QUANTUM ASPECTS OF CHAOTIC LATTICE FIELD THEORIES

EMMANUEL FLORATOS

ACADEMY OF ATHENS

INPP DEMOKRITOS

PROF EMERITUS
NUCLEAR AND PARTICLE PHYSICS DEPARTMENT
FACULTY OF PHYSICS UO ATHENS

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PLAN OF THE PRESENTATION

- **DEFINITION OF THE PROBLEM –MOTIVATION**

1) <u>INTRODUCTION TO CHAOTIC LATTICE FIELD THEORIES</u>

- TOROIDAL PHASE SPACE-EVOLUTION BY LINEAR HYPERBOLIC SYMPLECTIC MAPS
- COUPLING OF FIBONACI SEQUENCES
- ARNOLD CAT MAP LATTICE FIELD THEORIES IN d-DIM PERIODIC LATTICES

2) **<u>RESULTS : ERGODIC ,CHAOTIC AND MIXING PROPERTIES</u>**

- LIAPUNOV SPECTRA ,K-S ENTROPY, PERIODIC ORBITS - MIXING TIMES

3) **<u>QUANTIZATION OF ARNOLD CAT MAP LATTICE FIELD THEORIES</u>**

- MAGNETIC TRANSLATIONS AND THE METAPLECTIC REPRESENTATION OF THE SYMPLECTIC GROUP OVER Z/NZ: Sp[2n,ModN] n DOF
- UNITARY QM TIME EVOLUTION OPERATOR - SPECTRUM -CHAOTIC EIGENSTATES

4) **<u>BENCHMARKS OF QUANTUM CHAOTIC FIELD THEORIES</u>**

- HAYDEN-PRESKIL , SEKINO-SUSSKIND SCRAMBLING BOUND
- OTOC, OPERATOR SPREADING COMPLEXITY,ETH

5) **<u>CONCLUSIONS-OPEN QUESTIONS</u>**

ETH

- IF THE EIGENSTATES OF A CLOSED QM SYSTEM ARE RANDOM
- (RANDOM PHASES AND GAUSSIAN DISTRIBUTED AMPLITUDES)
- THEN ANY INITIAL PURE STATE OF A SUBSYSTEM THERMALIZES
- TO
- THE THERMAL DENSITY MATRIX OF THE SUBSYSTEM
- RELATION TO THE INFORMATION PARADOX
- 2013 SREDNICKI TALK TO KITP

CHAOTIC LATTICE FIELD THEORIES

- CHAOTIC LATTICE FIELD THEORIES ARE FIELD THEORIES OF INTERACTING CHAOTIC UNITS ,ONE AT EACH SITE OF A LATTICE ,WITH LOCAL OR NONLOCAL INTERACTIONS
- CHAOTIC UNITS ARE DYNAMICAL SYSTEMS WHICH LIVE ON A COMPACT PHASE SPACE AND WHICH ARE ERGODIC AND MIXING
- AS SUCH THEY ARE APPROPRIATE TO DESCRIBE NEW MANY BODY DYNAMICAL SYSTEMS
- WITH FASTER THERMALIZATION PROPERTIES AND FASTER MIXING OF INFORMATION

  THAN THE CONVENTIONAL FIELD THEORIES

  WHICH DESCRIBE HARMONIC

  OSCILLATORS WITH LOCAL OR NONLOCAL INTERACTIONS.

- CHAOTIC FIELD THEORIES APPEARED IN THE 80’s UNDER THE NAME

  OF COUPLED MAP LATTICES (KANEKO 1984) TO STUDY THE UNIVERSALITY CLASSES OF SPATIOTEMPORAL CHAOTIC BEHAVIOUR OF COUPLED CHAOTIC SYSTEMS

- CRUTCHFIELD, JAMES,P. AND KANEKO, KUNIHIKO (1987). PHENOMENOLOGY OF SPATIOTEMPORAL CHAOS (IN DIRECTIONS IN CHAOS, ED HAO BAI LIN) WORLD SCIENTIFIC, SINGAPORE.

- IN THE 90’s AND RECENTLY 2018-2020 THEY WERE PROPOSED BY CVITANOVIC TO STUDY THE COHERENT STRUCTURES OF WEAK TURBULENCE USING AS CHAOTIC UNITS LINEAR HYPERBOLIC MAPS ON A TOROIDAL PHASE SPACE

- CHAOTIC FIELD THEORIES EXHIBIT A RICH SPECTRUM OF SPATIOTEMPORAL PATTERNS AND RECENTLY HAVE BEEN USED TO STUDY A UNIVERSALITY CLASS OF SPECIAL STATES OF THESE SYSTEMS

- THE CHIMAIRA STATES

- A SUBSET OF OSCILLATORS HAVE A COHERENT DYNAMICAL BEHAVIOUR WHILE ALL THE OTHERS ARE RANDOM

- APPLICATIONS TO BOSE LOCALIZATION IN RANDOM MATERIALS
- MEMORY EFFECTS
- COGNITIVE PROCESSES
- MACHINE LEARNING(AI)

PREVIOUS WORK

1) Modular discretization of the AdS2/CFT1 Holography

M. Axenides, E. G. Floratos, S. Nicolis

JHEP 1402 (2014) 109

2) Chaotic Information Processing by Extremal Black Holes

M. Axenides, E.l Floratos, S. Nicolis

Int. J. Mod. Phys. D24 (2015) 15420122

3)The arithmetic geometry of AdS and its continuum limit

M.Axenides,E.Floratos,S.Nicolis

SIGMA 17 (2021), 004

4)The quantum cat map on the modular discretization of extremal black hole horizons

M.Axenides, E. Floratos, S. Nicolis

Eur. Phys. J. C (2018) 78:412

PREVIOUS WORK

- **5)Arnol'd cat map lattices**

  M. Axenides, E. Floratos, S. Nicolis

  Phys. Rev. E 107, 064206 (2023)

  6)Exponential mixing of all orders for Arnol'd cat map lattices

  M.Axenides,E.Floratos,S.Nicolis

  arXiv:2401.08521

  7)A Novel Finite Fractional Fourier Transform and its Quantum Circuit Implementation on Qudits

  Emmanuel Floratos, Archimedes Pavlidis

  arXiv:2409.05759

  8)Coupled Arnol'd cat maps on circulant graphs

  K. Manolas, E. Floratos

  arXiv:2605.00965

DEFINITION OF THE PROBLEM –MOTIVATION

- OUR MOTIVATION IS THE STUDY OF FAST THERMALIZATION PROPERTIES OF CLOSED CHAOTIC CLASSICAL AND QUANTUM SYSTEMS WITH MANY DOF -
- **SREDNICKI 1994- EIGENSTATE THERMALIZATION HYPOTHESIS -ETH**
- IF A CLOSED QUANTUM SYSTEM IS SUFFICIENTLY RANDOM (CHAOTIC) ANY OF ITS SUBSYSTEMS WILL THERMALIZE EVENTUALLY TO THE THERMAL DENSITY MATRIX WITH TEMPERATURE DEFINED BY THE TOTAL ENERGY OF THE SYSTEM.
- **WE ARE INTERESTED SPECIALLY FOR APPLICATIONS TO THE DYNAMICS OF BLACK HOLE HORIZONS**
- -**FAST SCRAMBLING** : HAYDEN PRESKILL (2007) -SEKINO SUSSKIND(2008)

  TBH=BH Scrambling Time=βLogS, β=1/κT, S=Blackhole Entropy
- **FOR ALL KNOWN CHAOTIC SYSTEMS Ts=β S^{a}>TBH a>0**

- TO OBTAIN THE FASTEST POSSIBLE THERMALIZATION

  WE ASSUME THAT THE INTERACTING CHAOTIC UNITS OF THE CLOSED LATTICE DYNAMICAL SYSTEM WITH MANY DOF MUST BE MAXIMALY CHAOTIC AND MIXING AND NOT HARMONIC OSCILLATORS WITH RANDOM NON LOCAL AND NONLINEAR INTERACTIONS AS IT IS USUALLY ASSUMED

  THESE NEW FIELD THEORIES ARE CALLED

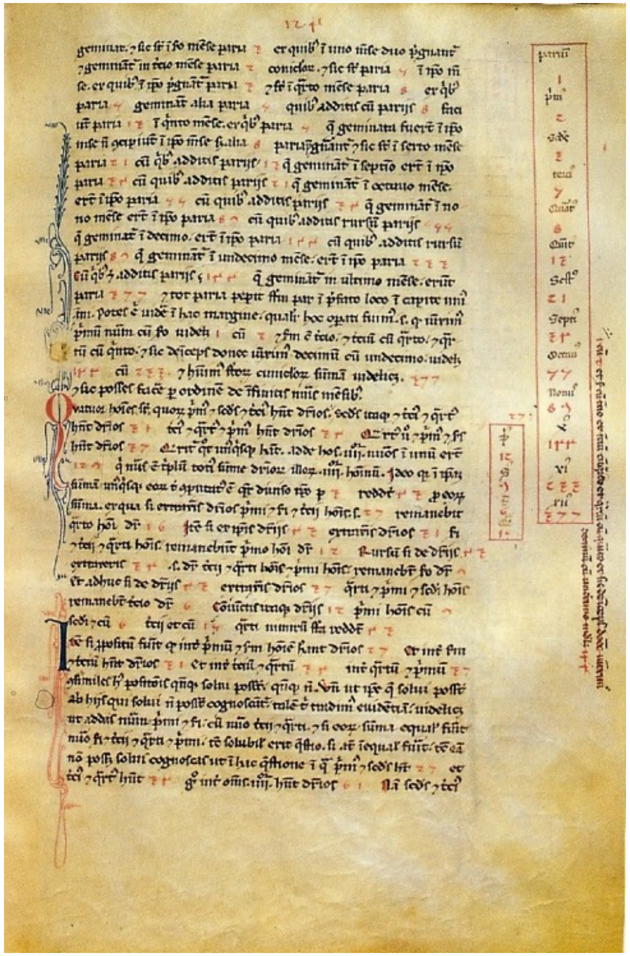
  CHAOTIC FIELD THEORIES

(KANEKO,CVITANOVIC)

Leonardo Pisano -Fibonacci PISA 1175 – 1240

Statue of Fibonacci (1863) by Giovanni Paganucci in the Camposanto di Pisa

Liber Abaci 1202 PISA
ΙΝΔΟΑΡΑΒΙΚΟ ΣΥΣΤΗΜΑ ΑΡΙΘΜΗΣΗΣ

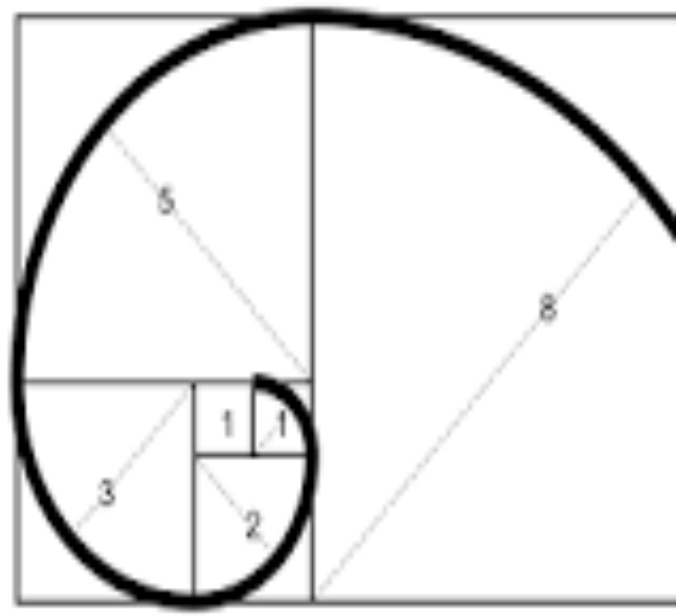


F0	F1	F2	F3	F4	F5	F6	F7	F8
F9	F10	F11	F12	F13	F14	F15	F16	F17
F18	F19							

0	1	1	2	3	5	8	13	21
34	55	89	144	233	377	610	987	
1597	2584	4181						

The Fibonacci numbers are defined by the recurrence relation[6]

F[n+1]=F[n]+F[n-1],,n=1,2,3,.. F[0]=0,F[1]=1



- ACM1

THE SIMPLEST CHAOTIC UNIT

THE ARNOLD'S CAT MAP

PHASE SPACE 2d TORUS

$[0,1) \times [0,1)$

$Q \qquad P$

LINEAR HYPERBOLIC MAP

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \quad - \text{ACM}$$

$$(Q,P)_{n+1} = (Q,P)_n \cdot A \mod 1$$

$$(Q,P)_n = (Q,P)_0 \cdot A^n \mod 1$$

$$A^n = \begin{pmatrix} F_{2n-1} & F_{2n} \\ F_{2n} & F_{2n+1} \end{pmatrix}$$

- ACM2

2

$$F_{n+1} = F_n + F_{n-1} \qquad , F_0 = 0, F_1 = 1.$$

FIBONACCI SEQUENCE OF INTEGERS

$$\rho^{n+1} = \rho^n + \rho^{n-1} \Rightarrow \rho_\pm = \frac{1 \pm \sqrt{5}}{2}$$

$$F_n = \frac{\rho_+^{\,n} - (-1)^n \rho_+^{-n}}{\sqrt{5}}$$

$$n \to \infty \qquad \rho_+^{\,n} \longrightarrow e^{n \log \rho_+}$$

$$F_n \xrightarrow[n\to\infty]{} \frac{1}{\sqrt{5}} \cdot e^{n \log \rho_+}$$

$$\frac{F_{n+1}}{F_n} \xrightarrow[n\to\infty]{} \rho_+ \qquad \text{GOLDEN RATIO}$$

- ACM3

③

ACM IS STRECHING THE PHASE SPACE ALONG TWO ORTHOGONAL DIRECTIONS

ALONG ρ_+ EXPANDING

ρ_- CONTRACTING

LIAPUNOV EXPONENTS

$\lambda_+ = 2 \log \rho_+$

$\lambda_- = -2 \log \rho_+$

A BELONGS TO $Sp_2(\mathbb{Z})$

HAMILTONIAN MAP

- ACM4

ALL THE PERIODIC ORBITS OF ACM

4

- ALL RATIONAL POINTS OF THE TORUS WITH THE SAME DENOMINATOR – N – BELONG TO PERIODIC ORBITS (FOR EVERY N)

$$\left(\frac{k}{N},\frac{\ell}{N}\right)\cdot A \mod 1$$

$$\Leftrightarrow (k,\ell)\cdot A \mod N$$

$$k,\ell = 0,1,\ldots,N-1$$

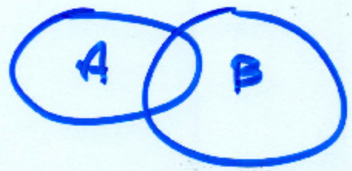
- ALL THE PERIODIC ORBITS ARE UNSTABLE
- ERGODICITY

- ACM5

BUT ALSO **STRONG MIXING!** ⑤

$A, B \subset T^2$ $\qquad \mu(T^2) = 1$

$d\mu = dQ\,dP$



$$\mu(A^n \cap B) \xrightarrow[n\to\infty]{} \mu(A)\cdot\mu(B)$$

A^n THE n-th ITERATE OF A UNDER **ACM**

ALSO

$$\mu(A^n \cap B) - \mu(A)\mu(B) \underset{n\to\infty}{\longrightarrow} c\cdot e^{-\lambda n}$$

$\Rightarrow$ **MIXING TIME** $= 1/\lambda$

- ACM6

COUPLING N-ACM's

$N = 2, 3, \ldots$ - INTEGERS -

$N=2$

$$F_{n+1} = a_1 F_n + b_1 F_{n-1} + c_1 G_n + d_1 G_{n-1}$$

$$G_{n+1} = a_2 G_n + b_2 G_{n-1} + c_2 F_n + d_2 F_{n-1}$$

$$\begin{pmatrix} F_n \\ G_n \\ F_{n+1} \\ G_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ b_1 & d_1 & a_1 & c_1 \\ d_2 & b_2 & c_2 & a_2 \end{pmatrix} \begin{pmatrix} F_{n-1} \\ G_{n-1} \\ F_n \\ G_n \end{pmatrix}$$

$$X_{n+1} = \begin{pmatrix} 0 & I \\ D & C \end{pmatrix} X_n \equiv \Lambda \cdot X_n$$

$$J = \begin{pmatrix} 0 & -I_{2\times 2} \\ I_{2\times 2} & 0 \end{pmatrix}$$ SYMPLECTIC FORM

- ACM7

DEMAND

7

$$\Lambda^T J \Lambda = -J$$

$$\Rightarrow \quad D = I \,, \quad C^T = C$$

$$\Rightarrow \Lambda = \begin{pmatrix} 0 & I \\ I & C \end{pmatrix}$$

$$\Rightarrow M = \Lambda^2 = \begin{pmatrix} I & C \\ C & I + C^2 \end{pmatrix}$$

$$\in Sp_4(\mathbb{Z})$$

$$M^T J M = J$$

- ACM8

(7b)

$$M : T^{2N} \to T^{2N}$$

2N- DIMENSIONAL UNIT TORUS

$$Q = (Q_1, \ldots, Q_N)$$

$$P = (P_1, \ldots, P_N)$$

$$(Q, P)_{n+1} = (Q, P)_n M$$

$$= (Q, P)_0 M^n$$

$$M = \begin{pmatrix} I & C \\ C & I + C^2 \end{pmatrix}$$

FIBONACCI POLYNOMIAL

$$M^n = \begin{pmatrix} F_{2n-1}(C) & F_{2n}(C) \\ F_{2n}(C) & F_{2n+1}(C) \end{pmatrix}$$

$$-F_{n+1}(x) = F_n(x) + F_{n-1}(x), \quad F_0(x) = 0, \quad F_1(x) = 1$$

- ACM9

ANY N-

$$\Lambda = \begin{pmatrix} 0_{n\times n} & I_{n\times n} \\ I_{n\times n} & C_{n\times n} \end{pmatrix}$$

C SYMMETRIC MATRIX INTEGER

- IMPOSING PERIODIC BC ON 1, 2, ..., N

THE LATTICE

CLOSED CHAIN

- NN-INTERACTION

$$C = K \cdot I_{n\times n} + G\,(P + P^T)$$

$$P = \begin{pmatrix} 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \\ 0 & \cdots & 1 & 0 \end{pmatrix}$$

TRANSLATION - SHIFT MATRIX ALONG THE CHAIN OF ACM'S

K, G, POSITIVE INTEGERS

- ACM10

NEXT TO NN INTERACTIO

$$C = k \cdot I_{n \times n} + \sum_{\ell=1}^{L} G_\ell (P^\ell + P^{T\ell})$$

$1 \leq L \leq \left[\frac{N-1}{2}\right]$.

$$M = \begin{pmatrix} I & C \\ C & I + C^2 \end{pmatrix}$$

POSITIVE SEMIDEFINITE

- ZERO MODES OF C

EIGENVALUES $= 1$ FOR M

ZERO LIAPUNOV EXPONENTS

- CONDITION: $k + \sum_{\ell=1}^{L} G_L (-1)^\ell = 0$

- ACM11

RESULTS

EXPLICIT LIAPUNOV SPECTRA

- EIGENVALUES OF C

$$D_J = K + 2 \sum_{\ell=1}^{L} G_\ell \cos \frac{2\pi \ell J}{N}$$

$$\lambda_{\pm J} = \log \rho_{\pm, J}$$

$$\rho_{\pm, J} = \frac{2 + D_J^2}{2} \pm \frac{|D_J|}{2} \sqrt{D_J^2 + 4}$$

- KOLMOGOROV-SINAÏ ENTROPY

$$KS = \sum_{J=1}^{N} \lambda_{+J}$$

- ACM12

11

- LINEARLY RISING WITH N

SLOPE - INCREASING FUNCTION OF THE RANGE L FOR FIXED G_e

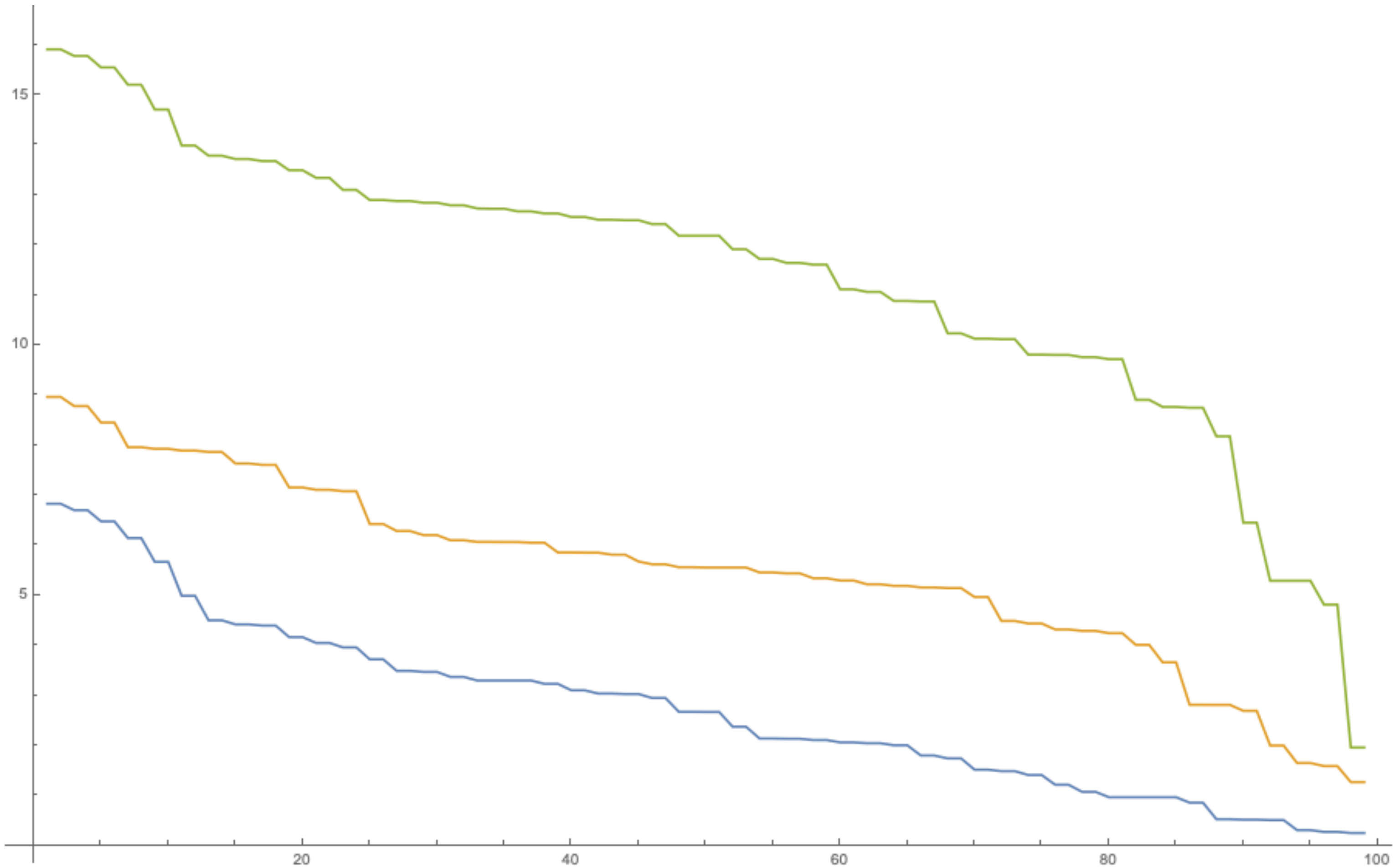
- INCREASING FUNCTION OF THE COUPLINGS G FOR FIXED RANGE L

- TUNABLE MIXING PROPERTIES

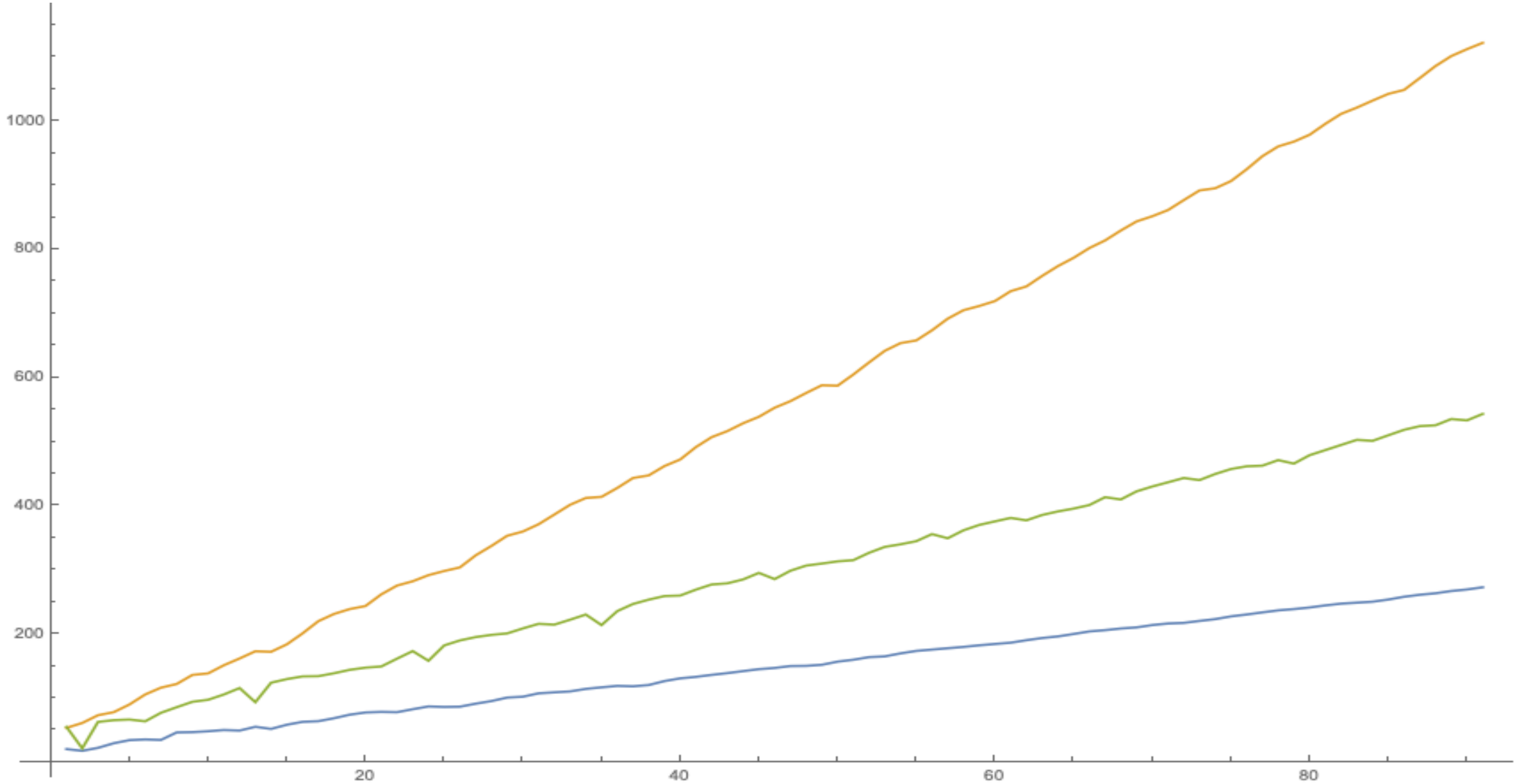
- FAST SCRAMBLING ?

SORTED LIAPUNOV SPECTRA

CONSTANT,INCREASING,DECREASING NNN COUPLINGS



KOLMOGOROV-SINAI ENTROPY OF ACML VERSUS NUMBER OF DOF



QUANTUM MECHANICS OF THE ARNOL'D CAT MAP n=1 DOF

- A PARTICLE ON THE DISCRETE CIRCLE OF N POINTS
- Q=Diag{1, ω, ω^2,.., ω^(N-1)} POSITION –CLOCK OPERATOR
- ω=Exp[2 π i/N], Planck constant , h=2 π/N
- P={{0,0,..,1},{1,0,0,..,0},{0,1,..,0},..{0,0,..,0,1,0}} MOMENTUM -SHIFT OPERATOR
- QP=ωPQ HEISENBERG-WEYL CR
- 1- QBIT PER MINIMUM PHASE SPACE AREA-
- Q-HALL EFFECT ,UNIT OF FLUX 1/N
- QF=FP, DISCRETE PARTICLE-WAVE DUALITY
- FINITE QUANTUM FOURIER TRANSFORM: FQFT , F, UNITARY NxN MATRIX,
- F(k,l)= ω^(k l) /Sqrt[N] k,l=0,1,2,…,N-1
- F^4=I,F^2=S INVERSION OPERATOR

QM TIME EVOLUTION OPERATORS

- U[A](k,l)=1/Sqrt[N] ω^[-1/2b (ak^2+d l^2-2 k l)], k,l,=0,1,2,..,N-1
- U[A]^n=U[A^n],
- EXAMPLES
- 1)HARMONIC OSCILLATOR   A={{0,-1},{1,0}}
- U[A](k,l)=1/Sqrt[N] ω^(kl)=F , Q-FOURIER=FFT
- HARMONIC OSCILLATOR GROUP SO[2,Z[N]]
- BALIAN-ITZYKSON 1986 FQM
- FLORATOS –ATHANASIU 1994 COHERENT STATES IN FQM
- FLORATOS-PAVLIDIS QUANTUM CIRCUITS FOR THE FRACTIONAL FOURIER TRANSFORM
- A={{a,-b},{b,a}}   a^2+b^2=1 MOD[N},a,b=0,1,2,..,N-1
- FOR N=prime integer=4 k+(-)1,ORDER OF THE GROUP 4k.

FAST QUANTUM MAPS

- N=N1xN2 ,N=Exp[R^2], R^2=>R1^2 + R2^2
- SL[2,Z[N]]=SL[2,Z[N1]]xSL[2,Z[N2]]
- A[N]=A[N1]A[N2]
- H[N]=H[N1]+H[N2], SCHWINGER HEIS-WEYL FACTORIZATION
- U[A[N]]=U[A[N1]]U[A[N2]]
- FAST QUANTUM MAPS N^2->N LogN
- S[N]=S[N1]+S[N2] ADDITIVITY OF COARSE GRAINED ENTROPIES

- **RESULTS FOR THE QUANTUM MECHANICS OF ONE ARNOLD CAT MAP A={{1,1},{2,1}}**

- EXACT CONSTRUCTION OF THE SPECTRUM AND EIGENSTATES FOR N=p,prime,

- LINEAR SPECTRUM

- RANDOM EIGENSTATES ->LINEAR COMBINATIONS OF MULTIPLICATIVE CHARACTERS OF
- GF[P]

- 1)RANDOM PHASES
- 2)GAUSSIAN RANDOM AMPLITUDES

- BUT SCARS(VOROS.,NONEMACHER)F
- FOR SEQUENCES OF N's WITH SHORT PERIODS

- QUANTUM CHAOS, STRONG MIXING

- FACTORIZATION FOR ARNOLD CAT MAPS IMPLIES LOGARITHMIC IMPROVEMENT FROM N^2->N LOGN

- USING QUANTUM CIRCUITS FOR THE IMPLEMENTATION OF THE QUANTUM MAP AND
- COUNTING THE NUMBER OF GATES NLOGN->(LOGN)^2

- EXACTLY AS FOR THE QUANTUM FOURIER FACTORIZATION ALGORITHM OF SHOR.

QUANTIZATION OF ARNOLD CAT MAP LATTICE FIELD THEORIES

LET M AN ELEMENT OF THE SYMPLECTIC GROUP Sp[2n,Z/NZ]

M={{A,B},{C,D}} ,2nx2n Integer matrix ACTING ON THE 2n Dim TORUS

FOR (Q,P) in ZN^nxZN^n

(Qm+1,Pm+1) =(Qm,Pm).M

CLASSICAL TIME EVOLUTION MAP

THE MAGNETIC TRANSLATIONS FOR n-DOF ARE

$\boldsymbol{J}_{\boldsymbol{rs}}=\boldsymbol{J}_{r1s1}\,\mathrm{X}\boldsymbol{J}_{r2s2}\ldots\mathrm{X}\boldsymbol{J}_{rnsn}$

M->U[M]: U[M]. $\boldsymbol{J}_{\boldsymbol{r},\boldsymbol{s}}$U^(-1)[M]$=\boldsymbol{J}_{(\boldsymbol{r},\boldsymbol{s})\boldsymbol{M}}$

PROPERTIES OF THE QEVOL OPERATOR

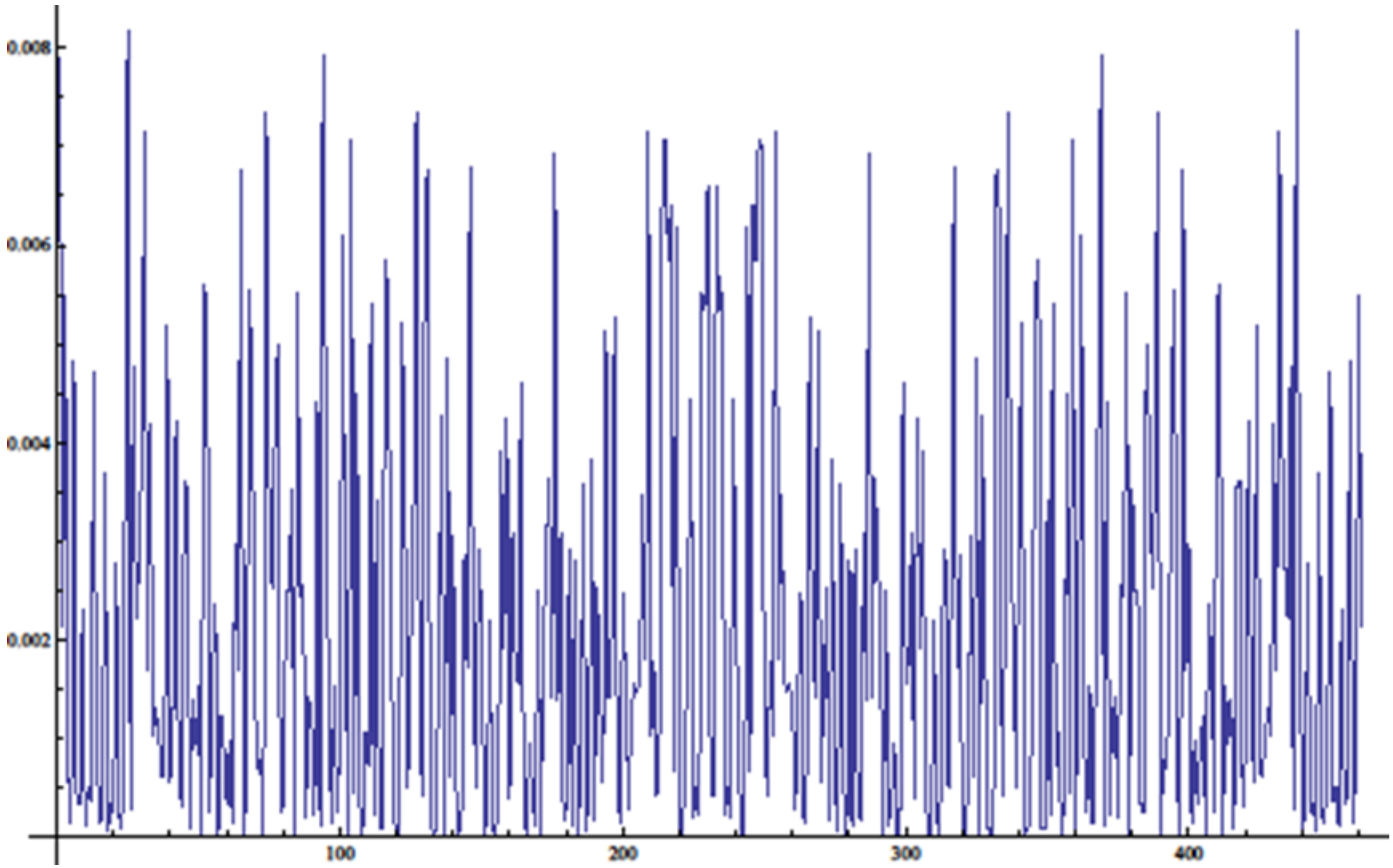
- $U[M](k,l)=Exp[2\pi i/N\ \Phi[k,l]]$
- $k=(k1,k2,..,kn)$, $l=(l1,l2,...,ln)$
- $ki=0,1,\ldots N-1, li=0,1,\ldots N-1)$

- THE PHASE Φ IS A QUADRATIC FORM IN k,l
- $M=\{\{A,B\},\{C,D\}\}$

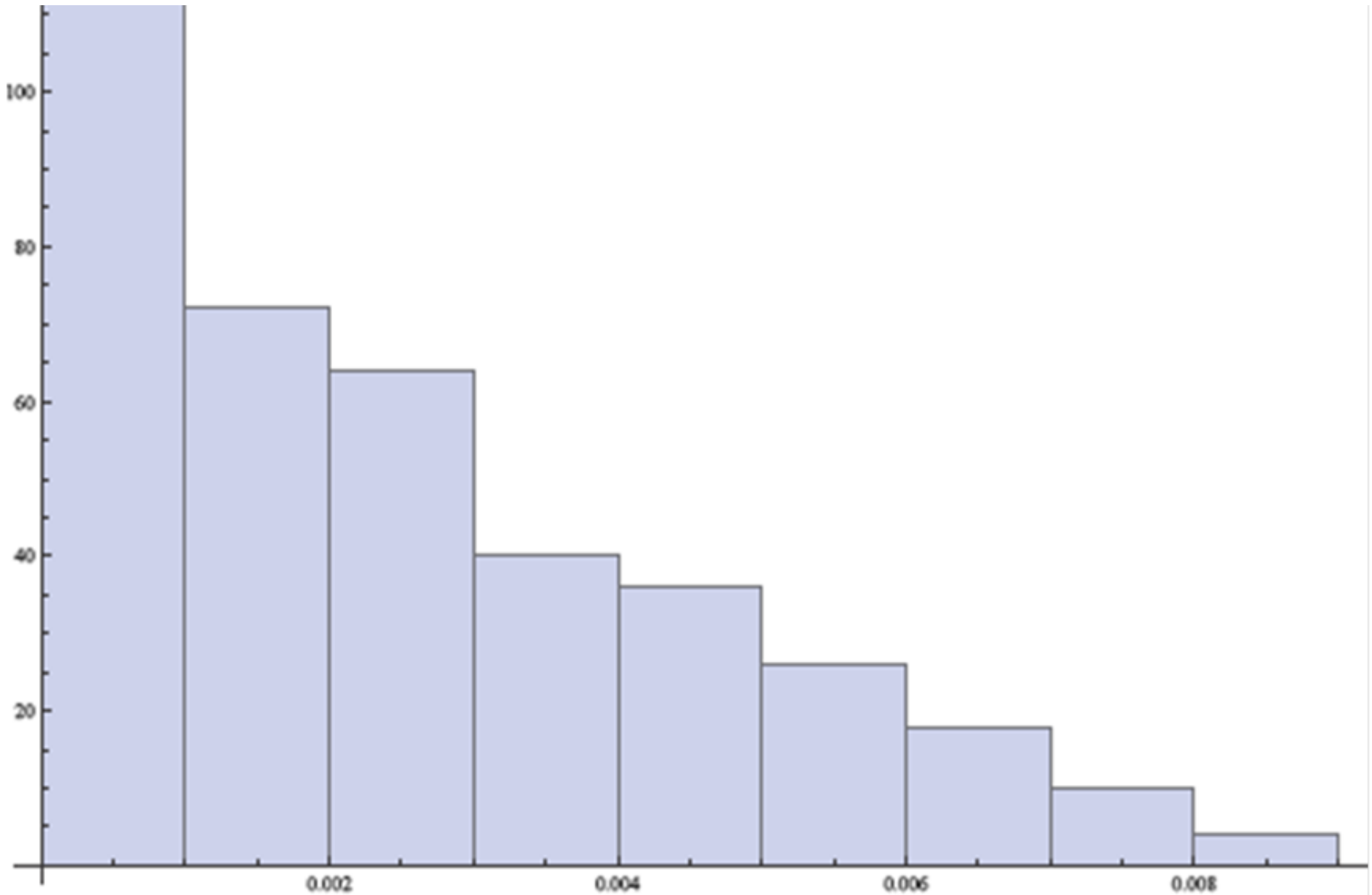
- $\Phi(k,l)=1/2\ [k.C^{\wedge}(-1).Dk-2\ k.C^{\wedge}(-1).l+l.A.C^{\wedge}(-1).l]$

- $U[M]$ IS A UNITARY $N^n x N^n$ MATRIX
- $U[M^m]=U[M]^m$
- TIME EVOLUTION IS KNOWN IF M^n IS KNOWN
- IN OUR CASE ACML FIELD THEORIES
- $M^m=$
- $\{\{F(2m-1)[C],F(2m)[C]\},\{F(2m)[C],F(2m+1)[C]$
- $F(m+1)[C]=C\ F(m)+F(m-1)$
- MATIRC FIBONACCI POLYNOMIALS
- EXPLICITLY KNOWN

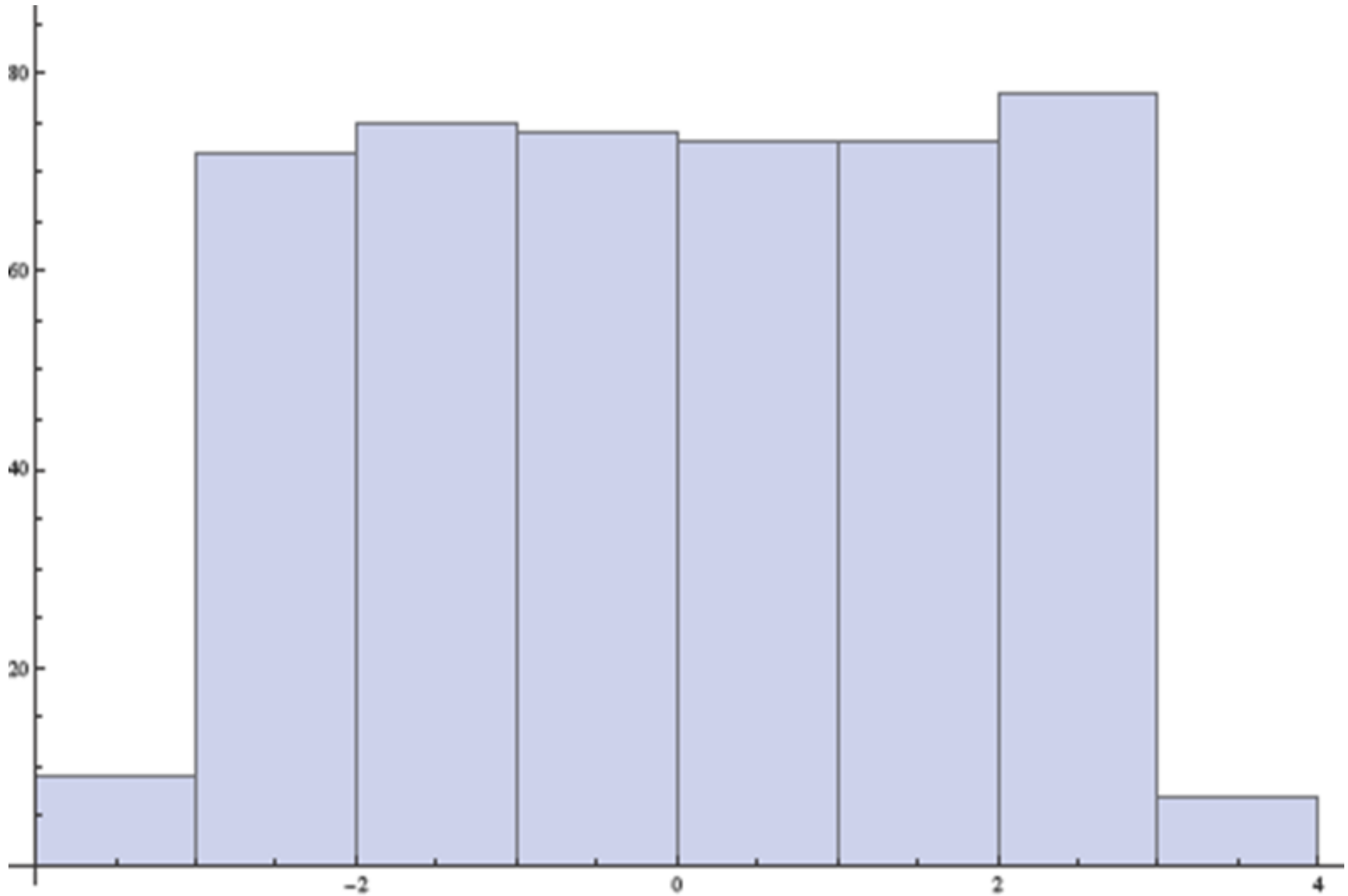
N=461,T[461]=23 ,GROUND STATE,PHI=0



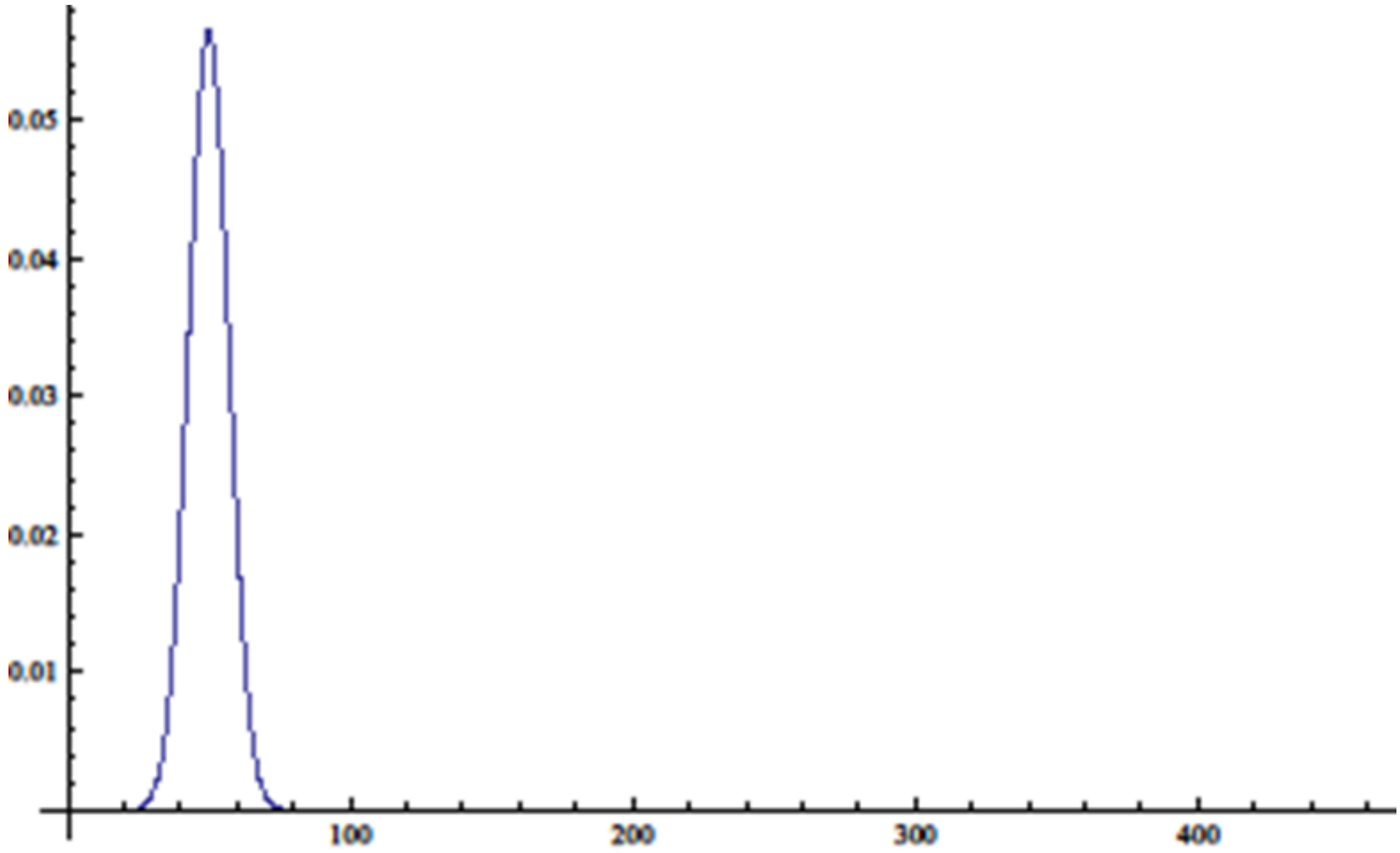
GROUND STATE AMPLSQUARE DF



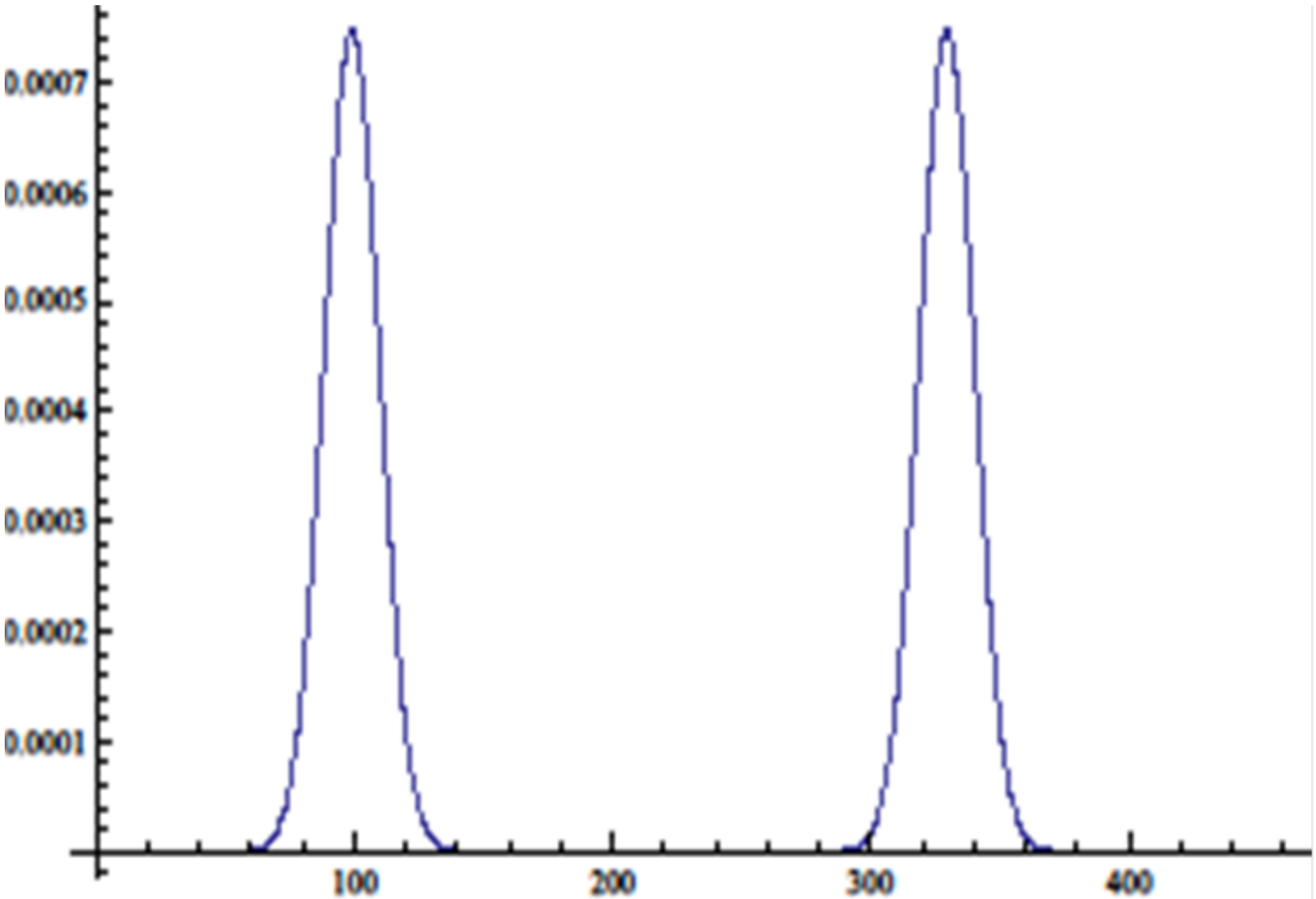
GROUND STATE PHASE DF



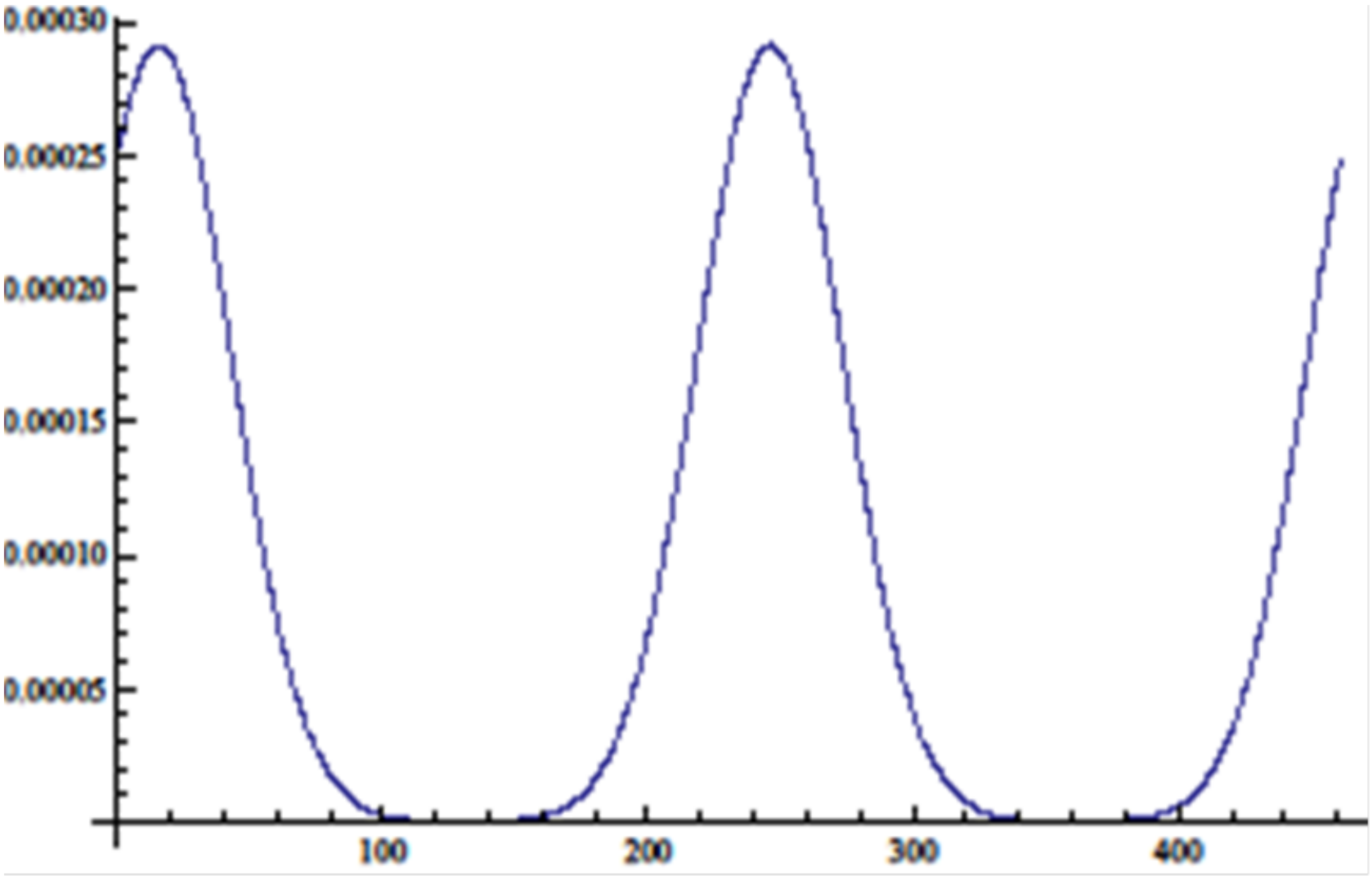
SCATTERING EXPERIMENT, N=p=461
GAUSSIAN WF AT T=0



TIME=1



T=3



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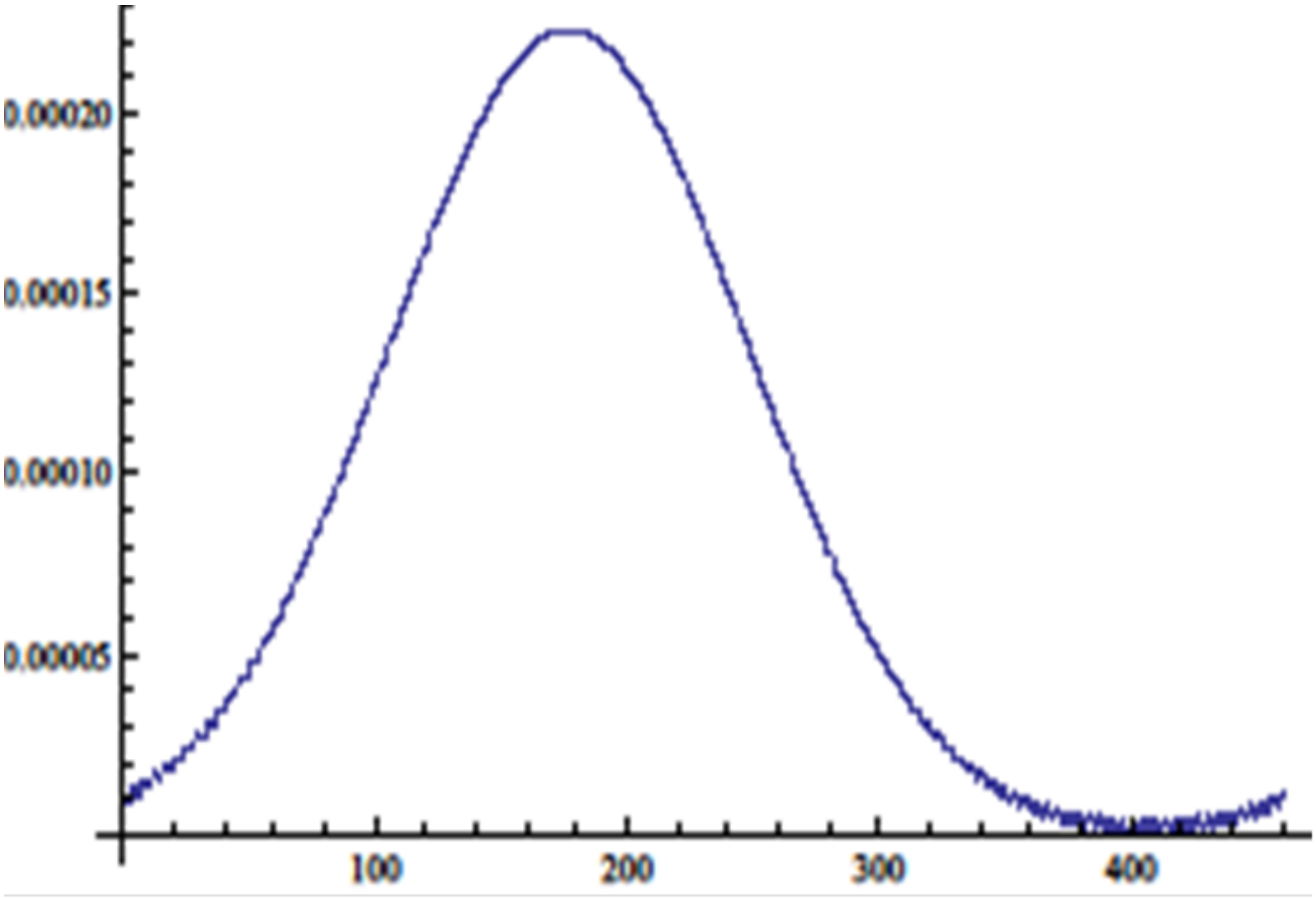

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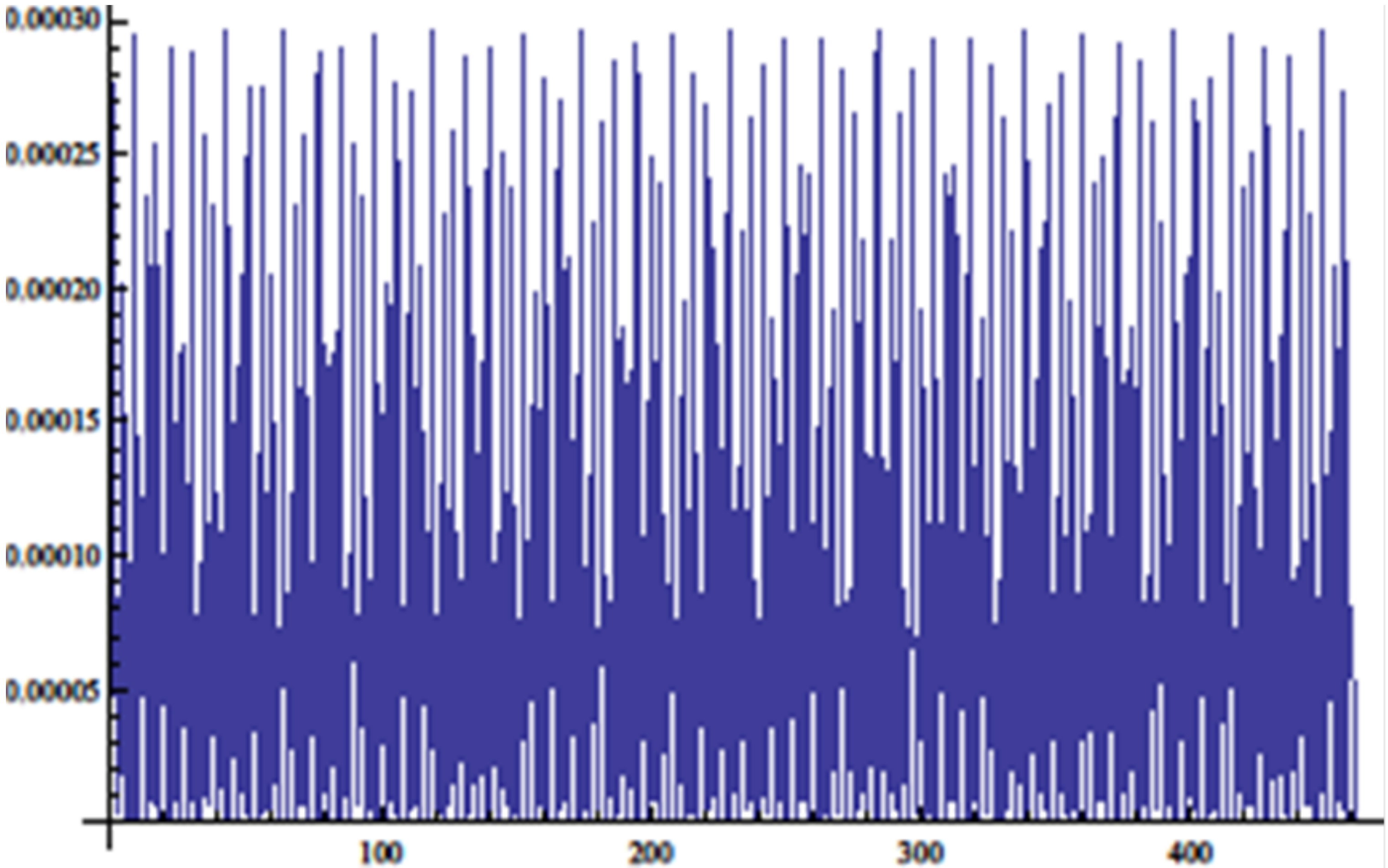

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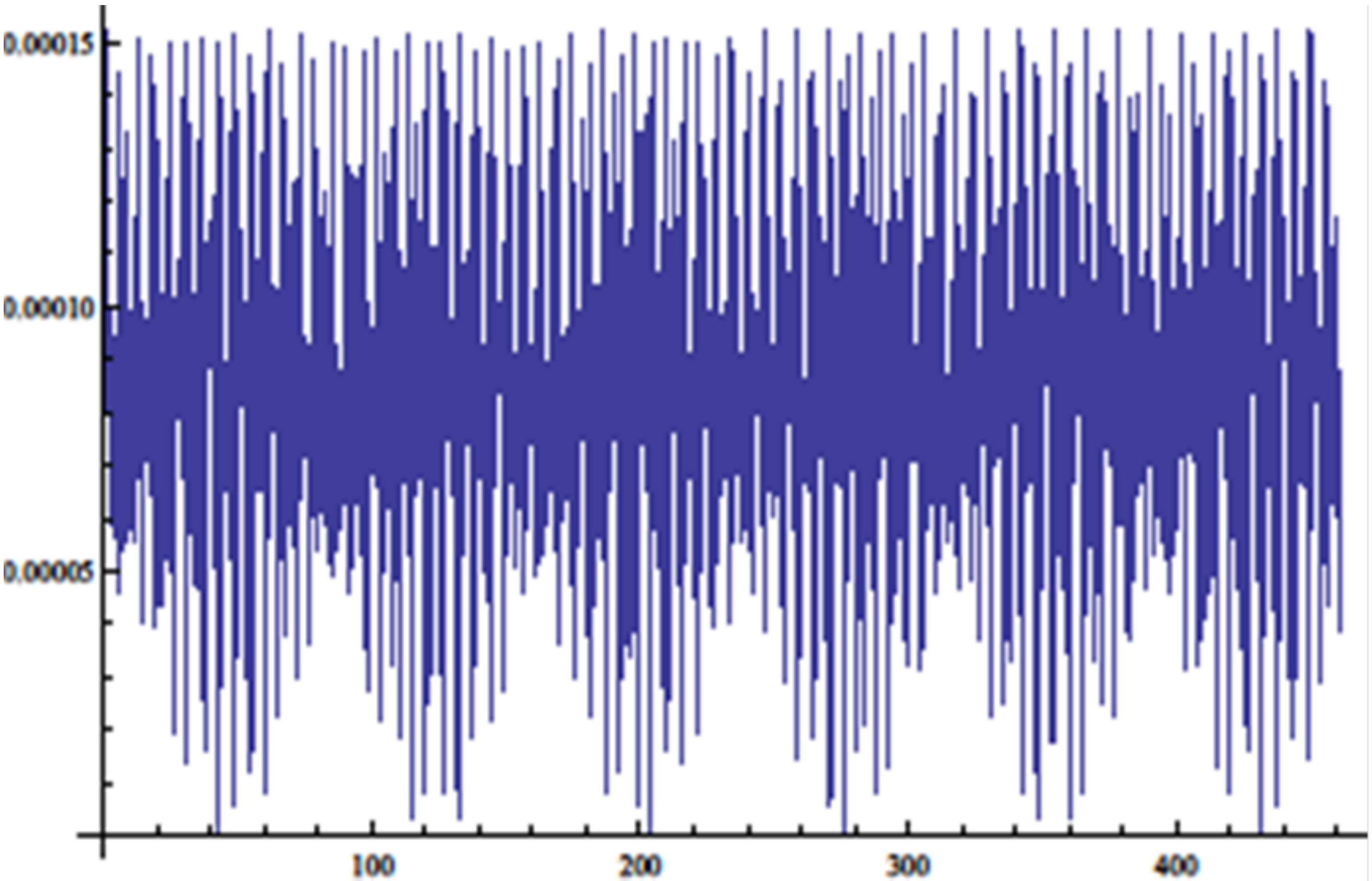
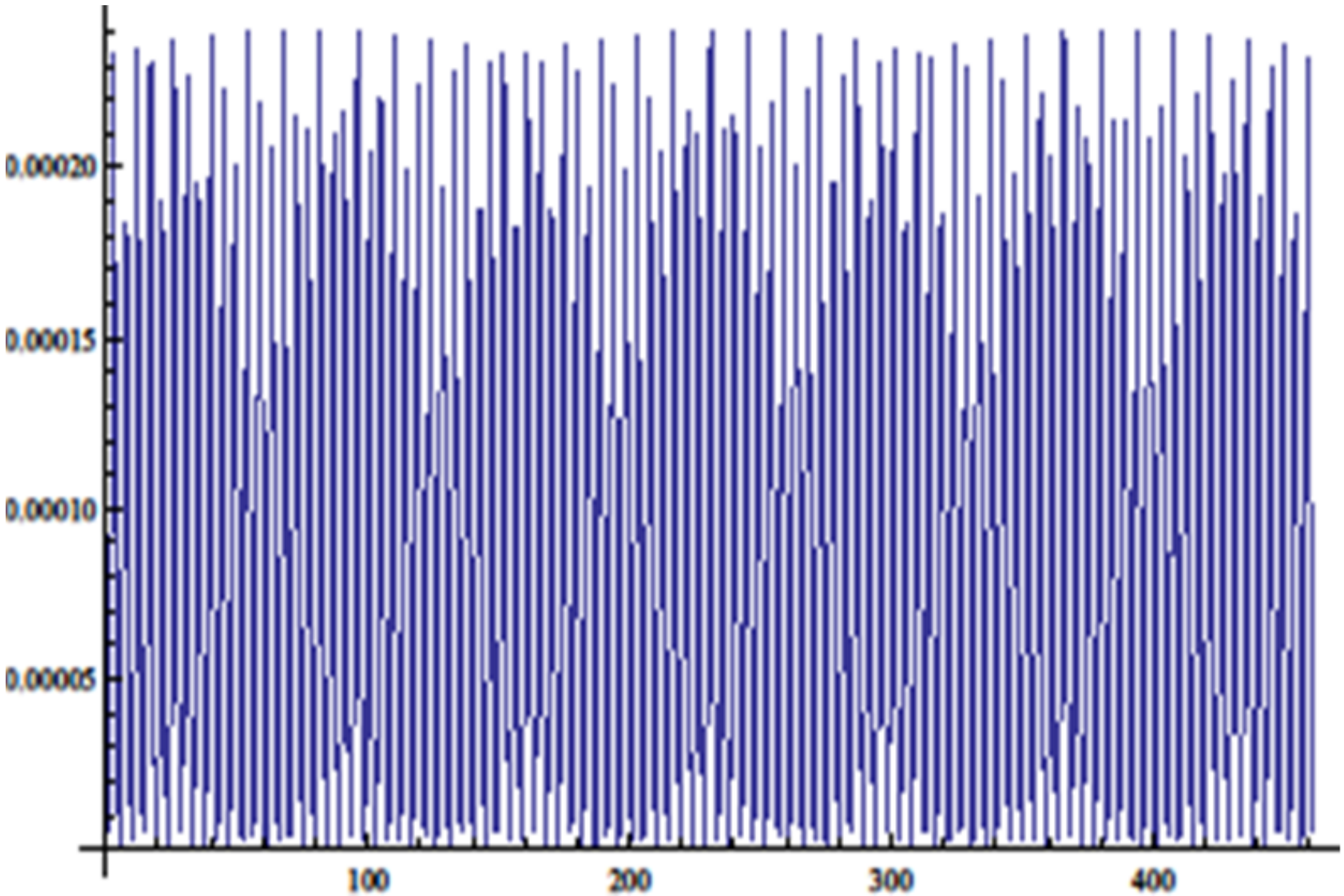


T=7



CONCLUSIONS

- **ACM LATTICE FIELD THEORIES ARE MAXIMALLY CHAOTIC DYNAMICAL SYSTEMS**
- **THEY ARE UNIQUELY CHARACTERIZED BY THE POSSIBILITY OF ANALYTIC CALCULATIONS FOR ALL OF THE RELEVANT PHYSICAL QUANTITIES AMONG THESE ARE →**
- TIME EVOLUTION DUE TO RELATIONS WITH FIBONACCI MATRIX POLYNOMIALS
- DETERMINATION OF ALL UNSTABLE PERIODIC ORBITS DENSE IN THE PHASE SPACE
- LIAPUNOV SPECTRA AND KOLMOGOROV-SINAI ENTROPY S-KS DUE TO TRANSLATIONAL INVARIANCE
- MIXING TIMES T=1/S
- EXPLICIT CONSTRUCTION OF THE QUANTUM MECHANICAL UNITARY TIME EVOLUTION OPERATOR
- ANALYTIC STUDY OF THE QUANTUM CHAOTIC PROPERTIES DONE FOR n=1
- QUANTUM ERGODICITY –QUANTUM MIXING DONE FOR n=1
- OTOC-OPERATOR SPREADING COMPLEXITY-ENTANGLEMENT IN PREPARATION IT IS CHECKED

THANK YOU

FOR YOUR ATTENTION!