Primordial black holes and induced gravitational waves: The case of thermal leptogenesis

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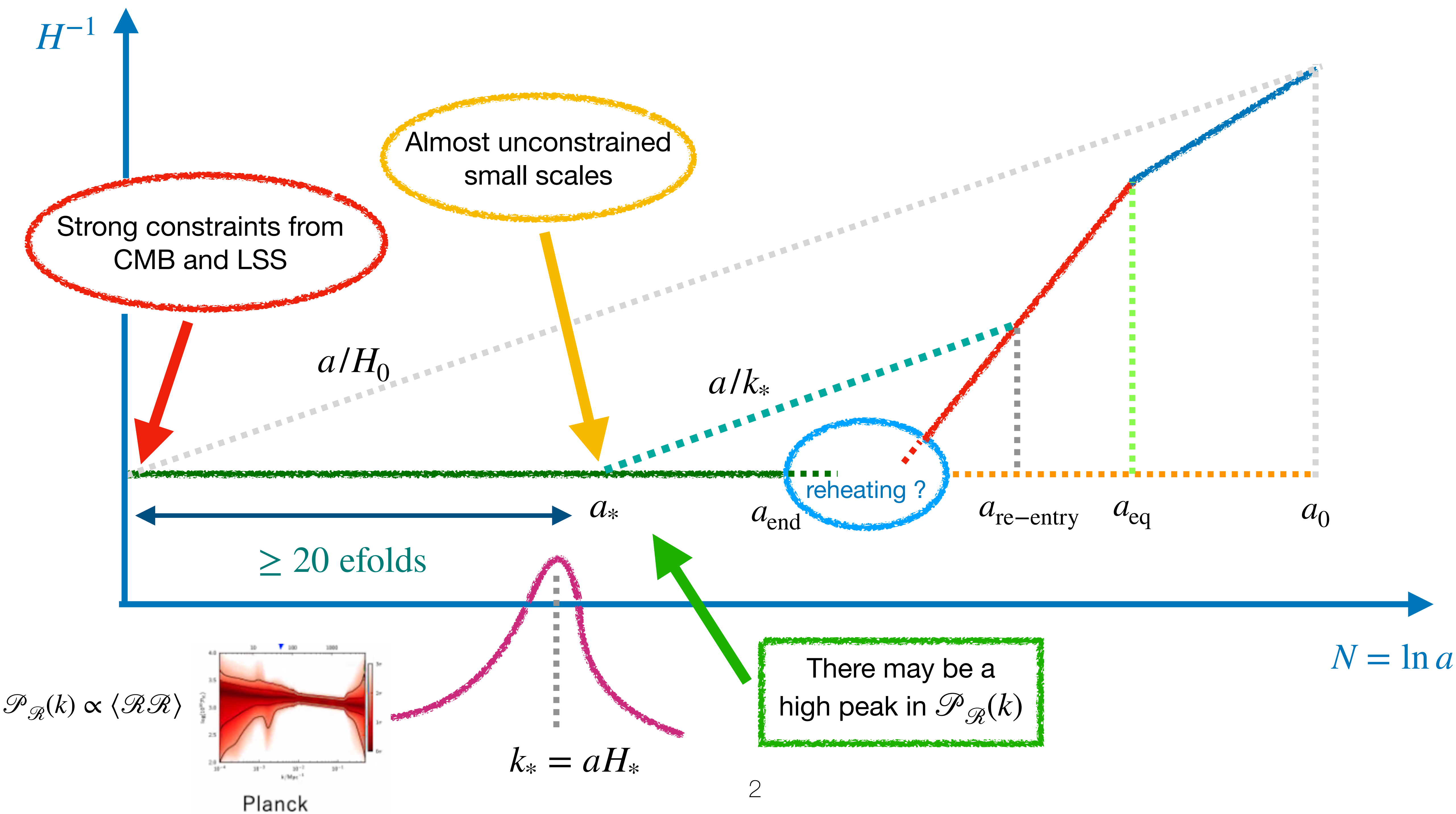
PBHs from collapse of primordial inhomogeneities

- Primordial Black Holes (PBHs) form in the early universe, before star formation, out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta > \delta_c(w \equiv p/\rho)$ [Carr - 1975].



Why we study PBHs?

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BBN ←	$t = 1\mathrm{s}$	$m_{\mathrm{PBH}} = 10^5 M_\odot$	→ SMBHs, LSS?
QCD Phase Transition ←	$t = 10^{-5}\mathrm{s}$	$m_{\mathrm{PBH}} = M_\odot$	→ LIGO/VIRGO Progenitors
			→ PBHs as DM ?
	$t = 10^{-23}\mathrm{s}$	$m_{\mathrm{PBH}} = 10^{15}\mathrm{g}$	→ PBHs evaporate today
	$t = 10^{-29}\mathrm{s}$	$m_{\mathrm{PBH}} = 10^{9}\mathrm{g}$	→ PBHs evaporate at BBN
End of Inflation ←	$t = 10^{-33}\mathrm{s}$	$m_{\mathrm{PBH}} = 10^{5}\mathrm{g}$	

See for reviews in [Carr et al.- 2020, Sasaki et al - 2018, Clesse et al. - 2017]

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4. Collapse through bubble collisions during a phase transition [M. Crawford and D. Schramm - 1982, S. Hawking et al. - 1982, H Kodoma et al. - 1982, I. Moss - 1994]

Observational Constraints on PBH abundances

$$t_{\rm evap} = 2 \times 10^{67}\text{years}\left(\frac{M}{M_\odot}\right)^3, \quad \text{with} \quad M_\odot \simeq 2 \times 10^{33} g \,.$$

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- For the **PBHs with masses** $m_{\text{PBH}} < 10^{15}\text{g}$**, which have evaporated by now,** the PBH constraints are obtained by accounting for the **effects of the products of PBH Hawking evaporation on:**

  a) the abundances of the light elements produced during **BBN**,
  b) the spectral shape of **CMB** radiation
  c) the galactic and extragalactic **cosmic ray background**.

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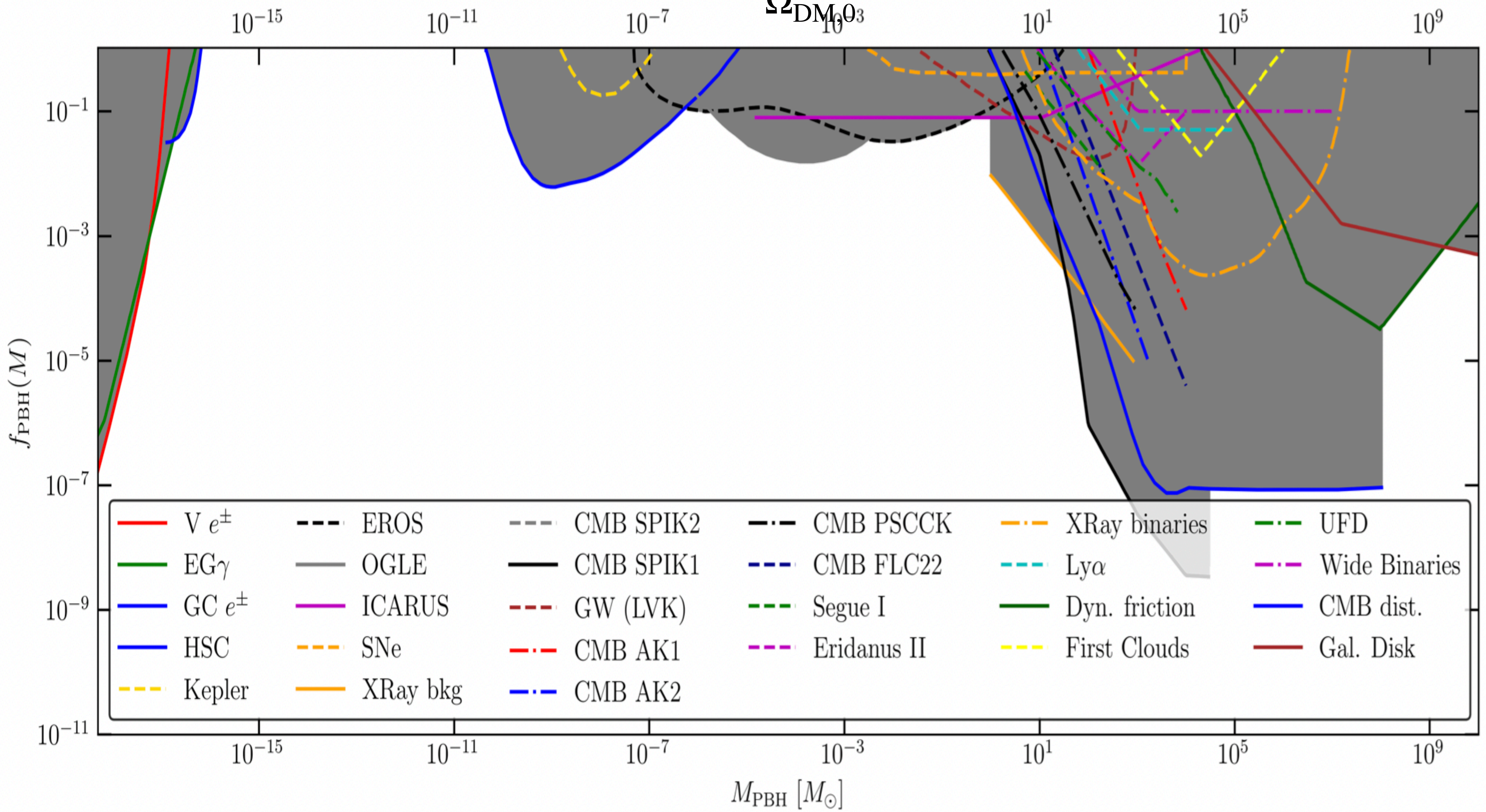
- For PBHs **with masses** $m_{\rm PBH} > 10^{15}$g**, which have not completed yet their evaporation**, we have constraints due to
  a) **gravitational lensing** effects,
  b) **dynamical effects** of a PBH onto an astrophysical system,
  c) **PBH accretion effects**,
  d) effects on the **large scale structure formation,**
  e) **gravitational wave (GW) production** associated to PBHs

Observational Constraints on PBH abundances

$$f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH,0}}{\Omega_{\rm DM,0}}$$



[Bagui et al. - Living Rev.Rel. 28 (2025) 1, 1]

PBHs and GWs

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- 1) **Primordial induced GWs** generated through second order gravitational effects: $\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2$, [Bugaev - 2009, Kohri & Terada - 2018].

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- 4) **GWs induced** at second order **by PBH number density fluctuations** [Papanikolaou et al. - 2020], abundantly produced during a PBH-dominated era.

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- 1) **Primordial induced GWs (SIGWs)** generated through second order gravitational effects: $\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2$, [Bugaev - 2009, Kohri & Terada - 2018].

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Induced Gravitational Waves

1

Inherent nonlinearity

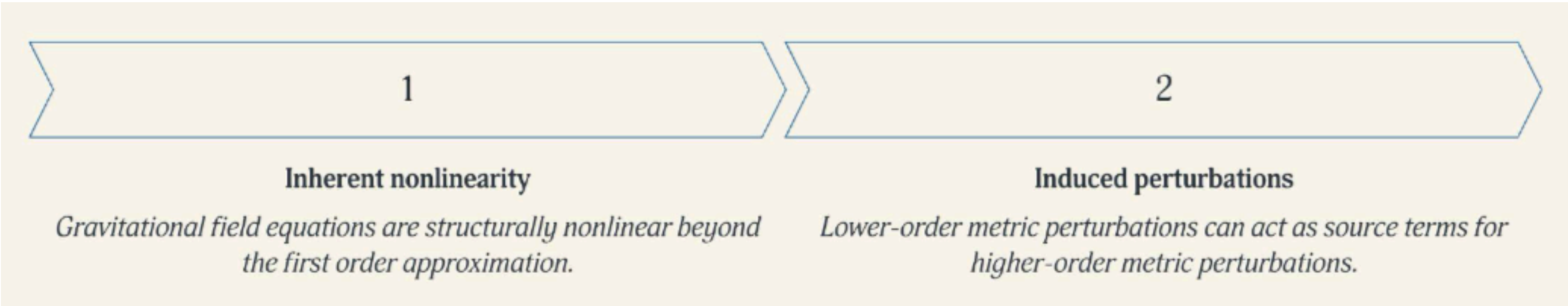
Gravitational field equations are structurally nonlinear beyond the first order approximation.

2

Induced perturbations

Lower-order metric perturbations can act as source terms for higher-order metric perturbations.

Induced Gravitational Waves



A few well-studied scenarios

- Scalar-induced gravitational waves (SIGWs) ⇒ Ananda et al (2007), Baumann et al (2007), Domenech (2021), etc.
- Tensor-induced scalar perturbations ⇒Bari et al (2022 & 2023).
- IGWs from scalar-tensor coupling ⇒Picard & Malik (2023), Bari et al (2023).
- Scalar-induced vector perturbations ⇒Mollerach et al (2003), Saga et al.(2015).
- Tensor-induced tensor perturbations ⇒Picard & Malik (2023), Gorji & Sasaki (2023).

The Source of Induced Gravitational Waves

$$\mathrm{d}s^2 = a^2(\eta)\left[-(1+2\Phi)\,\mathrm{d}\eta^2 - 2B_i\mathrm{d}\eta\mathrm{d}x^i + \left\{\left(1-2\Psi\right)\delta_{ij} + h_{ij} + \frac{h_{ij}^{(2)}}{2!}\right\}\mathrm{d}x^i\mathrm{d}x^j\right]$$

- Φ, Ψ: 1st order scalar metric perturbations (Bardeen gravitational potentials)
- B_i: 1st order vector metric perturbations
- h_{ij}: 1nd order tensor metric perturbations

The Source of Induced Gravitational Waves

- The equation of motion for the Fourier modes, $h_{\vec{k}}$, read as:

$$\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2 \Rightarrow \boxed{h_{ij}^{(2)''}(\boldsymbol{x},\eta) + 2\mathscr{H}(\eta) h_{ij}^{(2)'}(\boldsymbol{x},\eta) - \Delta h_{ij}^{(2)}(\boldsymbol{x},\eta) = \widehat{\mathscr{T}}_{ij}^{\ ab} S_{ab}(\boldsymbol{x},\eta),}$$

$$\begin{aligned}
S_{ij} =& - V_a\partial_a(\partial_i V_j + \partial_j V_i) + \partial_a V_i \partial_a V_j + \partial_i V_a \partial_j V_a + 2V_a \partial_i\partial_j V_a + \frac{\Delta V_i \Delta V_j}{6\mathcal{H}^2(1+w)} && \text{(C.1)}\\
&- 2\partial_i\phi\partial_j\phi - 6\partial_i\psi\partial_j\psi - 4\left(\phi\partial_i\partial_j\phi + \psi\partial_i\partial_j\psi\right) + 2\left(\partial_i\phi\partial_j\psi + \partial_j\phi\partial_i\psi\right)\\
&+ \frac{8}{3\mathcal{H}^2(1+w)}\left[\mathcal{H}^2\partial_i\phi\partial_j\phi + \mathcal{H}\left(\partial_i\phi\partial_j\psi' + \partial_j\phi\partial_i\psi'\right) + \partial_i\psi'\partial_j\psi'\right] && \text{(C.2)}\\
&+ 4h'_{ia}h'_{ja} - 2\partial_i h_{ab}\partial_j h_{ab} + 4h_{ab}\partial_b\left(\partial_i h_{aj} + \partial_j h_{ai}\right)\\
&+ 4\partial_b h_{ia}\left(\partial_a h_{bj} - \partial_b h_{aj}\right) - 4h_{ab}\left(\partial_i\partial_j h_{ab} + \partial_a\partial_b h_{ij}\right) + 12\mathcal{H}^2 w^{(1)} h_{ij} && \text{(C.3)}\\
&- \left(\phi' + \psi' + 4\mathcal{H}\phi\right)\left(\partial_i V_j + \partial_j V_i\right) - 2\phi\left(\partial_i V'_j + \partial_j V'_i\right) + 4\mathcal{H}\left(V_i\partial_j\psi + V_j\partial_i\psi\right)\\
&+ 2\left[(V_i\partial_j\psi)' + (V_j\partial_i\psi)'\right] - \frac{2}{3\mathcal{H}(1+w)}\left(\Delta V_i\partial_j + \Delta V_j\partial_i\right)\left(\phi + \frac{\psi'}{\mathcal{H}}\right) && \text{(C.4)}\\
&+ 4\phi\left(h''_{ij} + 2\mathcal{H}h'_{ij} + 6w\mathcal{H}^2 h_{ij}\right) - 2\left(\phi' - \psi'\right)h'_{ij} - 8\mathcal{H}\left(\phi' + 3\psi'\right)h_{ij}\\
&- 12\psi'' h_{ij} + 4\left(\psi\Delta h_{ij} - \Delta\phi h_{ij} + 2\Delta\psi h_{ij}\right) - 4\left(h_{ia}\partial_a\partial_j\psi + h_{ja}\partial_a\partial_i\psi\right)\\
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VV term

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$$\left.S_{ij} = -V_a\partial_a(\partial_i V_j + \partial_j V_i) + \partial_a V_i \partial_a V_j + \partial_i V_a \partial_j V_a + 2V_a\partial_i\partial_j V_a + \frac{\Delta V_i \Delta V_j}{6\mathcal{H}^2(1+w)}\right\} \text{VV term} \tag{C.1}$$

$$\left.\begin{aligned}& -2\partial_i\phi\partial_j\phi - 6\partial_i\psi\partial_j\psi - 4\left(\phi\partial_i\partial_j\phi + \psi\partial_i\partial_j\psi\right) + 2\left(\partial_i\phi\partial_j\psi + \partial_j\phi\partial_i\psi\right) \\ & + \frac{8}{3\mathcal{H}^2(1+w)}\left[\mathcal{H}^2\partial_i\phi\partial_j\phi + \mathcal{H}\left(\partial_i\phi\partial_j\psi' + \partial_j\phi\partial_i\psi'\right) + \partial_i\psi'\partial_j\psi'\right]\end{aligned}\right\} \text{SS term} \tag{C.2}$$

$$\left.\begin{aligned}& + 4h'_{ia}h'_{ja} - 2\partial_i h_{ab}\partial_j h_{ab} + 4h_{ab}\partial_b\left(\partial_i h_{aj} + \partial_j h_{ai}\right) \\ & + 4\partial_b h_{ia}\left(\partial_a h_{bj} - \partial_b h_{aj}\right) - 4h_{ab}\left(\partial_i\partial_j h_{ab} + \partial_a\partial_b h_{ij}\right) + 12\mathcal{H}^2 w^{(1)} h_{ij}\end{aligned}\right\} \text{TT term} \tag{C.3}$$

$$\left.\begin{aligned}& -\left(\phi' + \psi' + 4\mathcal{H}\phi\right)\left(\partial_i V_j + \partial_j V_i\right) - 2\phi\left(\partial_i V'_j + \partial_j V'_i\right) + 4\mathcal{H}\left(V_i\partial_j\psi + V_j\partial_i\psi\right) \\ & + 2\left[(V_i\partial_j\psi)' + (V_j\partial_i\psi)'\right] - \frac{2}{3\mathcal{H}(1+w)}\left(\Delta V_i\partial_j + \Delta V_j\partial_i\right)\left(\phi + \frac{\psi'}{\mathcal{H}}\right)\end{aligned}\right\} \text{SV term} \tag{C.4}$$

$$\begin{aligned}& + 4\phi\left(h''_{ij} + 2\mathcal{H}h'_{ij} + 6w\mathcal{H}^2 h_{ij}\right) - 2\left(\phi' - \psi'\right)h'_{ij} - 8\mathcal{H}\left(\phi' + 3\psi'\right)h_{ij} \\ & - 12\psi'' h_{ij} + 4\left(\psi\Delta h_{ij} - \Delta\phi h_{ij} + 2\Delta\psi h_{ij}\right) - 4\left(h_{ia}\partial_a\partial_j\psi + h_{ja}\partial_a\partial_i\psi\right) \\ & + 2\partial_a h_{ij}\partial_a\left(\phi + 3\psi\right) - 2\left(\partial_i h_{aj} + \partial_j h_{ia}\right)\partial_a\left(\phi + \psi\right)\end{aligned} \tag{C.5}$$

$$\begin{aligned}& + 4V_a\partial_a h'_{ij} + 2\left(V'_a + 4\mathcal{H}V_a\right)\partial_a h_{ij} - 2\left(h'_{ia}\partial_a V_j + h'_{aj}\partial_a V_i\right) \\ & - 2\left[(V_a\partial_i h_{aj})' + (V_a\partial_j h_{ia})'\right] - 4\mathcal{H}V_a\left(\partial_i h_{aj} + \partial_j h_{ia}\right)\ ,\end{aligned} \tag{C.6}$$

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$$+\frac{8}{3\mathcal{H}^2(1+w)}\left[\mathcal{H}^2\partial_i\phi\partial_j\phi + \mathcal{H}\left(\partial_i\phi\partial_j\psi' + \partial_j\phi\partial_i\psi'\right) + \partial_i\psi'\partial_j\psi'\right] \quad \text{SS term} \tag{C.2}$$

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Basics of Scalar Induced Gravitational Waves

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- Choosing as the gauge for the GW frame the Newtonian gauge, the metric is written as
$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\Phi)\mathrm{d}\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

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- The source term, $S_{\vec{k}}$ can be recast as:

$$S^s_{\vec{k}} = \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^{3/2}} e^s_{ij}(\vec{k}) q_i q_j \left[2\Phi_{\vec{q}}\Phi_{\vec{k}-\vec{q}} + \frac{4}{3(1+w)}(\mathcal{H}^{-1}\Phi'_{\vec{q}} + \Phi_{\vec{q}})(\mathcal{H}^{-1}\Phi'_{\vec{k}-\vec{q}} + \Phi_{\vec{k}-\vec{q}})\right]$$

The Energy Density of GWs

- Considering here only sub-horizon scales, where a flat spacetime approximation can be applied, the energy density of GWs reads as [M. Maggiore - 2000]

$$\rho_{\mathrm{GW}}(\eta, \vec{x}) = \frac{M_{\mathrm{Pl}}^2}{8} \overline{\left(\underbrace{\partial_t h_{\alpha\beta} \partial_t h^{\alpha\beta}}_{\text{Kinetic Energy (KE)}} + \underbrace{\partial_i h_{\alpha\beta} \partial_i h^{\alpha\beta}}_{\text{Gradient Energy (GE)}} \right)} \, .$$

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$$\mathcal{P}_h(\eta,k) \propto \int \mathrm{d}k_1 \int \mathrm{d}k_2 \left(\int f(k_1,k_2,\eta)\mathrm{d}\eta\right)^2 \mathcal{P}_\Phi(k_1)\mathcal{P}_\Phi(k_2),$$

where $f(k_1,k_2,\eta)$ is a function containing information about the epoch at which the PBH forms.

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Induced Gravitational Waves as Cosmic Tracers of Leptogenesis

[M. Chianese, G. Domenech, **T. Papanikolaou**, R. Samanta, N. Saviano,
2504. 20135, Accepted in JCAP]

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- **Scales** $M_N > \mathcal{O}(\mathrm{TeV})$ are **not reachable by colliders.**

Distinct Flavor Regimes of High-Scale Leptonesis: $|L_i> = A_{i\alpha}|L_\alpha>$, where $\alpha = e, \mu, \tau$

$M_N<10^9$ GeV	$10^9 < M_N<10^{12}$ GeV	$>10^{12}$ GeV
Three-flavor (3FL)	**Two-flavor (2FL)**	**One-flavor (1FL) /vanilla regime**

M_N

the rates of processes mediated by the τ and μ Yukawa coupling are non negligible—> effects of lepton flavors must be taken into account

no charged lepton flavour effects—> no sensitivity to the neutrino mixing matrix: mixing angles and low-energy CP phases.

While the 1FL regime is not sensitive to neutrino mixing parameters, the 2FL and 3FL regimes can offer low-energy neutrino phenomenology, which however may not be unequivocal, unless the flavor structure of the theory is constrained by invoking symmetries.

cannot be tested by terrestrial experiments !!

It is therefore necessary to find other evidence to test the neutrino sector.

[Credits to N. Saviano]

Our leptogenesis scenario

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- UV realisation of Type I seesaw mechanism based on $U(1)_{B-L}$ symmetry with gauge coupling g', naturally embedded to GUTs.

$$-\Delta\mathscr{L} \subset \frac{1}{2} y_N \overline{N_R} \Phi N_R^C + y_D \overline{N_R} \tilde{H}^\dagger L + \frac{1}{4} \lambda_{H\Phi} H^2 \Phi^2 + V(\Phi, T)$$

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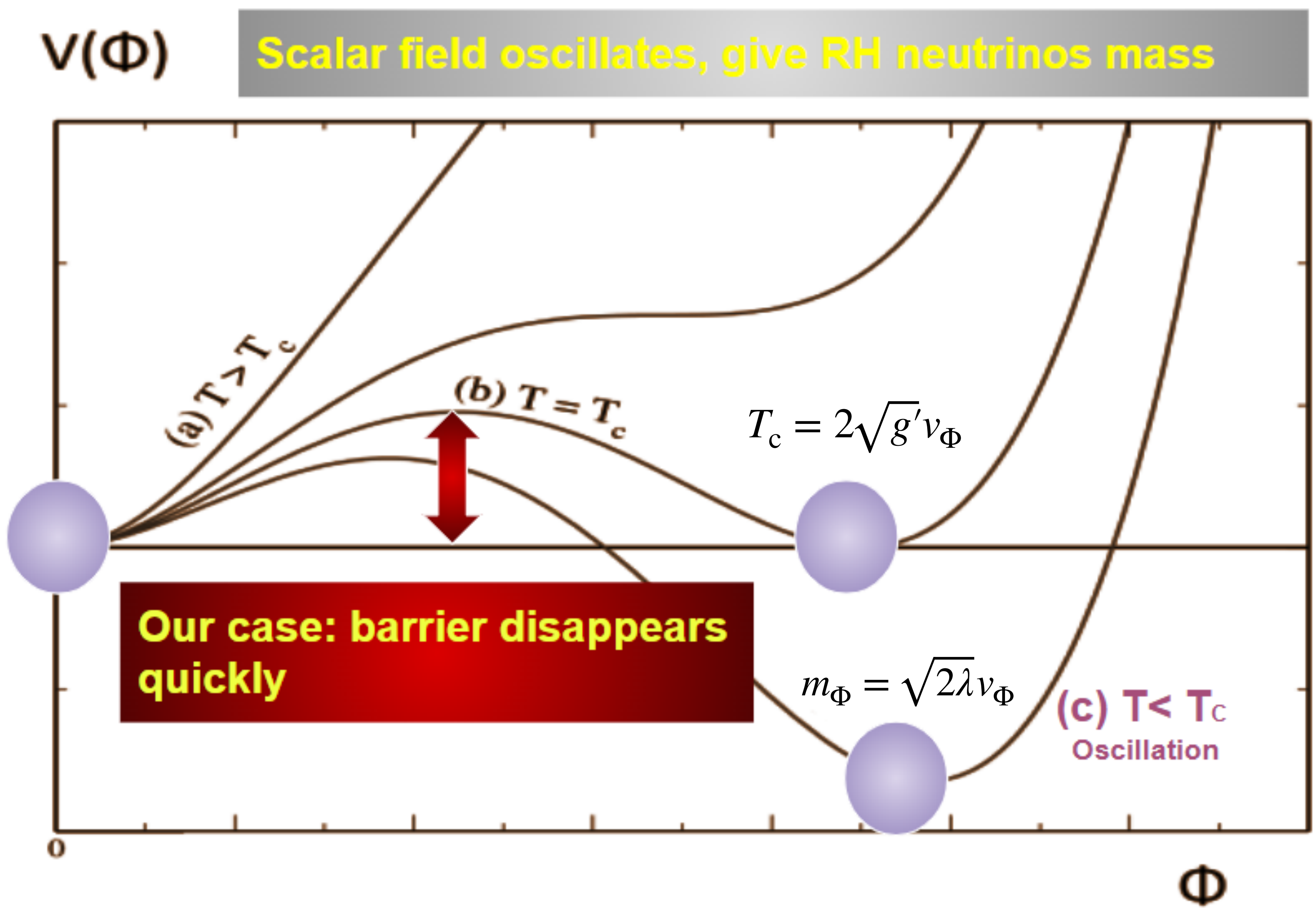

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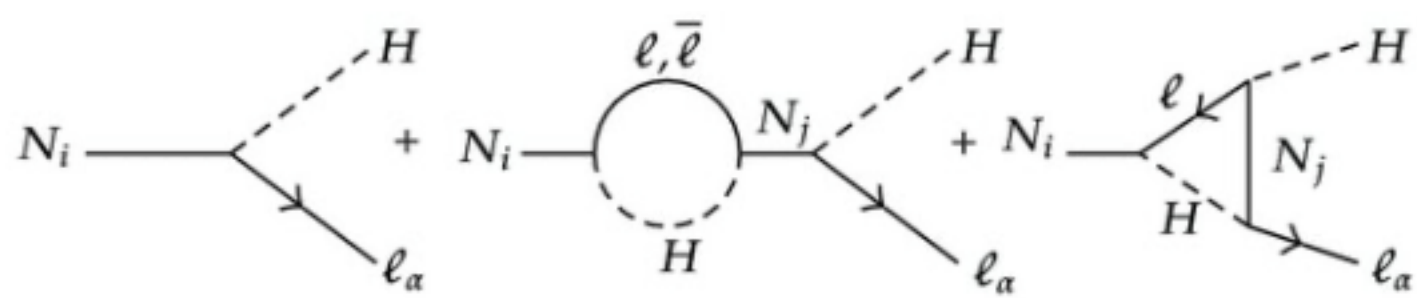
- At that time, **right-handed neutrinos (RHNs) become massive** with $M_N = y_N v_\Phi$, decaying **later CP-asymmetrically into lepton doublets and Higgs bosons** and producing the **lepton asymmetry around $T_{\text{lepto}} \sim M_N < T_c$.**



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- **Once the SM Higgs takes its vev** $v_h = 174\mathrm{GeV}$ at the electroweak (EW) phase transition, **light neutrinos become massive** $m_{\nu_i} \sim y_D^2 v_h^2 / M_N$ via the Type-I seesaw mechanism.

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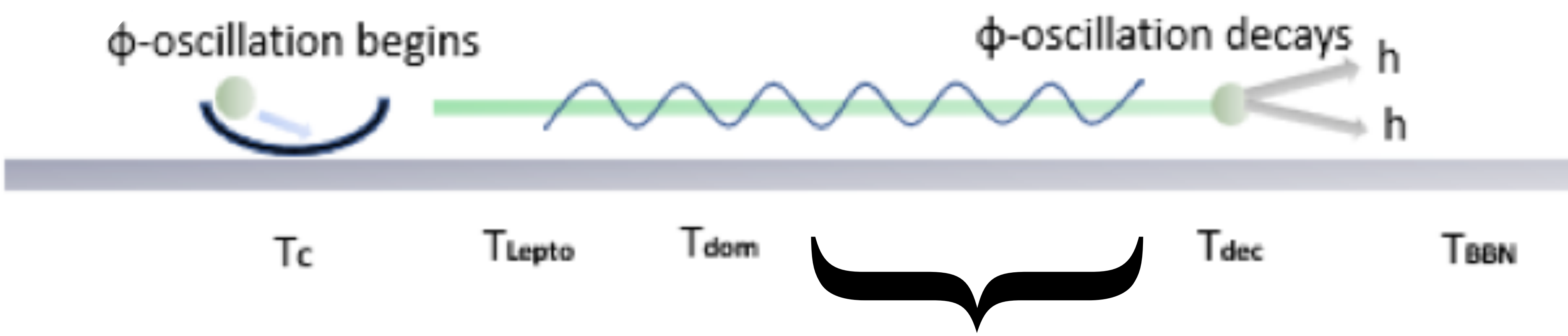
- **Once the SM Higgs takes its vev** $v_h = 174\mathrm{GeV}$ at the electroweak (EW) phase transition, **light neutrinos become massive** $m_{\nu_i} \sim y_D^2 v_h^2 / M_N$ via the Type-I seesaw mechanism.

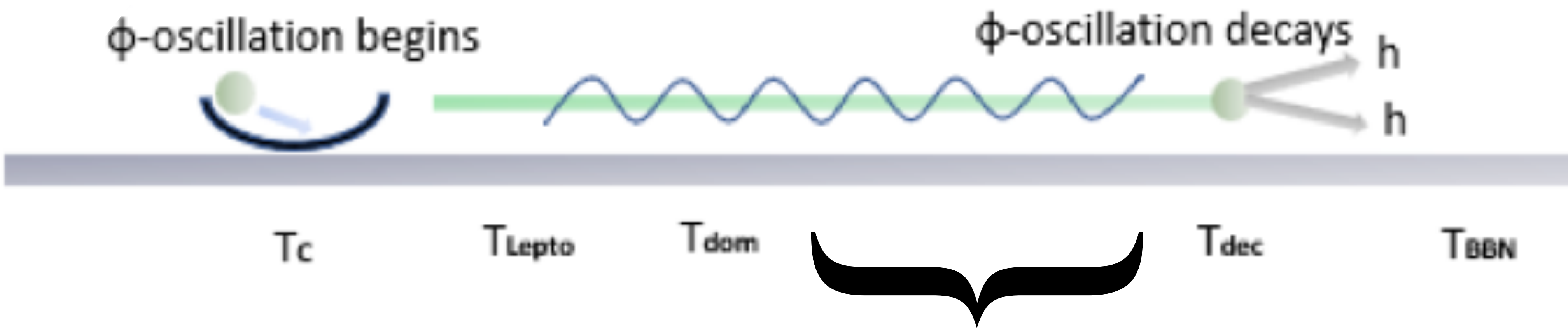
- Finally, the early matter-dominated **(eMD) era triggered by Φ ends through the decay channel $\Phi \to hh$.**

The overall picture



Φ-driven eMD era

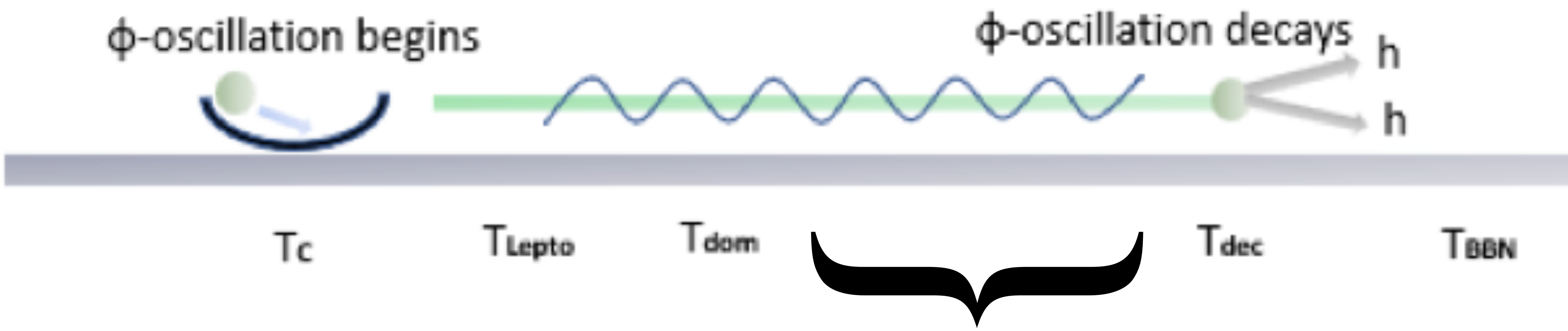
The overall picture



Φ-driven eMD era

What is the GW phenomenology of such an eMD era?

The overall picture



Φ-driven eMD era

What is the GW phenomenology of such an eMD era?

↓

We will focus on IGWs.

The parameter space (M_N, y_N, g')

- In order to make Φ long lived one should require that $\Phi \to Z'Z'$ and $\Phi \to NN$ are kinematically forbidden leading to

$$M_{Z'} = \sqrt{2}g'v_\Phi > m_\Phi \Rightarrow \boxed{g' < 1} \quad \text{and} \quad M_N = y_N v_\Phi > m_\Phi = \sqrt{2\lambda}v_\Phi \Rightarrow \boxed{y_N > \sqrt{2g'^3}\,.}$$

- One should also account for the competitive channel $\Phi \to f\bar{f}V$ requiring that $\boxed{\Gamma_\Phi^{hh}(M_N, y_N, g') \geq \Gamma_\Phi^{f\bar{f}V}(M_N, y_N, g')\,.}$

- Finally, one should require the lepton asymmetry takes place at $T_{\text{lepto}} \sim M_N < T_c = 2\sqrt{g'}v_\Phi \Rightarrow \boxed{y_N < 2\sqrt{g'}\,.}$

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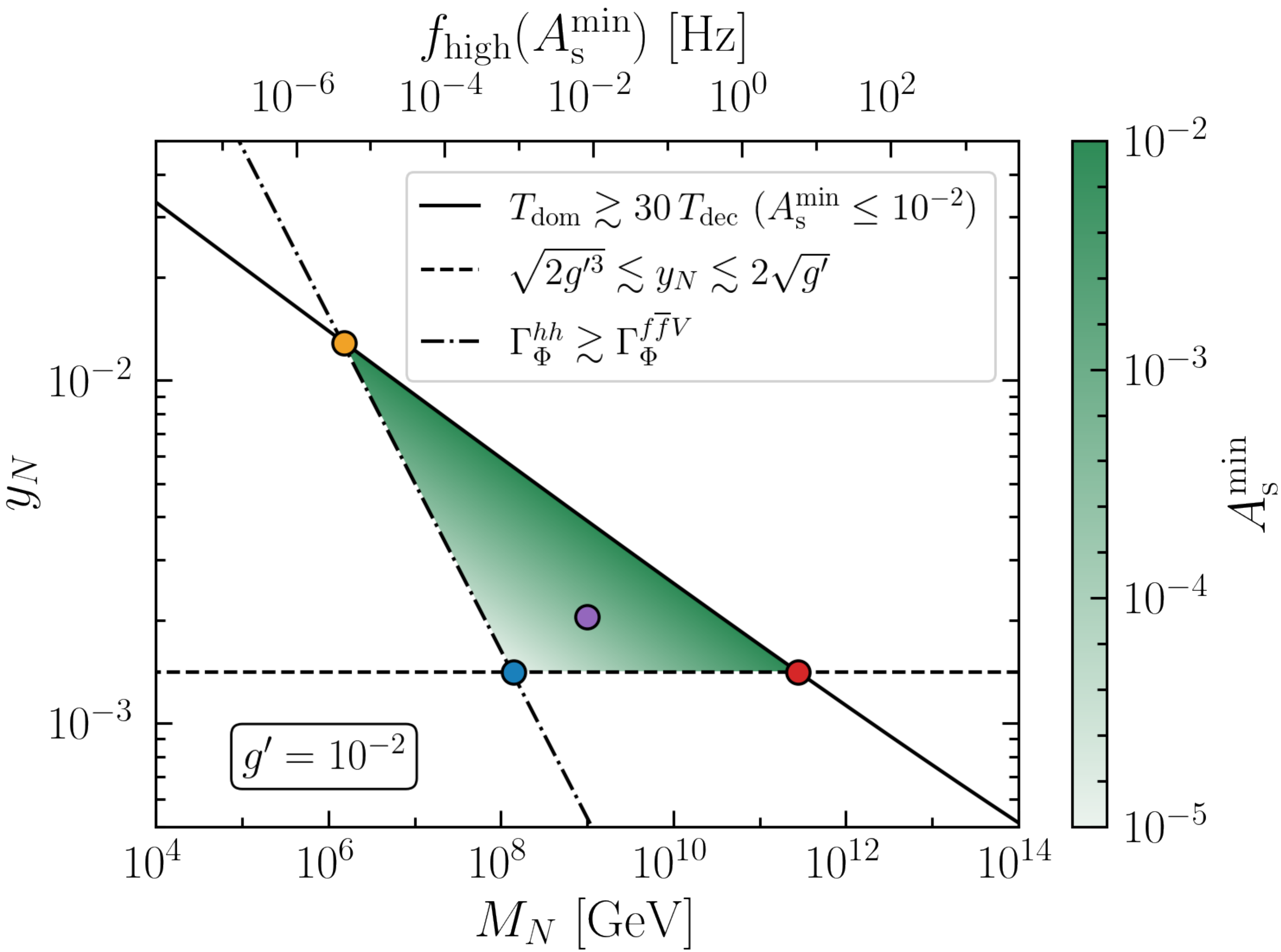

IGWs from an eMD era

- During an eMD phase, **sub-horizon density perturbations grow, i.e.** $\delta_k \propto a$**,** as well as velocity flows. One then expects **structure (Φ halo) formation** in the **non-linear regime** [Barenboim & Rasero - 2013].

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- **The scale of non-linearities** at a given conformal time η is estimated as [Assadullahi & Wands - 2009, Kohri & Terada - 2018]

$$k_{\rm NL}(\eta) \sim \alpha A_{\rm s}^{-1/4}\mathcal{H}(\eta)\,.$$

where we assumed a scale-invariant curvature power spectrum with amplitude $A_{\rm s}$ and $\alpha \sim 1.7$. **Modes with** $k > k_{\rm NL}$ **are in the non-linear regime, i.e.** $\mathcal{P}_\delta(k > k_{\rm NL}) > 1$.

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- **IGWs on linear scales $k < k_{\rm NL}$ are negligible. IGW generation** is abundant on non-linear scales and **is dominated by the largest structures that form at the end of the eMD era** [Fernandez et al. - 2024].

IGWs from an eMD era

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$$\text{with} \quad T_{\rm dom} = \frac{\rho_\Phi(T_c)}{\rho_{\rm r}(T_c)} T_c \quad \text{and} \quad T_{\rm dec} = \tilde{T}_{\rm dec} \ln\left(\frac{\Lambda_{\rm GUT}}{\mu}\right),$$

$$\text{where} \quad \tilde{T}_{\rm dec} = \left(\frac{72}{5\pi^2 g_*}\right)^{1/4} 10^{-2} \left[M_{\rm Pl}\Gamma_0 \frac{y_N^3}{\sqrt{g'^3}} \left(\frac{M_N}{10^{13}\text{GeV}}\right)^3\right]^{1/2}.$$

[Chianese et al. - 2024]

The non-linear IGW spectrum

- Borrowing results of numerical simulations [Eggemeier et al. - 2023, Fernandez et al. - 2024, Dalianis & Kouvaris - 2024], the GW signal can be fitted on non-linear scales quite well as

$$\boxed{\begin{gathered}\Omega_{\rm GW}(k) \simeq 0.05 A_{\rm s}^{7/4} \left(\frac{k}{\mathscr{H}_{\rm dec}}\right)^{3/2} \quad \text{for} \quad k_{\rm low} \leq k \leq k_{\rm high}, \\ \text{where} \quad k_{\rm low} = 15\mathscr{H}_{\rm dec} \quad \text{and} \quad k_{\rm high} = 9k_{\rm NL} = 14\mathscr{H}_{\rm dec}/A_{\rm s}^{1/4}\,.\end{gathered}}$$

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$$\boxed{f_{\rm high} \equiv \frac{k_{\rm high}}{2\pi} = \frac{14\mathcal{H}_{\rm dec}A_{\rm s}^{-1/4}}{2\pi} \propto \frac{M_N^3 y_N^3}{\sqrt{g'^3}} A_s^{-1/4}}$$

- One can establish **a remarkable connection between leptogenesis and IGWs** since $\Gamma_{\Phi}^{hh}(y_N, M_N, g') = \mathcal{H}_{\rm dec}/a_{\rm dec}$.

The non-linear IGW spectrum

- Working with frequencies f instead of wavenumbers k, one can recast $\Omega_{\rm GW}(k)$ as

$$\boxed{\begin{gathered}\Omega_{\rm GW,0}h^2 = 4.2\times 10^{-5}\,A_s^{11/8}\left(\frac{f}{f_{\rm high}}\right)^{3/2}, \\ \text{where}\quad f_{\rm high}\simeq 6.4\times 10^{-5}{\rm Hz}\left(\frac{A_{\rm s}}{10^{-5}}\right)^{-1/4}\left(\frac{T_{\rm dec}}{10\,{\rm GeV}}\right)\quad\text{and} \\ f_{\rm low}\simeq 3.8\times 10^{-6}{\rm Hz}\left(\frac{T_{\rm dec}}{10{\rm GeV}}\right)\end{gathered}}$$

Results

Benchmark parameter values

	g'	M_N [GeV]	y_N	A_s
BP1	10^{-2}	1.4×10^{8}	1.4×10^{-3}	1.18×10^{-5}
BP2	10^{-2}	2.8×10^{11}	1.4×10^{-3}	9.96×10^{-3}
BP3	10^{-2}	1.5×10^{6}	1.8×10^{-2}	1.00×10^{-2}
BP4	10^{-2}	1.0×10^{9}	2.0×10^{-3}	4.30×10^{-4}
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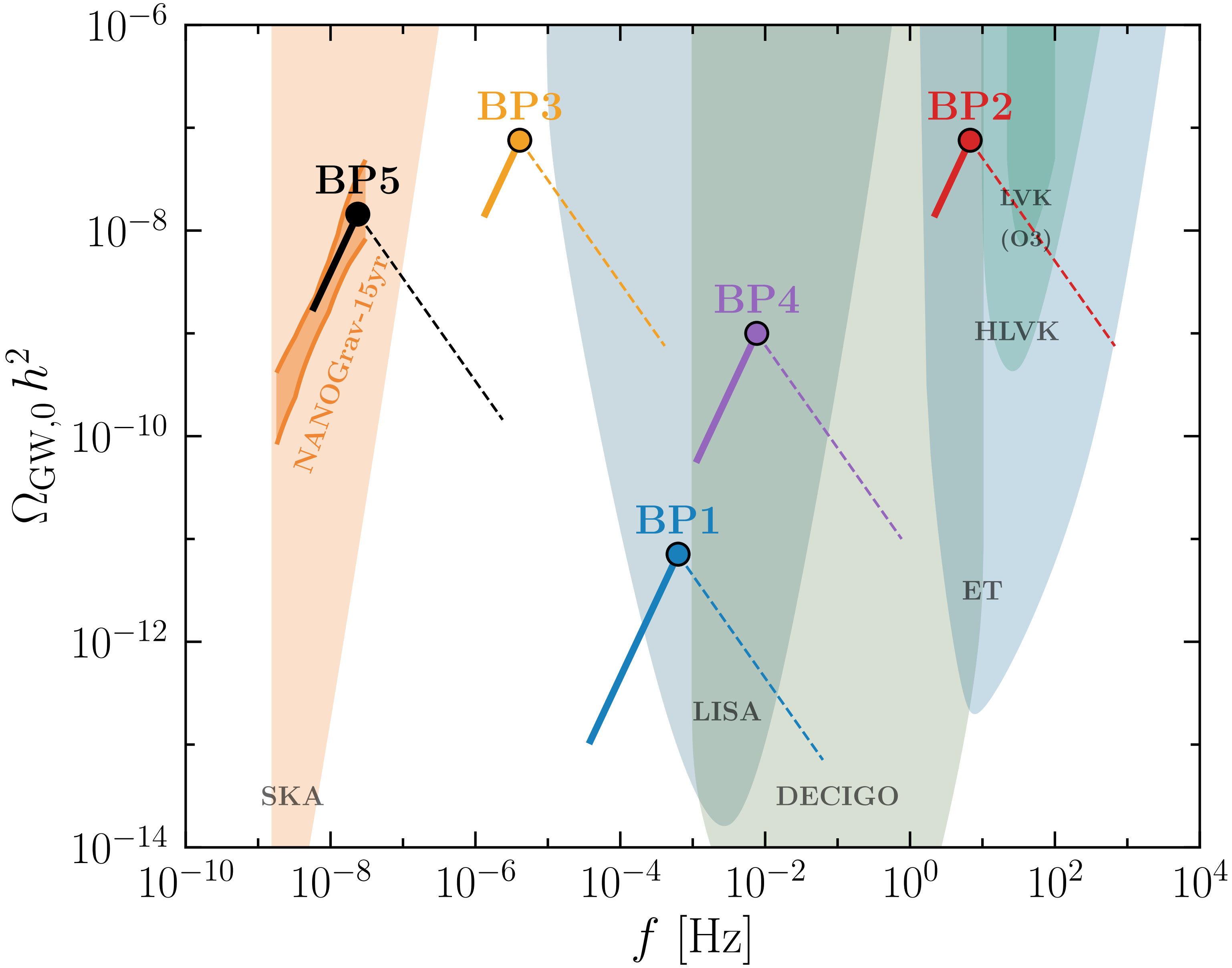
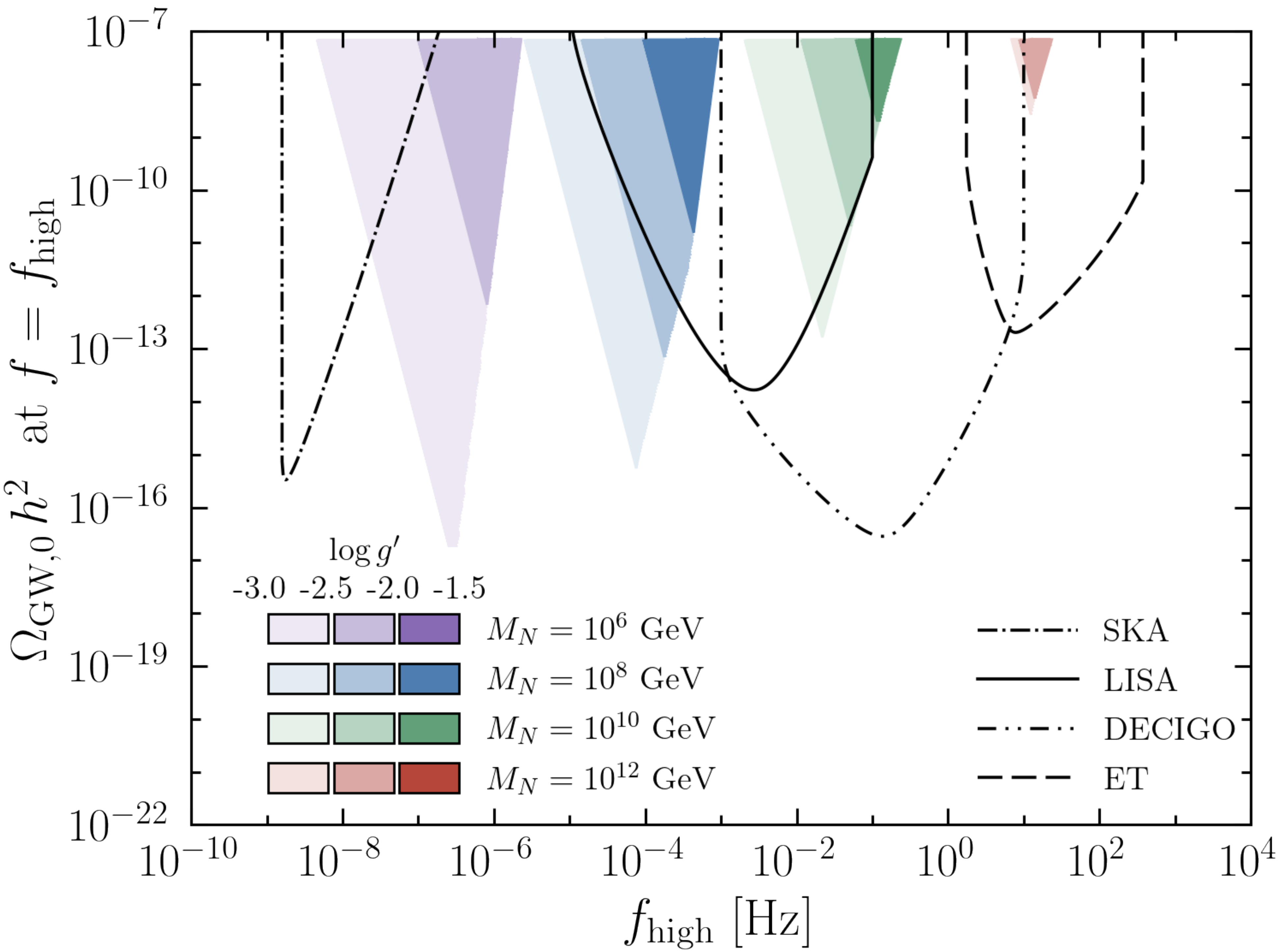
$$\Omega_{\rm GW} \propto \begin{cases} f^{3/2} & \text{for} \quad f_{\rm low} \leq f \leq f_{\rm high} \\ 1/f & \text{for} \quad f > f_{\rm high} \end{cases}$$



Detectability by GW observatories

$$y_N \in [\sqrt{2g'^3}, 2\sqrt{g'}], \quad A_s \in [\max(10^{-9}, A_s^{\min}), 10^{-2}]$$



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__Future Perspectives__

- **eMD eras favor** the formation of PBHs. Even with $A_{\rm s} \sim 10^{-4}$, one can produce **substantial PBH abundances** [Harada et al - 2016, Ballesteros et al. - 2019].

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- **Any BSM physics framework** involving an **eMD epoch triggered by long lived particles** can be readily explored through **IGWs.**

Thank you for your attention!

Appendix

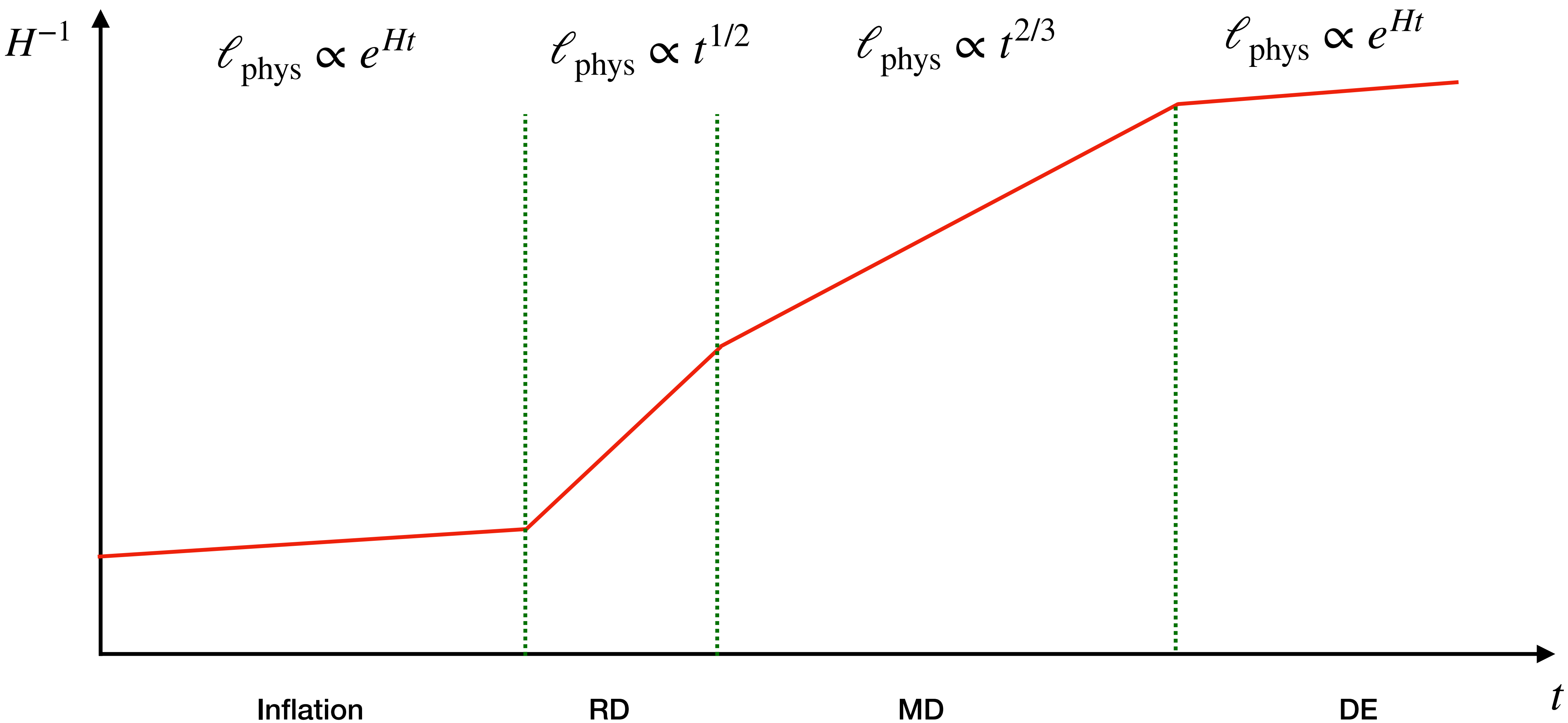
Cosmology Basics

- According to the cosmological principle, the **universe** is **homogeneous** and **isotropic on large scales** $\sim 100\mathrm{Mpc}$ and is expanding with its geometry being described by the FLRW metric:

$$\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{1-Kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right)\right],$$

where K the spatial curvature signature of the metric. According to observations, $K \simeq 0$.

- One can then define a **characteristic scale**, $H^{-1} \equiv a/\dot{a}$ **(Horizon/Hubble scale)** below which causal contact has been established.



Introducing inhomogeneities

SVT Decomposition

$\Longrightarrow$ FLRW metric with scalar, vector and tensor perturbations

$$\mathrm{d}s^2 = a^2(\tau)\left[-(1+2\Phi)\,\mathrm{d}\tau^2 - 2B_i\mathrm{d}\tau\mathrm{d}x^i + \left\{\left(1-2\Psi\right)\delta_{ij} + h_{ij}\right\}\mathrm{d}x^i\mathrm{d}x^j\right]$$

- Φ, Ψ: 1st order scalar metric perturbations (Bardeen gravitational potentials)
- B_i: 1st order vector metric perturbations
- h_{ij}: 2nd order tensor metric perturbations

Comoving curvature perturbation

$$\mathscr{R} \equiv \Psi\,\big|_{T_{i0}=0}$$

$$\langle \mathscr{R}_k(\tau)\mathscr{R}_{k'}(\tau)\rangle = \delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\mathscr{P}_{\mathscr{R}}(k)$$

Quasi-Degeneracy of RHNs

$$M_N > T_{\rm dom} \Rightarrow \Delta \simeq T_{\rm dec}/T_{\rm dom} \Rightarrow \eta_{\rm B} \simeq 10^{-3}\left(\frac{3m_\nu M_{N_i}}{8\pi v_h^2\delta}\right)\Delta$$

$$\text{where } \delta = \frac{\rm M_{N_i} - M_{N_j}}{\rm M_{N_i}} \text{ is the quasi} - \text{degenarcy among RHNs}\,.$$

$$\eta_B \simeq 6.3\times 10^{-10} \Rightarrow \delta \simeq 0.2\left(\frac{M_{N_i}}{10^9{\rm GeV}}\right)\sqrt{A_{\rm s}^{\rm min}}\,.$$

- Strong (weak) amplitude GWs are associated with weakly (strongly) quasi-degenerate RHNs.