matter is non-baryonic.



- **Cosmology:** Measurements of the cosmic microwave background and large-scale structure require a cold dark matter component $\Omega_{\text{DM}}h^2 = 0.12 \pm 0.0012$



$$\Omega_{\text{tot}} \sim 1.0$$

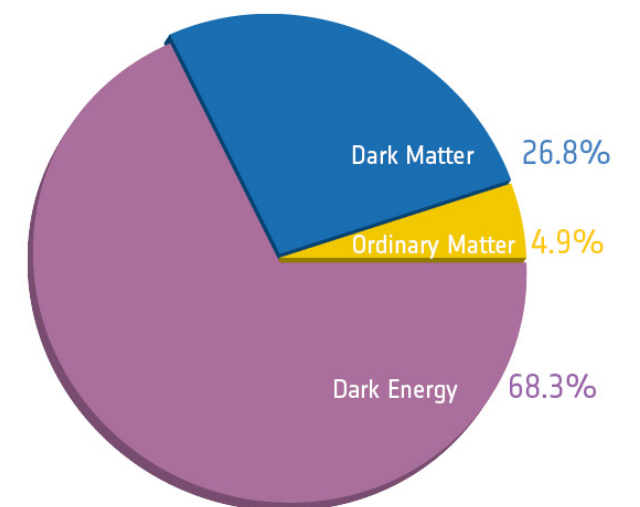
$$\Omega_{\text{M}} = 0.315 \pm 0.016$$

$$\Omega_{\Lambda} = 0.685 \pm 0.017$$

$$\Omega_{\text{B}} = 0.0499 \pm 0.0022 \quad \Rightarrow$$

$$\Omega_{\text{DM}} = \Omega_{\text{M}} - \Omega_{\text{B}} = 0.265 \pm 0.011$$

$$\Omega_{3/2}h^2 = 0.12 \pm 0.0012$$



PDG (2025)

The WIMP Miracle

- **Thermal Freeze-Out**

Dark matter particles are initially in thermal equilibrium with the Standard Model plasma. As the Universe expands and cools, the annihilation rate drops below the Hubble expansion rate and the DM abundance freezes out.

- **The Miracle**

The observed relic abundance is reproduced for a weak-scale annihilation cross section,

$$\langle \sigma v \rangle \simeq 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1},$$

which is naturally obtained for particles with masses of order 100 GeV–1 TeV and electroweak interactions.

- **Experimental Consequences**

The same interactions responsible for freeze-out imply

- production at colliders,
- direct detection through scattering with nuclei,
- indirect detection through annihilation products.

No convincing signal has been observed so far.

Freeze-In Dark Matter

- **Basic Idea**

Dark matter particles interact so feebly with the Standard Model that they never attain thermal equilibrium in the early Universe.

- **Production Mechanism**

The DM abundance is gradually built up through rare scatterings and particle decays in the thermal plasma,

$$\text{SM} + \text{SM} \rightarrow \text{DM} + \text{DM}, \quad \text{SM} \rightarrow \text{DM} + \dots$$

until the interaction rate becomes negligible.

- **Freeze-In Condition**

$$\Gamma_{\text{int}} \ll H,$$

throughout the thermal history of the Universe.

- **Characteristics**

- Extremely feeble interactions.
- No thermal equilibrium.
- Naturally evades collider, direct- and indirect-detection constraints.

Prominent examples: axions and gravitinos.

This Work

- Gravitino freeze-in dark matter.
- Thermal production after reheating.
- Peak reheating temperature:

$$T_{\text{reh}}^{\text{peak}}$$

- Introduce an additional scalar component ϕ .
- Can entropy dilution increase $T_{\text{reh}}^{\text{peak}}$?

Main result:

$$T_{\text{reh}}^{\text{peak},\phi} \gg T_{\text{reh}}^{\text{peak}}$$

for matter-like ϕ .

Freeze out mechanism

Boltzmann Equation

$$\mathbf{L}[f] = \mathbf{C}[f]$$

$$g_1 \int C[f_1] \frac{d^3 p_1}{(2\pi)^3} = - \sum_{\text{spins}} \int [f_1 f_2 (1 \pm f_3)(1 \pm f_4) |\mathcal{M}_{12 \rightarrow 34}|^2 - f_3 f_4 (1 \pm f_1)(1 \pm f_2) |\mathcal{M}_{34 \rightarrow 12}|^2] \\ \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 ,$$

$$\sum_{\text{spins}} \int |\mathcal{M}_{ij \rightarrow kl}|^2 \times (2\pi)^4 \delta^4(p_i + p_j - p_k - p_l) d\Pi_k d\Pi_l = 4 g_i g_j \sigma_{ij} \sqrt{(p_i \cdot p_j)^2 - (m_i m_j)^2} ,$$

$$g_1 \int C[f_1] \frac{d^3 p_1}{(2\pi)^3} = - \int \{ (\sigma v_{\text{Mø1}})_{12} dn_1 dn_2 - (\sigma v_{\text{Mø1}})_{34} dn_3 dn_4 \}$$

$$(v_{\text{Mø1}})_{ij} = \frac{\sqrt{(p_i \cdot p_j)^2 - (m_i m_j)^2}}{E_i E_j}$$

Gondolo, Gelmini, Nucl. Phys. B (1991)

Freeze out mechanism

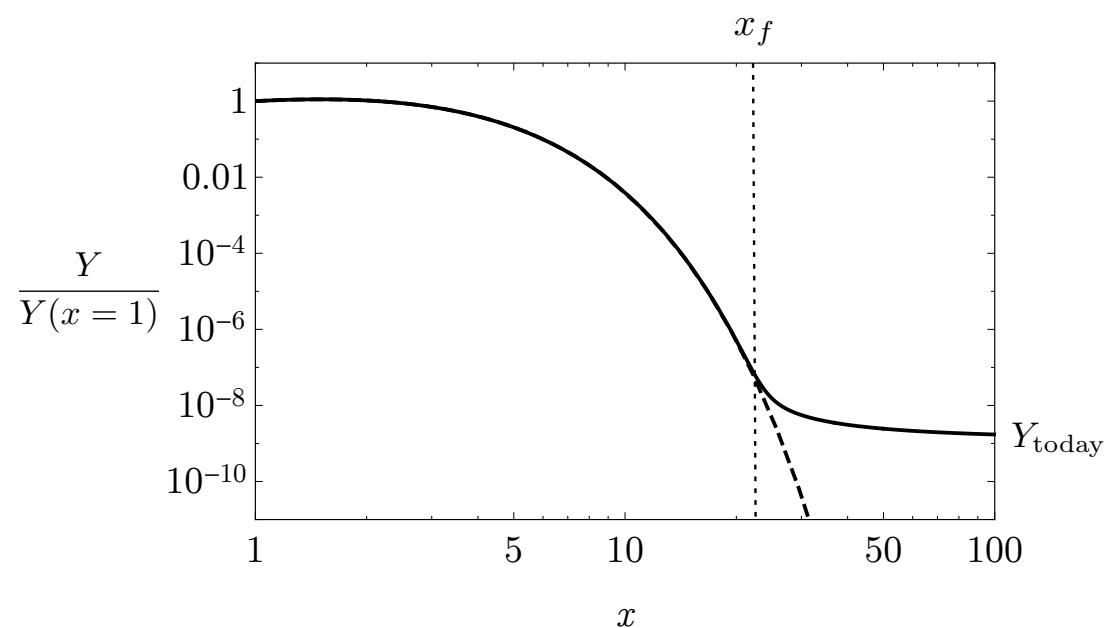
$$\dot{n}_1 + 3Hn_1 = -\langle\sigma v_{\text{M}\emptyset\text{l}}\rangle_{12}n_1n_2 + \langle\sigma v_{\text{M}\emptyset\text{l}}\rangle_{34}n_3n_4$$

$$\dot{n} + 3Hn = \langle\sigma v\rangle (n_{\text{eq}}^2 - n^2)$$

$$\langle\sigma v\rangle = \frac{\int \sigma v dn_1^{\text{eq}} dn_2^{\text{eq}}}{\int dn_1^{\text{eq}} dn_2^{\text{eq}}} = \frac{\int \sigma v e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}{\int e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}.$$

$$Y = n/s \qquad \frac{dY}{dx} = -\frac{xs\langle\sigma v\rangle}{H(m)} (Y^2 - Y_{\text{eq}}^2)$$

$$x = m/T$$



**Notable example of WIMP
Neutralino**

Freeze in mechanism

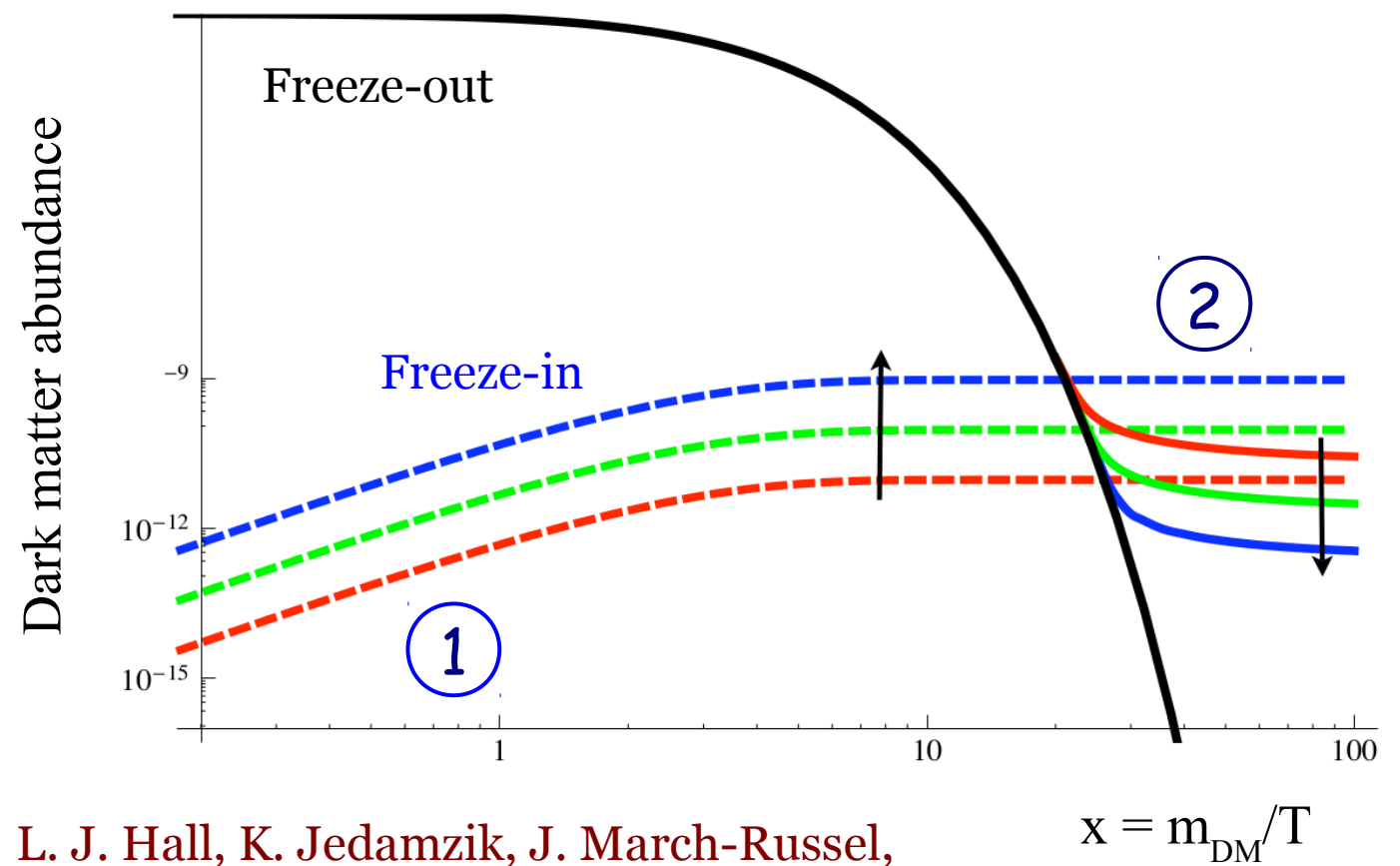
Freeze-in production of dark matter is based on two basic premises

- Dark matter interacts *very* weakly with the thermal bath.
- It has a negligible initial abundance: $(1 \pm f_\chi) \sim 1$.

dark matter could be produced directly from $2 \rightarrow 2$ processes, annihilations of Standard Model (or other bath) particles: $1 + 2 \rightarrow X + \chi$

$$\dot{n}_\chi + 3Hn_\chi = \int \prod_{i=1,2,X,\chi} \frac{d^3 |\vec{p}_i|}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4(P_1 + P_2 - P_X - P_\chi) |\mathcal{M}|^2 f_1 f_2 (1 \mp f_X)$$

Freeze in mechanism



L. J. Hall, K. Jedamzik, J. March-Russel,
S. M West, arXiv:0911.1120

Notable examples of FIMP
Axion, Gravitino

The setup of the calculation

The Braaten-Yuan prescription

[Braaten, Pisarski, Yuan (1990,1991)]

$$\gamma = \gamma|_{\text{hard}}^{k^* < k} + \gamma|_{\text{soft}}^{k^* > k}$$

where $gT \ll k^* \ll T$ assuming $g \ll 1$

Hard part is calculated from squared matrix elements

$$|\mathcal{M}(a b \rightarrow c \tilde{G})|^2$$

Soft part is calculated from Imaginary part of the gravitino self-energy

$$\gamma|_{\text{hard}}^{k^* < k} = A_{\text{hard}} + B \ln \left(\frac{T}{k^*} \right) \quad \text{and} \quad \gamma|_{\text{soft}}^{k^* > k} = A_{\text{soft}} + B' \ln \left(\frac{k^*}{m_{\text{thermal}}} \right)$$

Thus

$$\gamma_{\text{BY}} = \frac{3\zeta(3)}{16\pi^3} \frac{T^6}{M_{\text{P}}^2} \sum_{N=1}^3 c'_N g_N^2 \left(1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) \ln \left(\frac{k'_N}{g_N} \right)$$

$$c'_N = (11, 27, 72) \quad , \quad k'_N = (1.266, 1.312, 1.271) \quad [\text{Pradler, Steffen (2007)}]$$

Analytical result, but valid only for $g \ll 1$

where $\gamma|_{\text{soft}}$ is calculated in the Hard Thermal Loop (HTL) approx

The condition $g(T) \ll 1$ is not satisfied in the whole temperature range
especially if $g = g_3$

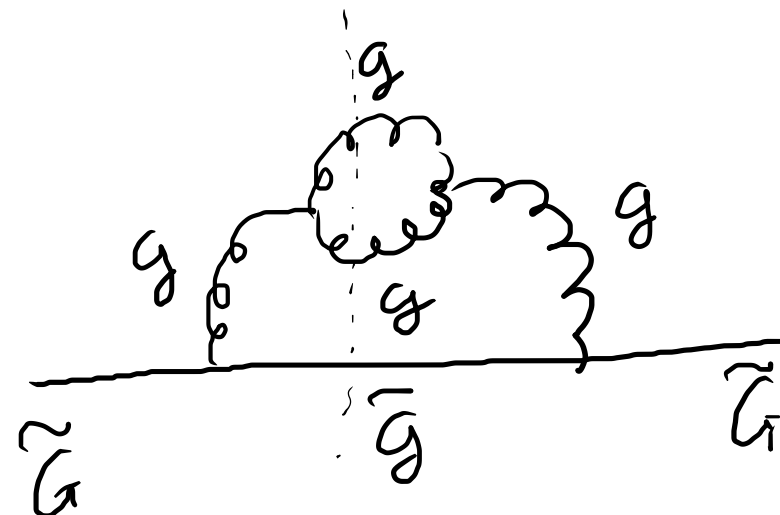
Beyond the HTL approx

- ▶ **Calculate the full 1-loop gravitino self-energy beyond HTL approximation**
- ▶ **Calculate the so-called subtracted part of the $|\mathcal{M}|^2$** [Rychkov, Strumia (2007)]

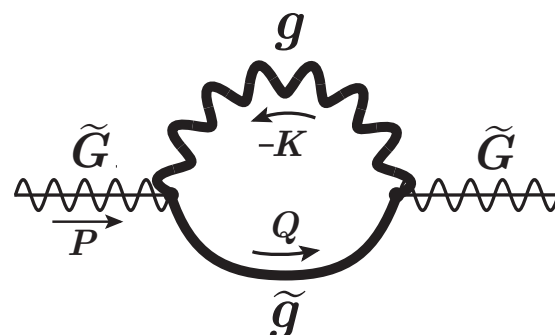
The subtracted part of the squared amplitude is this that cannot be part of the gravitino self-energy

For example if $X: gg \rightarrow \tilde{g}\tilde{G}$

$$|\mathcal{M}_{X,s}|^2 = \left| \text{Diagram} \right|^2 \quad \text{related to}$$



which is part of



(D-graph)

where thick lines denote resumed thermal propagators

Thus $\gamma_{3/2} = \gamma_{\text{sub}} + \gamma_{\text{D}} + \gamma_{\text{top}}$

X	process	$ \mathcal{M}_{X,\text{full}} ^2$	$ \mathcal{M}_{X,\text{sub}} ^2$
A	$gg \rightarrow \tilde{g}\tilde{G}$	$4C_3(s + 2t + 2t^2/s)$	$-2sC_3$
B	$g\tilde{g} \rightarrow g\tilde{G}$	$-4C_3(t + 2s + 2s^2/t)$	$2tC_3$
C	$\tilde{q}g \rightarrow q\tilde{G}$	$2sC'_3$	0
D	$gq \rightarrow \tilde{q}\tilde{G}$	$-2tC'_3$	0
E	$\tilde{q}q \rightarrow g\tilde{G}$	$-2tC'_3$	0
F	$\tilde{g}\tilde{g} \rightarrow \tilde{g}\tilde{G}$	$8C_3(s^2 + t^2 + u^2)^2/(stu)$	0
G	$q\tilde{g} \rightarrow q\tilde{G}$	$-4C'_3(s + s^2/t)$	0
H	$\tilde{q}\tilde{g} \rightarrow \tilde{q}\tilde{G}$	$-2C'_3(t + 2s + 2s^2/t)$	0
I	$q\tilde{q} \rightarrow \tilde{g}\tilde{G}$	$-4C'_3(t + t^2/s)$	0
J	$\tilde{q}\tilde{q} \rightarrow \tilde{g}\tilde{G}$	$2C'_3(s + 2t + 2t^2/s)$	0

Squared matrix elements for gravitino production in $SU(3)_c$ in terms of $g_3^2 Y_3/M_{\text{P}}^2$

$$Y_3 = 1 + m_{\tilde{g}}^2/(3m_{3/2}^2), \quad C_3 = 24 \text{ and } C'_3 = 48$$

$$|\mathcal{M}_{X,\text{full}}|^2 = |\mathcal{M}_{X,s} + \mathcal{M}_{X,t} + \mathcal{M}_{X,u} + \mathcal{M}_{X,x}|^2$$

$$|\mathcal{M}_{X,D}|^2 = |\mathcal{M}_{X,s}|^2 + |\mathcal{M}_{X,t}|^2 + |\mathcal{M}_{X,u}|^2$$

$$|\mathcal{M}_{X,\text{sub}}|^2 = |\mathcal{M}_{X,\text{full}}|^2 - |\mathcal{M}_{X,D}|^2$$

$$\gamma_{\text{sub}}$$

$$\gamma = \frac{1}{(2\pi)^8} \int \frac{d^3\mathbf{p}_a}{2E_a} \frac{d^3\mathbf{p}_b}{2E_b} \frac{d^3\mathbf{p}_c}{2E_c} \frac{d^3\mathbf{p}_{\tilde{G}}}{2E_{\tilde{G}}} |\mathcal{M}|^2 f_a f_b (1 \pm f_c) \times \delta^4(P_a + P_b - P_c - P_{\tilde{G}}) \quad f_{B|F} = \frac{1}{e^{\frac{E}{T}} \mp 1}$$

$$|\mathcal{M}_{A,\text{sub}}|^2 + |\mathcal{M}_{B,\text{sub}}|^2 = \frac{g_N^2}{M_{\text{P}}^2} \left(1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) C_N(-s+2t) \quad \text{as taken from the Table with the amplitudes}$$

Performing numerical integration

$$\gamma_{\text{sub}} = \frac{T^6}{M_{\text{P}}^2} \sum_{N=1}^3 g_N^2 \left(1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) C_N \left(-\mathcal{C}_{\text{BBF}}^s + 2\mathcal{C}_{\text{BFB}}^t \right)$$

$$\mathcal{C}_{\text{BBF}}^s = 0.25957 \times 10^{-3}$$

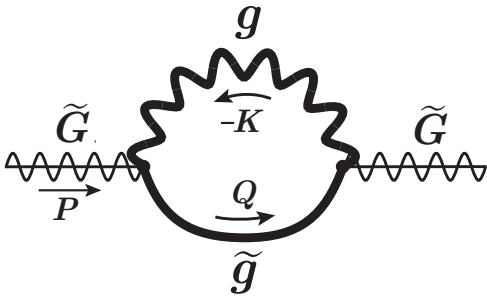
$$\mathcal{C}_{\text{BFB}}^t = -0.13286 \times 10^{-3}.$$

$$\gamma_{\text{D}}$$

$$\Pi^{<}(P)=\frac{1}{16M_{\text{P}}^2}\sum_{N=1}^3n_N\left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\int\frac{\text{d}^4K}{(2\pi)^4}\text{Tr}\left[\not{P}[\not{K},\gamma^\mu]^*S^{<}(Q)[\not{K},\gamma^\nu]^*D_{\mu\nu}^{<}(K)\right]$$

$$^*S^{<}(Q)=\frac{f_F(q_0)}{2}\left[(\gamma_0-\boldsymbol{\gamma}\cdot\mathbf{q}/q)\,\rho_+(Q)+(\gamma_0+\boldsymbol{\gamma}\cdot\mathbf{q}/q)\,\rho_-(Q)\right]$$

$$^*D_{\mu\nu}^{<}(K)=f_B(k_0)\left[\Pi_{\mu\nu}^T\,\rho_T(K)+\Pi_{\mu\nu}^L\,\frac{k^2}{K^2}\,\rho_L(K)+\xi\frac{K_\mu K_\nu}{K^4}\right]$$



$$\gamma_D=\int\frac{\text{d}^3\mathbf{p}}{2p_0(2\pi)^3}\,\Pi^{<}(p)$$

$$\begin{aligned}\gamma_{\text{D}}&=\frac{1}{4(2\pi)^5M_{\text{P}}^2}\sum_{N=1}^3n_N\left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\int_0^\infty\text{d}p\int_{-\infty}^\infty\text{d}k_0\int_0^\infty\text{d}k\int_{|k-p|}^{k+p}\text{d}q\,k\,f_B(k_0)\,f_F(q_0)\\&\times\left[\rho_L(K)\,\rho_-(Q)\,(p-q)^2\big((p+q)^2-k^2\big)+\rho_L(K)\,\rho_+(Q)\,(p+q)^2\big(k^2-(p-q)^2\big)\right.\\&+\,\rho_T(K)\,\rho_-(Q)\,\big(k^2-(p-q)^2\big)\Big((1+k_0^2/k^2)\big(k^2+(p+q)^2\big)-4k_0(p+q)\Big)\\&\left.+\,\rho_T(K)\,\rho_+(Q)\,\big((p+q)^2-k^2\big)\Big((1+k_0^2/k^2)\big(k^2+(p-q)^2\big)-4k_0(p-q)\Big)\right],\end{aligned}$$

$$\gamma_{\text{top}}$$

$$\gamma_{\text{top}}=\frac{T^6}{M_{\text{P}}^2}\,72\,C_{\text{BBF}}^s\,\lambda_t^2\left(1+\frac{A_t^2}{3m_{3/2}^2}\right)\qquad C_{\text{BBF}}^s=0.25957\times10^{-3}.$$

$$\rho_{L,T}(K) = 2\pi \left\{ Z_{L,T}(k) \left[\delta(k_0 - \omega_{L,T}(k)) - \delta(k_0 + \omega_{L,T}(k)) \right] + \rho_{L,T}^{\text{cont}}(K) \right\}$$

$$\rho_{\pm}(Q) = 2\pi \left\{ Z_{\pm}(q) \delta(q_0 - \omega_{\pm}(q)) + Z_{\mp}(q) \delta(q_0 + \omega_{\mp}(q)) + \rho_{\pm}^{\text{cont}}(Q) \right\}.$$

$$Z_L(k) = \frac{\omega_L(k)(\omega_L^2(k) - k^2)}{k^2(k^2 + 2m^2 - \omega_L^2(k))} \ , \ Z_T(k) = \frac{\omega_T(k)(\omega_T^2(k) - k^2)}{2m^2\omega_T^2(k) - (\omega_T^2(k) - k^2)^2} \ , \ Z_{\pm}(q) = \frac{\omega_{\pm}(q)^2 - q^2}{2m_f^2}$$

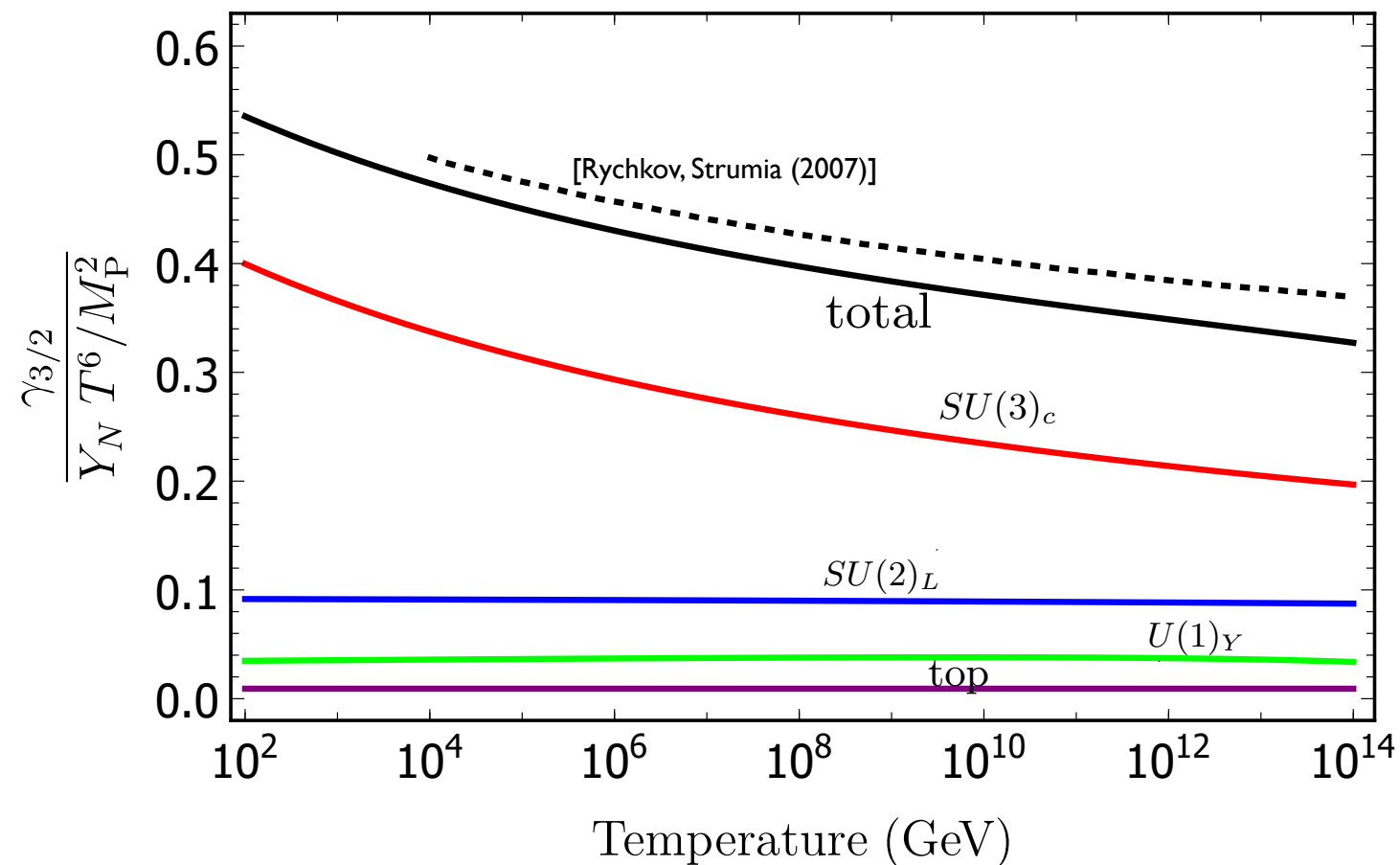
Background of the calculation

- **Effective theory of light gravitinos, only $1/2$ goldstino components**
[Ellis, Kim, Nanopoulos (1984); Moroi, Murayama, Yamagushi, Kawasaki (1993,1994)]
- **Use of the Braaten, Pisarski, Yan method including $3/2$ components**
[Ellis, Nanopoulos, Olive, Rey (1996), Bolz, Buchmuller, Plumacher, Brandenburger (1998,2001); Pedlar, Steffen (2007)]
- **1-loop calculation beyond the HTL approximation** [Rychkov, Strumia (2007)]
- **Our calculation: error corrections and proper parametrization of the result** [Eberl, Gialamas, VCS (2021)]
- **Improved calculation and reheating temperature** [Eberl, Gialamas, VCS (2024)]

Result and cosmological consequences

$$\gamma_{\text{sub}} + \gamma_{\text{D}} = \frac{3\zeta(3)}{16\pi^3} \frac{T^6}{M_{\text{P}}^2} \sum_{N=1}^3 c_N g_N^2 \left(1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) \ln \left(\frac{k_N}{g_N} \right)$$

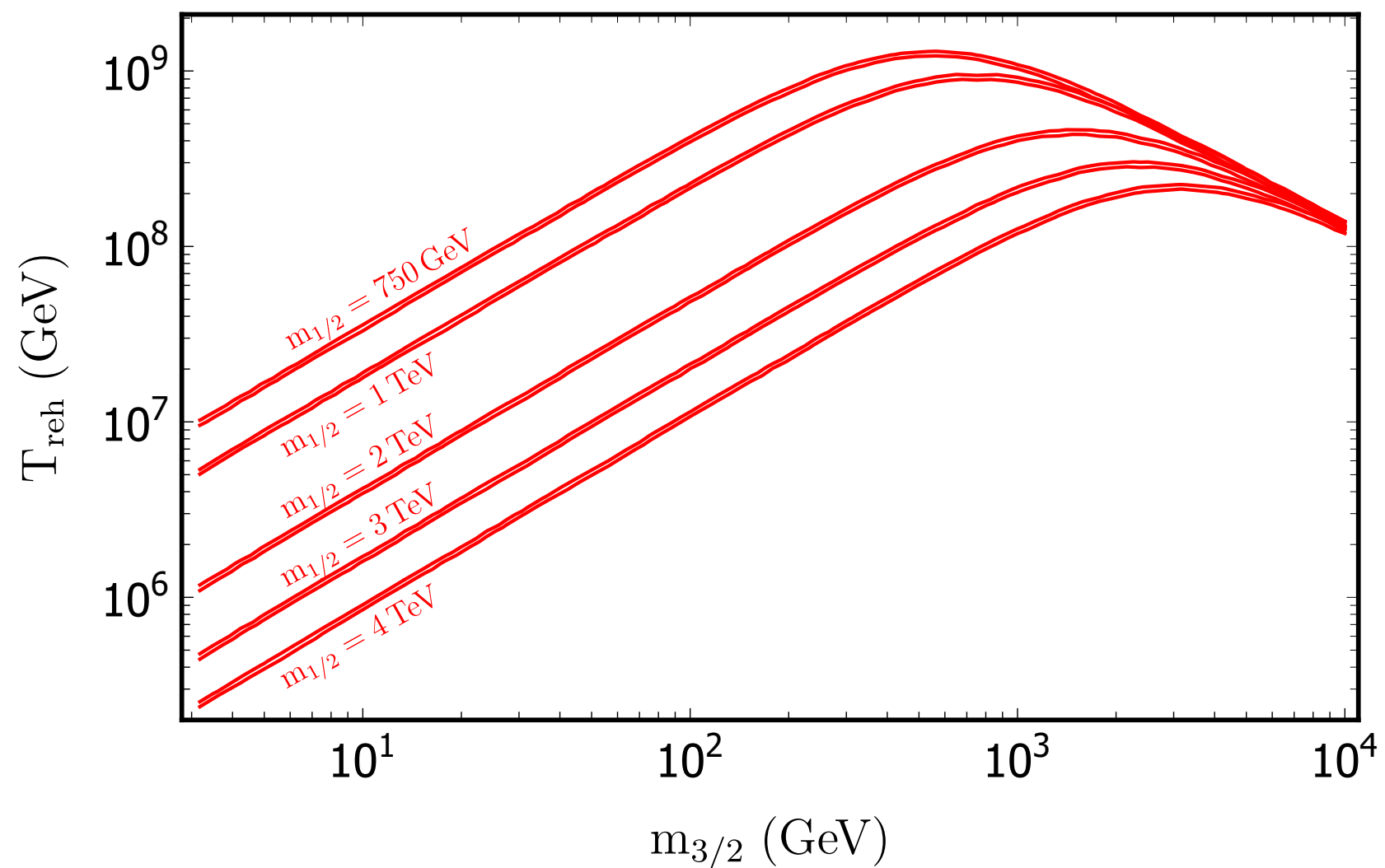
Gauge group	c_N	k_N
$U(1)_Y$	41.937	0.824
$SU(2)_L$	68.228	1.008
$SU(3)_c$	21.067	6.878



Gravitino abundance

$$Y_{3/2}(T) \simeq \frac{\gamma_{3/2}(T_{\text{reh}})}{H(T_{\text{reh}}) n_{\text{rad}}(T_{\text{reh}})} \frac{g_{*s}(T)}{g_{*s}(T_{\text{reh}})}$$

$$\Omega_{\text{DM}} h^2 = \frac{\rho_{3/2}(t_0) h^2}{\rho_{\text{cr}}} = \frac{m_{3/2} Y_{3/2}(T_0) n_{\text{rad}}(T_0) h^2}{\rho_{\text{cr}}} \simeq 1.33 \times 10^{24} \frac{m_{3/2} \gamma_{3/2}(T_{\text{reh}})}{T_{\text{reh}}^5}$$



Thermal Production of Gravitinos

- Thermal production rate

$$\gamma_{3/2} = \sum_{N=1}^3 \gamma_N \propto \sum_{N=1}^3 c_N g_N^2 \left(1 + \frac{M_N^2}{3m_{3/2}^2} \right) T^6.$$

- Two contributions

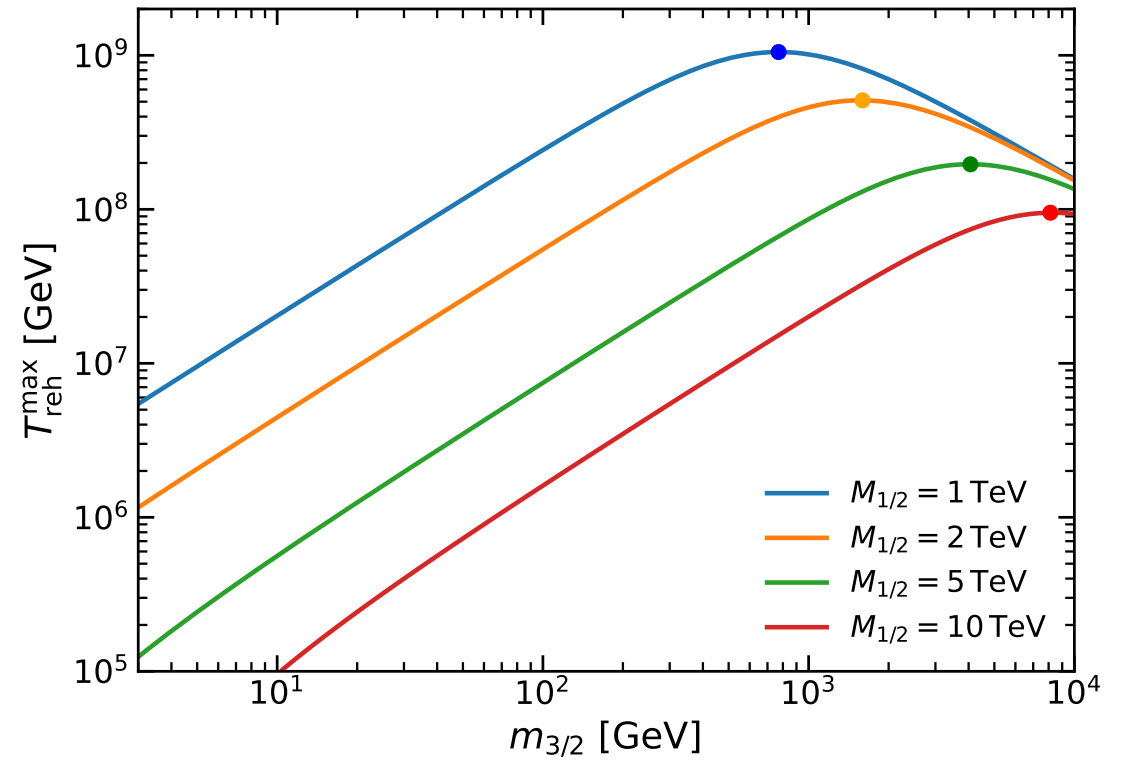
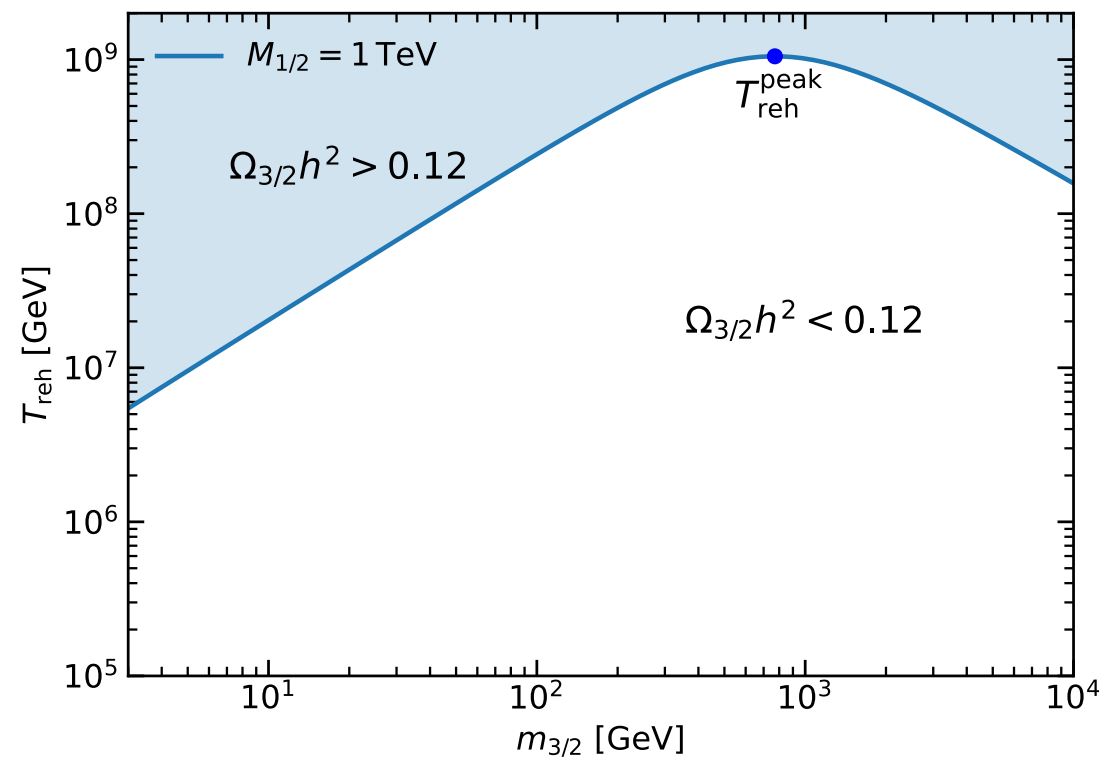
- Transverse spin-3/2 gravitino
- Goldstino (spin-1/2) component

SM group	c_N	k_N
U(1) _Y	35.56	0.85
SU(2) _L	35.33	1.38
SU(3) _c	29.40	3.07

- Approximate relic abundance

$$\Omega_{3/2} h^2 \simeq 0.016 \left(\frac{m_{3/2}}{1 \text{ TeV}} \right) \frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \sum_N c_N g_N^2 \left(1 + \frac{M_N^2}{3m_{3/2}^2} \right) \ln \left(\frac{k_N}{g_N} \right).$$

- The transition from Goldstino to transverse production is responsible for the appearance of the peak reheating temperature, $T_{\text{reh}}^{\text{peak}}$.



$M_{1/2} (\text{ TeV})$	$m_{3/2} (\text{ GeV})$	$T_{\text{reh}}^{\text{peak}} (\text{ GeV})$
1	771	1.05×10^9
2	1596	5.10×10^8
5	4065	1.96×10^8
10	8124	9.51×10^7

Motivation

- **Gravitino Freeze-In**

The reheating temperature compatible with $\Omega_{3/2} h^2 = 0.12$ exhibits a maximum, $T_{\text{reh}}^{\text{peak}}$.

- **Collider Constraints**

Increasing lower bounds on the gluino mass imply

$$M_{1/2} \uparrow \quad \Longrightarrow \quad T_{\text{reh}}^{\text{peak}} \downarrow .$$

- **Thermal Leptogenesis**

Successful thermal leptogenesis typically requires

$$T_{\text{reh}} \gtrsim 2 \times 10^9 \text{ GeV}.$$

- **Tension**

Future collider bounds may further reduce $T_{\text{reh}}^{\text{peak}}$, increasing the tension between thermal gravitino production and thermal leptogenesis.

Can a non-standard cosmological history relax this bound?

An Additional Scalar Component

- **Possible Origin**

Additional scalar fields arise naturally in supergravity and string-inspired models.

Examples include

- moduli,
- flat directions,
- saxions,
- hidden-sector scalars.

- **Model-Independent Description**

Rather than specifying the microscopic origin, we describe the scalar phenomenologically through

$$(w_\phi, \Gamma_\phi, r_\phi)$$

where

- w_ϕ : equation-of-state parameter,
- Γ_ϕ : decay width,
- $r_\phi = \rho_\phi / \rho_R|_{T_{\text{reh}}}$: initial abundance.

Question: How do these parameters affect Δ_ϕ and $T_{\text{reh}}^{\text{peak}, \phi}$?

Boltzmann Equations

- Three-component cosmological system

$$\begin{aligned}\dot{\rho}_\phi &= -3(1 + w_\phi)H \rho_\phi - \Gamma_\phi \rho_\phi, \\ \dot{\rho}_R &= -4H \rho_R + \Gamma_\phi \rho_\phi, \\ \dot{\rho}_{3/2} &= -3H \rho_{3/2} + m_{3/2} \gamma_{3/2}(T).\end{aligned}$$

- Expansion rate

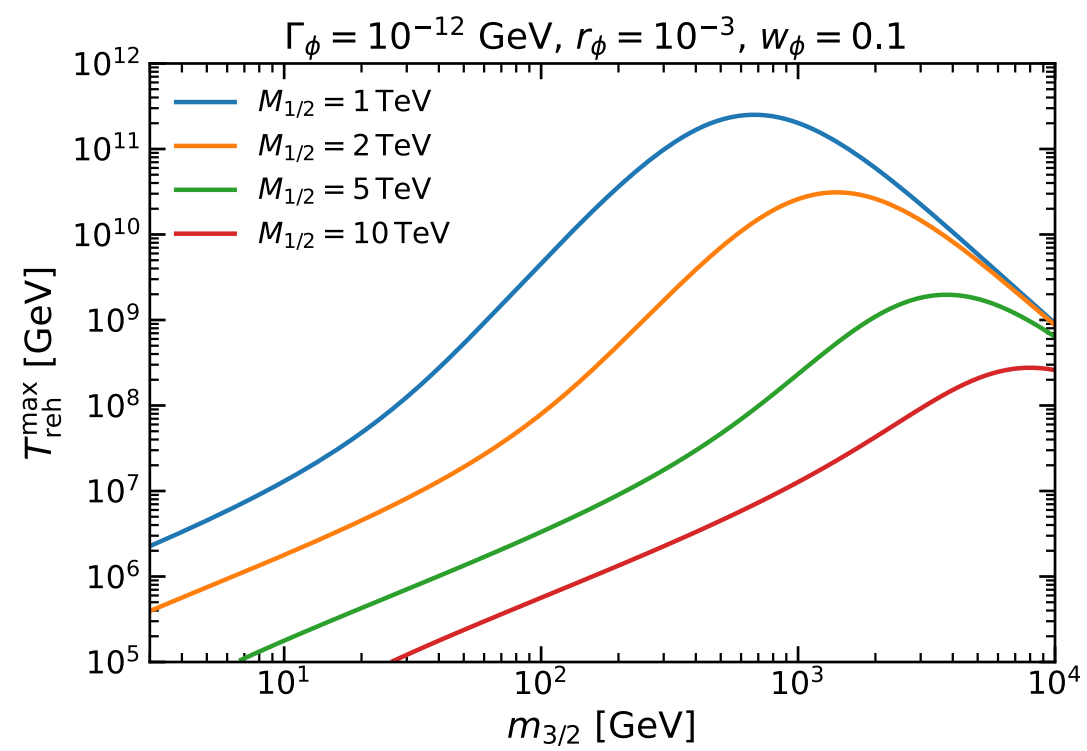
$$H^2 = \frac{1}{3M_P^2} (\rho_\phi + \rho_R).$$

- Initial conditions

At $T = T_{\text{reh}}$,

$$\rho_\phi = r_\phi \rho_R, \quad \rho_{3/2} = 0.$$

- Numerical integration from T_{reh} down to $T \simeq 10^{-4} \text{ GeV}$.



$M_{1/2}$ (TeV)	$m_{3/2}$ (GeV)	$T_{\text{reh}}^{\text{peak}, \phi}$ (GeV)	Δ_ϕ
1	675	2.51×10^{11}	187.6
2	1472	3.11×10^{10}	49.6
5	3856	1.97×10^9	8.7
10	7777	2.76×10^8	2.6

Understanding the Dilution

- Dilution factor

$$\Delta_\phi \equiv \frac{Y_{3/2}}{Y_{3/2}^\phi}.$$

- Matter-like scalar ($w_\phi = 0$)

$$\frac{\rho_\phi}{\rho_R} \simeq r_\phi \frac{T_{\text{reh}}}{T}, \quad T_{\text{dom}} \simeq r_\phi T_{\text{reh}}.$$

- Decay temperature

$$T_\phi \propto \sqrt{\Gamma_\phi M_P}.$$

- Scaling estimate

$$\Delta_\phi \sim \frac{T_{\text{dom}}}{T_\phi} \sim \frac{r_\phi T_{\text{reh}}}{T_\phi}.$$

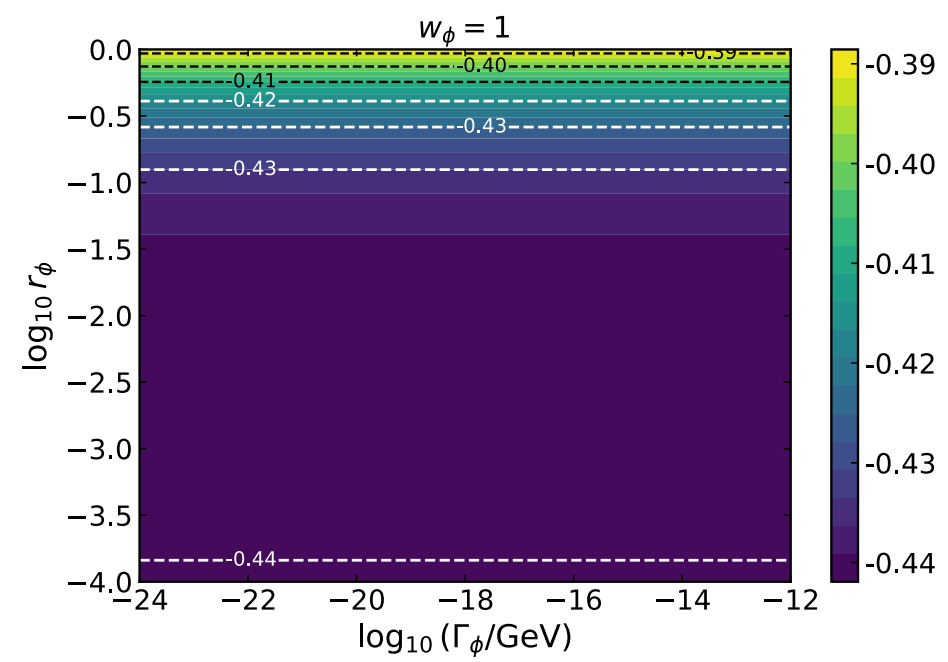
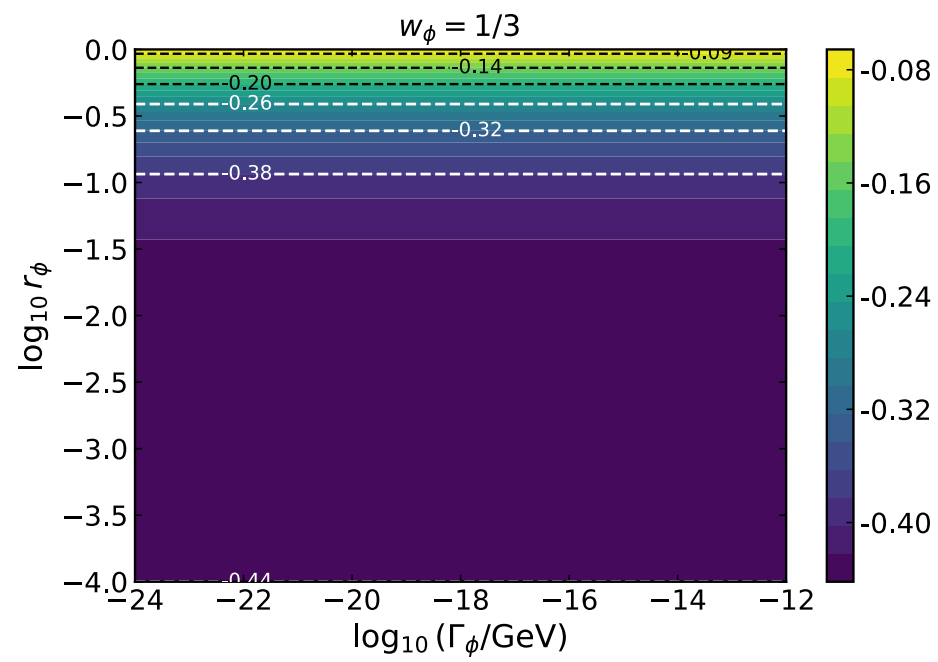
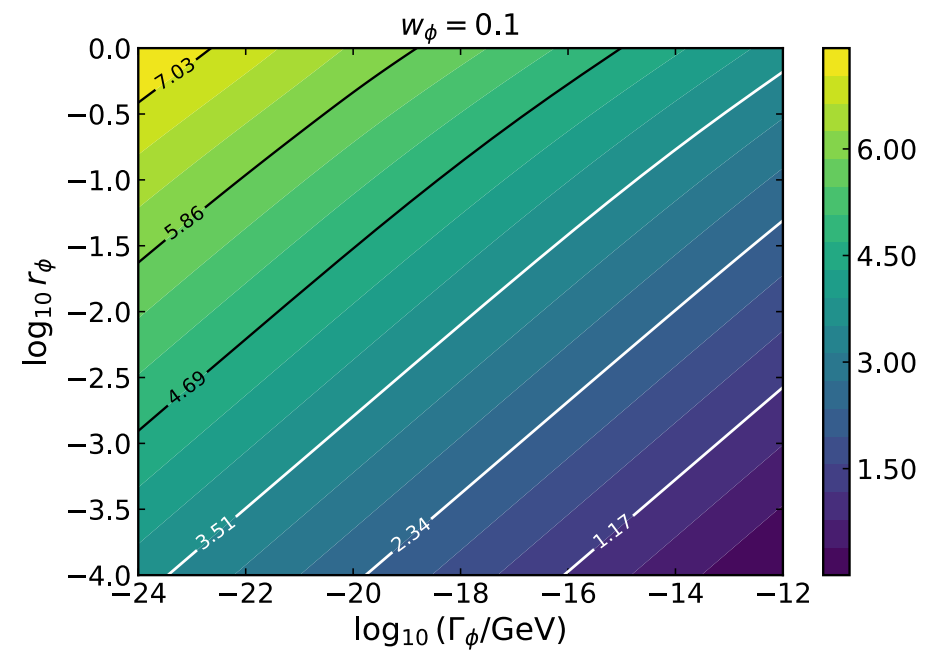
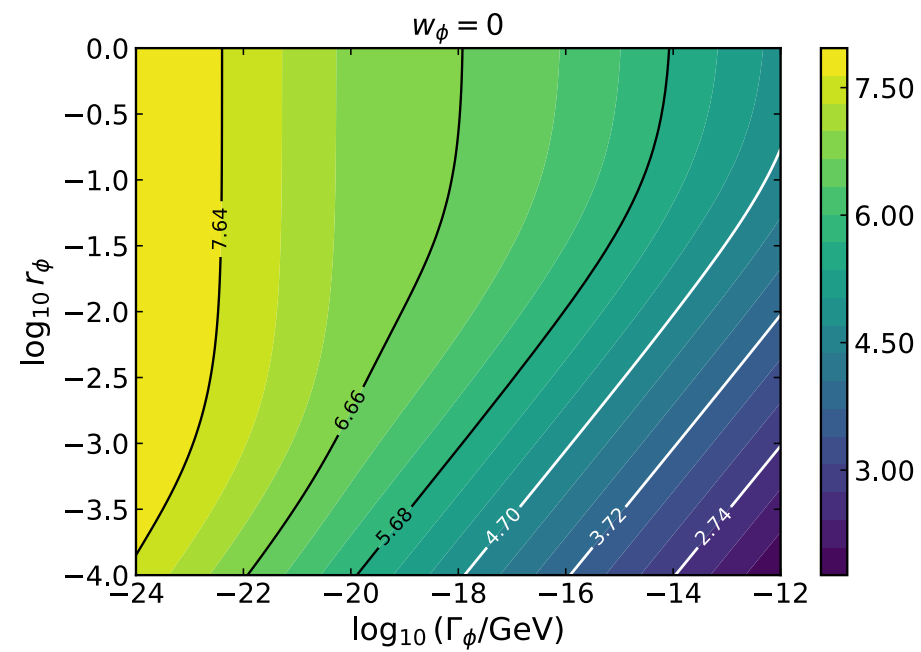
- Consequence

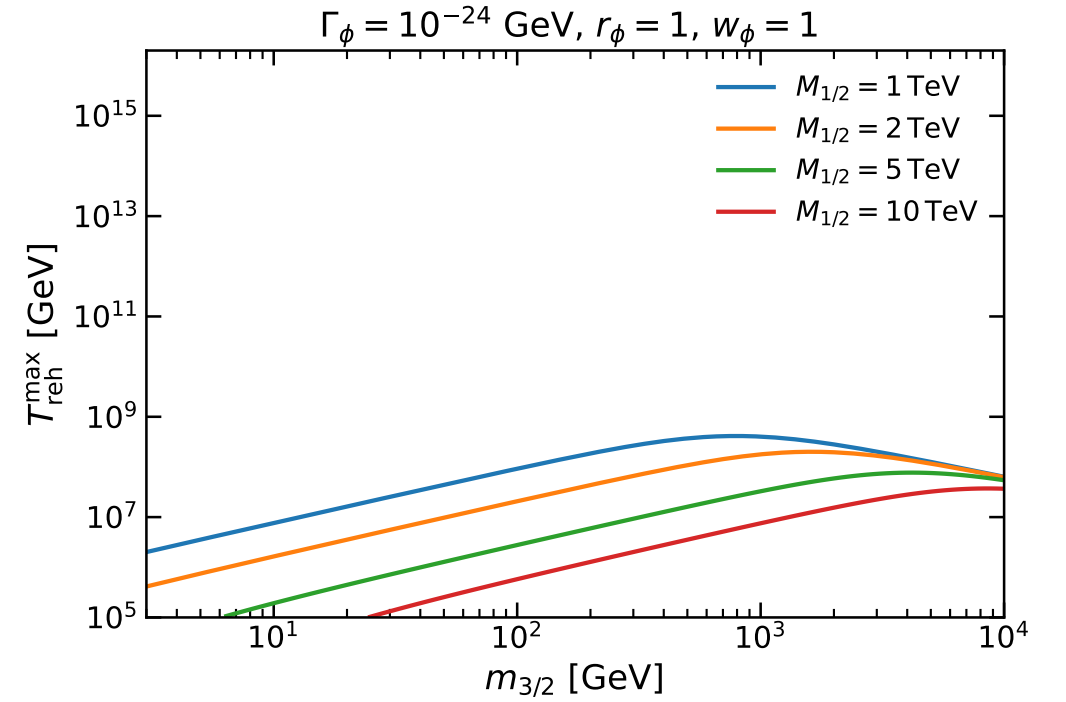
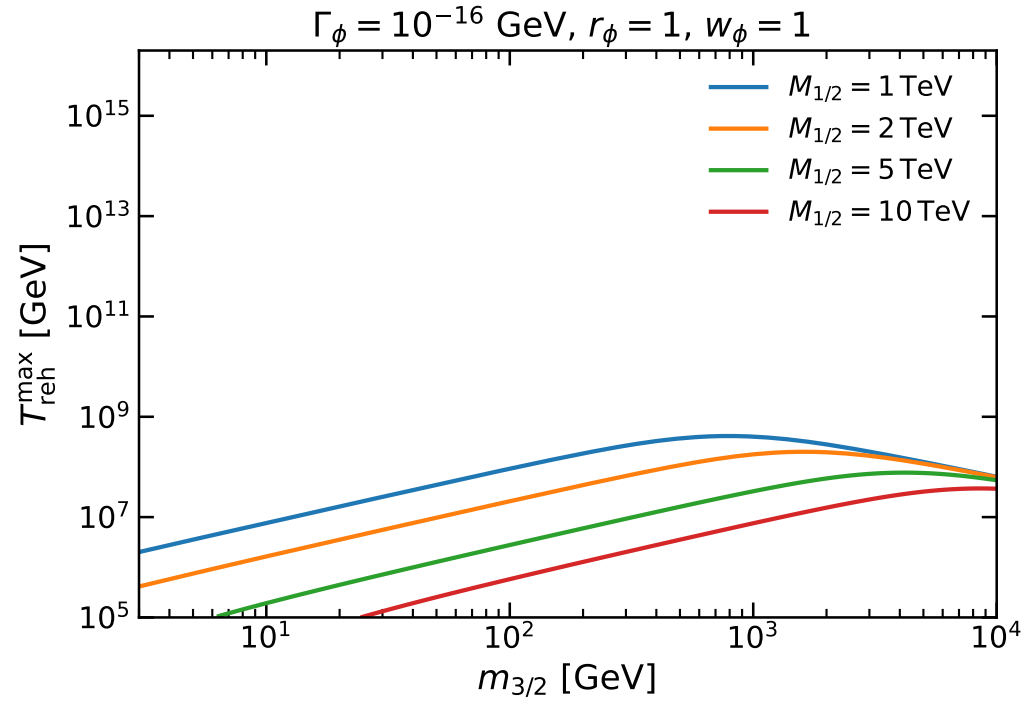
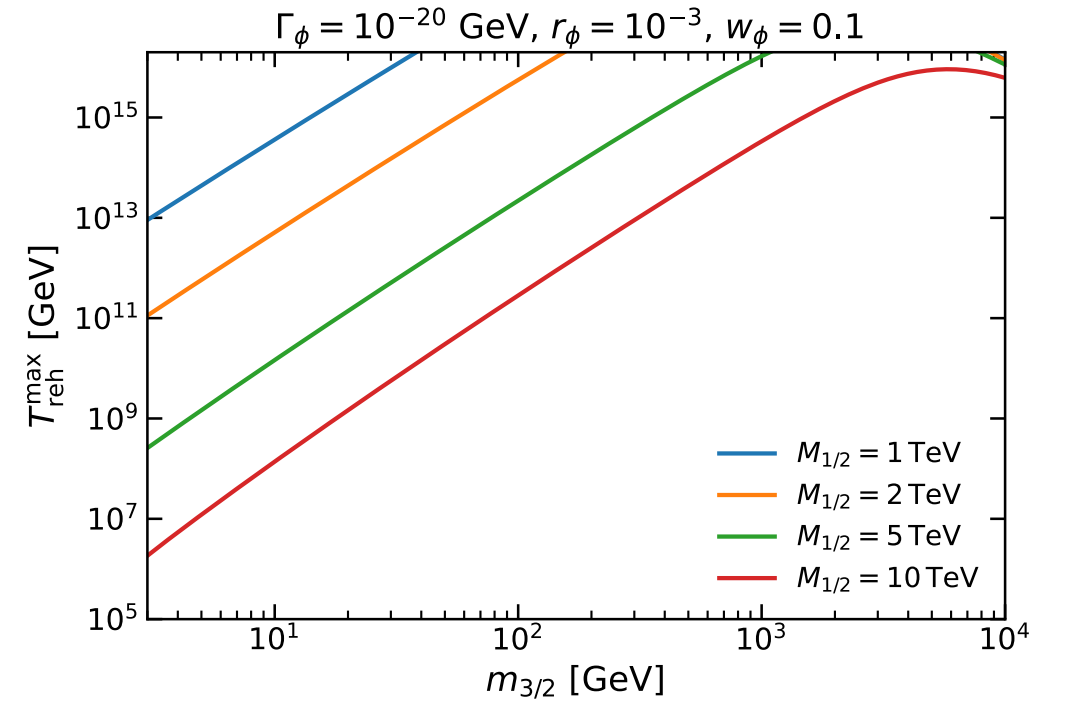
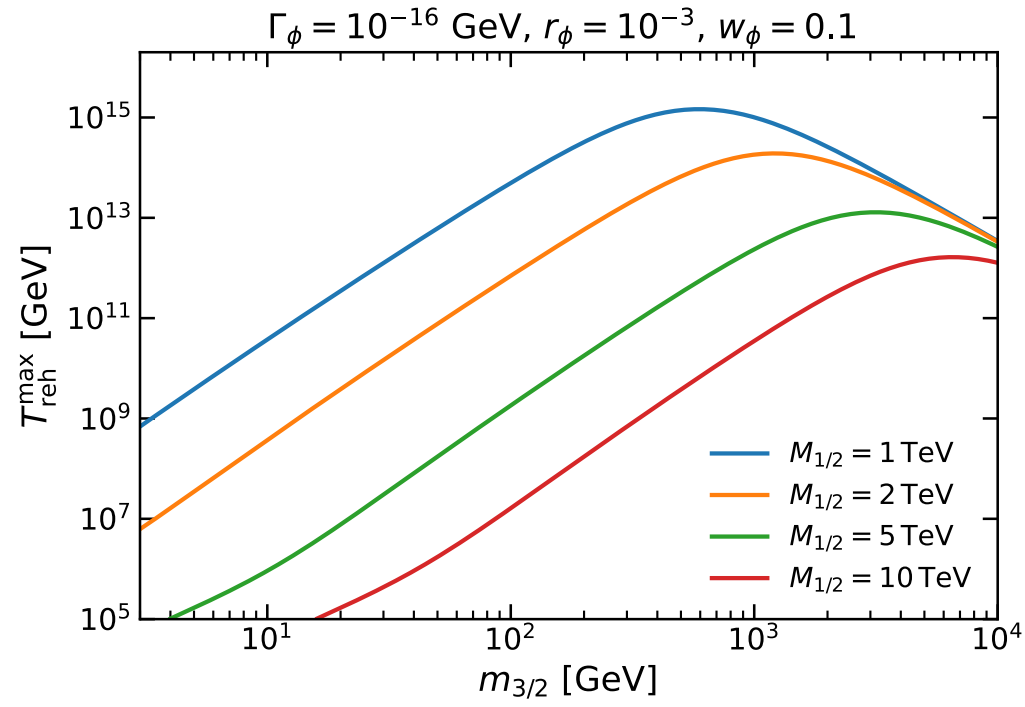
$$T_{\text{reh}}^{\text{peak},\phi} \sim \Delta_\phi T_{\text{reh}}^{\text{peak}}.$$

Physical Interpretation

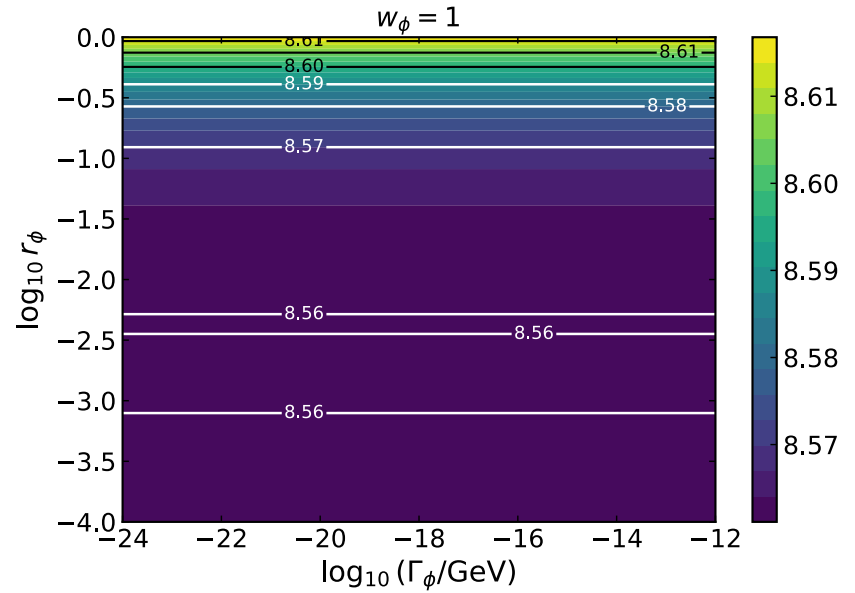
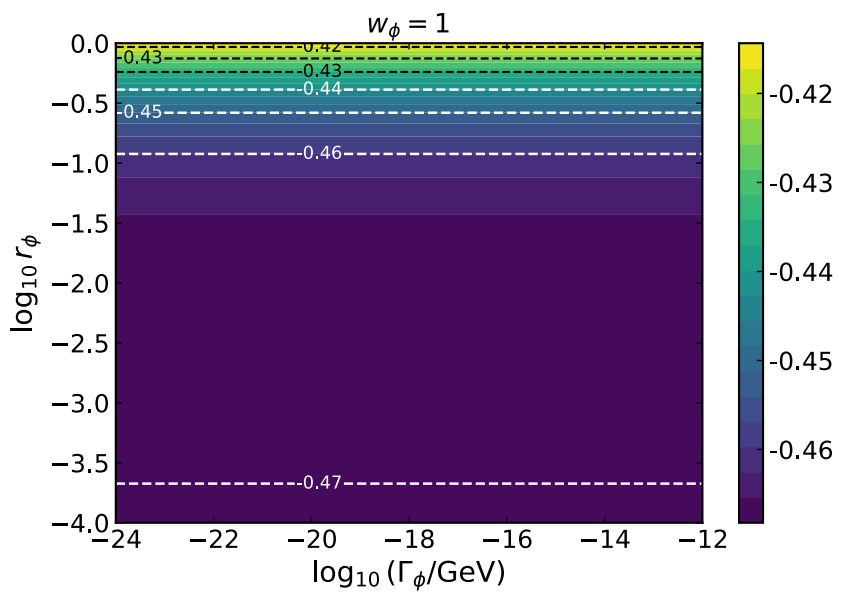
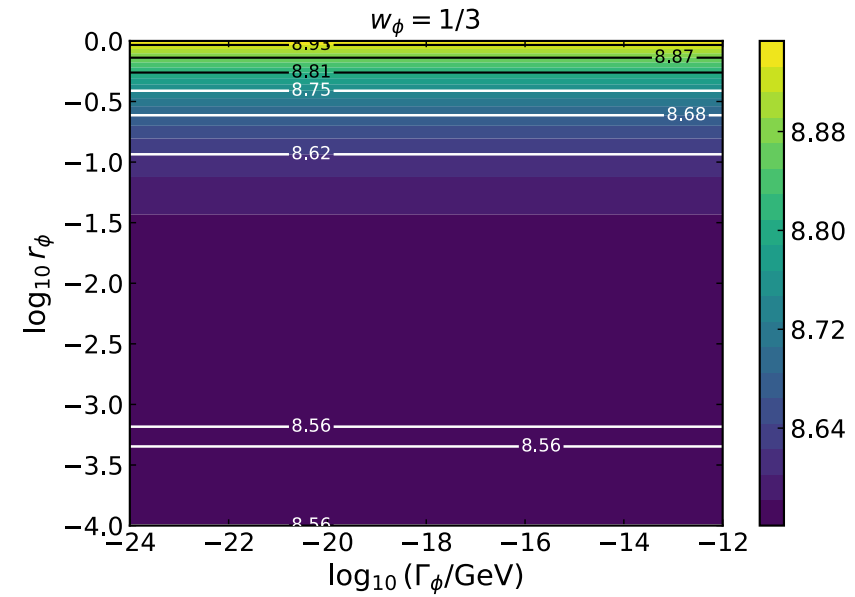
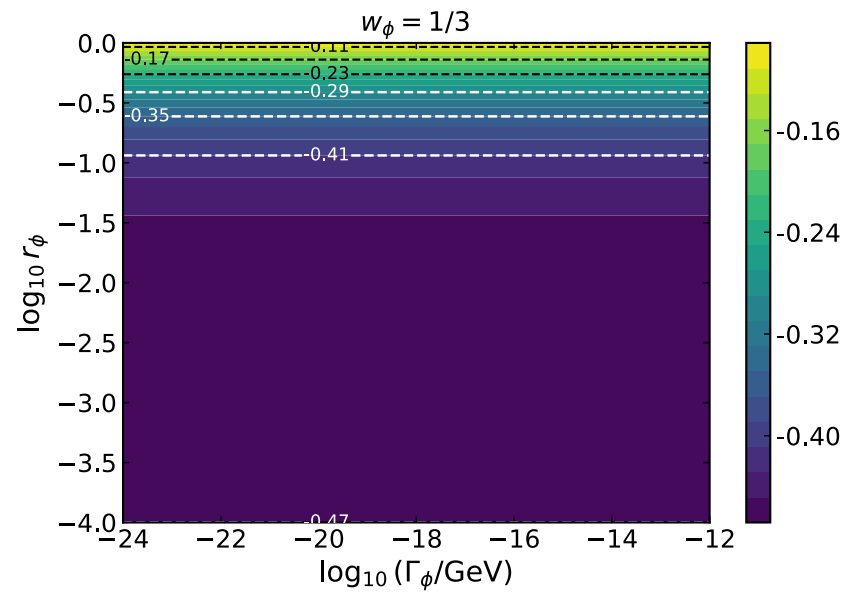
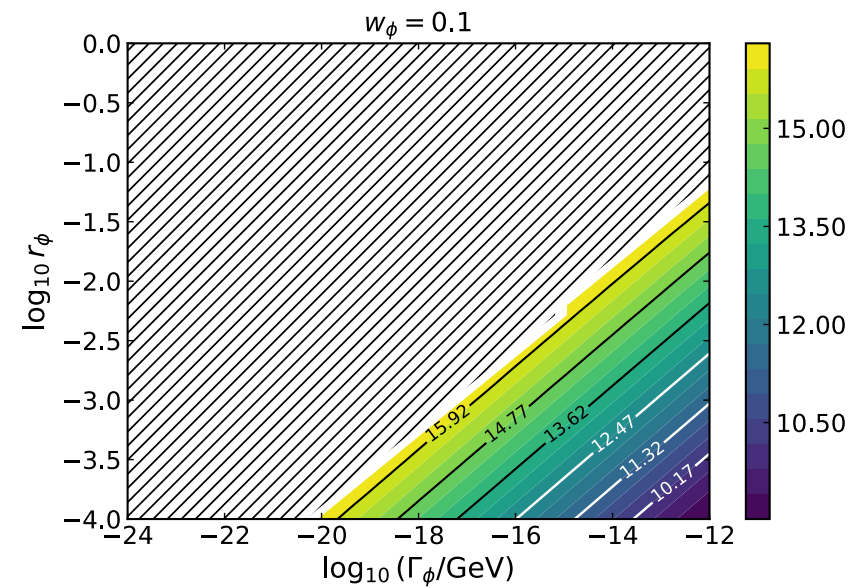
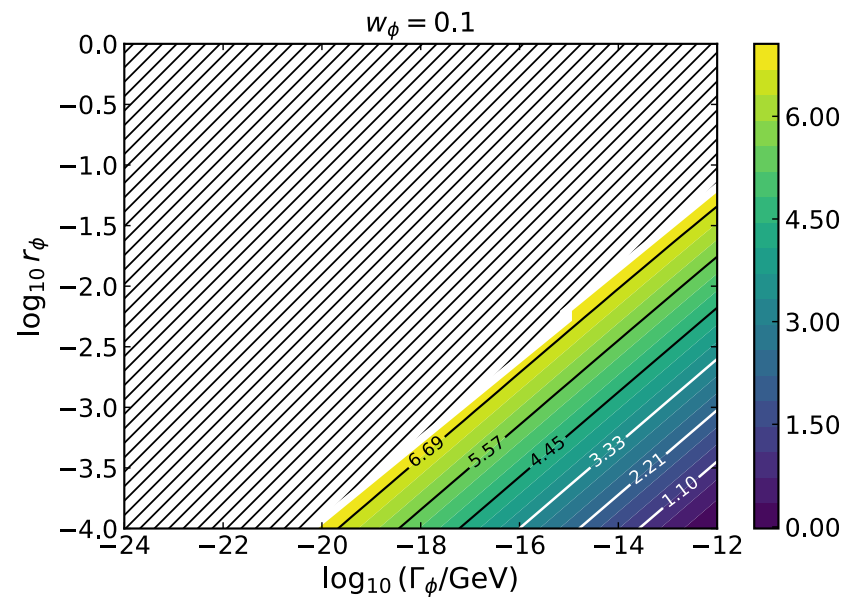
- Increasing $r_\phi \implies$ earlier ϕ domination $\implies$ larger Δ_ϕ
- Decreasing $\Gamma_\phi \implies$ later decay $\implies$ larger Δ_ϕ .
- Equation of state
 - $w_\phi < 1/3$: matter redshifts slower than radiation $\implies$ dilution.
 - $w_\phi > 1/3$: component redshifts faster than radiation $\implies$ enhancement.

The numerical results follow directly from these simple scaling arguments





$M_{1/2}(\text{TeV})$	$w_\phi = 0.1, \Gamma_\phi = 10^{-16} \text{ GeV}$			$w_\phi = 1, \Gamma_\phi = 10^{-12} \text{ GeV}$		
	$m_{3/2}(\text{GeV})$	$T_{\text{reh}}^{\text{peak},\phi}(\text{GeV})$	Δ_ϕ	$m_{3/2}(\text{GeV})$	$T_{\text{reh}}^{\text{peak},\phi}(\text{GeV})$	Δ_ϕ
1	645	1.45×10^{15}	8.80×10^5	763	3.64×10^8	3.40×10^{-1}
2	1441	1.84×10^{14}	2.37×10^5	1584	1.77×10^8	3.40×10^{-1}
5	3826	1.22×10^{13}	4.20×10^4	3896	6.77×10^7	3.39×10^{-1}
10	7748	1.58×10^{12}	1.14×10^4	7931	3.27×10^7	3.38×10^{-1}



Summary

- We studied gravitino freeze-in dark matter in the presence of an additional scalar component ϕ .
- The non-standard cosmological evolution is characterized by three phenomenological parameters:

$$(w_\phi, \Gamma_\phi, r_\phi).$$

- For $w_\phi < 1/3$, entropy production dilutes the gravitino abundance, allowing substantially larger values of $T_{\text{reh}}^{\text{peak},\phi}$.
- For $w_\phi \geq 1/3$, the gravitino abundance is enhanced ($\Delta_\phi < 1$), leading to smaller values of $T_{\text{reh}}^{\text{peak},\phi}$.
- Matter-like configurations can increase the allowed reheating temperature by several orders of magnitude.
- The gravitino upper bound on T_{reh} is therefore significantly relaxed, alleviating its tension with high-scale baryogenesis scenarios.

Thank you for your attention!

Backup slides

$$Y_{3/2} = \frac{n_{3/2}}{s}, \quad \frac{dY_{3/2}}{dt} = \frac{\gamma_{3/2}}{s}. \quad \frac{dY_{3/2}}{dT} = -\frac{\gamma_{3/2}}{H T s} \left[1 + \frac{T}{3} \frac{d \ln g_{*s}}{dT} \right].$$

$$Y_{3/2}(T) = Y_{3/2}(T_{\text{reh}}) - \int_{T_{\text{reh}}}^T d\tau \frac{\gamma_{3/2}(\tau)}{H(\tau) s(\tau) \tau} \left[1 + \frac{\tau}{3} \frac{d \ln g_{*s}(\tau)}{d\tau} \right].$$

$$Y_{3/2}(T) \xrightarrow{T \ll T_{\text{reh}}} \frac{\gamma_{3/2}(T_{\text{reh}})}{H(T_{\text{reh}}) s(T_{\text{reh}})}.$$

Moreover, using the definitions

$$\Omega_{3/2} = \frac{\rho_{3/2}}{\rho_{\text{cr}}}, \quad \rho_{3/2} = n_{3/2} m_{3/2},$$

and the Eqs (3) and (8), one gets $\Omega_{3/2} h^2$ as

$$\Omega_{3/2} h^2 = \frac{\rho_{3/2}(t_0) h^2}{\rho_{\text{cr}}} = \frac{m_{3/2} Y_{3/2}(T_0) s(T_0) h^2}{\rho_{\text{cr}}} \simeq 1.33 \times 10^{24} \frac{m_{3/2} \gamma_{3/2}(T_{\text{reh}})}{T_{\text{reh}}^5}.$$

$$\Omega_{3/2} h^2 \simeq 0.016 \left(\frac{m_{3/2}}{1 \text{ TeV}} \right) \frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \sum_{N=1}^3 c_N g_N^2 \left(1 + \frac{M_N^2}{3m_{3/2}^2} \right) \ln \left(\frac{k_N}{g_N} \right).$$

$$w_\phi=\frac{P_\phi}{\rho_\phi}=\frac{\dot{\phi}^2/2-V(\phi)}{\dot{\phi}^2/2+V(\phi)}\,.$$

$$\begin{aligned}\dot{\rho}_\phi &= -3(1+w_\phi)H\,\rho_\phi - \Gamma_\phi\,\rho_\phi\,,\\ \dot{\rho}_R &= -4H\,\rho_R + \Gamma_\phi\,\rho_\phi\,,\\ \dot{\rho}_{3/2} &= -3H\,\rho_{3/2} + m_{3/2}\,\gamma_{3/2}(T)\,,\end{aligned}$$

$$H^2=\frac{1}{3M_P^2}\left(\rho_\phi+\rho_R\right)\,.$$

$$\rho_R(T)=\frac{\pi^2}{30}\,g_*(T)\,T^4$$

$$s(T)=\frac{2\pi^2}{45}\,g_{*s}(T)\,T^3\,,$$

$$N=\ln a\,,\,\,$$

$$\begin{aligned}\frac{d\rho_\phi}{dN}&=-3(1+w_\phi)\rho_\phi-\frac{\Gamma_\phi}{H}\rho_\phi,\\ \frac{d\rho_R}{dN}&=-4\rho_R+\frac{\Gamma_\phi}{H}\rho_\phi,\\ \frac{d\rho_{3/2}}{dN}&=-3\rho_{3/2}+\frac{m_{3/2}\gamma_{3/2}(T)}{H}\,.\end{aligned}$$

$$\begin{aligned}\rho_R(0) &= \frac{\pi^2}{30} g_*(T_{\rm reh}) T_{\rm reh}^4\,,\\ \rho_\phi(0) &= r_\phi\,\rho_R(0)\,,\\ \rho_{3/2}(0) &= 0\,,\end{aligned}$$

$$r_\phi\equiv\left.\frac{\rho_\phi}{\rho_R}\right|_{T=T_{\rm reh}}$$

$$N\simeq\ln\left(\frac{T_{\rm reh}}{T}\right)\,,$$

$$N_{\rm final}^{\rm (RD)}\simeq\ln\left(\frac{T_{\rm reh}}{T_{\rm final}}\right)\,.$$

$$Y_{3/2}^\phi=\frac{n_{3/2}}{s}=\frac{\rho_{3/2}}{m_{3/2}\,s(T)}\,.$$

$$\Omega_{3/2}^\phi h^2=2.742\times 10^8\left(\frac{m_{3/2}}{\rm GeV}\right)Y_{3/2}^\phi\,.$$

$$\Delta_\phi=\frac{\Omega_{3/2}}{\Omega_{3/2}^\phi}\,.$$

$$\Delta_\phi=\frac{Y_{3/2}}{Y_{3/2}^\phi},$$

$$T_\phi\simeq\left(\frac{90}{\pi^2g_*(T_\phi)}\right)^{1/4}\sqrt{\Gamma_\phi M_P}\,.$$

For a matter-like scalar component, $w_\phi = 0$, the ratio of scalar to radiation energy densities evolves as

$$\frac{\rho_\phi}{\rho_R} \simeq r_\phi \frac{T_{\text{reh}}}{T}, \quad (28)$$

prior to the decay of ϕ . Consequently, the scalar component starts to dominate the energy density at the temperature

$$T_{\text{dom}} \simeq r_\phi T_{\text{reh}}. \quad (29)$$

If a period of ϕ domination is realized, one expects the dilution factor to scale parametrically as

$$\Delta_\phi \sim \frac{T_{\text{dom}}}{T_\phi} \sim \frac{r_\phi T_{\text{reh}}}{T_\phi}. \quad (30)$$

Equation (30) explains the main trends observed in our numerical analysis. The dilution increases with the initial abundance r_ϕ and decreases with the decay temperature T_ϕ , or equivalently with increasing decay width Γ_ϕ , since $T_\phi \propto \sqrt{\Gamma_\phi}$. As a result, larger values of r_ϕ and smaller values of Γ_ϕ lead to a stronger suppression of the gravitino abundance.

For a fixed relic density, this suppression can be compensated by increasing the reheating temperature, suggesting the approximate scaling

$$T_{\text{reh}}^{\text{peak},\phi} \sim \Delta_\phi T_{\text{reh}}^{\text{peak}}, \quad (31)$$

$$\begin{aligned}
\psi_\mu(\mathbf{k}, \frac{3}{2}) &= u(\mathbf{k}, \frac{1}{2}) \epsilon_\mu(\mathbf{k}, 1) , \\
\psi_\mu(\mathbf{k}, \frac{1}{2}) &= \sqrt{\frac{1}{3}} u(\mathbf{k}, -\frac{1}{2}) \epsilon_\mu(\mathbf{k}, 1) + \sqrt{\frac{2}{3}} u(\mathbf{k}, \frac{1}{2}) \epsilon_\mu(\mathbf{k}, 0) \\
\psi_\mu(\mathbf{k}, -\frac{1}{2}) &= \sqrt{\frac{1}{3}} u(\mathbf{k}, \frac{1}{2}) \epsilon_\mu(\mathbf{k}, -1) + \sqrt{\frac{2}{3}} u(\mathbf{k}, -\frac{1}{2}) \epsilon_\mu(\mathbf{k}, 0) \\
\psi_\mu(\mathbf{k}, -\frac{3}{2}) &= u(\mathbf{k}, -\frac{1}{2}) \epsilon_\mu(\mathbf{k}, -1) ,
\end{aligned}$$

Our starting point is the Lagrangian that describes the gravitino interactions in the MSSM [28],

$$\begin{aligned}
\mathcal{L}_{\tilde{G}, \text{int}}^{(\alpha)} &= -\frac{i}{\sqrt{2}M_{\text{P}}} \left[\mathcal{D}_\mu^{(\alpha)} \phi^{*i} \tilde{\overline{G}}_\nu \gamma^\mu \gamma^\nu \chi_L^i - \mathcal{D}_\mu^{(\alpha)} \phi^i \tilde{\overline{\chi}}_L^i \gamma^\nu \gamma^\mu \tilde{G}_\nu \right] \\
&\quad - \frac{i}{8M_{\text{P}}} \tilde{\overline{G}}_\mu [\gamma^\nu, \gamma^\rho] \gamma^\mu \lambda^{(\alpha) a} F_{\nu\rho}^{(\alpha) a} ,
\end{aligned} \tag{3}$$

with the covariant derivative given by

$$\mathcal{D}_\mu^{(\alpha)} \phi^i = \partial_\mu \phi^i + i g_\alpha A_\mu^{(\alpha) a} T_{a, ij}^{(\alpha)} \phi^j , \tag{4}$$

and the field strength tensor $F_{\mu\nu}^{(\alpha) a}$ reads

$$F_{\mu\nu}^{(\alpha) a} = \partial_\mu A_\nu^{(\alpha) a} - \partial_\nu A_\mu^{(\alpha) a} - g_\alpha f^{(\alpha) abc} A_\mu^{(\alpha) b} A_\nu^{(\alpha) c} . \tag{5}$$

$$\begin{aligned}
\gamma^\mu \psi_\mu(\mathbf{k}, \lambda) &= 0 , \\
k^\mu \psi_\mu(\mathbf{k}, \lambda) &= 0 , \\
(\not{k} - m_{\tilde{G}}) \psi_\mu(\mathbf{k}, \lambda) &= 0 ,
\end{aligned}$$