Two-loop amplitudes for pp→bbH

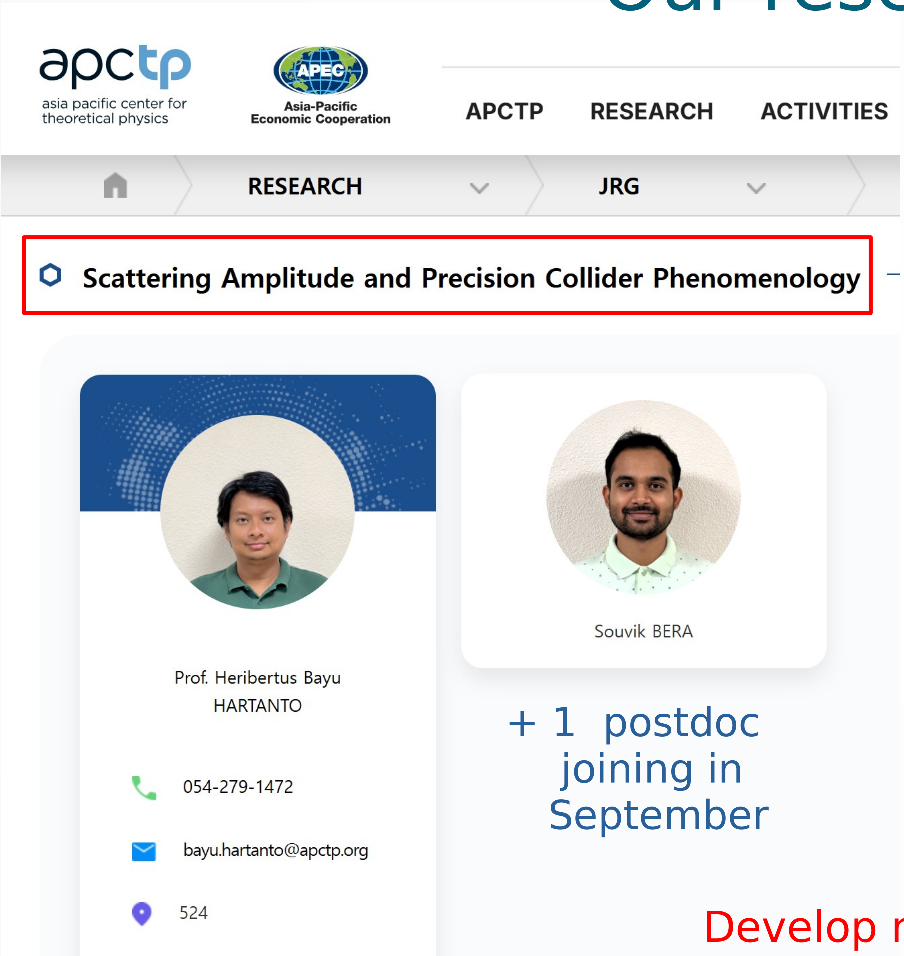
Heribertus Bayu Hartanto

Asia Pacific Center for Theoretical Physics (APCTP) Pohang, South Korea

3rd INPP Demokritos-APCTP Workshop

July 2nd, 2026

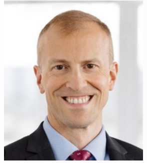
Our research group



+ 1 postdoc joining in September

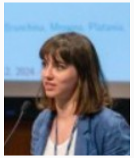
Collaborators


Simon Badger (Torino)


Michal Czakon (RWTH)


Simone Zoia (UZH)


Rene Poncelet (IFJ PAN)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Jim Canko (Bologna)


Mattia Pozzoli (Bologna)


Matteo Becchetti (Bologna)


Yang Zhang (USTC)

Zihao Wu (SCNU)

Develop methods and tools for higher-order calculation within pQFT framework → enable precision studies at colliders

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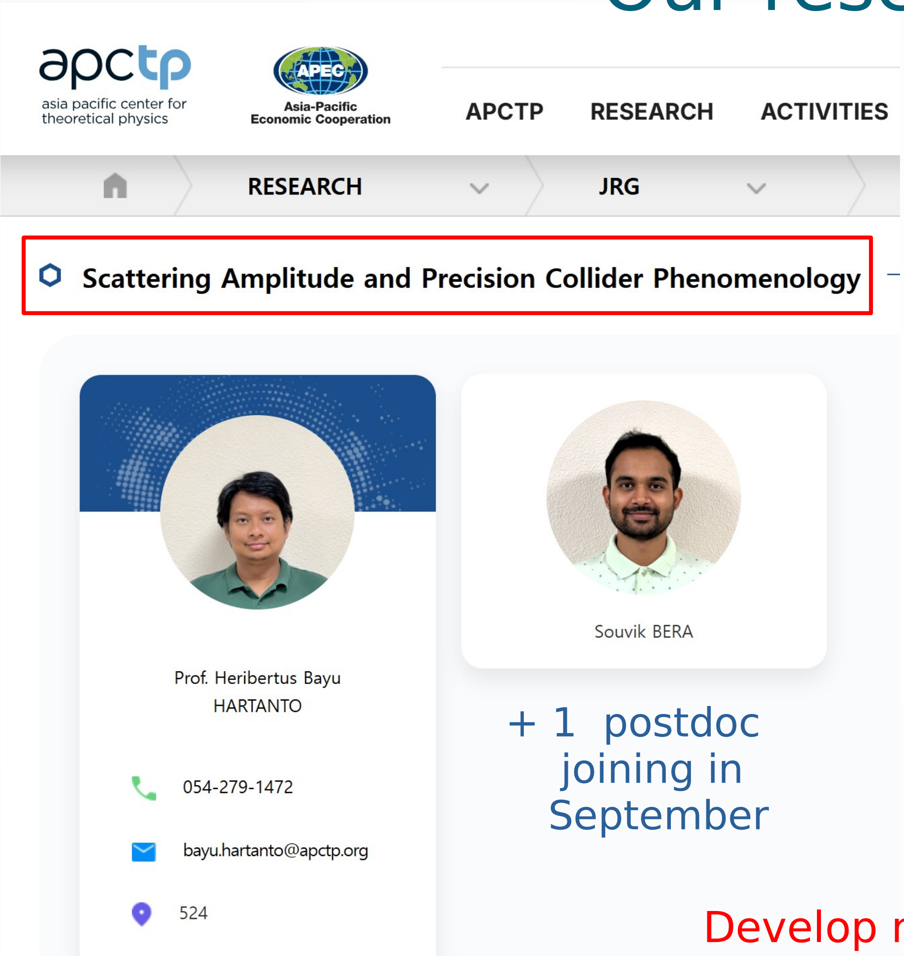

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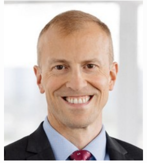
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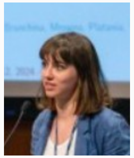
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also previous collaboration with Giuseppe

2015-2022

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Two-loop amplitudes for pp→bbH

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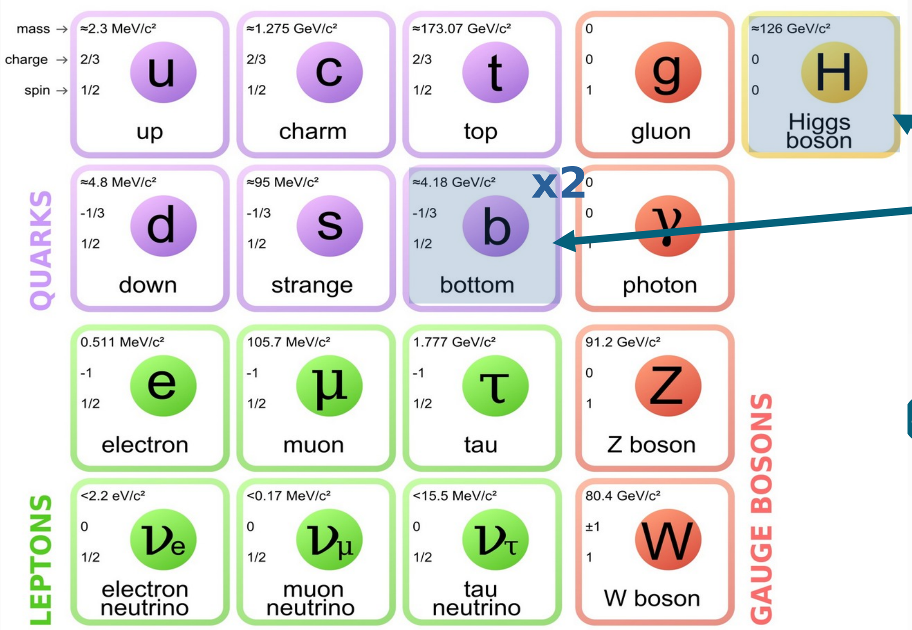
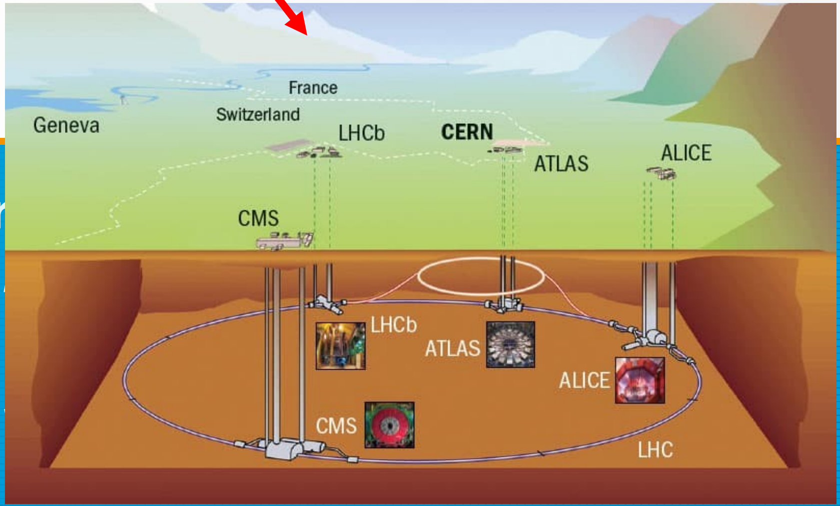


es for pp→bbH



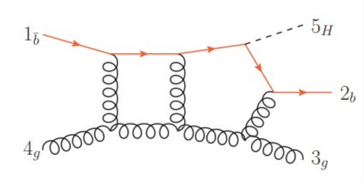
Heribertus Bayu Har

Asia Pacific Center for Theoretical Physics (A

3rd INPP Demokritos-APCTP

July 2nd, 2026

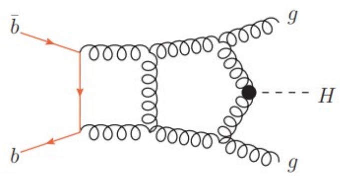
based on



Full-colour double-virtual amplitudes for associated production of a Higgs boson with a bottom-quark pair at the LHC

Simon Badger (INFN, Turin), Heribertus Bayu Hartanto (APCTP, Pohang and POSTECH), Rene Poncelet (Cracow, INP), Zihao Wu (HIAS, UCAS, Hangzhou and Unlisted, CN), Yang Zhang (Hefei, CUST and PCFT, Hefei) et al. (Dec 9, 2024)

Published in: *JHEP* 03 (2025) 066 • e-Print: 2412.06519 [hep-ph]



Top-Yukawa contributions to $pp \to b\bar{b}H$: two-loop leading-colour amplitudes

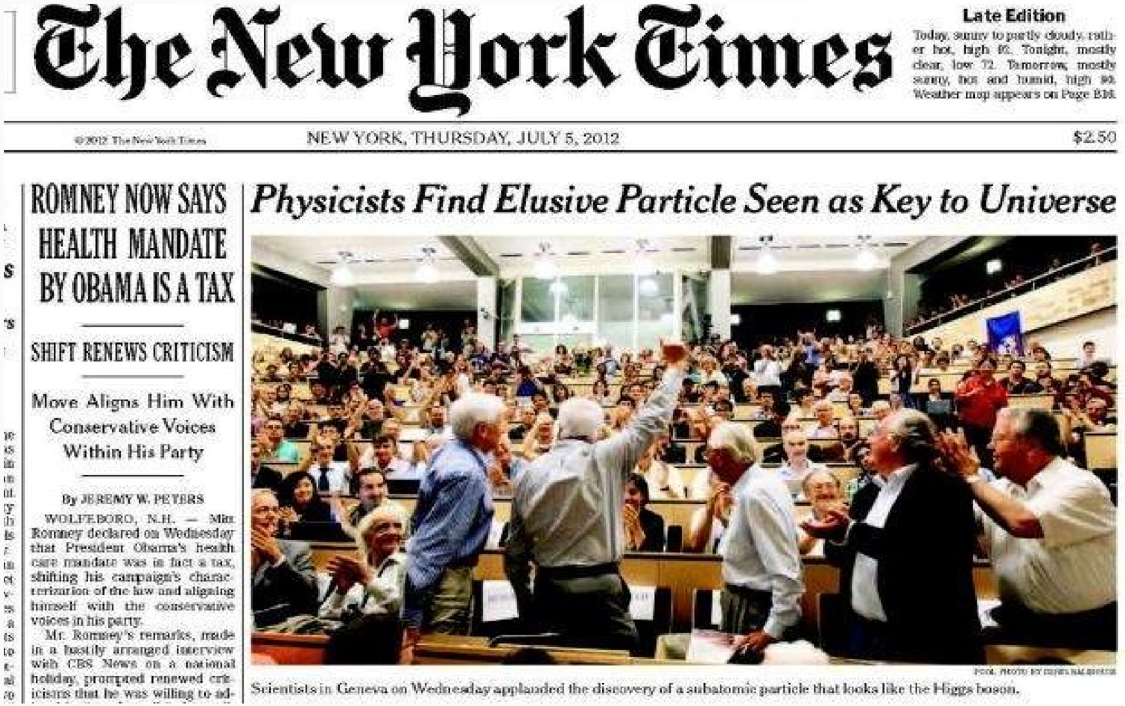
Heribertus Bayu Hartanto (APCTP, Pohang and POSTECH), Rene Poncelet (Cracow, INP) (Mar 31, 2026)

e-Print: 2603.29480 [hep-ph]

Higgs Physics at the LHC

Higgs boson discovery in 2012 → from discovery to precision measurements

study its fundamental properties
explore connection to Beyond Standard Model (BSM) physics


The New York Times

Late Edition

Today, sunny to partly cloudy, rather hot, high 92. Tonight, mostly clear, low 72. Tomorrow, mostly sunny, hot and humid, high 90. Weather map appears on Page B10.

NEW YORK, THURSDAY, JULY 5, 2012

$2.50

ROMNEY NOW SAYS HEALTH MANDATE BY OBAMA IS A TAX

SHIFT RENEWS CRITICISM

Move Aligns Him With Conservative Voices Within His Party

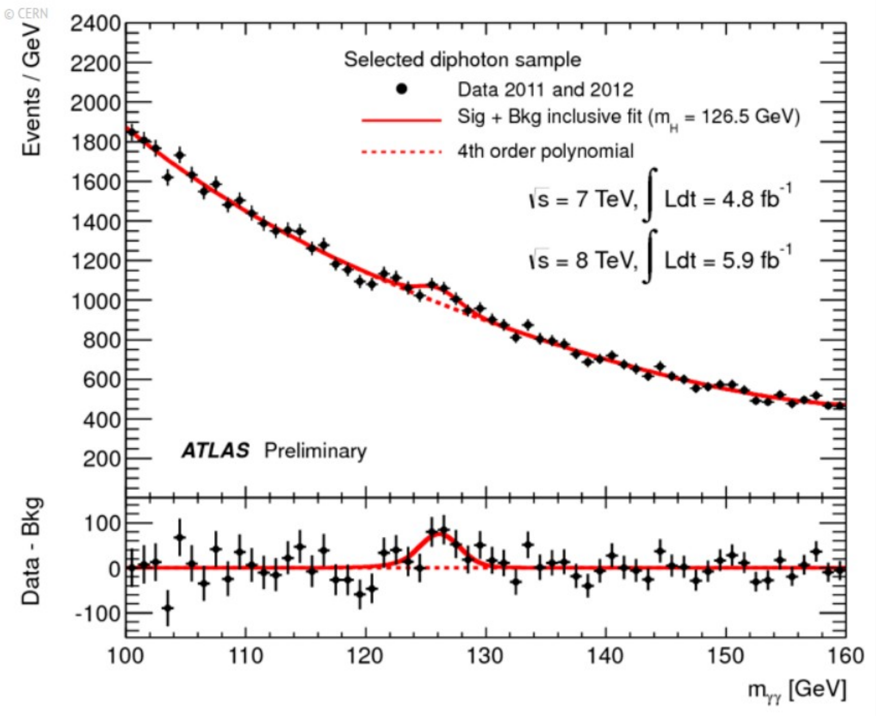
By JEREMY W. PETERS

WOLFEBORO, N.H. — Mitt Romney declared on Wednesday that President Obama's health care mandate was in fact a tax, shifting his campaign's characterization of the law and aligning himself with the conservative voices in his party.

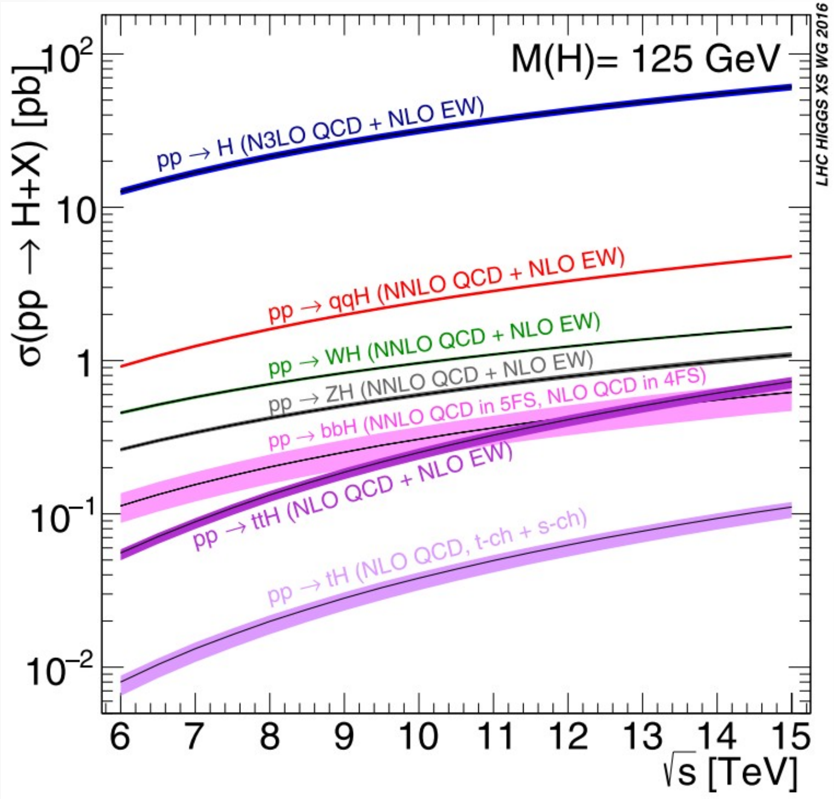
Mr. Romney's remarks, made in a hastily arranged interview with CBS News on a national holiday, prompted renewed criticisms that he was willing to ad-

Physicists Find Elusive Particle Seen as Key to Universe

Scientists in Geneva on Wednesday applauded the discovery of a subatomic particle that looks like the Higgs boson.



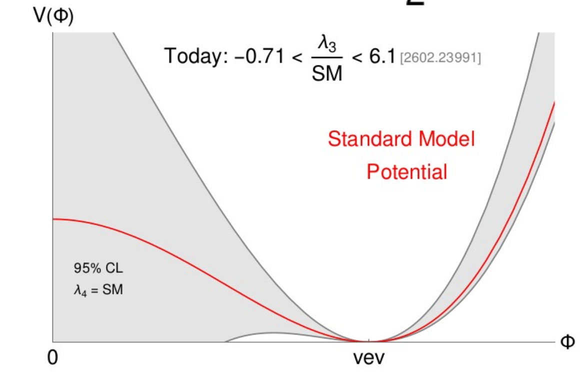
bbH production at the LHC

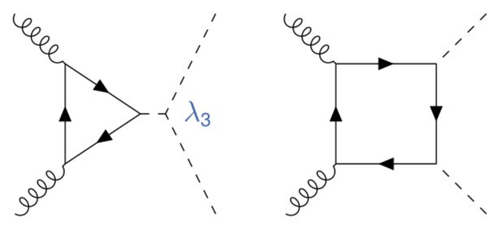


arXiv:1610.07922 [hep-ph]

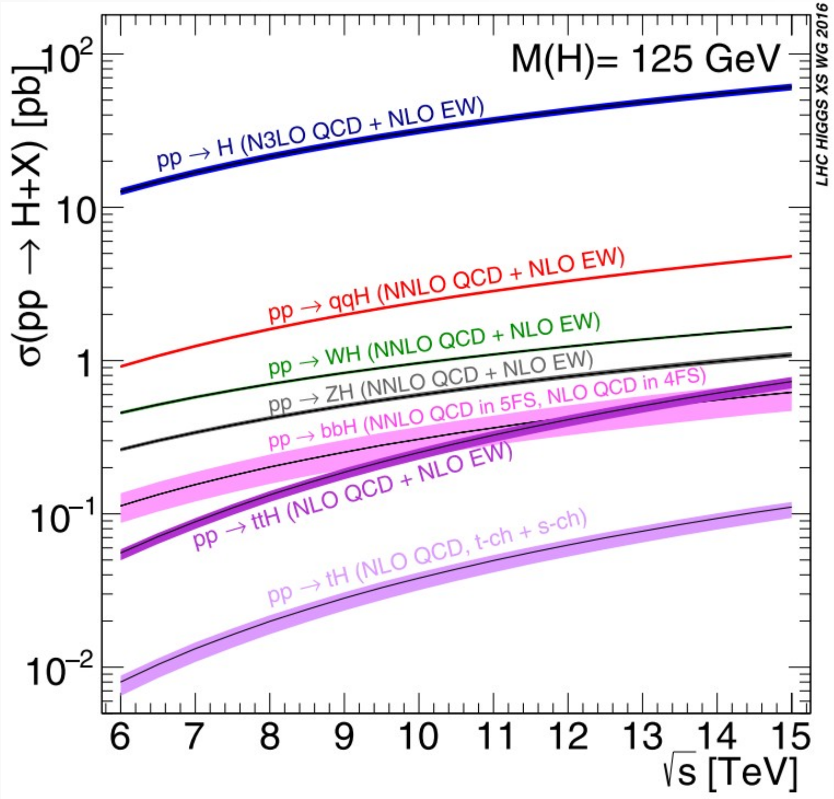
- One of the main single Higgs production mode
  → contributes at ~1% to $\sigma_{H,SM}$
- b-tagging kills the production rate
  → yet to be observed
- Bottom-Yukawa enhancement: 2HDM/MSSM
- Irreducible background to HH searches
  ⇒ $(H\to bb)(H\to\tau\tau)$, $(H\to bb)(H\to\gamma\gamma)$ channels

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2}\left(\partial_\mu H\right)\left(\partial^\mu H\right) + \frac{1}{2}m^2H^2 + \lambda_3 H^3 + \lambda_4 H^4$$





bbH production at the LHC



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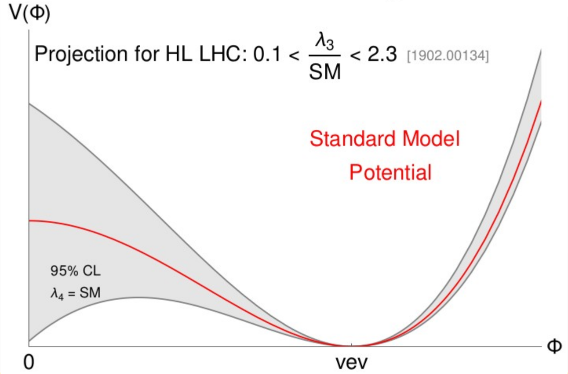
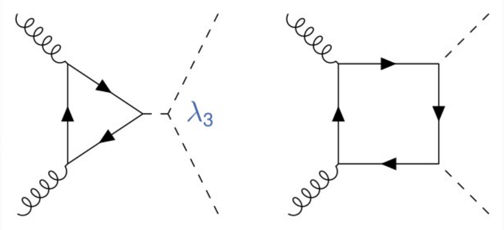


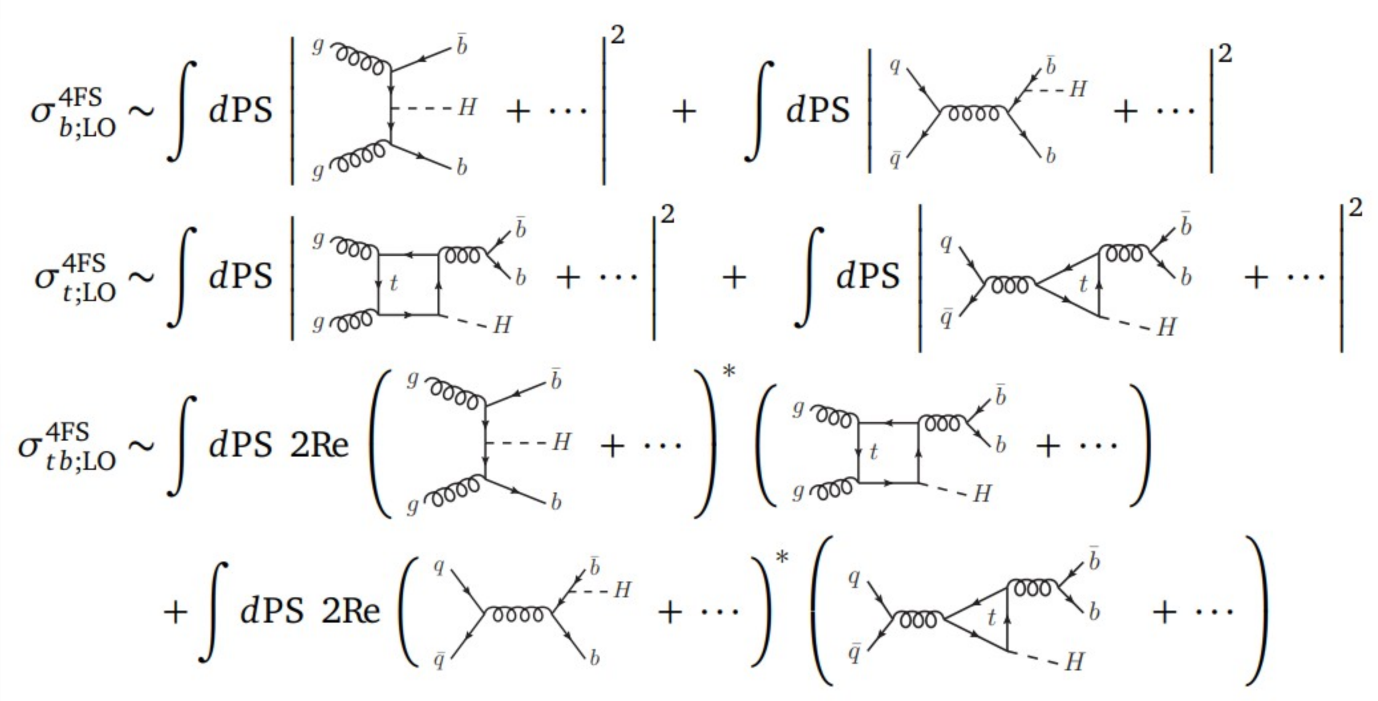


pp→bbH cross sections

Contributions from different Yukawa couplings

[Hamilton,Nason,Re,Zanderighi(2013)]
[Deutschmann,Maltoni,Wiesemann,Zaro(2018)]
[Manzoni,Mazzeo,Mazzitelli,Wiesemann,Zaro(2023)]
[Biello,Mazzitelli,Sankar,Wiesemann,Zanderighi(2025)]

$$\sigma_{b\bar{b}H} = y_b^2\,\sigma_b + y_b y_t\,\sigma_{tb} + y_t^2\,\sigma_t$$



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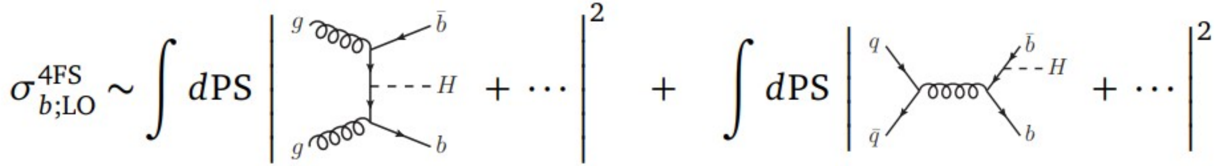
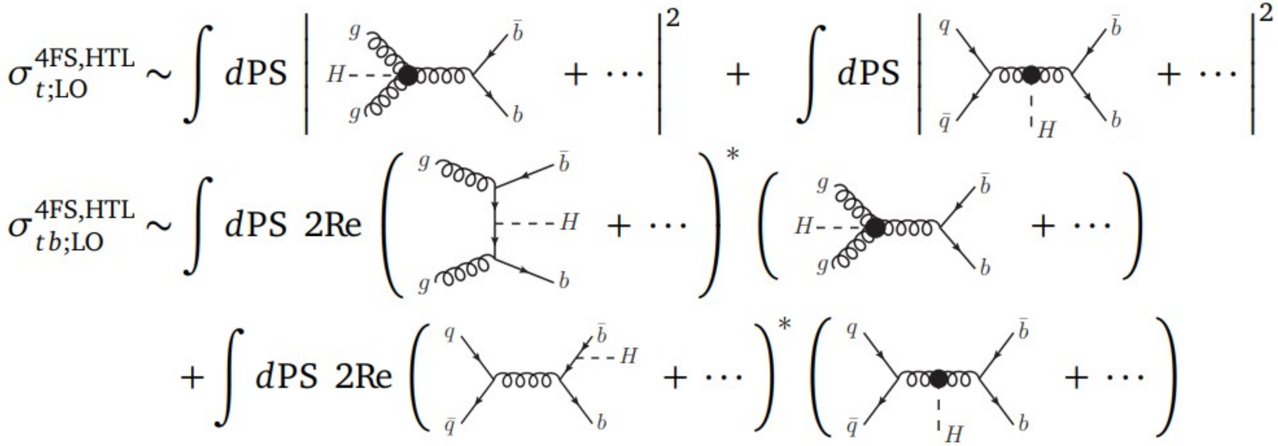




Work in
Heavy-top limit

$$\mathcal{L} = -\frac{1}{4} C_1 H G^a_{\mu\nu} G^{a\mu\nu}$$

$$C_1 = \frac{\alpha_s}{3\pi}\frac{y_t}{\sqrt{2}m_t} + \mathcal{O}(\alpha_s^2)$$

pp→bbH cross sections

$$d\hat{\sigma} = \underbrace{d\hat{\sigma}^{(0)}}_{\mathrm{LO}} + \underbrace{\alpha\, d\hat{\sigma}^{(1)}}_{\delta\mathrm{NLO}} + \underbrace{\alpha^2\, d\hat{\sigma}^{(2)}}_{\delta\mathrm{NNLO}} + \ldots$$

Fiducial region	y_t^2 NLO (NLO+PS)	y_t^2 LO (ggF NNLOPS)	y_t^2 LO $\cancel{g\to b\bar{b}}$ (ggF NNLOPS)	y_b^2 NNLO ($b\bar{b}H$ MiNNLO$_{\mathrm{PS}}$)
No cut	$1696^{+62\%}_{-35\%}$ ab	$7574^{+8\%}_{-8\%}$ ab	$2774^{+7\%}_{-7\%}$ ab	$1180^{+19\%}_{-15\%}$ ab
Fid. cuts	$18^{+55\%}_{-33\%}$ ab	$31.6^{+18\%}_{-18\%}$ ab	$20.5^{+18\%}_{-18\%}$ ab	$5.6^{+2.8\%}_{-10\%}$ ab
Fid. cuts + $m^{\star}_{2b2\gamma} < 500\,\mathrm{GeV}$	$12^{+57\%}_{-33\%}$ ab	$23.3^{+20\%}_{-20\%}$ ab	$15.4^{+20\%}_{-20\%}$ ab	$5.5^{+2.8\%}_{-10\%}$ ab
Fid. cuts + $m^{\star}_{2b2\gamma} < 350\,\mathrm{GeV}$	$5.5^{+60\%}_{-34\%}$ ab	$11.5^{+20\%}_{-20\%}$ ab	$7.76^{+20\%}_{-20\%}$ ab	$4.8^{+3.1\%}_{-10\%}$ ab

Table 5: Cross sections in ab for the y_t^2 and y_b^2 contributions to $pp \to b\bar{b}H$ with $H \to \gamma\gamma$ decay at $\sqrt{s} = 13.6$ TeV.

Biello etal. Modelling bbH production for the LHC at 13.6 TeV, arXiv:2510.18815[hep-ph]

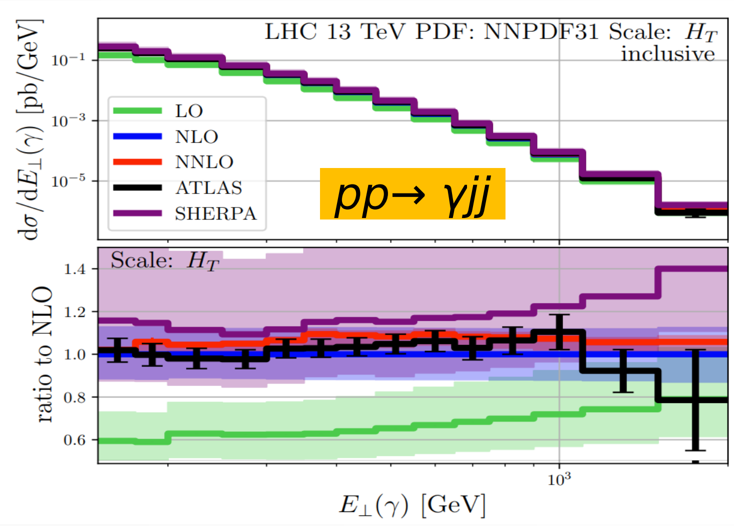
NNLO(+PS) HEFT prediction for y_t^2 contribution is desirable

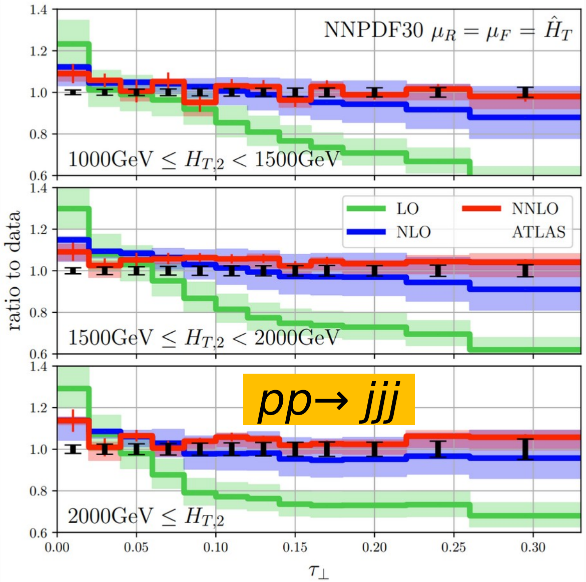
pp→bbH: 2→3 scattering process, actively studied in the last couple of years

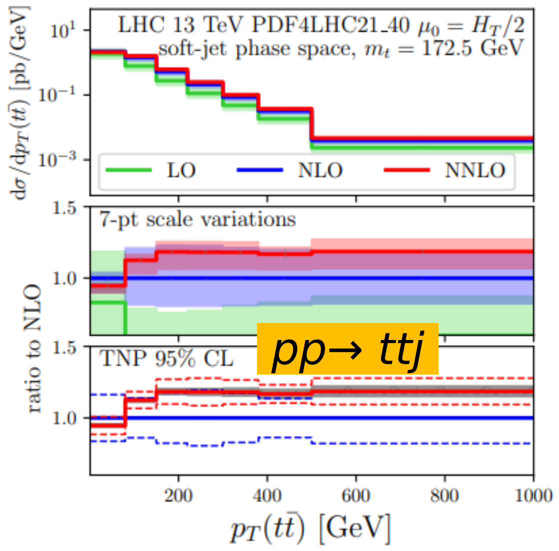
[Badger,C`zakon,**HBH**,Moodie,Peraro,Poncelet,Zoia(2023)]

One of the precision calculation frontiers at the LHC:

NNLO QCD for 2→3 scattering process







[Czakon,Mitov,Poncelet(2021)][Chen,Gehrmann, Glover,Huss,Marcoli(2022)]Alvarez,Cantero, Czakon,Llorente,Mitov,Poncelet(2023)] [Czakon,Poncelet(2025)]

[Badger,Brancaccio,Becchetti,Czakon, **HBH**,Poncelet,Zoia(2025)]

pp→γγγ, pp→γγj,pp→Wbb,
pp→Wγγ, pp→Zbb, pp→Hbb

approximated two-loop amplitudes (massification) for massive b quark calculation

[Chawdhry,Czakon,Mitov,Poncelet(2019)]
[Kallweit,Sotnikov,Wiesemann(2020)]
[Czakon,Mitov,Poncelet(2020)]
[**HBH**,Poncelet,Popescu,Zoia(2022)]
[Buonocore,Devoto,Kallweit,Mazzitelli,Rottoli,Savoini(2022)]
[Mazzitelli,Sotnikov,Wiesemann(2024)]
[Buccioni,Chen,Feng,Gehrmann,Huss,Marcolli(2025)]
[Garbarino,Grazzini,Kallweit,Savoini(2025)]
[Biello,Mazzitelli,Sankar,Wiesemann,Zanderighi(2025)]

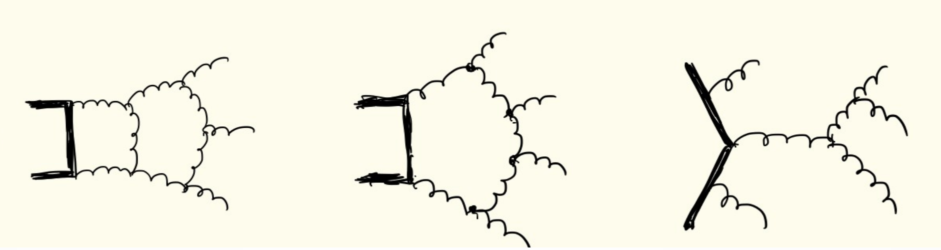
pp→ttW, pp→ttH

approximated two-loop amplitudes (soft approximation and massification) & fully numerical approach

[Buonocore,Devoto,Grazzini,Kallweit,Mazzitelli,Rottoli,Savoini(2022)]
[Catani,Devoto,Grazzini,Kallweit,Mazzitelli,Savoini(2020)][Biello,Savoini, Signorile-Signorile,Wiesemann(2026)][Becchetti,Canko,Chen,Chestnov, Delto,Ditsch,Grazzini, Kallweit,Peraro,Pozzoli,Savoini,Tancredi,Zoia(2026)]

NNLO QCD corrections: ingredients

δNNLO:



Double-Virtual (VV) Real-Virtual (RV) Double-Real (RR)

NNLO subtraction scheme to cancel IR singularities

applied to 2→3 process: STRIPPER, Antenna Subtraction, q_T-subtraction

[Czakon(2010)][Gehrmann,Gehrmann-de Ridder,Glover(2005)][Catani,Grazzini(2007)]

NNLO+PS applied to 2→3 process: MiNNLOPS (massive coloured pair+colourless)

[Monni,Nason,Re,Wiesemann,Zanderighi(2022)]

One-loop amplitudes highly automatized: OpenLoops,Recola,MadLoop,Helac-1loop,etc

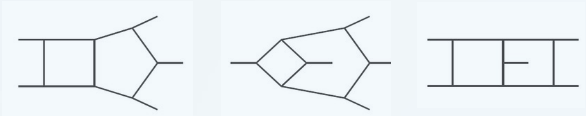
Remaining bottleneck: two-loop five-point amplitudes

$$\text{loop amplitude} = \sum(\text{rational coefficients}) \times (\text{integral/special functions})$$

highly non-trivial for multi-scale process!!

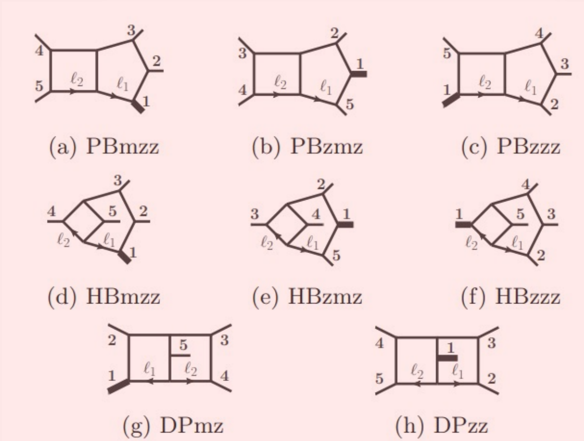
State-of-the art: two-loop five-point integrals

massless



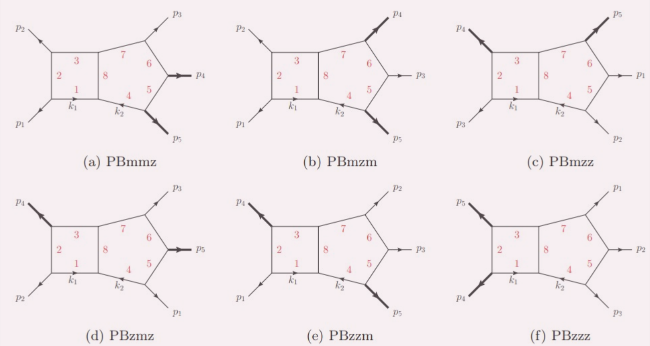
[Abreu,Chicherin,Dixon,Gehrmann,Henn, Herrmann,LoPresti,Papadopoulos,Page, Sotnikov,Tomassini,Wasser,Wever,Zeng, Zhang,Zoia(2015-2020)]

one external mass



[Abreu,Canko,Chicherin,Ita,Kardos,Moriello, Page,Papadopoulos,Smirnov,Sotnikov,Syrrakos, Tomassini,Tschernow,Wever,Zeng,Zoia (2015-2023)]

two external masses (planar)

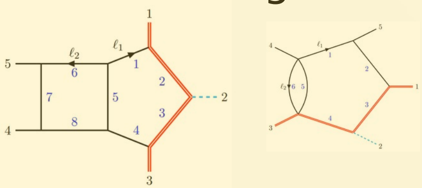


[Jiang,Liu,Xu,Yang(2024)]
[Abreu,Chicherin,Sotnikov,Zoia(2024)]

pentagon functions for massless and one-external mass

[Gehrmann,Henn,Lo Presti(2018)] [Chicherin,Sotnikov(2020)][Chicherin, Sotnikov,Zoia(2021)][Abreu,Chicherin,Ita, Page,Sotnikov,Tschernow,Zoia(2023)]

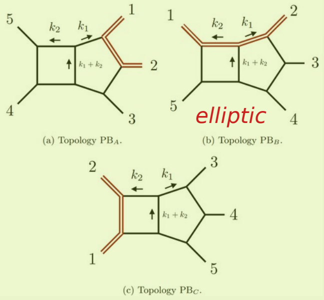
pp→ttH: leading colour, N_f



[Febres Cordero,Figueiredo,Kraus, Page,Reina(2023)]

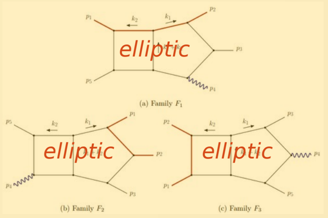
pp→ttj: leading colour

[Badger,Becchetti,Chaubey, Marzucca(2022)]
[Badger,Becchetti,Giraudo, Zoia(2024)]
[Becchetti,Dlapa,Zoia(2025)]



pp→ttW: leading colour

[Becchetti,Canko, Chestnov,Peraro, Pozzoli,Zoia(2025)]



State-of-the art: analytic two-loop five-point amplitudes

massless
all full colour

- $pp \to \gamma\gamma\gamma$ [Abreu,Page,Pascual,Sotnikov(2020)][Chawdhry,Czakon,Mitov,Poncelet(2021)] [Abreu,De Laurentis,Ita,Klinkert,Page,Sotnikov(2023)]
- $pp \to \gamma\gamma j$ [Agarwal,Buccioni,von Manteuffel,Tancredi(2021)][Chawdhry,Czakon,Mitov,Poncelet(2021)] [Badger,Brönnum-Hansen,Chicherin,Gehrmann,**HBH**,Henn,Marcoli,Moodie,Peraro,Zoia(2021)]
- $pp \to \gamma jj$ [Badger,Czakon,**HBH**,Moodie,Peraro,Poncelet,Zoia(2023)]
- $pp \to jjj$ [Abreu,Febres Cordero,Ita,Page,Sotnikov(2021)][De Laurentis,Ita,Klinkert,Sotnikov(2023)] [Agarwal,Buccioni,Devoto,Gambuti,von Manteuffel,Tancredi(2023)][De Laurentis,Ita,Sotnikov(2023)]

1 external mass

- $pp \to Wbb$ (leading colour, massless b) [Badger,**HBH**,Zoia(2021)] [**HBH**,Poncelet,Popescu,Zoia(2022)]
- $pp \to Wjj$ (leading colour) [Abreu,Febres Cordero,Ita,Klinkert,Page,Sotnikov(2022)] [De Laurentis,Ita,Page,Sotnikov(2025)]
- $pp \to Hbb$ (y_b-term, full colour, massless b) [Badger,**HBH**,Krys,Zoia(2021)] [Badger,**HBH**,Poncelet,Wu,Zhang,Zoia(2024)]
- $pp \to W\gamma j$ (leading colour) [Badger,**HBH**,Krys,Zoia(2022)]
- $pp \to W\gamma\gamma$ (analytic l.c., numerical s.l.c.) [Badger,**HBH**,Wu,Zhang,Zoia(2024)]
- $pp \to Hbb$ (y_t-term, leading colour, $m_b=0$, HTL) [**HBH**,Poncelet(2026)]
- $pp \to Hjj$ (leading colour, HTL) [DeLaurentis,Ita,Kuschke,Ruf,Sotnikov(2026)]

State-of-the art: analytic two-loop five-point amplitudes

Internal & external mass

- $pp \to t\bar{t}j$ (leading colour) [Badger,Brancaccio,Becchetti,Czakon,**HBH**,Poncelet,Zoia(2025)]
- $pp \to Wb\bar{b}$ (leading colour, massive b) [Buonocore,Devoto,Kallweit,Mazzitelli,Rottoli,Savoini(2022)]
- $pp \to Zb\bar{b}$ (leading colour, massive b) [Mazzitelli,Sotnikov,Wiesemann(2024)]
- $pp \to Hb\bar{b}$ (leading colour, massive b) [Biello,Mazzitelli,Sankar,Wiesemann,Zanderighi(2025)]
- $pp \to Ht\bar{t}$ (leading colour) [Devoto,Grazzini,Kallweit,Mazzitelli,Savoini(2024)] [Biello,Savoini,Signorile-Signorile,Wiesemann(2026)]
- $pp \to Wt\bar{t}$ (leading colour) [Buonocore,Devoto,Grazzini,Kallweit,Mazzitelli, Rottoli,Savoini(2022)]
- $pp \to Wt\bar{t}$ (leading colour, fully numerical) [Becchetti,Canko,Chen,Chestnov,Delto,Ditsch,Grazzini, Kallweit,Peraro,Pozzoli,Savoini,Tancredi,Zoia(2026)]
  see Jim Canko's talk

State-of-the art: analytic two-loop five-point amplitudes

Internal & external mass

- *pp→ttj* (leading colour) [Badger,Brancaccio,Becchetti,Czakon,**HBH**,Poncelet,Zoia(2025)]
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- *pp→Zbb* (leading colour, massive *b*) [Mazzitelli,Sotnikov,Wiesemann(2024)]
- *pp→Hbb* (leading colour, massive *b*) [Biello,Mazzitelli,Sankar,Wiesemann,Zanderighi(2025)]
- *pp→Htt* (leading colour) [Devoto,Grazzini,Kallweit,Mazzitelli,Savoini(2024)] [Biello,Savoini,Signorile-Signorile,Wiesemann(2026)]
- *pp→Wtt* (leading colour) [Buonocore,Devoto,Grazzini,Kallweit,Mazzitelli, Rottoli,Savoini(2022)]
- *pp→Wtt* (leading colour, fully numerical) [Becchetti,Canko,Chen,Chestnov,Delto,Ditsch,Grazzini, Kallweit,Peraro,Pozzoli,Savoini,Tancredi,Zoia(2026)]
  see Jim Canko's talk

capture leading contributions in m_Q/Q using **massification** procedure [Mitov,Moch(2007)]

$$\mathcal{M}_2^m = \mathcal{M}_2^{m=0} + Z^1_{[q]}\mathcal{M}_1^{m=0} + Z^2_{[q]}\mathcal{M}_0^{m=0}$$

$$Z^l_{[q]} = f\left(\epsilon, \log m_b^2/Q^2\right)$$

power supressed terms neglected

$Z_{[q]}$: universal, perturbative factor, obtained from the ratio of massive to massless $\gamma^* qq$ form factors

Multi-loop scattering amplitudes

Finite remainder → subtract universal UV/IR singularities [Catani][Becher,Neubert][Gardi,Magnea]

$$\mathcal{F}^{(2)} = \mathcal{A}^{(2)} - \mathcal{P}^{(2)}_{\mathrm{UV+IR}} = \sum_i r_i(\{p\})\,\mathrm{mon}_i(f) + \mathcal{O}(\epsilon)$$

⇒ analytic pole cancellation, simplest representation of the amplitude

<u>Challenges</u>:

- **Analytic** → obtaining canonical basis of Feynman master integrals
  → determination of special functions: f_i = ln, Li_n, Elliptic
  → intricate branch cuts
  → efficient evaluation for physical kinematics

- **Algebraic** → expression swell in the intermediate stage
  → high-degree rational functions in r_i for multiscale process

A basis of special functions: *pentagon function* approach

[Gehrmann,Henn,Lo Presti(2018)][Chicherin,Sotnikov(2020)][Chicherin,Sotnikov,Zoia(2021)]
[Abreu,Chicherin,Ita,Page,Sotnikov,Tschernow,Zoia(2023)]

Why? analytic cancellation of UV+IR poles

simplification of finite remainders

efficient numerical evaluation

$$\frac{d\mathrm{MI}_i}{dx} = B_{ij}(x,\epsilon)\mathrm{MI}_j \quad \Rightarrow \quad \boxed{\frac{d\mathrm{MI}'_i}{dx} = \epsilon \sum_\alpha \frac{\tilde{B}^\alpha_{ij}(x,\epsilon)}{x - x_\alpha}\mathrm{MI}'_j} \quad \Rightarrow \quad \mathrm{MI}'_i(x,\epsilon) = \sum_{j=0}^{4}\sum_k d_{jk,i}\,\epsilon^j\,\mathrm{mon}_{jk}\boxed{(f)}$$

"canonical" basis

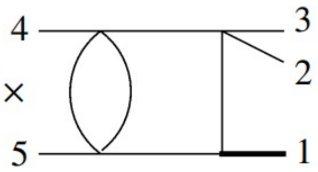
basis of special functions

see Mattia Pozzolli's and Jim Canko's talks

<u>Inputs</u>:

✔ canonical DEs

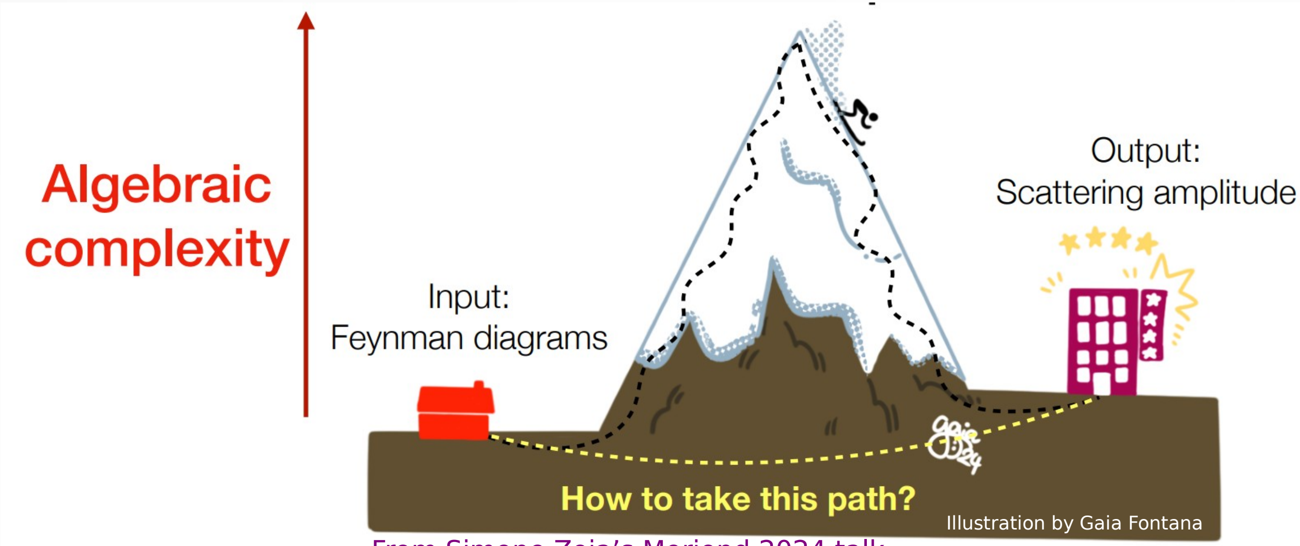
✔ high-precision boundary values of MIs at a random (boundary) point (AMFlow[Liu,Ma(2020)])

$\epsilon^3(1-2\epsilon)\sqrt{\Delta_3^{(1)}}\times$ 

$= \epsilon^2 f^{(2)}_{23} + \epsilon^3\left[\frac{1}{4}\left(f^{(1)}_1 - f^{(1)}_6\right)f^{(2)}_{23} + \frac{1}{2}f^{(3)}_3 - \frac{1}{2}f^{(3)}_{29}\right] + \epsilon^4 f^{(4)}_{47} + \mathcal{O}\left(\epsilon^5\right)$

Credit: Simone Zoia (https://www.physik.uzh.ch/dam/jcr:89cf407b-5202-463a-8985-e1947084478e/zoia_UZH_2024.pdf)

Taming algebraic complexity: analytics from numerics



From Simone Zoia's Moriond 2024 talk

$$A^{(2)}(\{p\},\epsilon) = \sum_i (\text{Feynman diagram})_i$$

$\downarrow$ map loop numerators to $\mathcal{I}$

$$A^{(2),h}(\{p\},\epsilon) = \sum_i c_i^h(\{p\},\epsilon)\, \mathcal{I}_i(\{p\},\epsilon)$$

$\downarrow$ IBP reduction

$$A^{(2),h}(\{p\},\epsilon) = \sum_i d_i^h(\{p\},\epsilon)\, \mathrm{MI}_i(\{p\},\epsilon)$$

$\downarrow$ map to special functions

$\downarrow$ subtract UV/IR poles

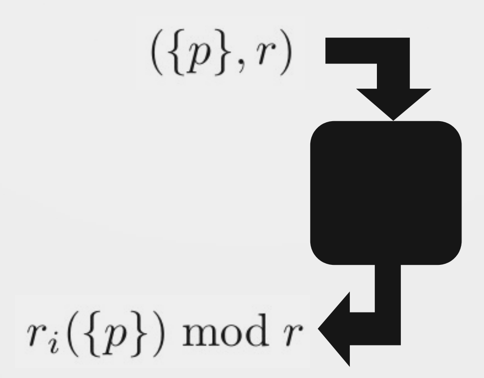
$\downarrow$ ϵ expansion

$$F^{(2),h}(\{p\}) = \sum_i r_i^h(\{p\})\, \mathrm{mon}_i(f) + \mathcal{O}(\epsilon)$$

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f)$$

Replace algebraic operations with numerical evaluation over finite fields

Reconstruct analytic expressions from numerical samples

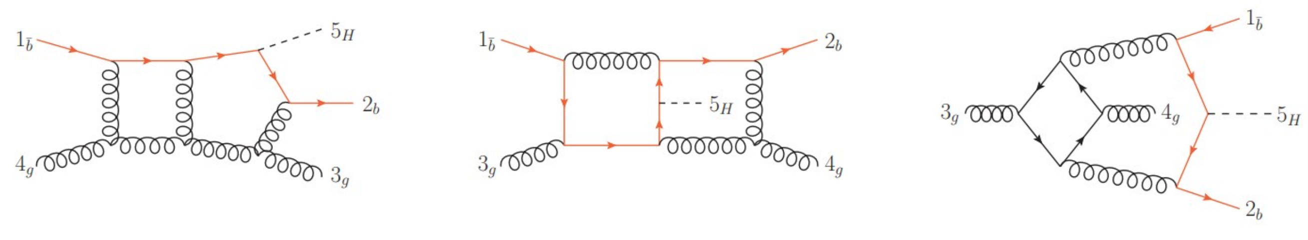


Reconstructed in $\dfrac{\#\text{ points} \times \text{eval. time}}{\#\text{ cores}}$

Two-loop amplitudes for pp→bbH (y_b contribution)

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

m_b=0
$y_b \neq 0$



Colour decomposition in terms of partial amplitudes

$$\mathcal{M}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) = n^L \bar{g}_s^2 \bar{y}_b \Big\{ (t^{a_3}t^{a_4})_{i_2}^{\;\bar{i}_1} \mathcal{A}_{34}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) + (t^{a_4}t^{a_3})_{i_2}^{\;\bar{i}_1} \mathcal{A}_{43}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) + \delta_{i_2}^{\;\bar{i}_1}\delta^{a_3a_4}\mathcal{A}_{\delta}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) \Big\}$$

(N_c,n_f) decomposition

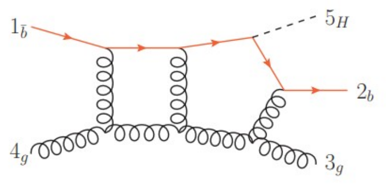
$$\mathcal{A}_{34}^{(2)} = N_c^2 A_{34}^{(2),N_c^2} + A_{34}^{(2),1} + \frac{1}{N_c^2} A_{34}^{(2),1/N_c^2} + N_c n_f A_{34}^{(2),N_c n_f} + \frac{n_f}{N_c} A_{34}^{(2),n_f/N_c} + n_f^2 A_{34}^{(2),n_f^2} \,,$$

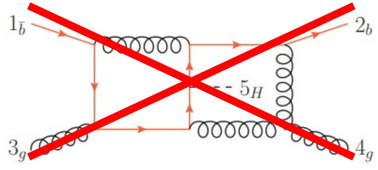
$$\mathcal{A}_{\delta}^{(2)} = N_c A_{\delta}^{(2),N_c} + \frac{1}{N_c} A_{\delta}^{(2),1/N_c} + n_f A_{\delta}^{(2),n_f} + \frac{n_f}{N_c^2} A_{\delta}^{(2),n_f/N_c^2} \,.$$

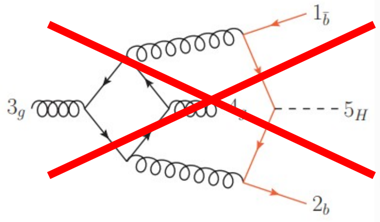
Two-loop amplitudes for pp→bbH (y_b contribution)

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

$m_b=0$
$y_b\neq 0$







Colour decomposition in terms of partial amplitudes

Leading-colour approximation

$$\mathcal{M}^{(L)}(1_{\bar b},2_b,3_g,4_g,5_H) = n^L \bar g_s^2 \bar y_b \Big\{ (t^{a_3}t^{a_4})_{i_2}^{\;\bar i_1} \mathcal{A}_{34}^{(L)}(1_{\bar b},2_b,3_g,4_g,5_H) + (t^{a_4}t^{a_3})_{i_2}^{\;\bar i_1} \mathcal{A}_{43}^{(L)}(1_{\bar b},2_b,3_g,4_g,5_H) + \cancel{\delta_{i_2}^{\;\bar i_1}\delta^{a_3a_4}\mathcal{A}_{\delta}^{(L)}(1_{\bar b},2_b,3_g,4_g,5_H)} \Big\}$$

(N_c,n_f) decomposition

$$\mathcal{A}_{34}^{(2)} = N_c^2 A_{34}^{(2),N_c^2} + \cancel{A_{34}^{(2),1}} + \cancel{\frac{1}{N_c^2}A_{34}^{(2),1/N_c^2}} + N_c n_f A_{34}^{(2),N_c n_f} + \cancel{\frac{n_f}{N_c}A_{34}^{(2),n_f/N_c}} + n_f^2 A_{34}^{(2),n_f^2}\,,$$

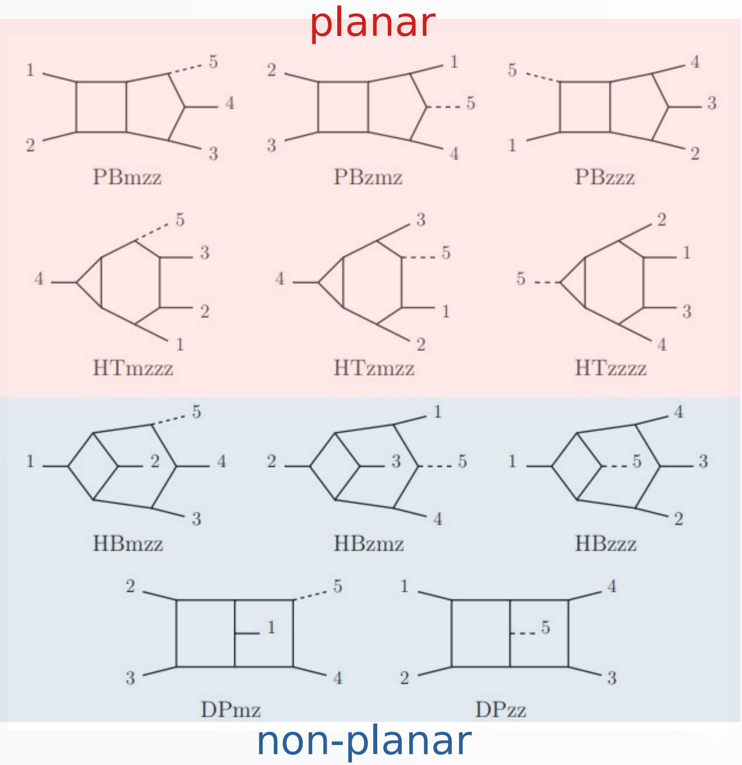
$$\mathcal{A}_{\delta}^{(2)} = \cancel{N_c A_{\delta}^{(2),N_c}} + \cancel{\frac{1}{N_c}A_{\delta}^{(2),1/N_c}} + \cancel{n_f A_{\delta}^{(2),n_f}} + \cancel{\frac{n_f}{N_c^2}A_{\delta}^{(2),n_f/N_c^2}}\,.$$

Two-loop amplitudes for pp→bbH (y_b contribution)

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

IBP reduction to master integrals: solving linear system of eqs for all integral families

Solve IBP systems numerically over FF, permute the solutions, chain to the next stage of computation



	$A_{34}^{(2),N_c^2}$	$A_{34}^{(2),1}$	$A_{34}^{(2),1/N_c^2}$	$A_{\delta}^{(2),N_c}$	$A_{\delta}^{(2),1/N_c}$	NeatIBP file size
# of scalar integrals	20905	147235	83731	215394	204375	
$N_{\mathrm{perm}}(\mathrm{PBmzz})$	2	16	7	8	8	8MB
$N_{\mathrm{perm}}(\mathrm{PBzmz})$	1	7	4	4	3	10MB
$N_{\mathrm{perm}}(\mathrm{PBzzz})$	2	4	7	12	24	5.9MB
$N_{\mathrm{perm}}(\mathrm{HTmzzz})$	2	8	12	24	24	1.9MB
$N_{\mathrm{perm}}(\mathrm{HTzmzz})$	2	14	8	24	20	3.8MB
$N_{\mathrm{perm}}(\mathrm{HTzzzz})$	1	4	4	12	12	4MB
$N_{\mathrm{perm}}(\mathrm{HBmzz})$	0	10	6	10	12	17MB
$N_{\mathrm{perm}}(\mathrm{HBzmz})$	0	5	3	5	5	13MB
$N_{\mathrm{perm}}(\mathrm{HBzzz})$	0	4	2	6	6	38MB
$N_{\mathrm{perm}}(\mathrm{DPmz})$	0	18	8	16	24	71MB
$N_{\mathrm{perm}}(\mathrm{DPzz})$	0	4	1	2	2	376MB

Two-loop amplitudes for pp→bbH (y_b contribution)

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

Analytic reconstruction data

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f)$$

Subleading colour terms:

- more complicated rational coeffs

- slower evaluation time

exploit relations of rational coefficients among different partial amplitudes

	$A^{(2),N_c^2}_{34}$	$A^{(2),1}_{34}$		$A^{(2),N_c}_{\delta}$		$A^{(2),1/N_c}_{\delta}$	
helicity set	$\vec{h}_g$	$\vec{h}'_g$		$\vec{h}_g$		$\vec{h}_g$	
$x_1 = 1$	133/132	176/175	176/175	265/269	265/269	214/207	214/207
ansatz for linear relations	-				$A^{(2),1}_{34}$ $A^{(2),1}$		$A^{(2),1}_{34}$ $A^{(2),1}_{43}$ $A^{(2),1/N_c^2}_{34}$ $A^{(2),1/N_c^2}_{43}$
linear relations	133/132	17					135/130
number of independent coefficients	162	572	216	582	151	581	258
denominator matching	133/0	176/0	110/0	233/0	119/0	141/0	135/0
univariate partial fraction	32/25	38/33	25/22	36/33	36/33	32/28	32/28
factor matching	28/0	38/0	24/0	36/0	36/0	32/0	32/0
number of sample points	16711	47608	10382	38221	37002	21150	20337
evaluation time per point	t_0	$100t_0$	$67t_0$	$106t_0$	$57t_0$	$74t_0$	$62t_0$

N/D: max polynomial degrees in the numerator/denominator

Two-loop amplitudes for pp→bbH (y_b contribution)

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

Analytic reconstruction data

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\,\mathrm{mon}_i(f)$$

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	$A_{34}^{(2),N_c^2}$	$A_{34}^{(2),1}$		$A_\delta^{(2),N_c}$		$A_\delta^{(2),1/N_c}$	
helicity set	$\vec{h}_g$	$\vec{h}'_g$		$\vec{h}_g$		$\vec{h}_g$	
$x_1 = 1$	133/132	176/175	176/175	265/269	265/269	214/207	214/207
ansatz for linear relations	-	-	$A_{34}^{(2),1/N_c^2}$	-	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$ $A_\delta^{(2),1/N_c}$	-	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$
linear relations	133/132	176/175	110/109	233/233	121/115	141/140	135/130
number of independent coefficients	162	572	216	582	151	581	258
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univariate partial fraction	32/25	38/33	25/22	36/33	36/33	32/28	32/28
factor matching	28/0	38/0	24/0	36/0	36/0	32/0	32/0
number of sample points	16711	47608	10382	38221	37002	21150	20337
evaluation time per point	t_0	$100t_0$	$67t_0$	$106t_0$	$57t_0$	$74t_0$	$62t_0$

Two-loop amplitudes for pp→bbH (y_b contribution)

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

Analytic reconstruction data

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f)$$

Subleading colour terms:

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	$A_{34}^{(2),N_c^2}$	$A_{34}^{(2),1}$		$A_{\delta}^{(2),N_c}$		$A_{\delta}^{(2),1/N_c}$	
helicity set	$\vec{h}_g$	$\vec{h}'_g$		$\vec{h}_g$		$\vec{h}_g$	
$x_1 = 1$	$133/132$	$176/175$	$176/175$	$265/269$	$265/269$	$214/207$	$214/207$
ansatz for linear relations	-	-	$A_{34}^{(2),1/N_c^2}$	-	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$ $A_{\delta}^{(2),1/N_c}$	-	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$
linear relations	$133/132$	$176/175$	$110/109$	$233/233$	$121/115$	$141/140$	$135/130$
number of independent coefficients	162	572	216	582	151	581	258
denominator matching	$133/0$	$176/0$	$110/0$	$233/0$	$119/0$	$141/0$	$135/0$
univariate partial fraction	$32/25$	$38/33$	$25/22$	$36/33$	$36/33$	$32/28$	$32/28$
factor matching	$28/0$	$38/0$	$24/0$	$36/0$	$36/0$	$32/0$	$32/0$
number of sample points	16711	47608	10382	38221	37002	21150	20337
evaluation time per point	t_0	$100t_0$	$67t_0$	$106t_0$	$57t_0$	$74t_0$	$62t_0$

Two-loop amplitudes for pp→bbH (y_b contribution)

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

Analytic structure of the full-colour amplitude

Planar scattering

V. Sotnikov, QCD meets Gravity 2023

Only linear or quadratic letters vanish in $\mathcal{P}_{\text{phys}}$, poles always canceled, i.e. $h^{(0)} = 0$

New feature of nonplanar scattering

Square roots of quartic polynomials $\sqrt{\Sigma_5}$ can vanish in $\mathcal{P}_{\text{phys}}$ $\implies$ new types of divergences

1. Integrable square root: $\mathrm{d}\log \frac{a+\sqrt{\Sigma_5}}{a-\sqrt{\Sigma_5}} \xrightarrow{\Sigma_5 \to 0} \frac{\mathrm{d}\Sigma_5}{a\sqrt{\Sigma_5}} \xrightarrow{t \to t^\star} \frac{C}{\sqrt{t-t^\star}} + \ldots$
2. Uncompensated poles: $\mathrm{d}\log \sqrt{\Sigma_5} \xrightarrow{\Sigma_5 \to 0} \frac{\mathrm{d}\Sigma_5}{2\Sigma_5} \xrightarrow{t \to t^\star} \frac{C}{t-t^\star} + \ldots \implies$ log divergence!

see G. Papathanasiou’s talk

- Choose basis functions to localize non-analytic behavior
- $\boxed{\text{Functions with type 2 divergence cancel out in physical results?}}$ YES!! (at the level of bare amplitude) !!! need to check for other processes !!!
- Numerical evaluation more challenging

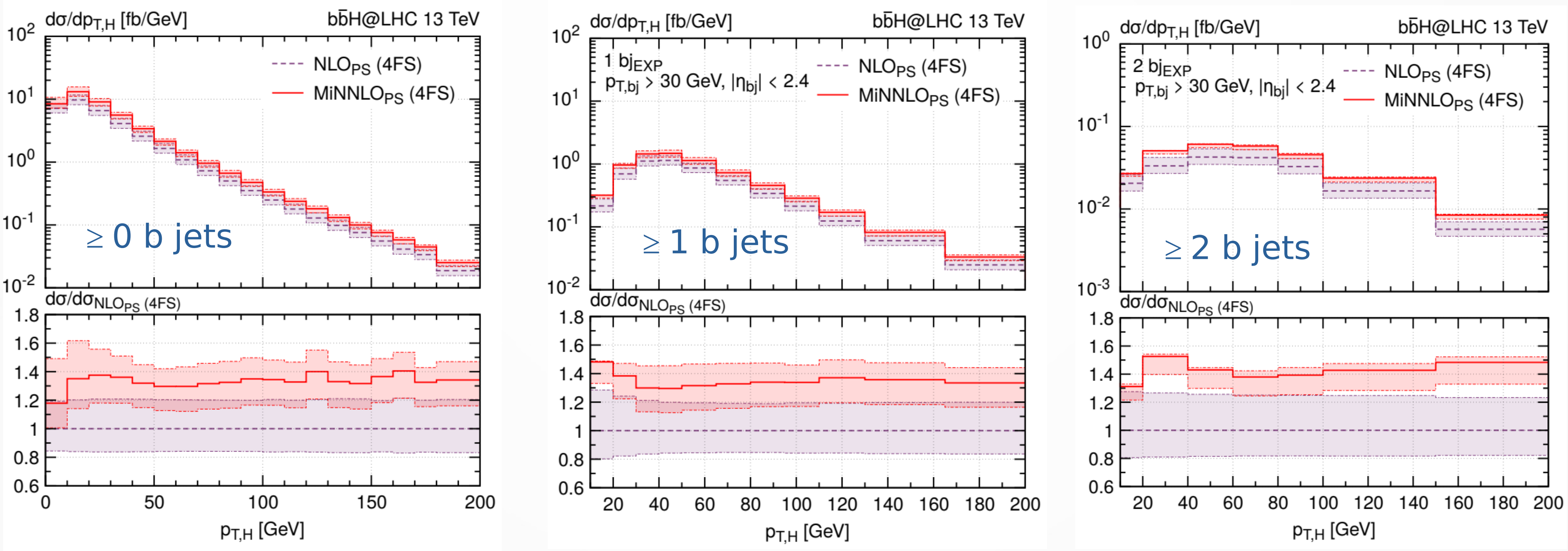
NNLO+PS predictions for pp→bbH (y_b contribution)

[Biello,Mazzitelli,Sankar,Wiesemann,Zanderighi;arXiv:2412.09510]

NNLO calculation with massive b matched to parton shower using MiNNLO+PS

→ our two-loop amplitudes with massless b + massification

Final state signatures: inclusive, at least 1 b jet, at least 2 b jets

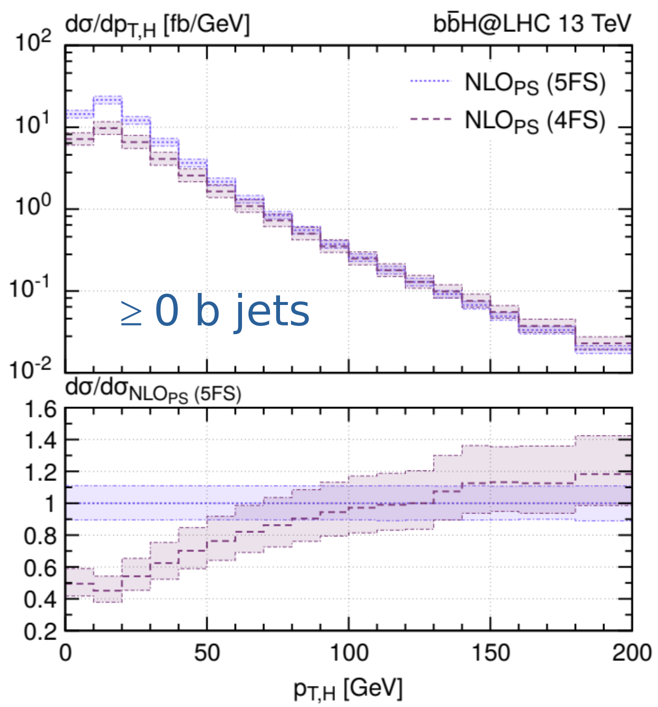


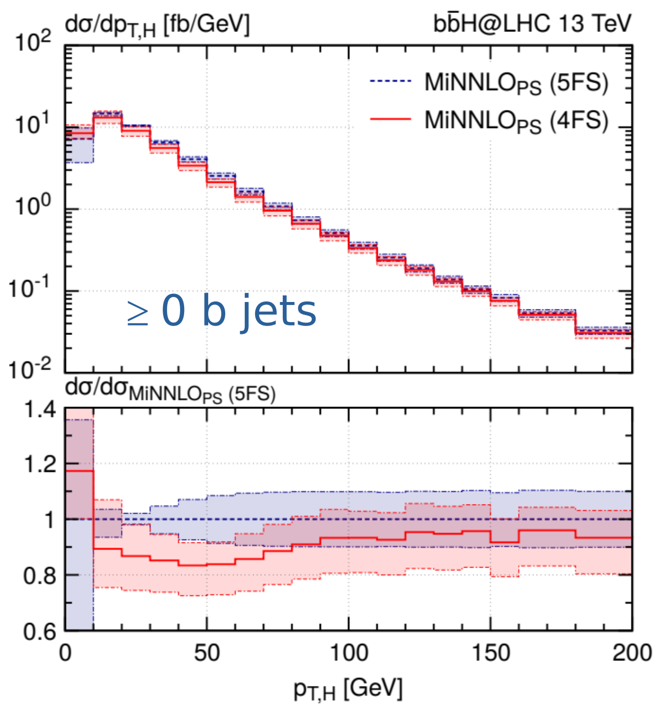
NNLO+PS predictions for pp→bbH (y_b contribution)

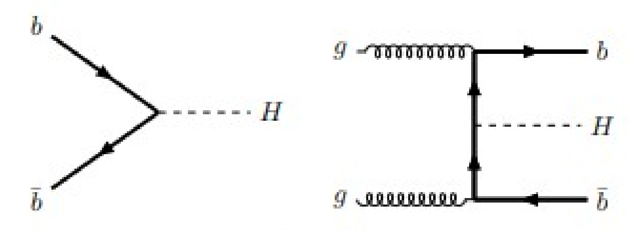
[Biello,Mazzitelli,Sankar,Wiesemann,Zanderighi;arXiv:2412.09510]

5 flavour scheme: typically $Q^2 \gg m_b^2$ → treat bottom quark as massless, active flavour in proton PDF, absorb $\log(Q^2/m_b^2)$ in b-quark PDF

4 flavour scheme: full bottom-quark mass dependence, only appear in the final state







(a) LO y_b diagram in 5FS (b) LO y_b diagram in 4FS

4FS vs 5FS

good agreement at

NNLO QCD

NNLO+PS predictions for pp→bbH (y_b contribution)

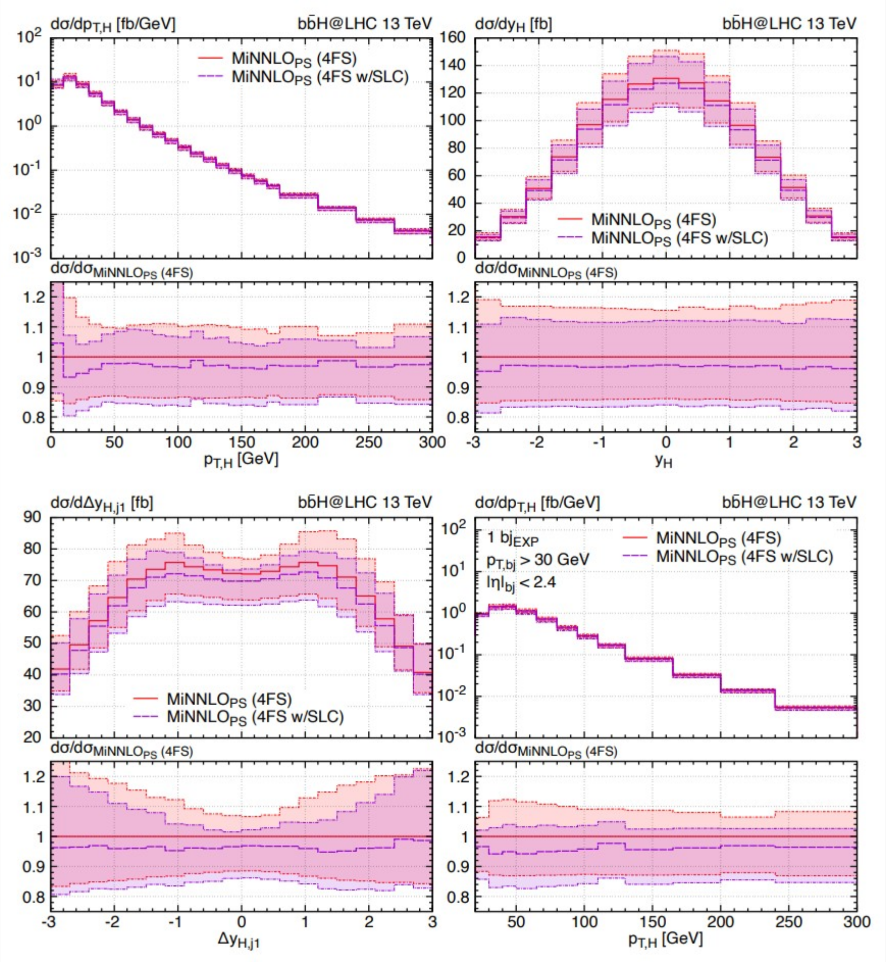
[Biello,Mazzitelli,Sankar,Wiesemann,Zanderighi;arXiv:2412.09510], Christian Biello's Dissertation (2025)

Assessment of two-loop subleading colour effects

size of corrections <5% but

~20x more expensive to evaluate

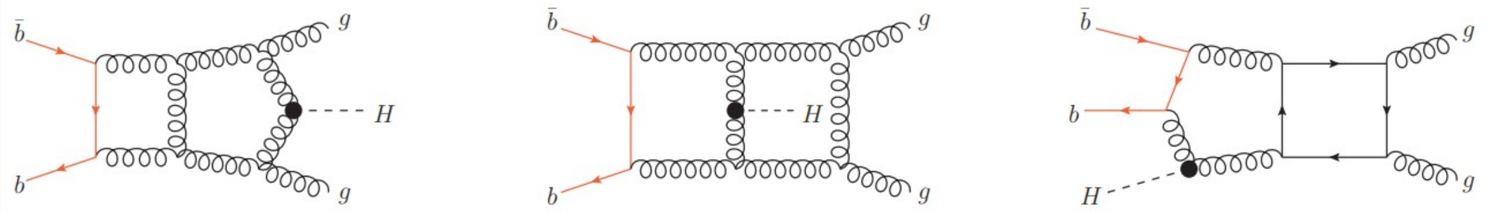
Fiducial region	Precision	$\sigma_{\text{integrated}}$ [fb]	Ratio to MiNNLO$_{\text{PS}}$ wo/SLC
$pp \to H$	MiNNLO$_{\text{PS}}$ wo/SLC	$466.(0)^{+16\%}_{-14\%}$	1.000
	MiNNLO$_{\text{PS}}$ w/SLC	$451.(3)^{+15\%}_{-14\%}$	0.968
$pp \to H + \leq 1\, b$ jets	MiNNLO$_{\text{PS}}$ wo/SLC	$92.(8)^{+9.8\%}_{-12\%}$	1.000
	MiNNLO$_{\text{PS}}$ w/SLC	$88.(6)^{+8.2\%}_{-11\%}$	0.955
$pp \to H + \leq 2\, b$ jets	MiNNLO$_{\text{PS}}$ wo/SLC	$6.5(4)^{+1.8\%}_{-9.2\%}$	1.000
	MiNNLO$_{\text{PS}}$ w/SLC	$6.5(1)^{+3.5\%}_{-9.8\%}$	0.995
$pp \to H + 0\, b$ jets	MiNNLO$_{\text{PS}}$ wo/SLC	$373.(2)^{+22\%}_{-20\%}$	1.000
	MiNNLO$_{\text{PS}}$ w/SLC	$362.(7)^{+21\%}_{-19\%}$	0.972



Two-loop amplitudes for pp→bbH (y_t)

[**HBH**,Poncelet;arXiv:2603.29480]

$m_b=0$



Colour decomposition in terms of partial amplitudes (exactly the same as y_t amplitude)

$$\mathcal{M}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) = \bar{n}^L \bar{g}_s^2 \bar{C}_1 \Big\{ (t^{a_3}t^{a_4})_{i_2}^{\;\bar{i}_1} \mathcal{A}_{34}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) + (t^{a_4}t^{a_3})_{i_2}^{\;\bar{i}_1} \mathcal{A}_{43}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) + \delta_{i_2}^{\;\bar{i}_1}\delta^{a_3a_4}\mathcal{A}_{\delta}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) \Big\}$$

(N_c,n_f) decomposition (exactly the same as y_b amplitude)

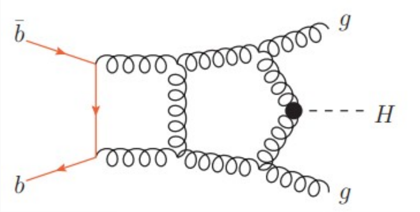
$$\mathcal{A}_{34}^{(2)} = N_c^2 A_{34}^{(2),N_c^2} + A_{34}^{(2),1} + \frac{1}{N_c^2} A_{34}^{(2),1/N_c^2} + N_c n_f A_{34}^{(2),N_c n_f} + \frac{n_f}{N_c} A_{34}^{(2),n_f/N_c} + n_f^2 A_{34}^{(2),n_f^2},$$

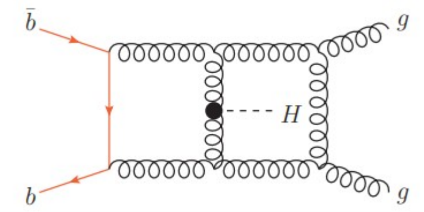
$$\mathcal{A}_{\delta}^{(2)} = N_c A_{\delta}^{(2),N_c} + \frac{1}{N_c} A_{\delta}^{(2),1/N_c} + n_f A_{\delta}^{(2),n_f} + \frac{n_f}{N_c^2} A_{\delta}^{(2),n_f/N_c^2}.$$

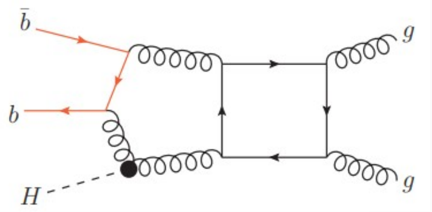
Leading colour two-loop amplitudes for pp→bbH (y_t)

[**HBH**,Poncelet;arXiv:2603.29480]

$m_b=0$







Colour decomposition in terms of partial amplitudes (exactly the same as y_b amplitude)

$$\mathcal{M}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) = \bar{n}^L \bar{g}_s^2 \bar{C}_1 \Big\{ (t^{a_3}t^{a_4})_{i_2}^{\bar{i}_1} \mathcal{A}_{34}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) + (t^{a_4}t^{a_3})_{i_2}^{\bar{i}_1} \mathcal{A}_{43}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H) + \cancel{\delta_{i_2}^{\bar{i}_1}\delta^{a_3a_4}\mathcal{A}_{\delta}^{(L)}(1_{\bar{b}},2_b,3_g,4_g,5_H)} \Big\}$$

(N_c,n_f) decomposition (exactly the same as y_b amplitude)

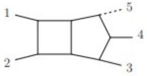
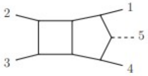
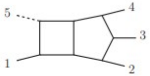
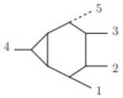
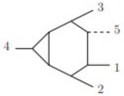
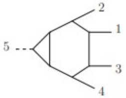
$$\mathcal{A}_{34}^{(2)} = N_c^2 A_{34}^{(2),N_c^2} + \cancel{A_{34}^{(2),1}} + \cancel{\frac{1}{N_c^2}A_{34}^{(2),1/N_c^2}} + N_c n_f A_{34}^{(2),N_c n_f} + \cancel{\frac{n_f}{N_c}A_{34}^{(2),n_f/N_c}} + n_f^2 A_{34}^{(2),n_f^2},$$

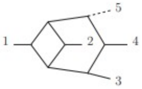
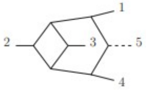
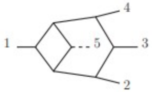
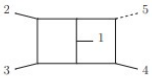
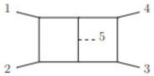
$$\mathcal{A}_{\delta}^{(2)} = \cancel{N_c A_{\delta}^{(2),N_c}} + \cancel{\frac{1}{N_c}A_{\delta}^{(2),1/N_c}} + \cancel{n_f A_{\delta}^{(2),n_f}} + \cancel{\frac{n_f}{N_c^2}A_{\delta}^{(2),n_f/N_c^2}}.$$

Leading colour two-loop amplitudes for pp→bbH (y_t)

[**HBH**,Poncelet;arXiv:2603.29480]

IBP reduction to master integrals

			$\mathcal{A}^{(2),N_c^2}_{0\to\bar{b}bggH}$	$\mathcal{A}^{(2),N_c n_f}_{0\to\bar{b}bggH}$	$\mathcal{A}^{(2),N_c^2}_{0\to\bar{b}b\bar{q}qH}$
# of scalar integrals			139180	62337	54042
PBmzz		N_{perms}	6	4	4
PBzmz		N_{perms}	3	1	2
PBzzz		N_{perms}	6	6	4
HTmzz		N_{perms}	6	6	4
HTzmz		N_{perms}	6	6	4
HTzzz		N_{perms}	4	3	4

			$\mathcal{A}^{(2),N_c^2}_{0\to\bar{b}bggH}$	$\mathcal{A}^{(2),N_c n_f}_{0\to\bar{b}bggH}$	$\mathcal{A}^{(2),N_c^2}_{0\to\bar{b}b\bar{q}qH}$
HBmzz		N_{perms}	-	-	-
HBzmz		N_{perms}	-	-	-
HBzzz		N_{perms}	4	4	4
DPmz		N_{perms}	-	-	-
DPzz		N_{perms}	2	1	2

Leading colour two-loop amplitudes for pp→bbH (y_t)

[**HBH**,Poncelet;arXiv:2603.29480]

Analytic reconstruction data

numerator degrees already quite high at leading colour

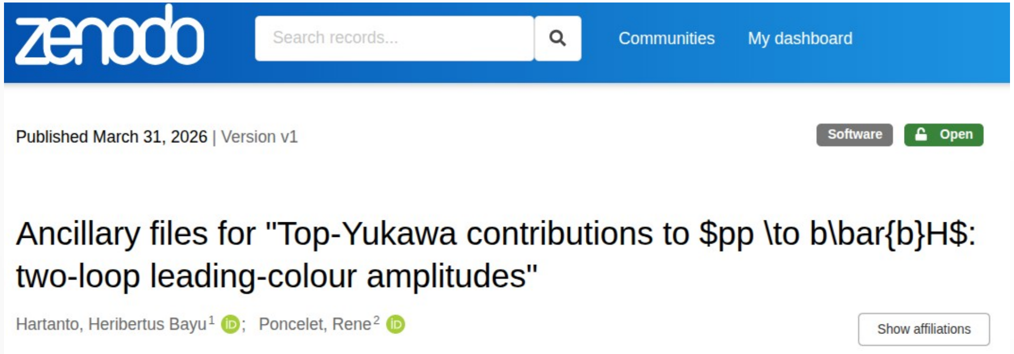
reconstructing subleading colour contributions will be significantly more challenging

	$\mathcal{A}^{(2),N_c^2}_{0\to\bar{b}bggH}$	$\mathcal{A}^{(2),N_c n_f}_{0\to\bar{b}bggH}$	$\mathcal{A}^{(2),N_c^2}_{0\to\bar{b}b\bar{q}qH}$
$x_1 = 1$	256/251	166/161	145/140
linear relations	187/183	157/153	118/114
denominator matching	187/0	157/0	118/0
univariate partial fraction in x_5	53/52	50/49	41/38
factor matching	53/0	50/0	41/0
number of sample points	157798	122037	60482
required prime fields	3	2	3
evaluation time per point	2050s	190s	841s

Leading colour two-loop amplitudes for pp→bbH (y_t)

[**HBH**,Poncelet;arXiv:2603.29480]

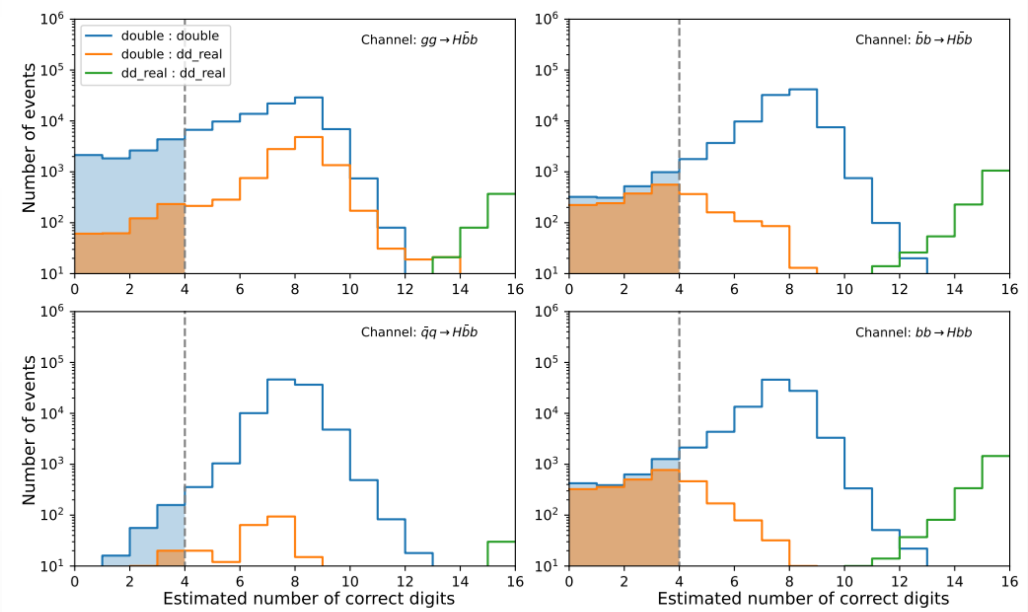
C++ implementation to compute
two-loop hard functions
(interfaced to PentagonFunctions++)



Ready for pheno application!!

Also available as pp→Hjj subprocesses
from [DeLaurentis,Ita,Kuschke,Ruf,Sotnikov(2026)]

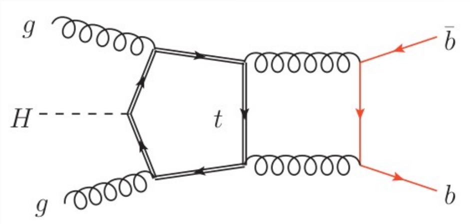
Rescue system: evaluate hard functions
at a rescaled point,
switch to higher precision if necessary



Summary

- Two-loop amplitude for pp→bbH: y_b (massless b, full colour)

  y_t (massless b, heavy-top-limit, leading colour)
- Enabled NNLO+PS bbH (y_b) computation (massive b via massification)
- y_t^2 bbH at NNLO+PS in HEFT is foreseen
- Beyond HEFT?



Analytic approach will be extremely challenging
→ numerical or semi-numerical approach

see C. Papadopoulos', A. Spourdalakis'
and D. Canko's talks

Summary

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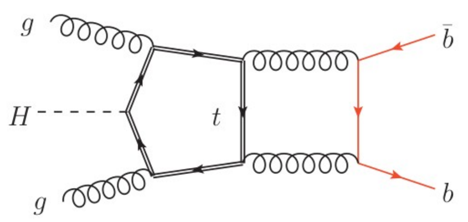

Analytic approach will be extremely challenging
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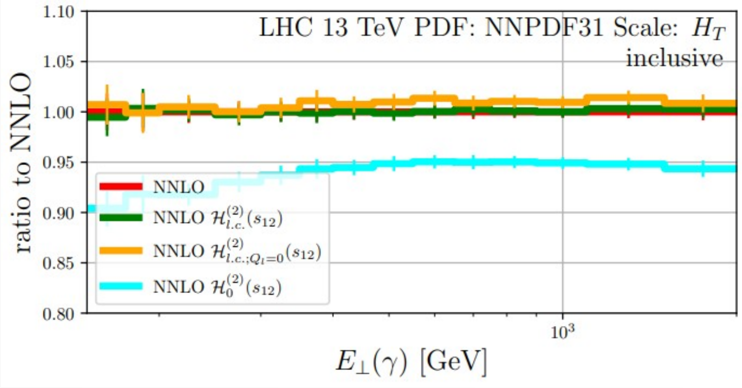
Thank you for your attention!!

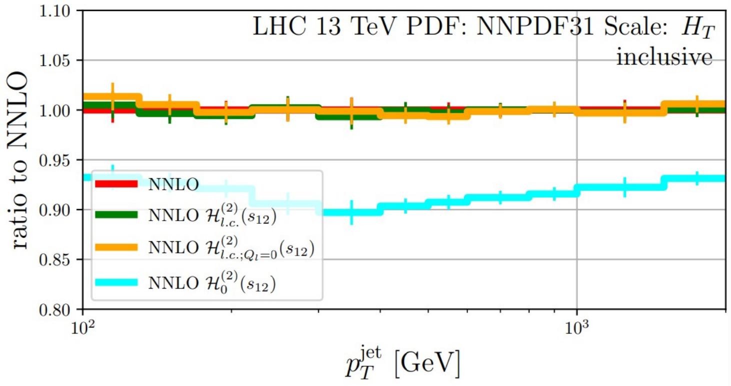
Back-up slides

Subleading colour contribution

pp→γjj

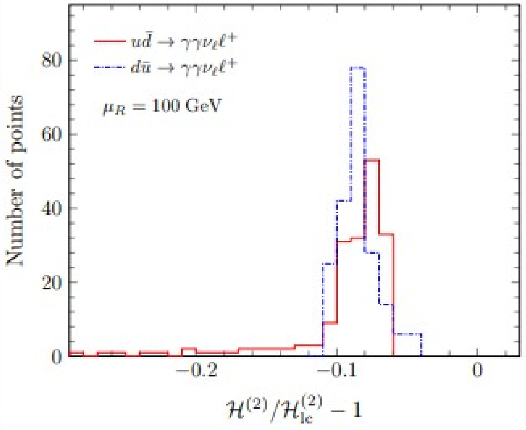
[Badger,Czakon,**HBH**,Moodie,Peraro,Poncelet,Zoia(2023)]

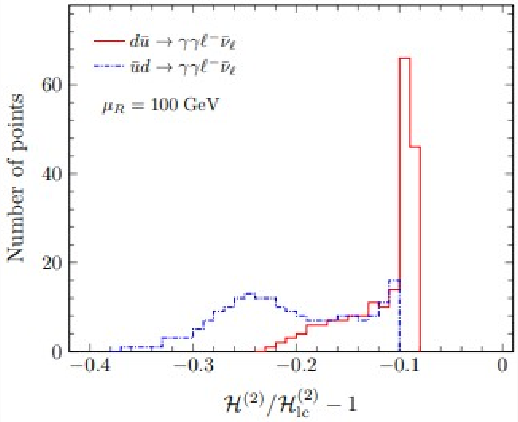




pp→Wγγ

[Badger,**HBH**,Wu,Zhang,Zoia(2024)]





Optimising analytic reconstruction

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f)$$

need to reduce the complexity of the rational coefficients!!

- set one of the kinematic variables to one ($s_{12} = 1$ or $x_1 = 1$)
- find linear relations among coefficients and reconstruct the simpler ones

$$\sum_i y_i r_i = 0,$$ we can supply ansatz → $$\sum_i y_i r_i + \sum_j \tilde{y}_j \tilde{r}_j = 0,$$

- guess the denominators → from letters
- on-the-fly univariate partial fraction → one fewer variable, lower degrees

$$f(x,y) = \frac{y^4 + 13xy^2 + x^2}{(y-x)(y+x)^2}, \quad \text{ansatz} \Rightarrow f(x,y) = \frac{u_{110}(x)}{y-x} + \frac{u_{210}(x)}{y+x} + \frac{u_{220}(x)}{(y+x)^2} + r(x) + v_1(x)y$$

- factor matching: letters, spinor products/strings
- reconstructed expressions can be further simplifed: *MultivariateApart, pfd-parallel*