Adaptive transitions in networks of coupled nonlinear oscillators

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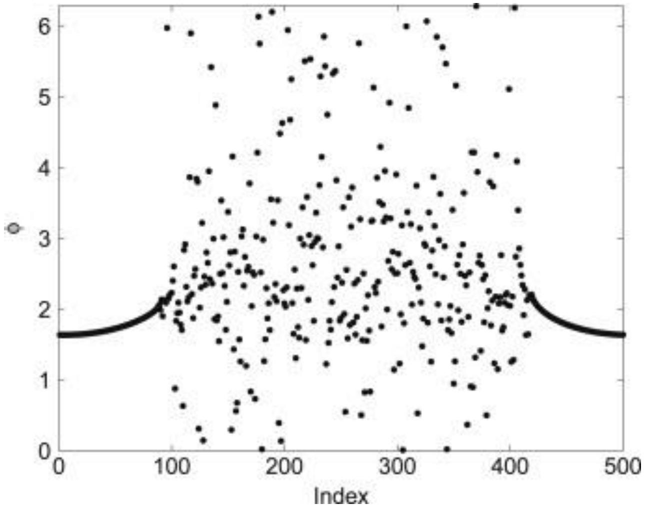
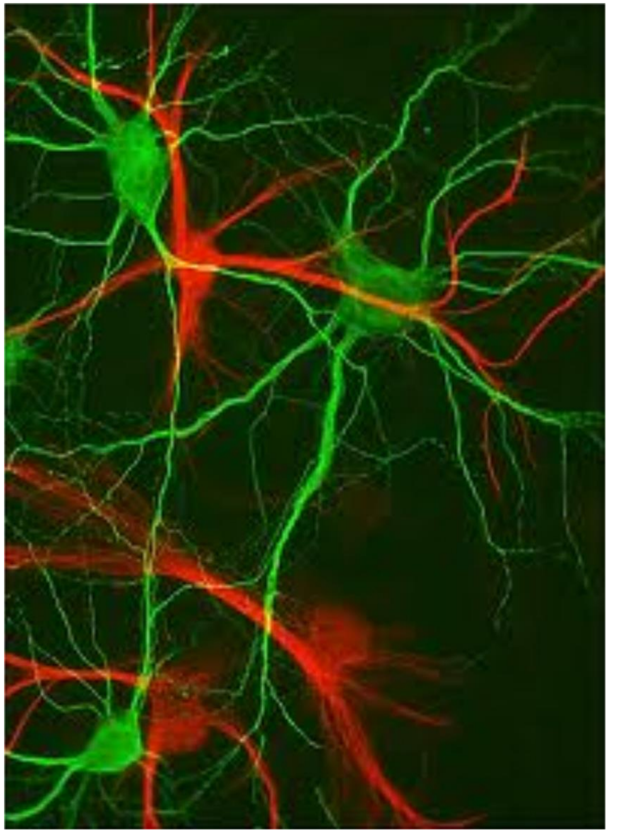


3rd INPP(Greece)-APCTP (Korea)Meeting
June 29th- July 3rd, 2026

Overview:



1. Introduction & Motivation
   - Temporal and Adaptive Networks
   - Simulations: coupled neural dynamics & synchronization phenomena
2. The Leaky Integrate-and-Fire (LIF) network
3. Chimera states as collective dynamical states
4. Effects of adaptive connectivity (Hebb-Oja)
5. The FitzHugh-Nagumo (FHN) network
6. Adaptive connectivity in FHN networks
7. Conclusions & Open Problems

-2002: Y. Kuramoto and D. Battogtokh, Nonlinear Phenomena in Complex Systems 5:380

-2004: D. M. Abrams and S. H. Strogatz, Phys. Rev. Lett., 93:174102

-2015: M. J. Panaggio and D. M. Abrams, Nonlinearity, 28:R67 (review)

-2016: E. Schöll, EPJ-ST, 225:891 (review)

-2018: O. E. Omel'chenko, Nonlinearity, 31:R121–R164 (review)

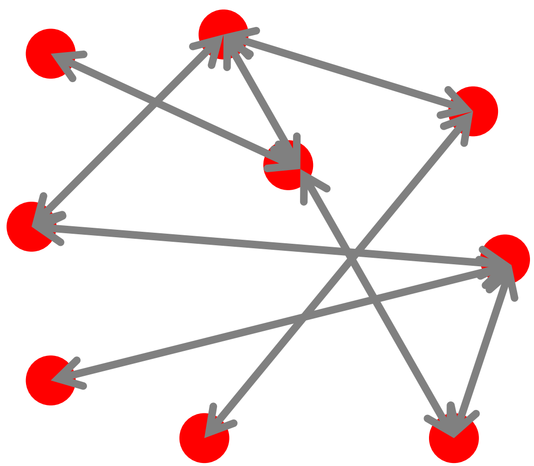
-2020: A. Zakharova, Chimera patterns in Networks, Springer, Berlin (book)

-2021: C. R. Laing, Chaos, 31:113116

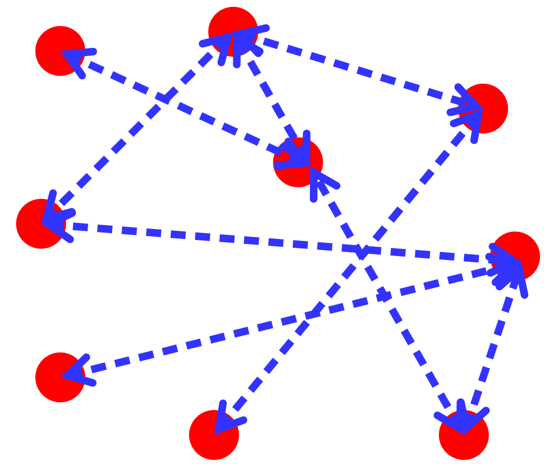
-2024: O. E. Omel'chenko, C. R. Laing, PRE, 110:034411

1. Introduction and Motivation

1.1 Modern approaches in network science turn attention to time dependent (temporal) and adaptive networks



Static: $\boldsymbol{\sigma_{ij}} = \text{constant}$



Temporal: $\boldsymbol{\sigma_{ij}(t)}$

$$\frac{du_i(t)}{dt} = \boldsymbol{f}(\boldsymbol{u_i(t)}) + \sum_{j=1}^{N} \boldsymbol{\sigma_{ij}}[\boldsymbol{u_j(t)} - \boldsymbol{u_i(t)}] + \cancel{\text{delay terms}}$$

$$\frac{d\sigma_{ij}}{dt} = \boldsymbol{g}\left(\boldsymbol{u_i(t)}, \boldsymbol{u_j(t)}, \boldsymbol{\sigma_{ij}(t)}\right) + \text{other factors}$$

1.2 Examples from brain dynamics and AI

- In brain dynamics, the strength of the synapses may change dynamically reaching any of many possible final states, depending on factors such as local potentials, local connectivity of neighboring cells, internal dynamics, external factors (medication) …

- Short-term plasticity: changes of σ_{ij} in millisecs-secs (recorded via fMRI, MEG, EEG techniques)

- Long-term plasticity (long-term potentiation & long-term depression) : changes of σ_{ij} in min-hours (recorded via 3D DTI-MRI techniques)

- Structural plasticity: changes of σ_{ij} in days-weeks+, changing, remodelling or even breaking of synapses (recorded via 3D DTI-MRI techniques)

&

- AI: The various AI models learn through adaptation of weights in different layers

- 2012: Kreutz M.R. and Sala C. (Eds), Synaptic Plasticity: Dynamics, Development and Disease, Springer.
- 2008: Morrison A., Diesmann M. and Gestner W., Biological Cybernetics 98, 458.
- 2008: Citri A. and Malenka R.C., Neuropsychopharamcology, 33, 18.
- 2004: Lamprecht R and LeDoux J., Nature Review Neuroscience 5, 45.
- 2012: Caroni P., Donato F. and Muller D., Nature Review Neuroscience 13, 478.

1.3 Learning...

- Adjustments of weights mediated by
  *local potential environment
  *local synaptic environment (average)
  *external factors

$\Rightarrow$ **In Biology**: in a “learning process”, the synaptic parameters (weights) are modified to optimize the organism’s ability to perform and survive.

$\Rightarrow$ **In AI:** optimize the “program” learning ability to recognize, categorize or predict

It all started in 1949 with the Hebbian learning rule: Fire together wire together principle (early introduction of adaptive weights)

=> Such ideas are currently borrowed and used in current Machine Learning and Artificial Intelligence [10,11]

- 1982: Hopfield J.J., Proceedings National Academy USA, 79, 2554 .
- 2019: Carleo G., et al., Reviews of Modern Physics 91, 045002.

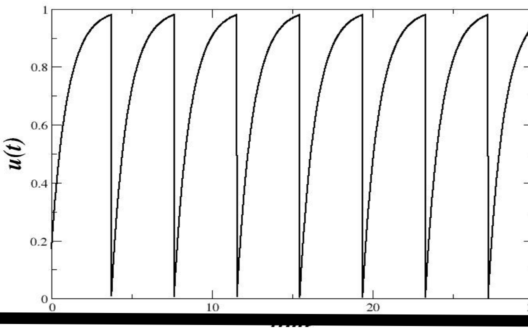
Here: Use adaptivity mediated by the local potential environment, following the Hebb-Oja learning rule (see later).

1.4 Nodal Dynamics: Typical nonlinear !neuron! oscillators

Leaky Integrate-and-Fire Model (Louis Lapicque, 1907)

$$\frac{du(t)}{dt} = \mu - u(t) + I(t)$$
$$u(t) \to 0 \quad \text{when} \quad u(t) > u_{th}$$

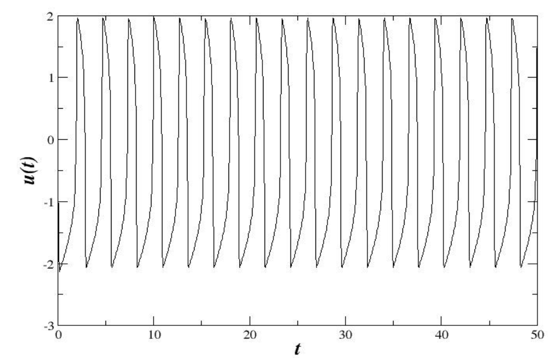
2 parameters



FitzHugh-Nagumo Model (1961-1962)

$$\epsilon\frac{du(t)}{dt} = u(t) - \frac{u^3(t)}{3} - v(t) +$$
$$\frac{dv(t)}{dt} = u(t) + \gamma$$

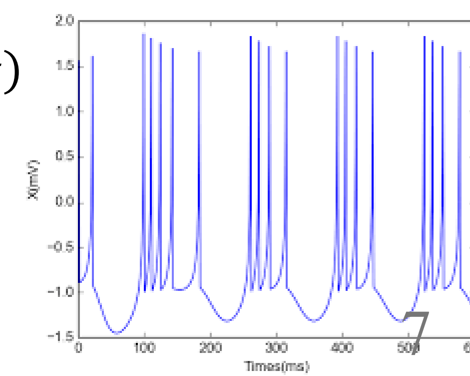
2 parameters



Hindmarsh-Rose Model (1984)

$$\frac{du(t)}{dt} = -au^3 + bu^2 + v - w + I(t)$$
$$\frac{dv(t)}{dt} = -du^2 - v + c$$
$$\frac{dw(t)}{dt} = r[s(u - u^R) - w]$$

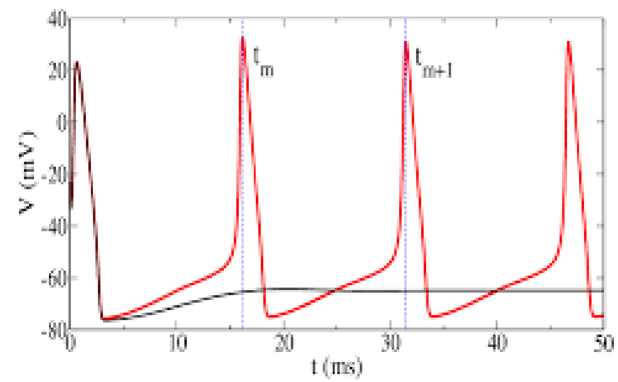
7 parameters



Typical models of nonlinear !neuron! oscillators

Hodgkin-Huxley Model
(1952)

5 equations
16 parameters



Morris-Lecar Model
(1981)

Theta Model
(1986)

Izhikevich Model
(2003)

….

- 1999: L.F. Abbott, "Lapique's ... (1907)". Brain Research Bulletin. **50,** 303–304
- 1952: A.L. Hodgkin, A.F. Huxley AF, The Journal of Physiology. **117**, 500–44.
- 1961: R. FitzHugh, Biophysical Journal **1,** 445-466.
- 1984: J. L. Hindmarsh, R. M. Rose, Proceedings of the Royal Society of London. Series B. Biological Sciences. **221,** 87–102.

2.0 The Leaky Integrate-and-Fire model (L. Lapique, 1907+)

[propagation of electrical signals in neurons, simple, popular in computational neuroscience]

$$\frac{du(t)}{dt} = \mu - u(t)$$

$$u(t) \to u_{\text{rest}}, \qquad \text{when } u(t) > u_{\text{th}}$$

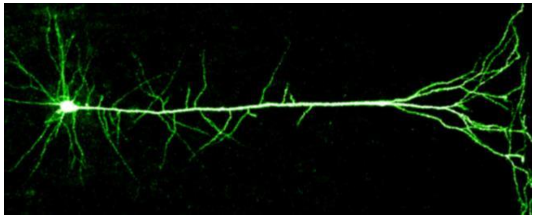
$\mathbf{u_{th}}$ is the threshold potential

$\mathbf{u_{rest}}$ is the rest state potential

$\boldsymbol{\mu}$ = parameter

$\mathbf{T_s}$ is the period of motion

$\mathbf{f_s}$ is the firing rate

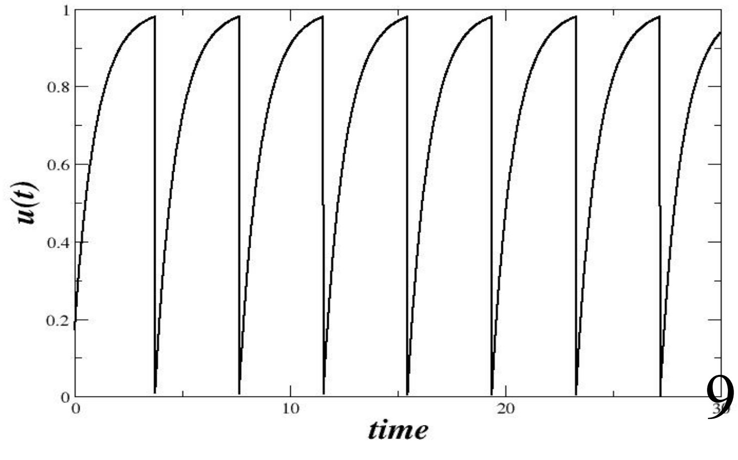


Solution

$$u(t) = \mu - (\mu - u_{\text{rest}})e^{-t} \quad \text{for} \quad u_{\text{rest}} < u(t) < u_{\text{th}}$$

$$\text{When } u(t) = u_{\text{th}}, \text{ then } t = T_s \text{ as :}$$

$$T_s = \ln\left[\frac{\mu - u_{\text{rest}}}{\mu - u_{\text{th}}}\right] \quad \text{and} \quad f_s = 1/T_s$$

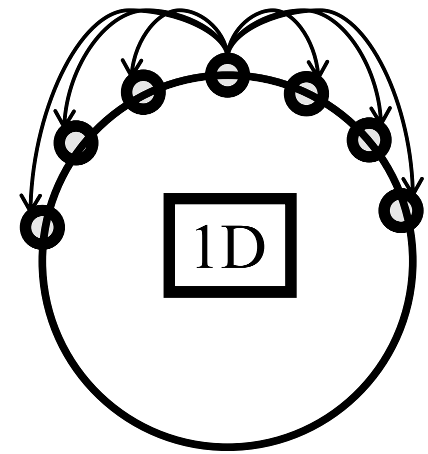


2.1 Bump states and chimera states as collective phenomena

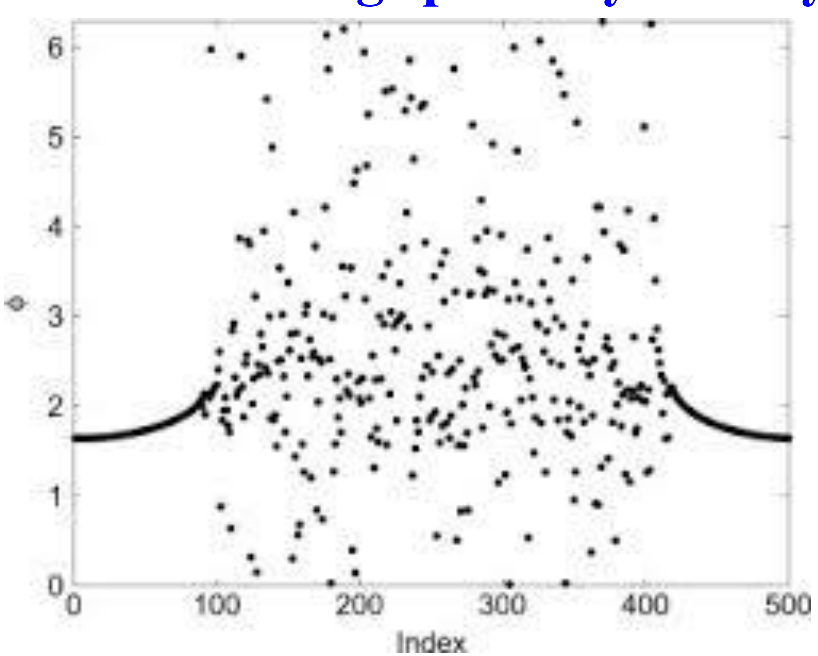
* Systems of coupled non-linear oscillators

* Use simplest nonlocal coupling in 1D ring

* Use simplest case with all elements identical
& all links identical



=> interesting spatial symmetry breaking takes place



-2009: C. R. Laing,
Physica D: Nonlinear Phenomena,
<u>238</u>, 1569-1588.

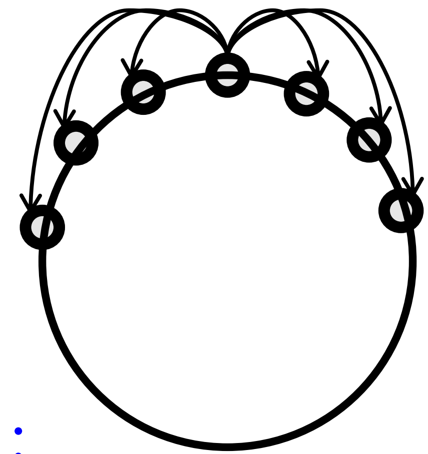
-2002: Y. Kuramoto and D. Battogtokh,
Nonlinear Phenomena in Complex Systems
<u>5</u>, 380

- 2004: D. M. Abrams and S. H. Strogatz,
Phys. Rev. Lett., <u>93</u>, 174102

3.0 Nonlocally coupled LIF oscillators in 1D (ring)

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} \left[u_j(t) - u_i(t)\right]$$

$$u_i(t) \to u_{\text{rest}} \quad \text{when} \quad u_i(t) > u_{\text{th}}$$



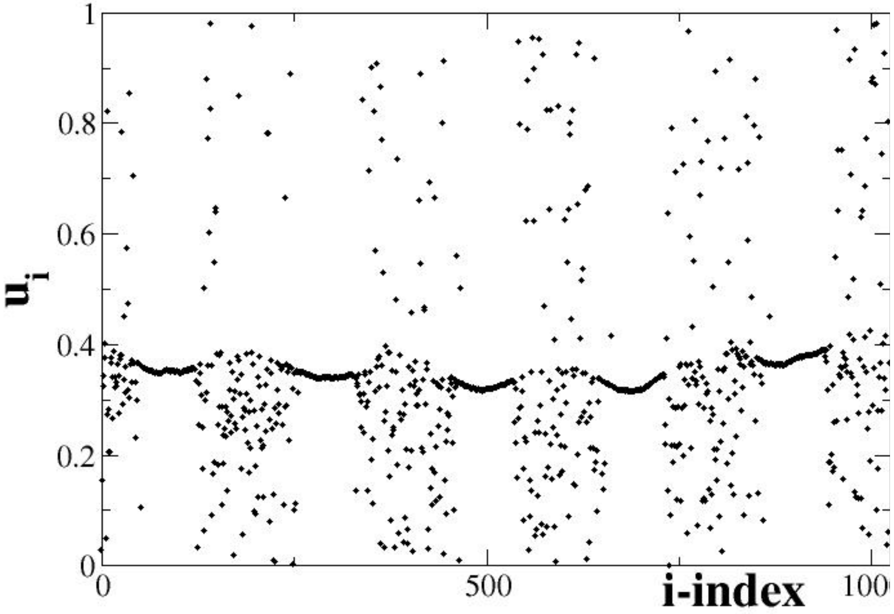
Note: In case R=1 the equation reduces to :

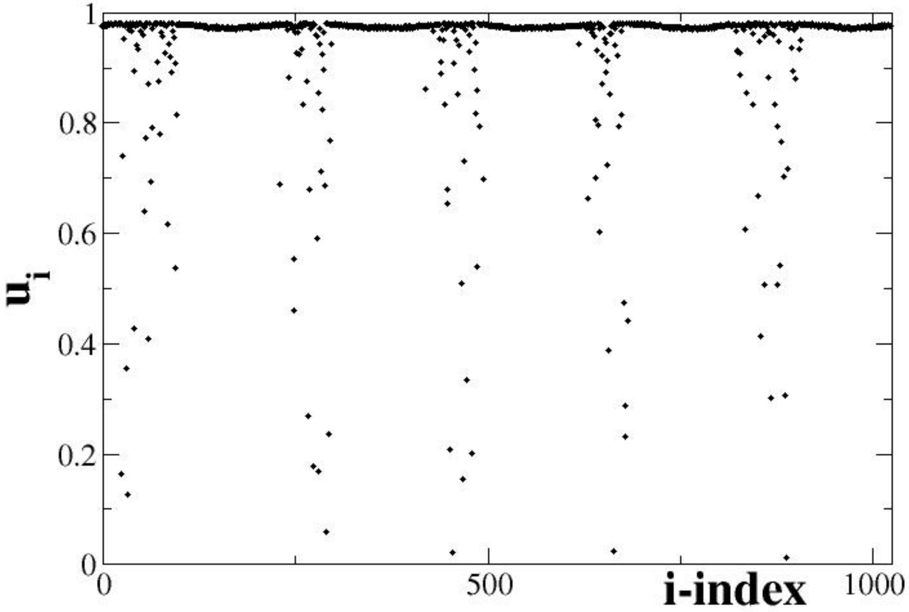
$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{\sigma}{2}\left[u_{i+1}(t) - 2u_i(t) + u_{i-1}(t)\right] \qquad \text{(discrete space)}$$

$$\frac{du(\mathsf{x},t)}{dt} = \mu - u(\mathsf{x},t) + D\nabla^2 u(\mathsf{x},t) \qquad \text{Reaction−Diffusion Equation}$$

- 2010: S. Olmi, A. Politi. A. Torcini, Europhysics Letts, 92, 60007
- 2017: N. Tsigkri-Desmedt, J. Hizanidis, E. Schoell, P. Hoevel, AP, EPJB, 90, 139
- 2018: A. Politi, E. Ullner, A. Torcini, EPJST, 227, 1185-1204 (2018)

3.1 Chimera states vs bump states





Chimera States **Bump States**

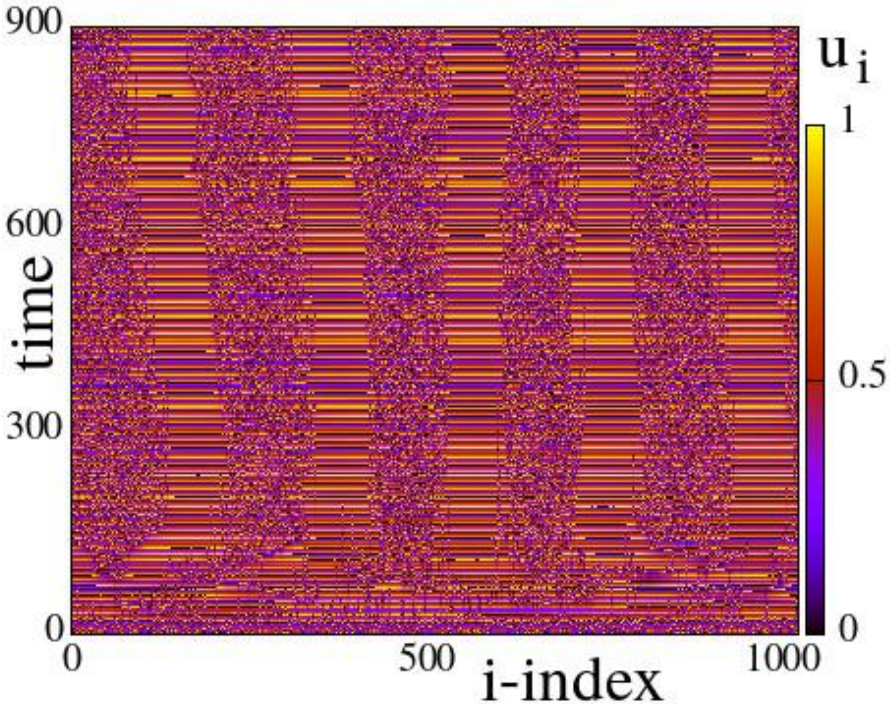
μ=1, u_{th}=0.98, u_{rest}=0

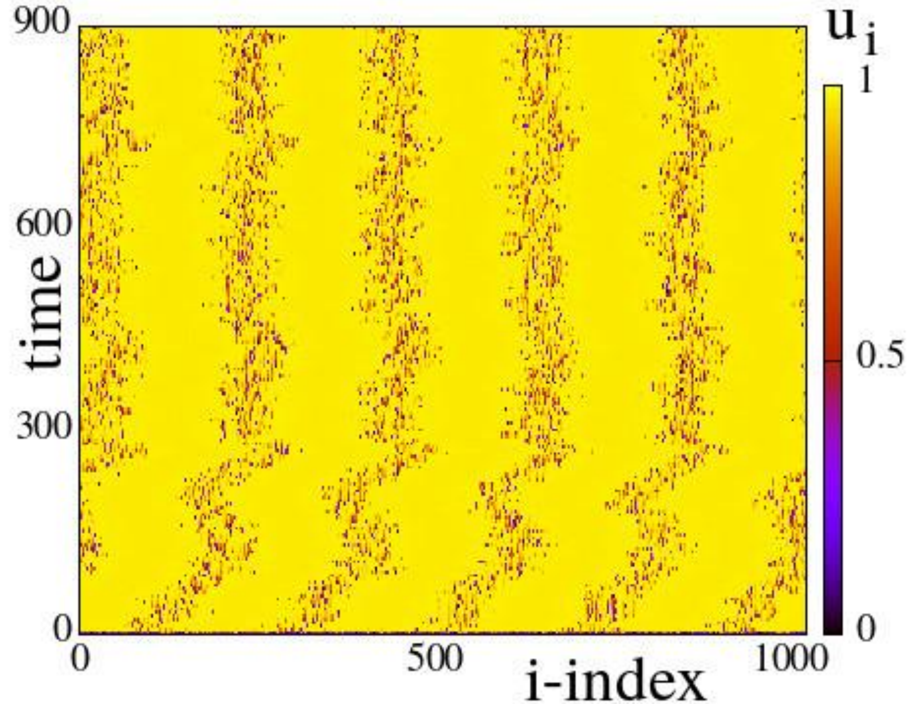
σ = -1.6 **σ = 0.7**

Typical snapshots in LIF netwotk

-2018: N. Tsigkri-Desmedt, I. Koulierakis, G. Karakos, AP, EPJB, 91, 305.

Chimera states vs bump states





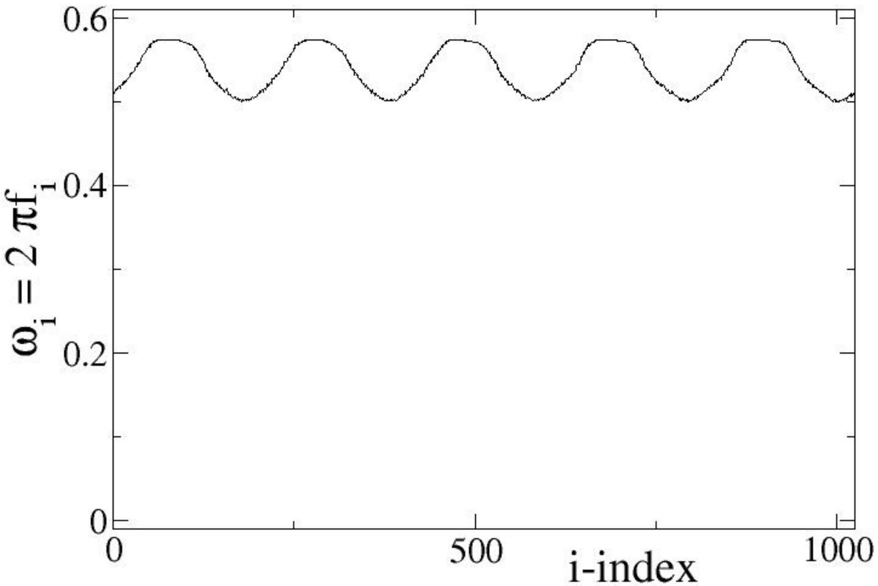
Chimera States **Bump States**

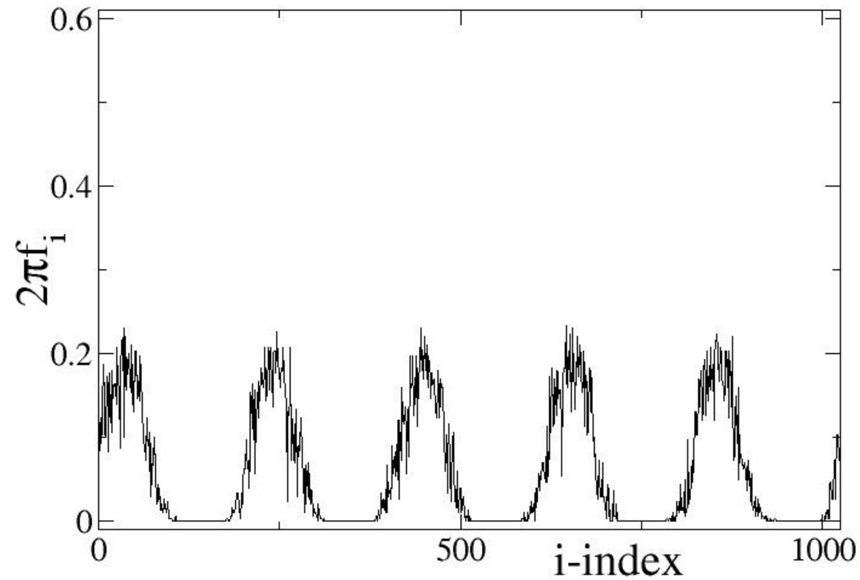
μ=1, u_{th}=0.98, u_{rest}=0

σ = -1.6 **σ = 0.7**

Typical spacetime plots in LIF model

Chimera states vs bump states





$$\boxed{\omega_i = 2\pi \frac{Q_i(\Delta T)}{\Delta T}}$$

Chimera State **Bump State**

μ=1, u_{th}=0.98, u_{rest}=0

σ = -1.6 **σ = 0.7**

Typical firing rate (mean phase velocity) plots in LIF model

3.2 Possible mean field approach to nonlocally coupled LIF oscillators

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{\sigma}{2R}\sum_{j=i-R}^{i+R}\left[u_j(t) - u_i(t)\right] \qquad \text{(1a)}$$

$$u_i(t) \to u_{rest}, \qquad \text{when} \quad u_i(t) > u_{th} \qquad \text{(1b)}$$

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{\sigma}{2R}\sum_{j=i-R}^{i+R}\left[u_j(t)\right] - \frac{\sigma}{2R}\sum_{j=i-R}^{i+R}\left[u_i(t)\right] \qquad \text{(2a)}$$

$$\frac{du_i(t)}{dt} = \mu - (1+\sigma)u_i(t) + \boxed{\frac{\sigma}{2R}\sum_{j=i-R}^{i+R}\left[u_j(t)\right]} \rightarrow \boxed{\sigma C} \qquad \text{(3a)}$$

$$u_i(t) \to u_{rest}, \qquad \text{when} \quad u_i(t) > u_{th} \qquad \text{(3b)}$$

$$u* = \frac{\mu + \sigma C}{1+\sigma} \qquad \text{fixed point with} \quad u_{\mathrm{rest}} \le u* < u_{\mathrm{th}}$$

$$\frac{du_i(t)}{dt} = \mu - (1+\sigma)u_i(t) + \sigma \langle u(t) \rangle_i \qquad (4a)$$

$$u_i(t) \rightarrow u_{rest} \quad \text{when} \quad u_i(t) > u_{th} \qquad (4b)$$

Mean field assumption : $\langle u(t)_i \rangle$ = constant = C

Furthermore, since $u_{rest} \leq u_i(t) < u_{th} \rightarrow u_{rest} \leq C < u_{th}$

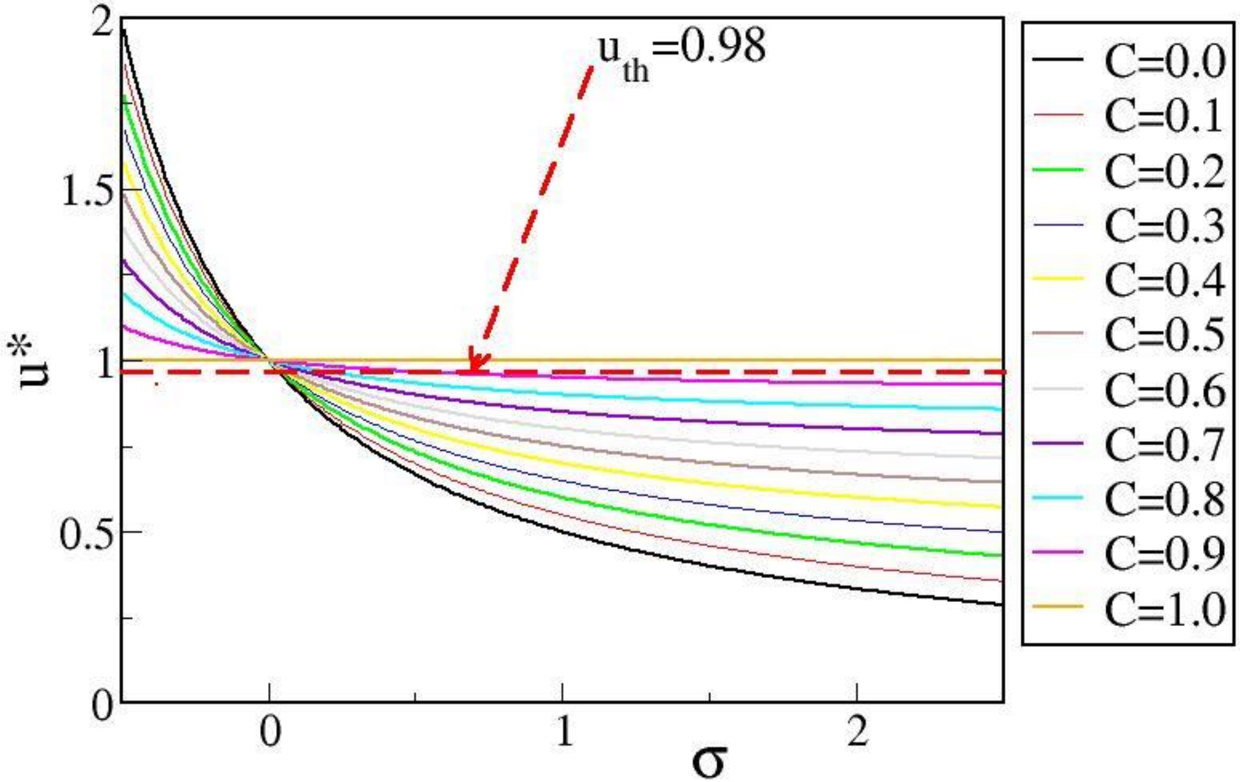
$$\frac{du(t)}{dt} = \mu - (1+\sigma)u(t) + \sigma C \qquad (5a)$$

$$u(t) \rightarrow u_{rest} \quad \text{when} \quad u(t) > u_{th} \qquad (5b)$$

$$u* = \frac{\mu + \sigma C}{1+\sigma} \qquad \text{fixed point} \qquad (6)$$

where $u_{rest} \leq u* < u_{th}$

Crude estimation for single LIF : $C = \langle u \rangle = \frac{1}{T}\int_0^T u\,(t)dt$ (=0.749)



- for $\sigma < 0$ the values of C have fixed points above threshold, i.e., $u^* > u_{th}$. All trajectories reach u_{th} and they get reset. Here chimera states are possible.

The mean field assumption shows that:
- for $\sigma > 0$ ALL values of C have a fixed point below threshold, i.e., $u^* < u_{th}$. Then some trajectories never reach u_{th} and they get trapped at u^* , bellow u_{th} and never reset. This is how the bump states arise.

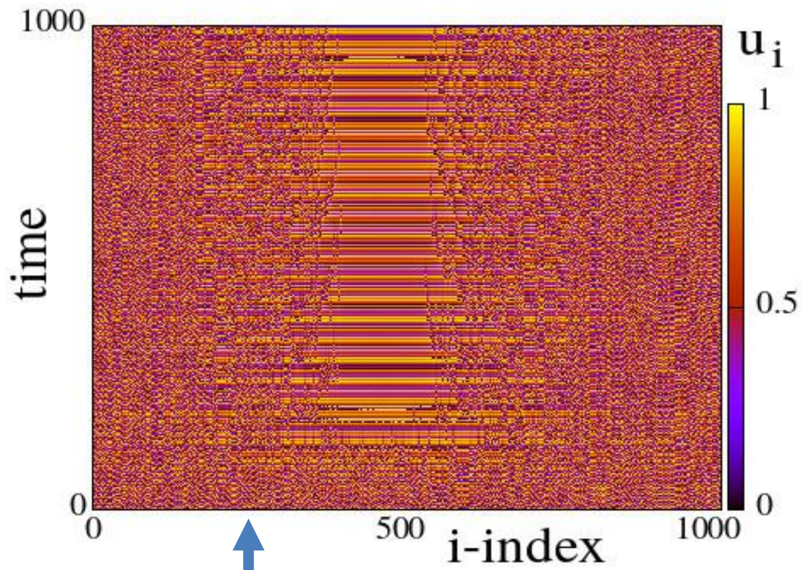
Curious observations:

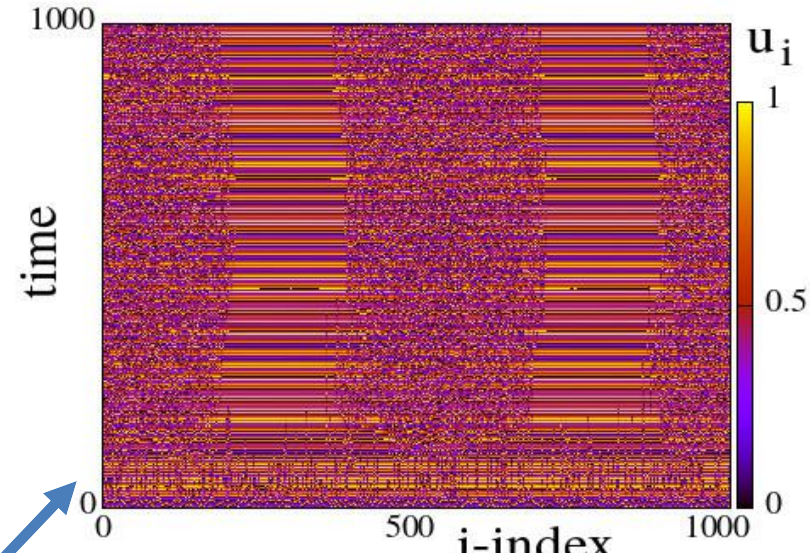
- The single LIF model presents **one** frequency $f_s=1/T_s$ (for the given parameters, $\mu=1$, $u_{th}=0.98$, $u_{rest}=0$, $T_s=3.91$ and $f_s=0.2558$)

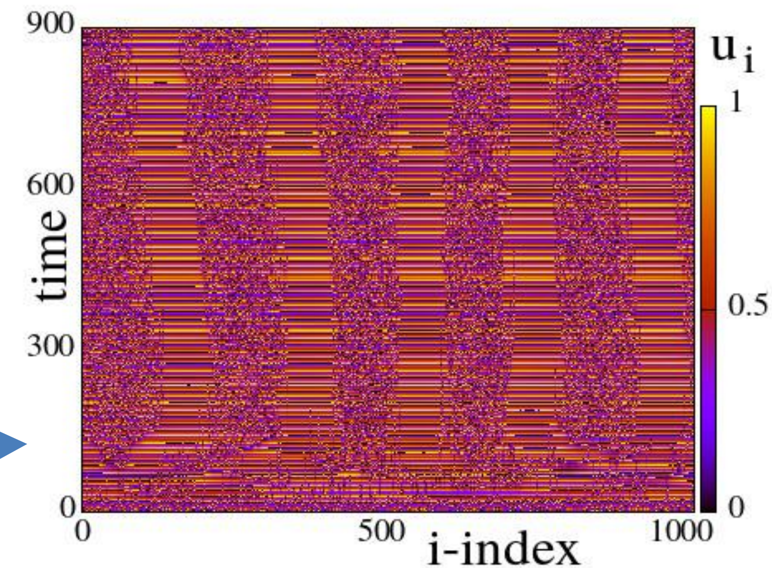
- The coupled LIFs develop **many** frequencies
  coherent and **incoherent domains** when $\sigma < 0$
  (e.g., $\sigma = -1.6$, inhibitory coupling, chimera states)
  and
  incoherent domains + **subthreshold domains** when $\sigma > 0$
  (e.g., $\sigma = +0.7$, excitatory coupling, bump states)

Where does this additional complexity/frequencies come from?

3.3 Other chimeras to be used lateron





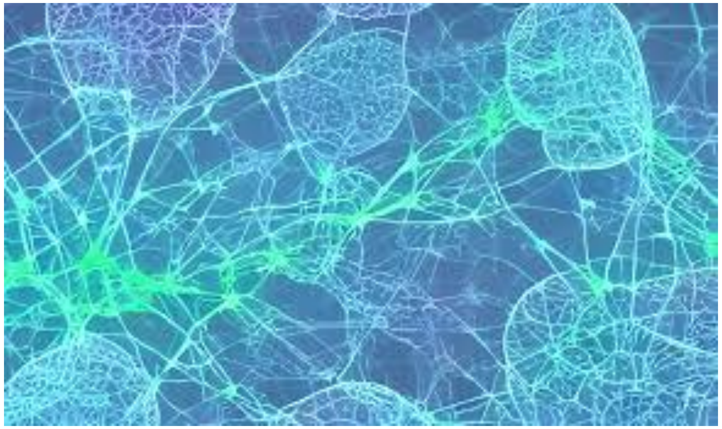


σ=-0.7, R=350

σ=-1.6, R=350

σ=-1.6, R=150

4.0 The role of synaptic plasticity: Oja rule



In neuron networks, synapses undergo dynamical changes due to:
- growth at young ages
- learning *
- aging
- neurodegenerative disorders

* Hebbian learning rule: Fire together wire together principle (unstable for all neuron models)

* Oja learning rule: stable

Other rules ...

-1949: D. O. Hebb, The organization of behaviour New York, Wiley and Sons

-1982: E. Oja, Journal of Mathematical Biology **15,** 267-273

-1989: E. Oja, International Journal of Neural Systems **1**, 61-68

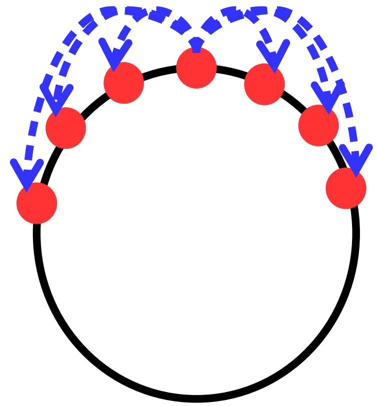
-2023: B. Juttner and E. A. Martens, Chaos, **33**, 053106

4.1 Realization of synaptic plasticity in LIF

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{\sigma_c}{2R}\sum_{j=i-R}^{i+R} \sigma_{ij}(t)\left[u_j(t) - u_i(t)\right] \quad (7a)$$

$$u_i(t) \to u_{\text{rest.}}, \qquad \text{when } u_i(t) > u_{\text{th}} \quad (7b)$$

$$\tau_\sigma \frac{d\sigma_{ij}(t)}{dt} = u_i u_j - \alpha u_i u_i \sigma_{ij}(t) \quad (7c)$$



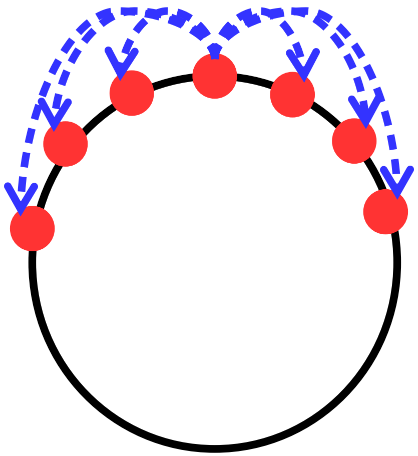
σ_c = is the coupling control parameter
τ_σ = is the plasticity time scale parameter
α = is the Oja parameter ($\boldsymbol{\alpha} > 0$)

We note that in Hebbian Plasticity : $\tau_H \frac{d\sigma_{ij}(t)}{dt} = u_i(t)u_j(t)$

Realization of synaptic plasticity in LIF (cont.)

$$\tau_\sigma \frac{d\sigma_{ij}(t)}{dt} = u_i u_j - \alpha u_i u_i \sigma_{ij}(t) \qquad (7c)$$

Under "normal conditions", Eq. (7c) has a fixed point in the domain of σ, as: $(\sigma_{ij}*) \sim 1/\alpha$ provided that $u_i = u_j$ at the steady state



How is this realizable in the case of
- Bump states and
- Chimera states

Realization of synaptic plasticity in LIF (cont.)

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{\sigma_c}{2R}\sum_{j=i-R}^{i+R} \sigma_{ij}(t)\left[u_j(t) - u_i(t)\right] \qquad (7a)$$

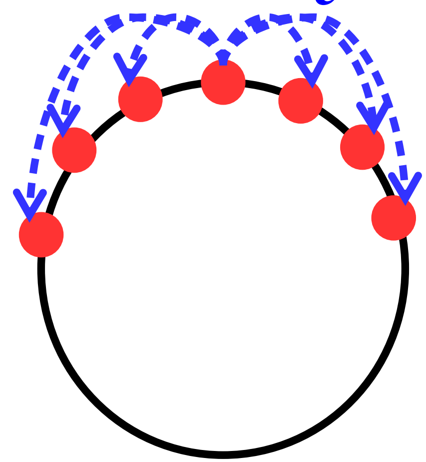
$$u_i(t) \to u_{rest}, \qquad \text{when } u_i(t) > u_{th} \qquad (7b)$$

$$\tau_\sigma \frac{d\sigma_{ij}(t)}{dt} = u_i u_j - \alpha u_i u_i \sigma_{ij}(t) \qquad (7c)$$

Effective weights: $\sigma_{ij}^{eff} = \sigma_c \sigma_{ij}$ ***and*** $\sigma^{eff} = \sigma_c <\sigma_{ij}>$

when $\alpha = 1$ we expect at $t \to \infty$:

- **chimera states for $\sigma_{ij}^{eff} = s < 0$**
- **bump states for $\sigma_{ij}^{eff} = s > 0$**



Importance of τ_σ

Quantitative indices

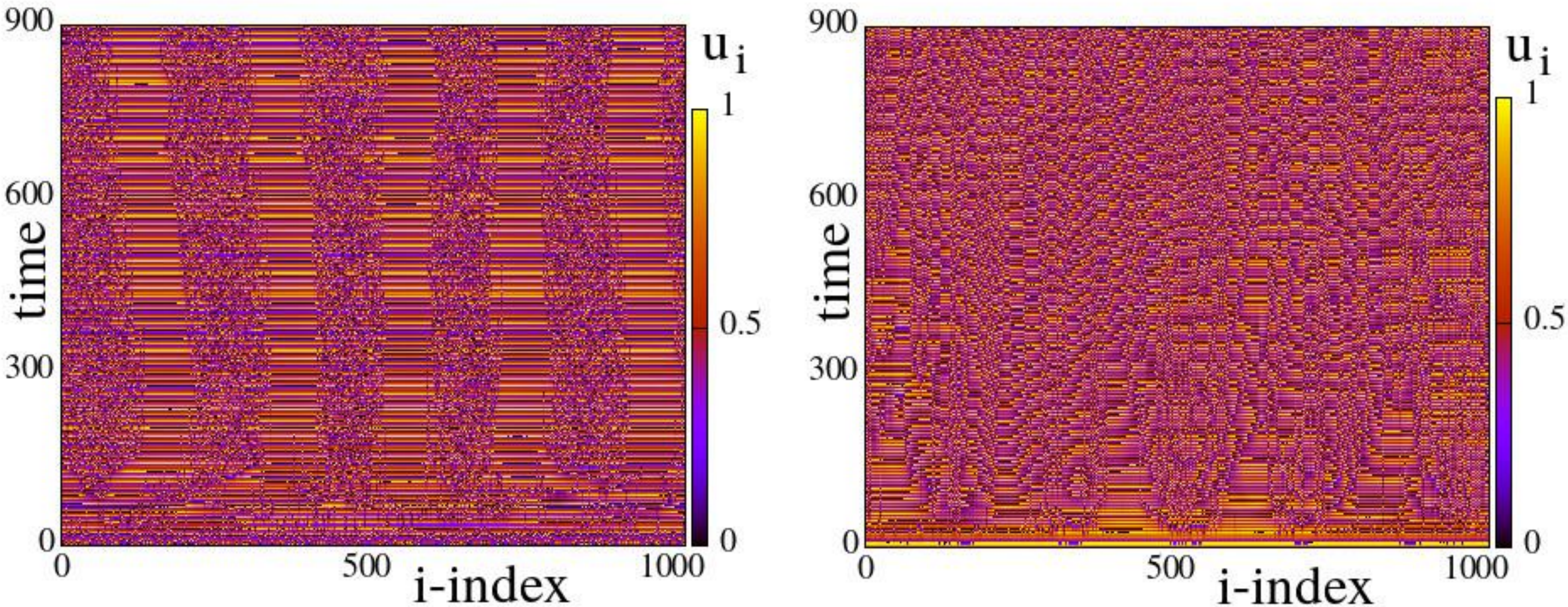
$$Mean\,Phase\,Velocity:\ \omega_i = \frac{Q_i(\Delta T)}{\Delta T},$$

$$Kuramoto\,Order\,Parameter: r = \|Z\| = \left\| \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j} \right\|,$$

$$Effective\,coupling:\quad <\sigma^{eff}(t)> = \frac{\sigma_c}{2RN}\sum_{j=1}^{N}\sum_{k=1}^{N}\sigma_{jk}(t) \quad k \neq j\,,$$

$$Coupling\,Fluctuations: D_\sigma^2(t) = \frac{1}{2RN}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(\sigma_c\sigma_{jk}(t) - <\sigma^{eff}(t)>\right)$$

4.2 Didactic examples of the effects of synaptic plasticity

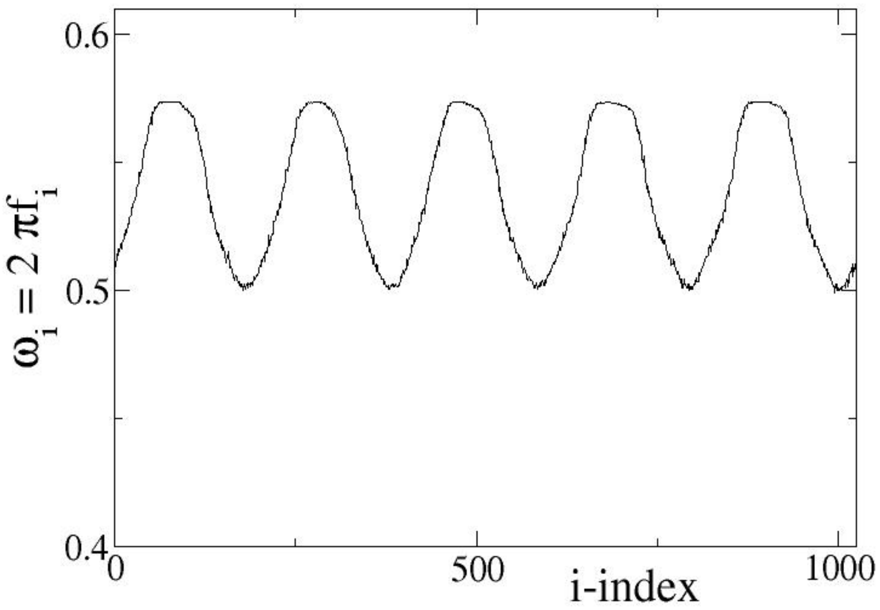


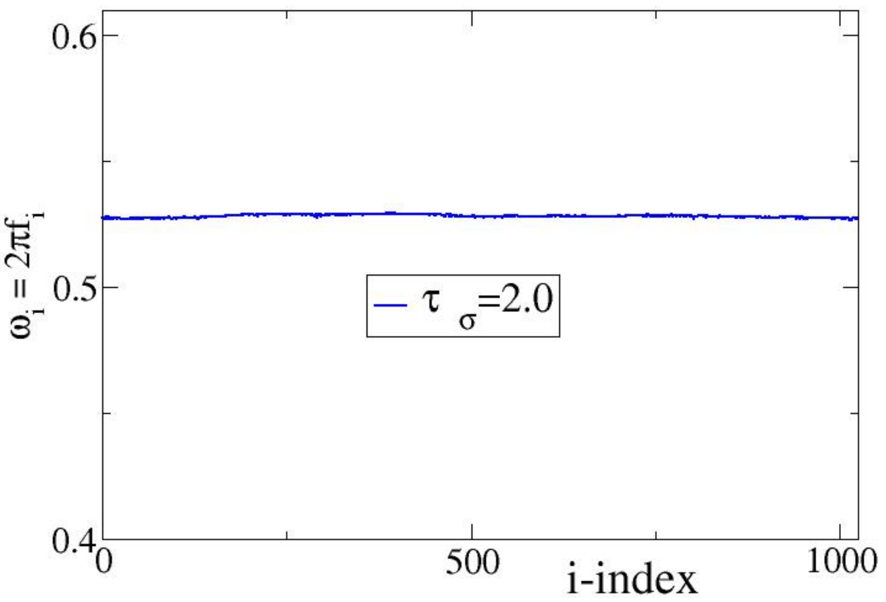
Without plasticity **Chimeras** **With plasticity**

$\alpha=1$, $\tau_\sigma=2.0$

$\mu=1$, $u_{th}=0.98$, $u_{rest}=0$, $R=150$, $\sigma_c=-1.6$

Didactic examples of the effects of synaptic plasticity (cont.)



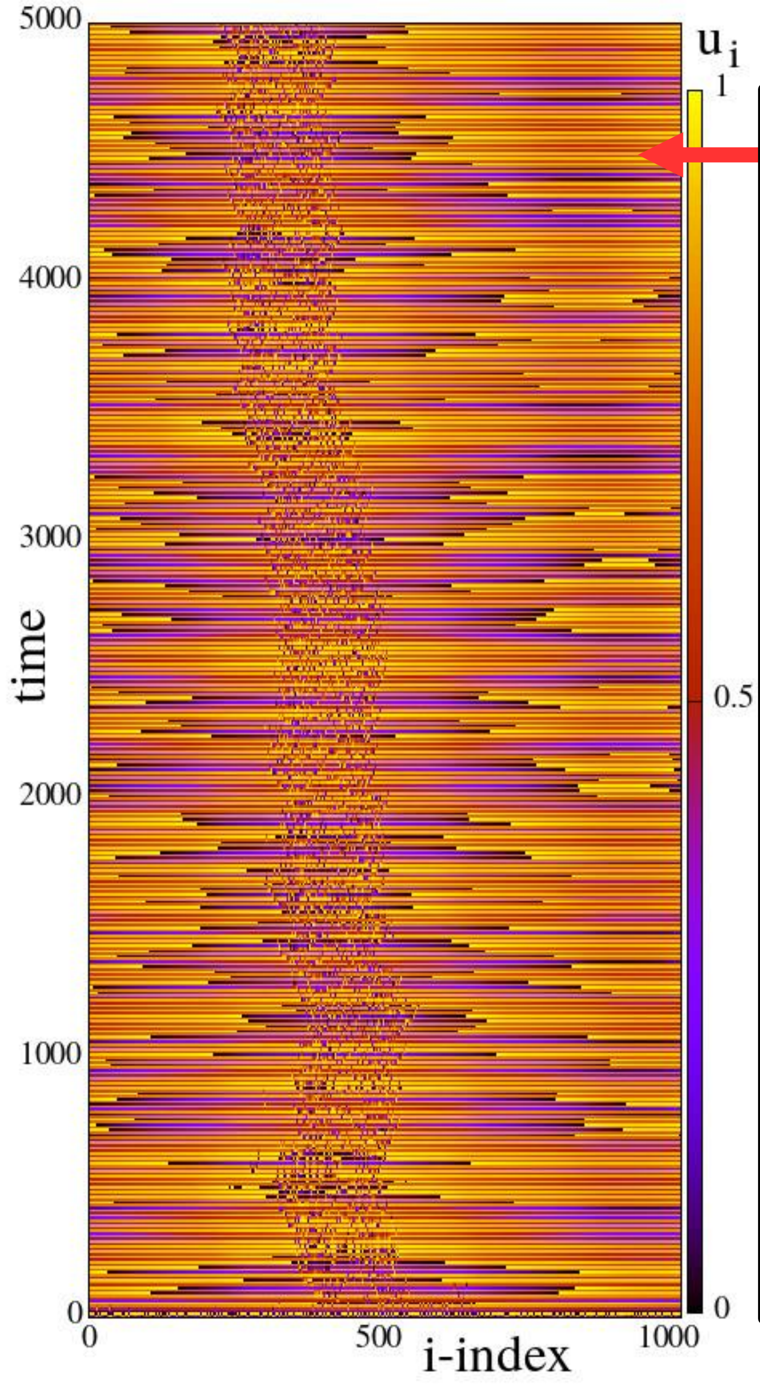


Without plasticity **Chimeras** **With plasticity**

α=1, τ_σ=2.0

μ=1, u_{th}=0.98, u_{rest}=0, R=150, σ_c=-1.6

AP, Y. Almirantis, W. Li (submitted 2024)



$\tau_\sigma = 2$

$\tau_\sigma = 1000$

Effects of time scales

$\alpha = 1$

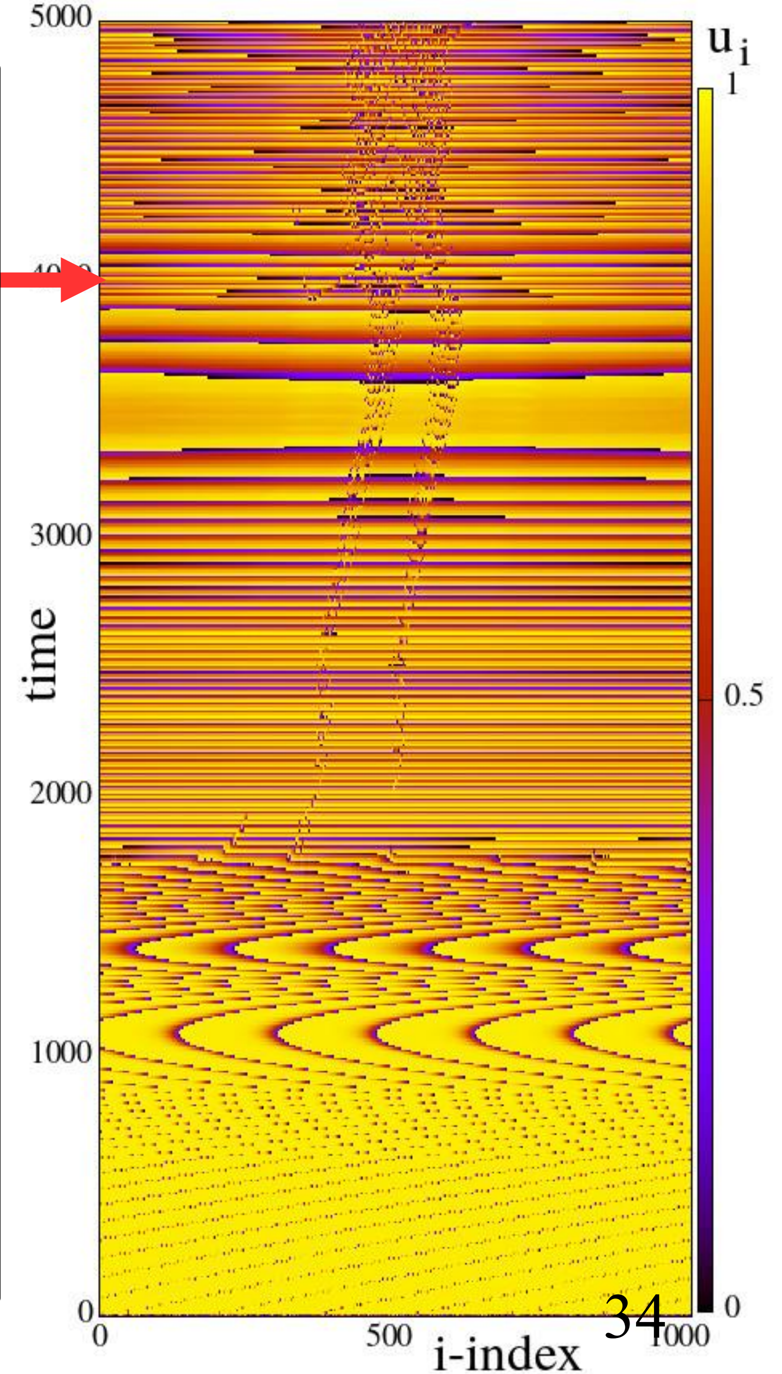
$R = 150$

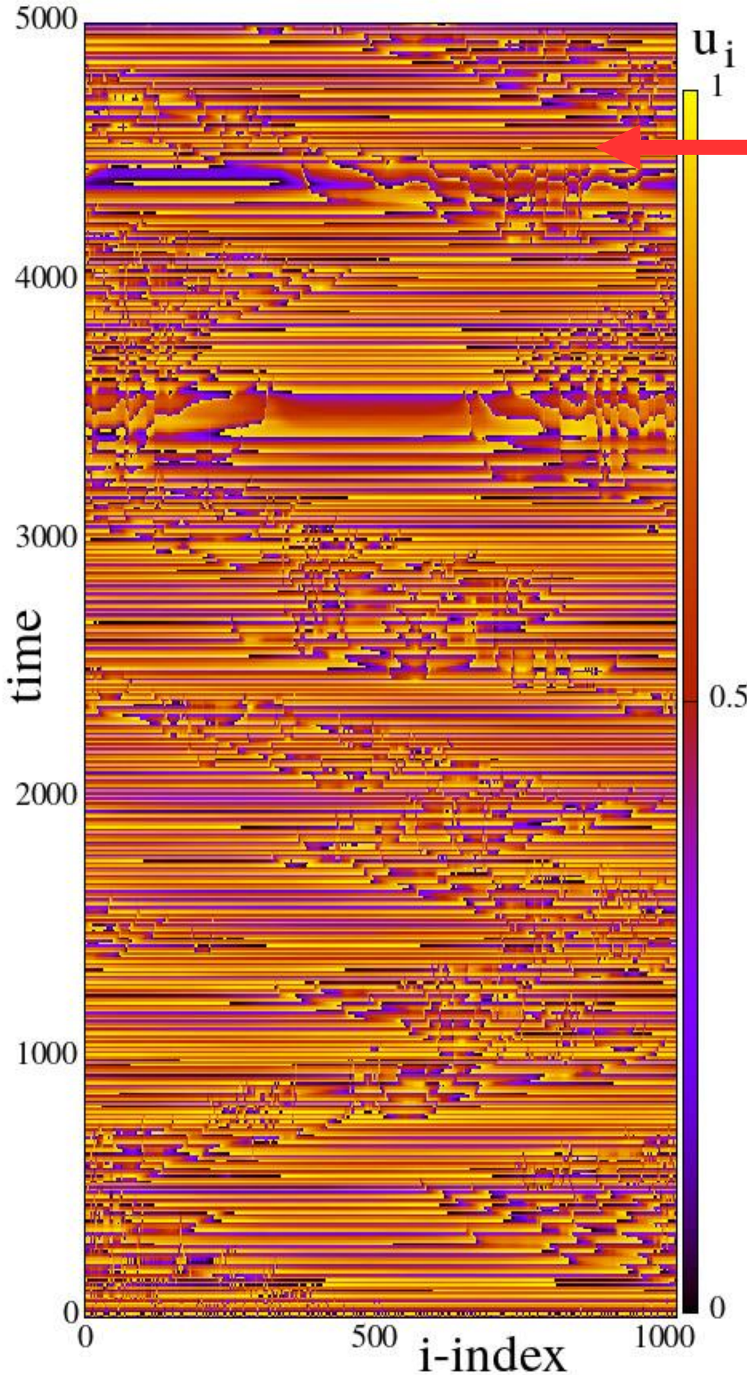
$\mu = 1$

$u_{th} = 0.98$

$\sigma_c = -0.7$

$\sigma_{ij}(t=0) = -2$





τ_σ=2

τ_σ=1000

Effects of time scales

α=1

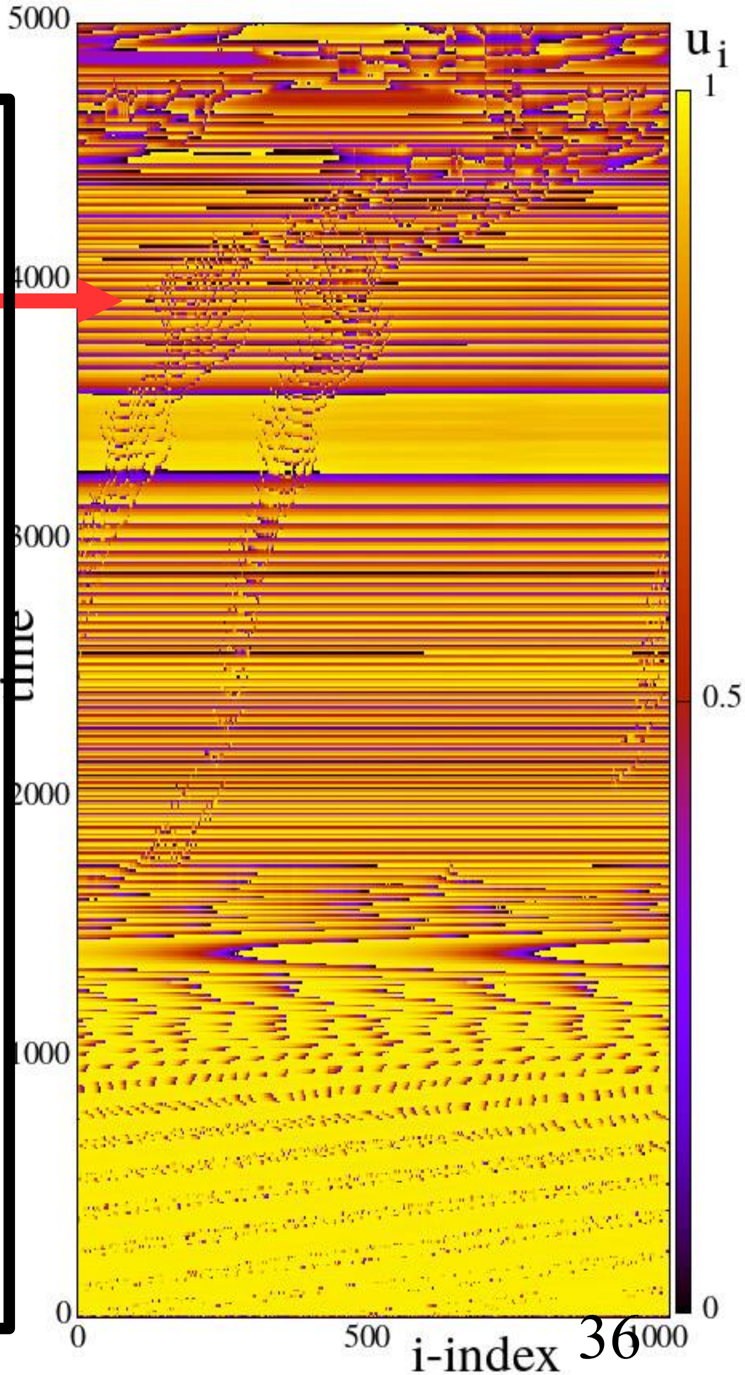
R=350

μ=1

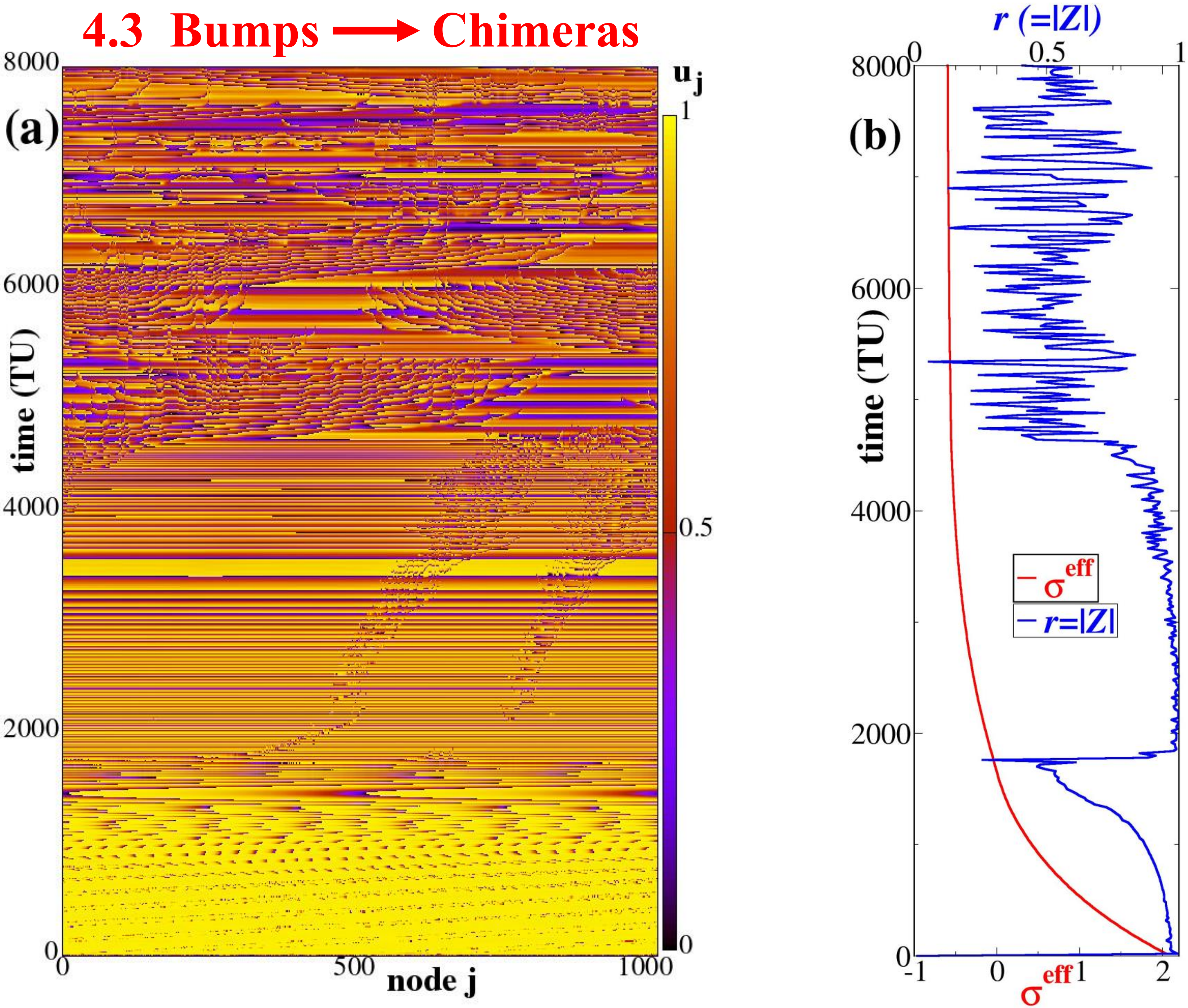
u_{th}=0.98

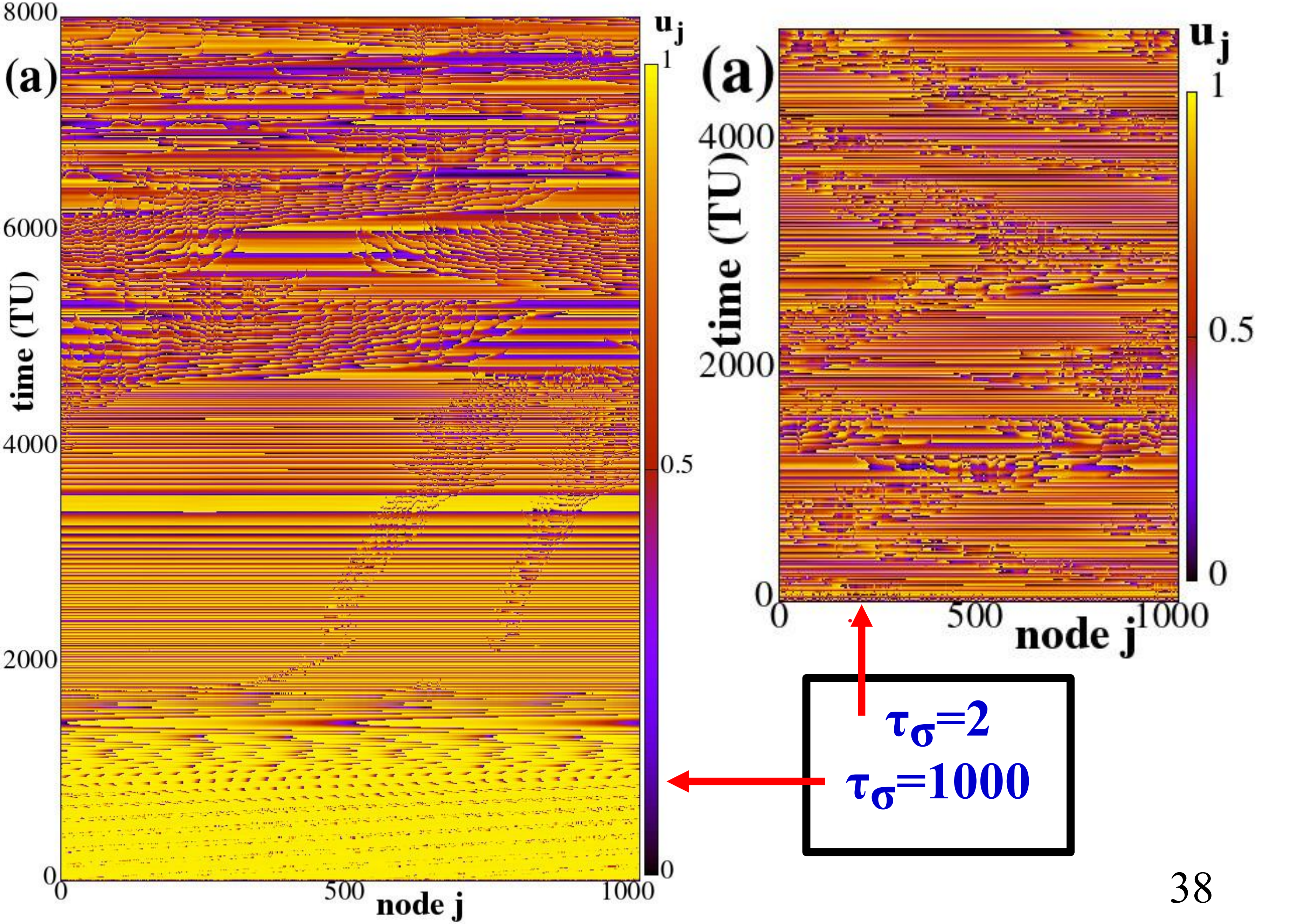
σ_c=- 0.7

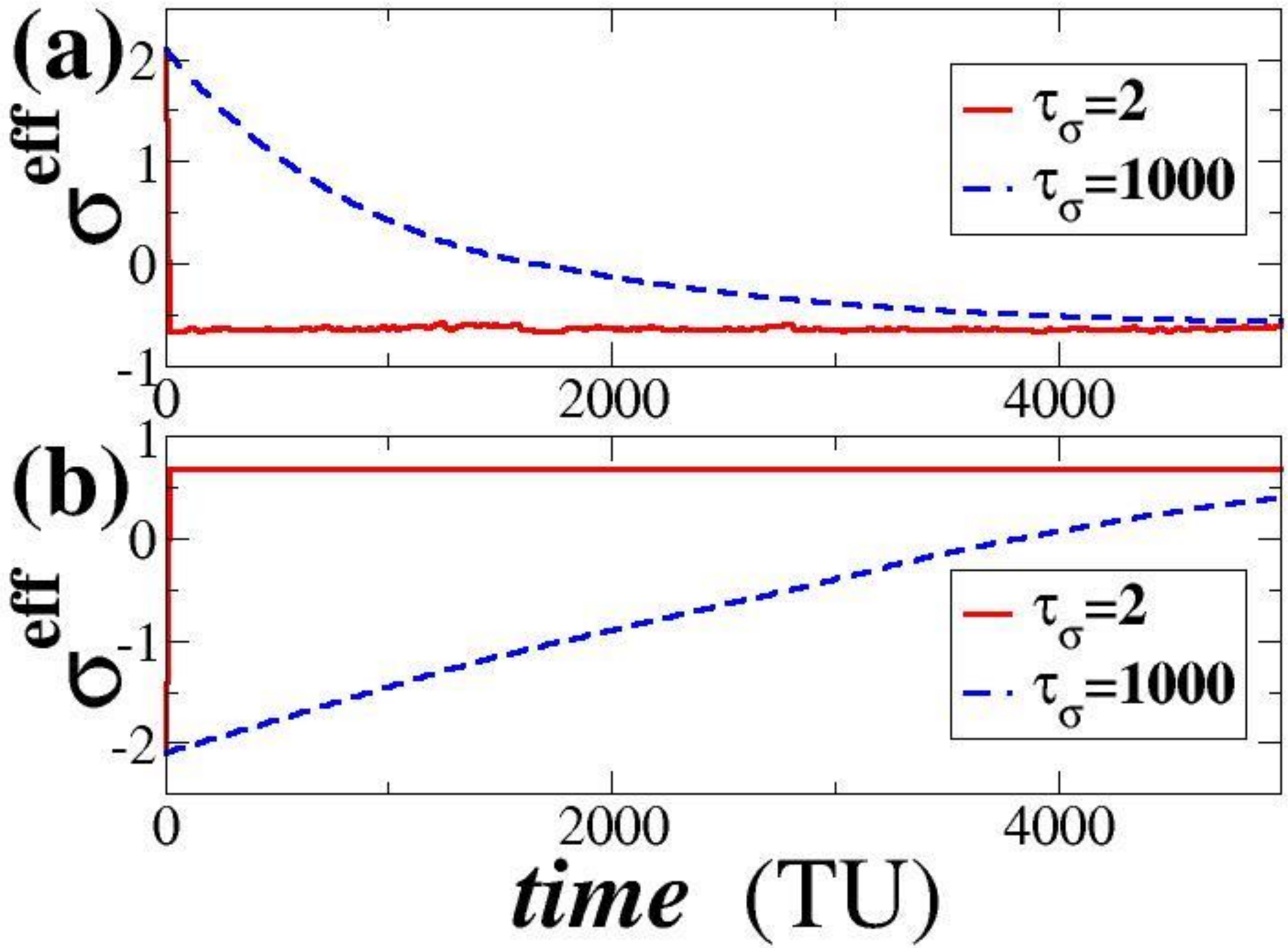
$\sigma_{ij}(t=0)$=-2



4.3 Bumps ⟶ Chimeras

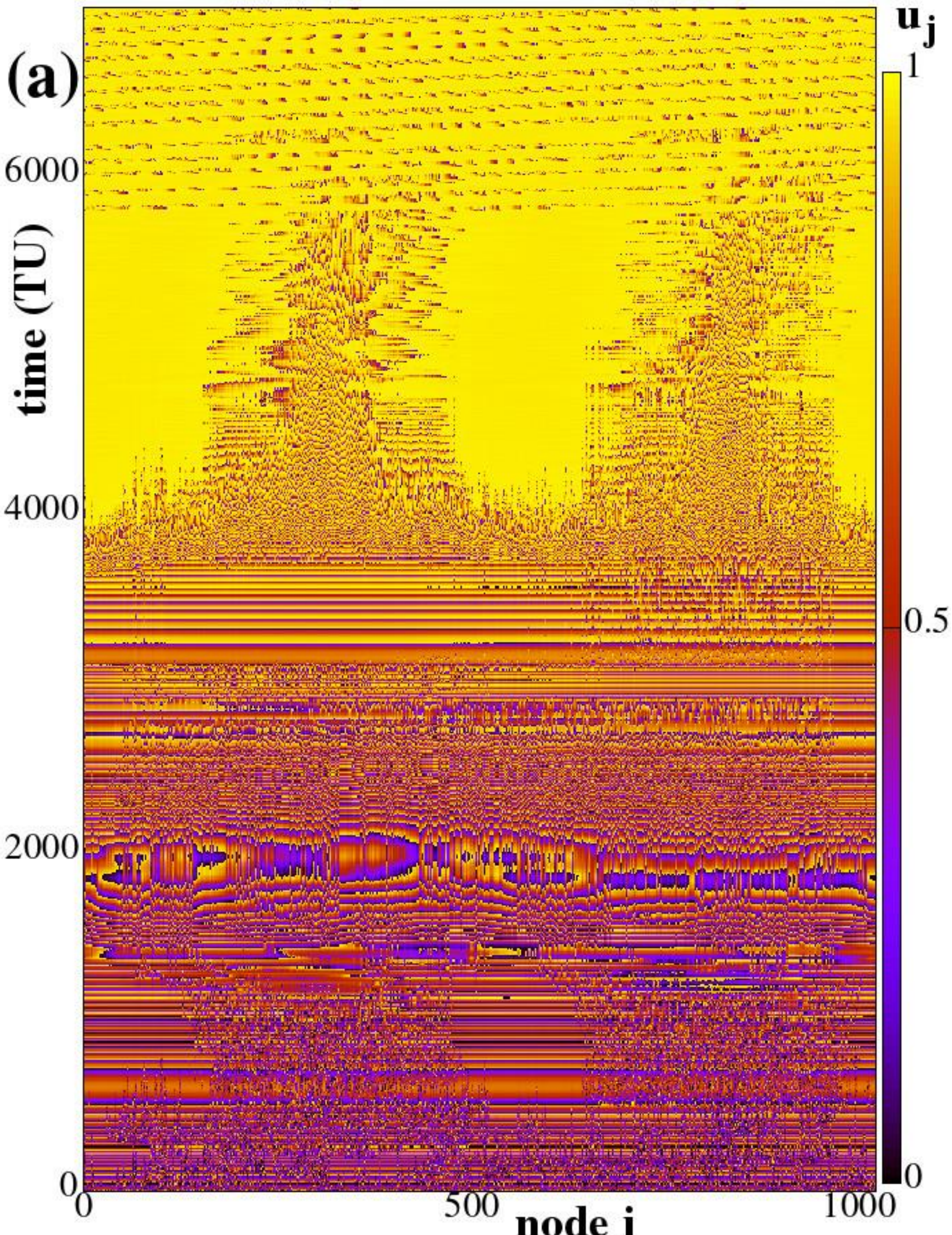


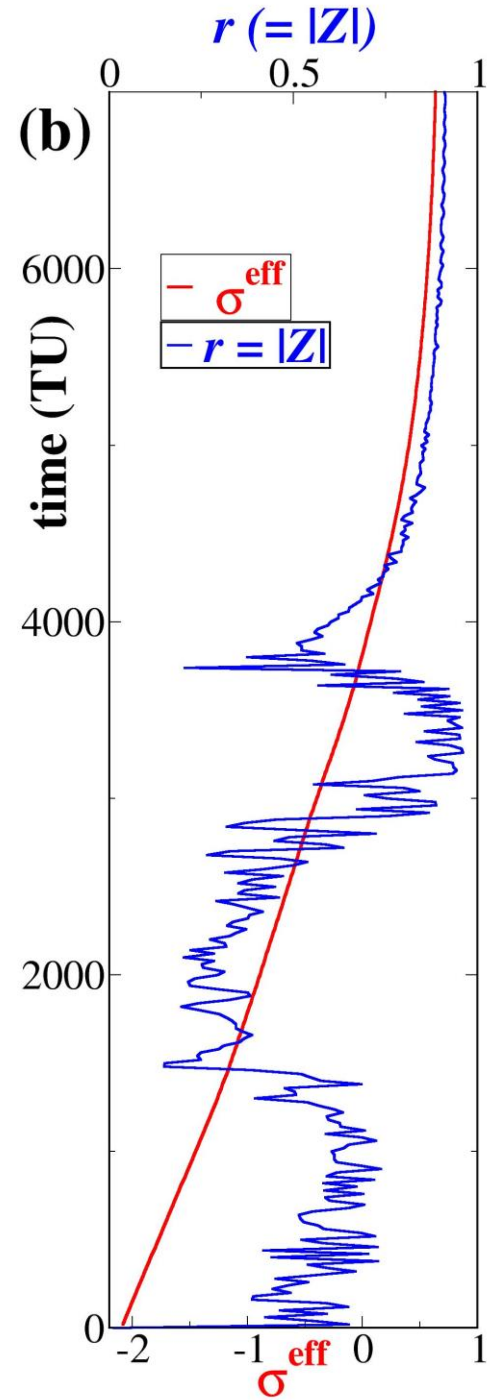

(a)
8000
6000
4000
2000
0
0
500
1000
time (TU)
node j
u_j
1
0.5
0
(a)
4000
2000
0
0
500
1000
time (TU)
node j
u_j
1
0.5
0
τ_σ=2
τ_σ=1000



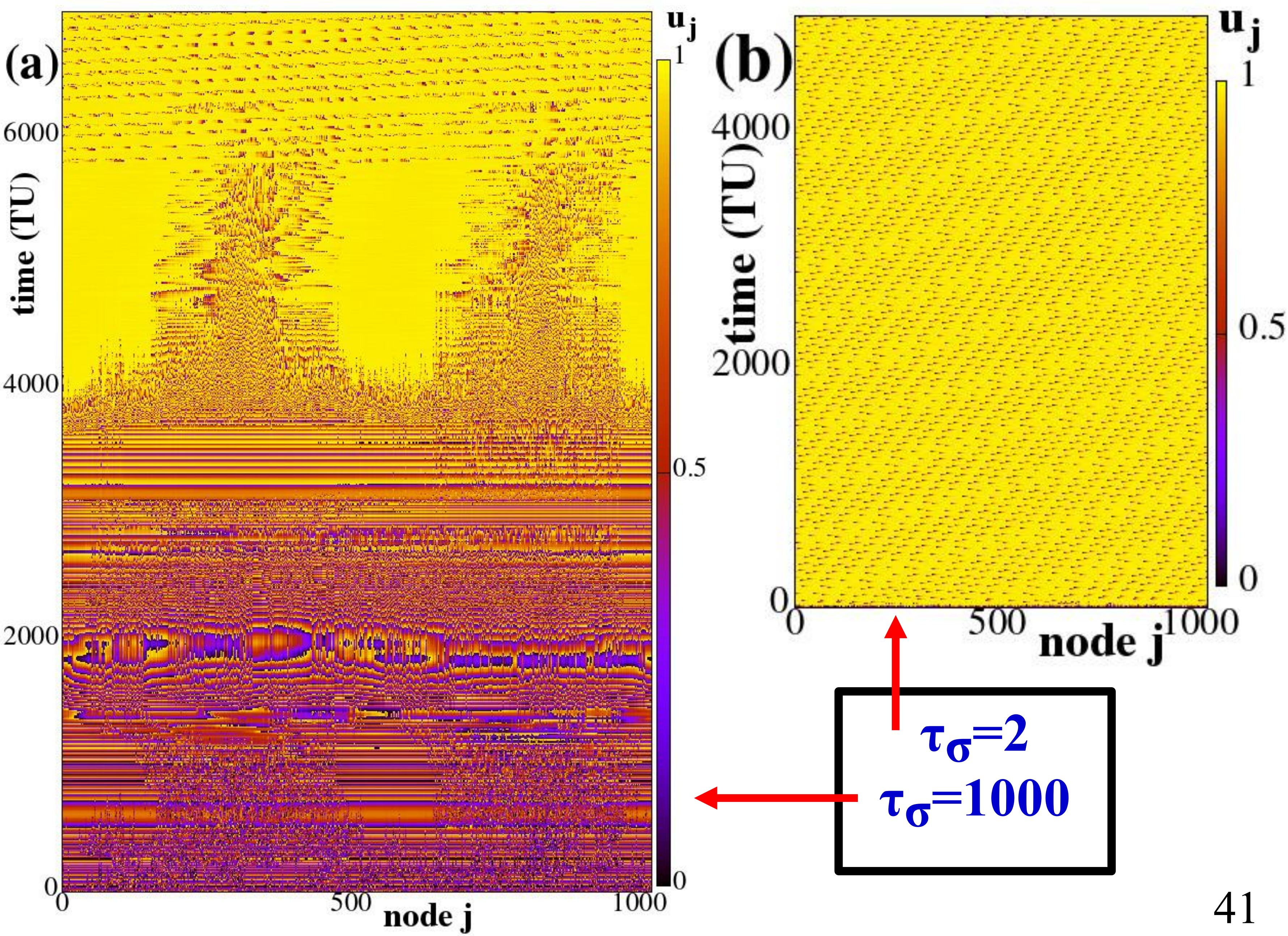
(a)
(b)
σ^{eff}
$\tau_\sigma=2$
$\tau_\sigma=1000$
2
1
0
-1
-2
0
2000
4000
time (TU)

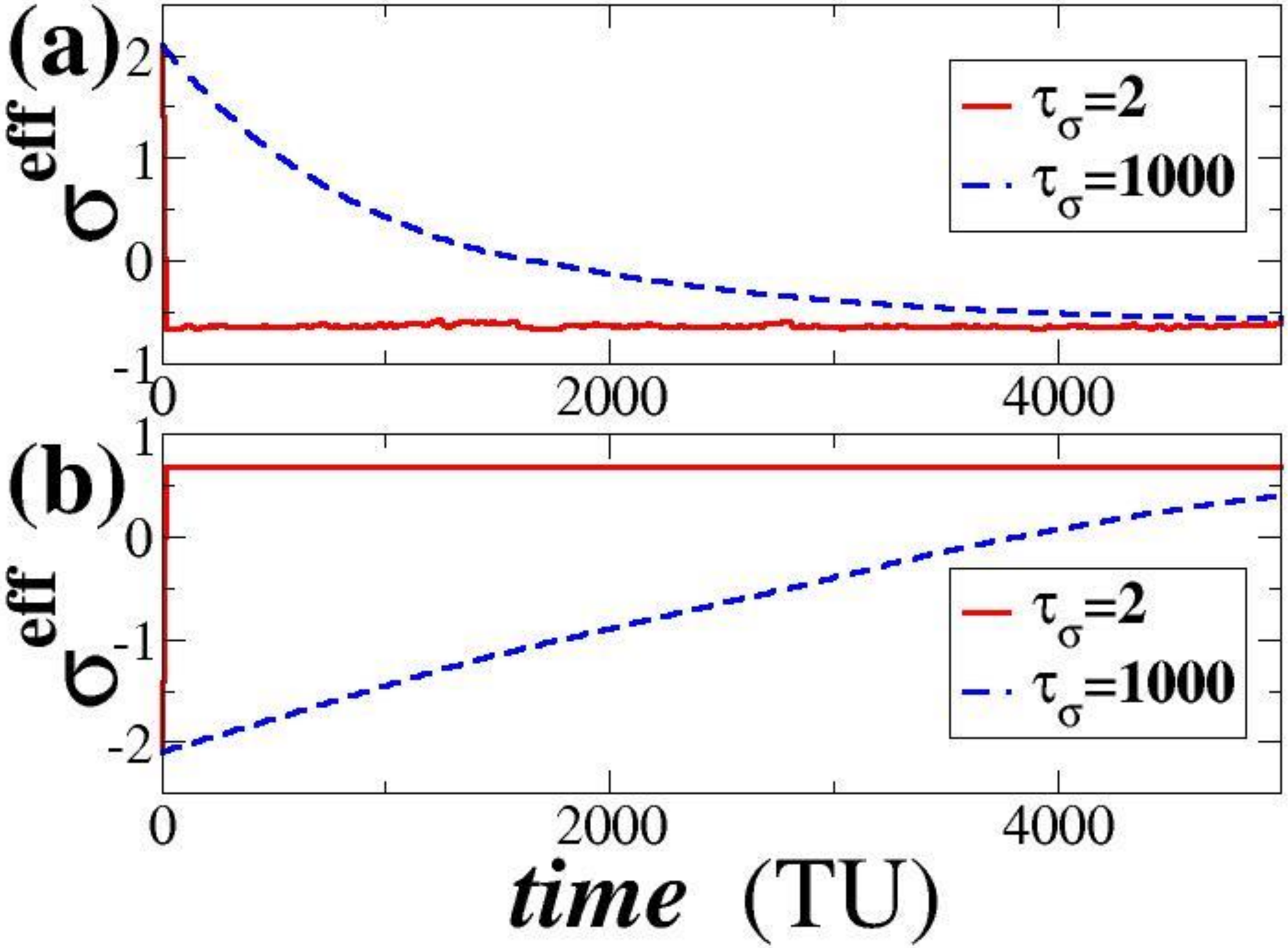
4.4 Chimeras ⟶ Bumps



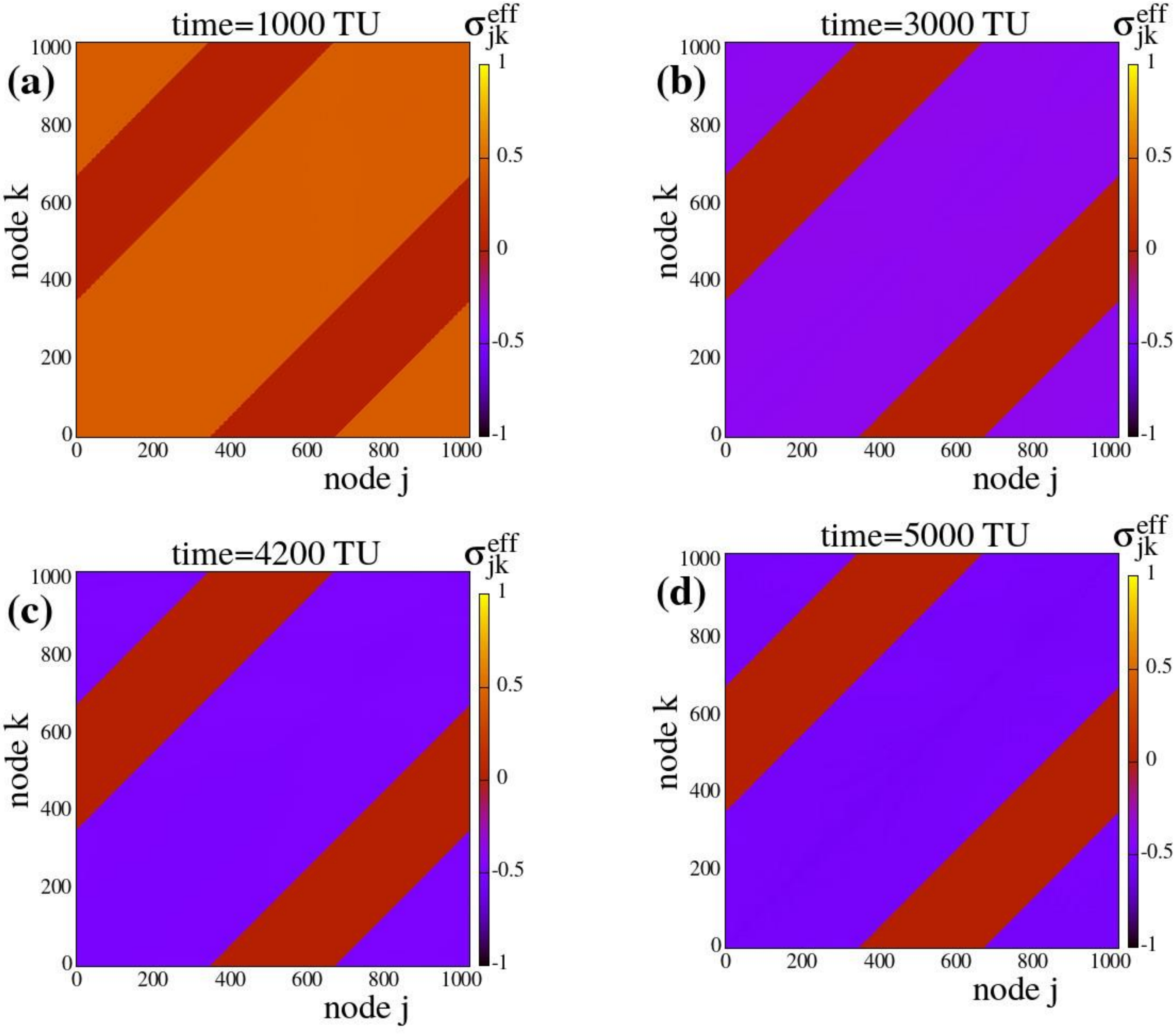


4.4 Chimeras ⟶ Bumps



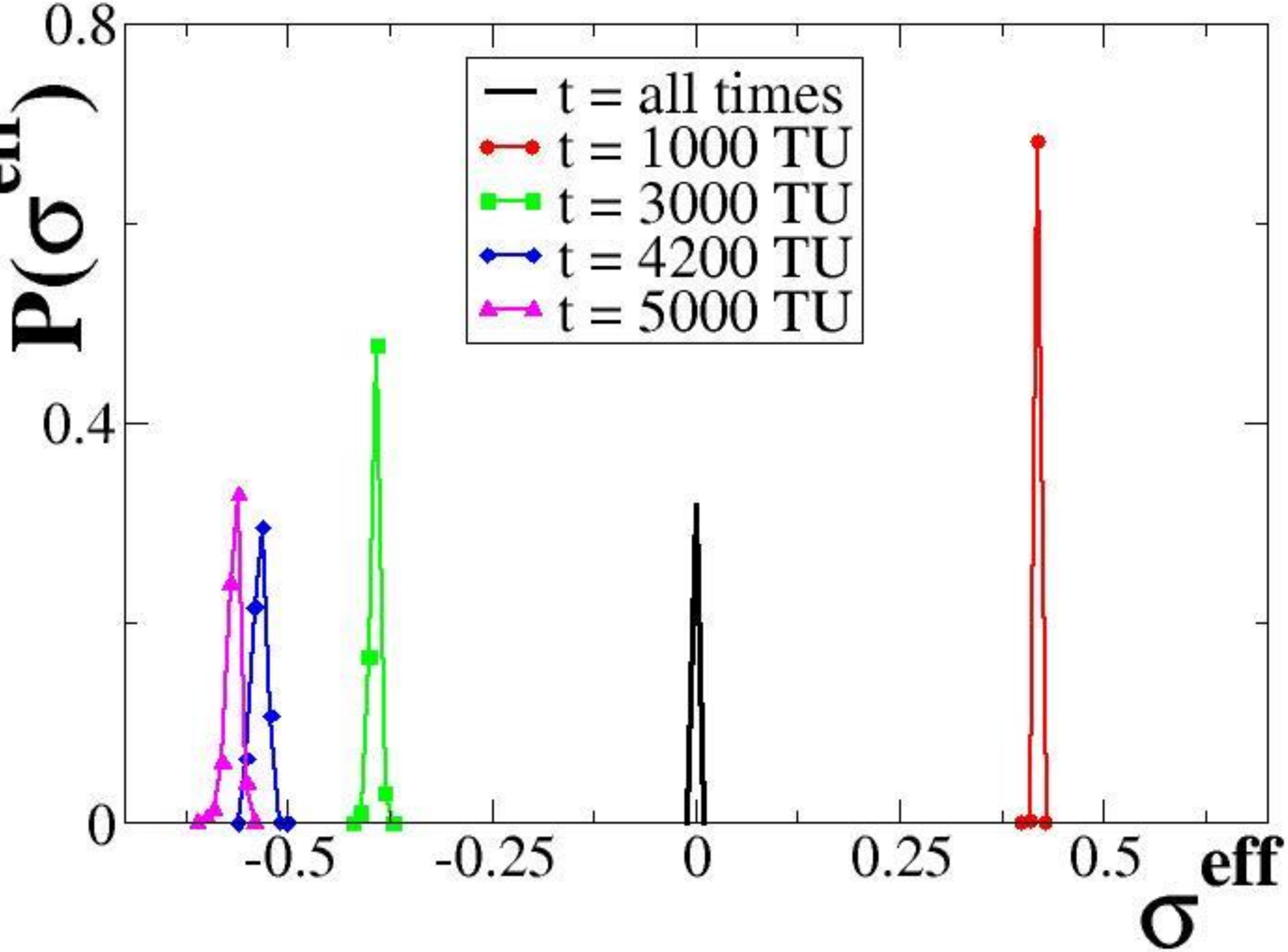

(a)
(b)
σ^{eff}
τ_σ=2
τ_σ=1000
-1
0
1
2
-2
2000
4000
time (TU)

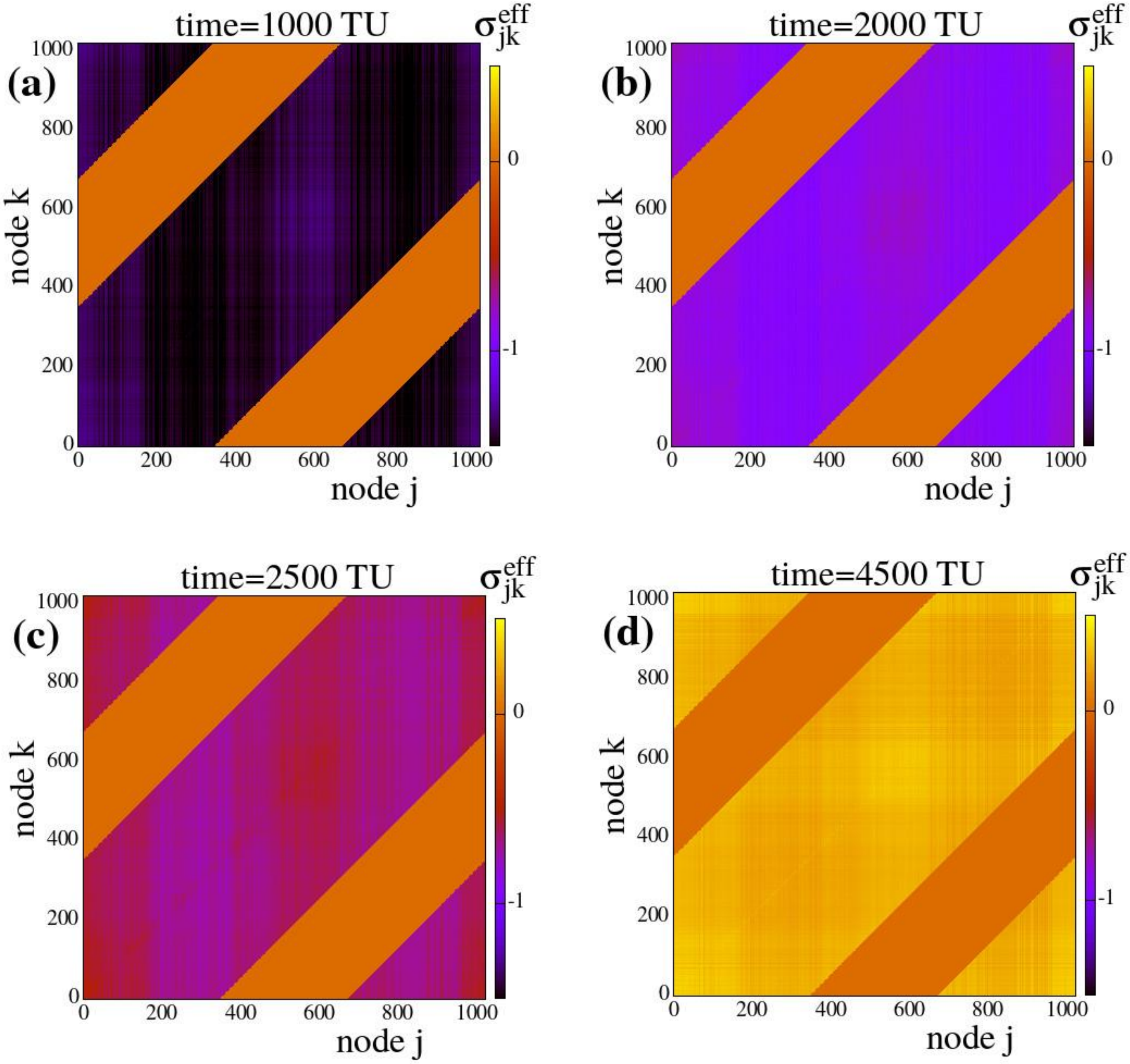
4.5 Other properties as the coupling strengths adapt to the local potential environment:



Bumps to Chimeras

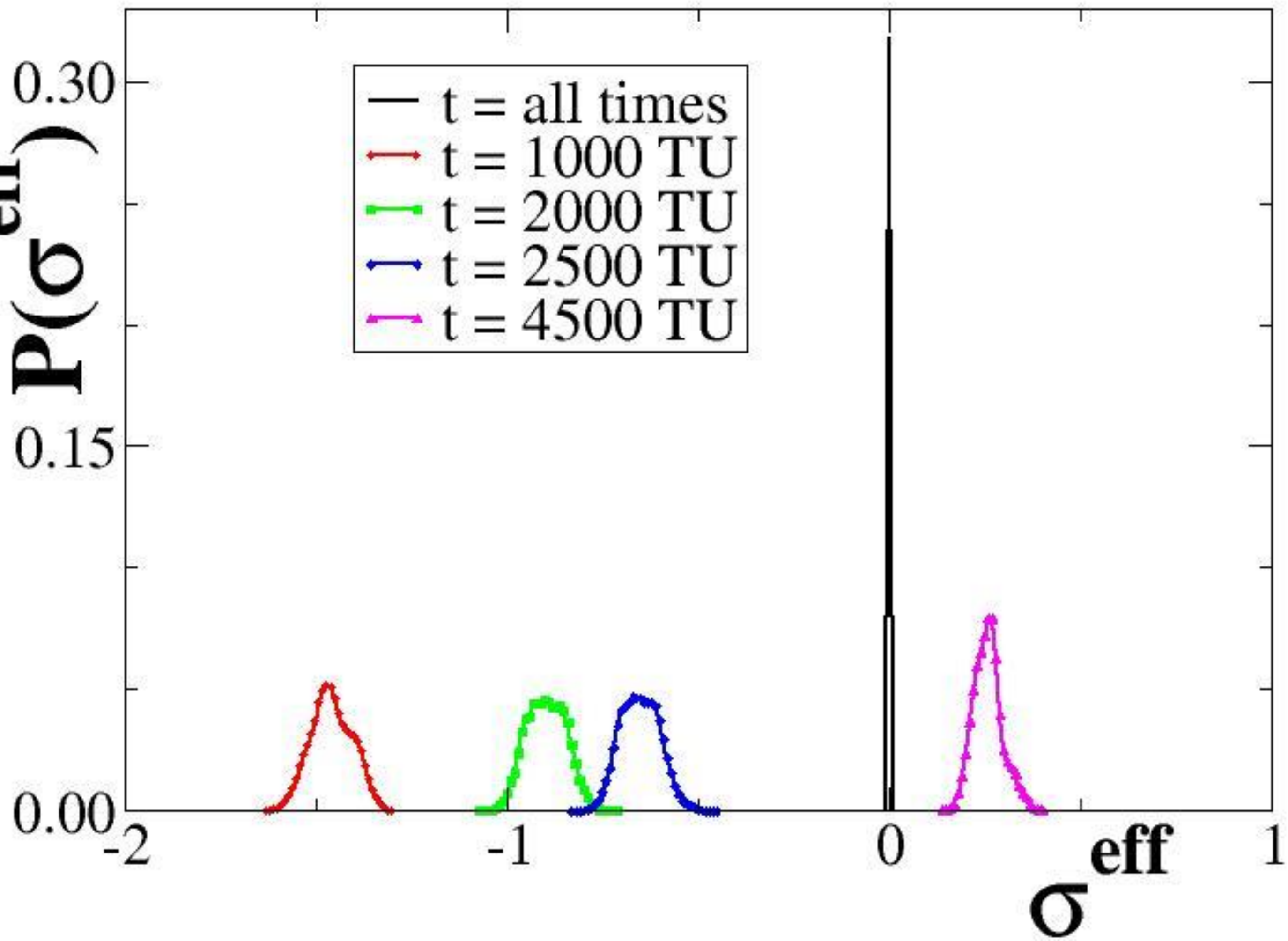
All couplings develop homogeneously in time


0.8
0.4
0
$P(\sigma^{eff})$
t = all times
t = 1000 TU
t = 3000 TU
t = 4200 TU
t = 5000 TU
-0.5
-0.25
0
0.25
0.5
σ^{eff}



Chimeras
to
Bumps

Couplings develop some inhomogeneities in time

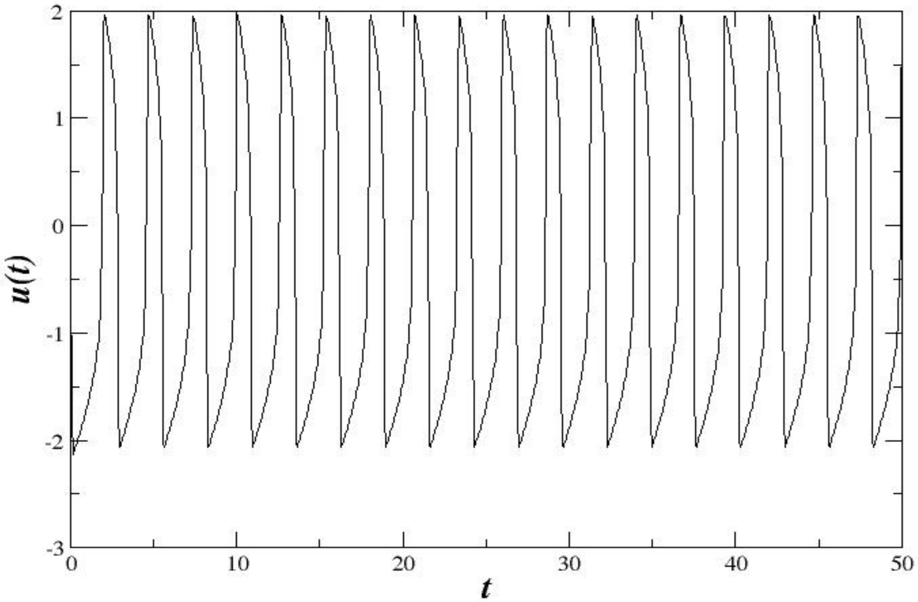


0.30
0.15
0.00
-2
-1
0
1
$P(\sigma^{eff})$
σ^{eff}
t = all times
t = 1000 TU
t = 2000 TU
t = 2500 TU
t = 4500 TU

5.0 The FitzHugh-Nagumo (FHN) nonlinear oscillators

$$\epsilon \frac{du(t)}{dt} = u(t) - \frac{u^3(t)}{3} - v(t) + I(t)$$

$$\frac{dv(t)}{dt} = u(t) + \gamma$$



u(t) is the membrane potential

v(t) is the recovery potential

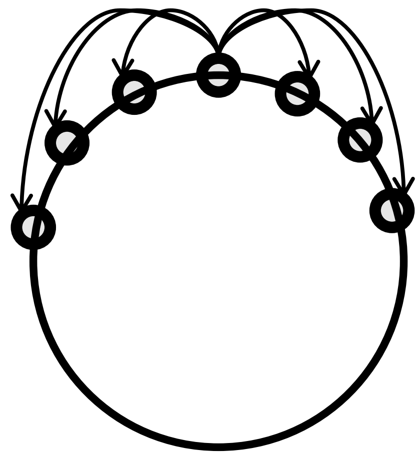
ε is time scale separation between membrane and recover potentials.

I(t) is Input

α is bifurcation parameter

5.1 Nonlocally coupled FHN Oscillators in 1D

Non-local connectivity



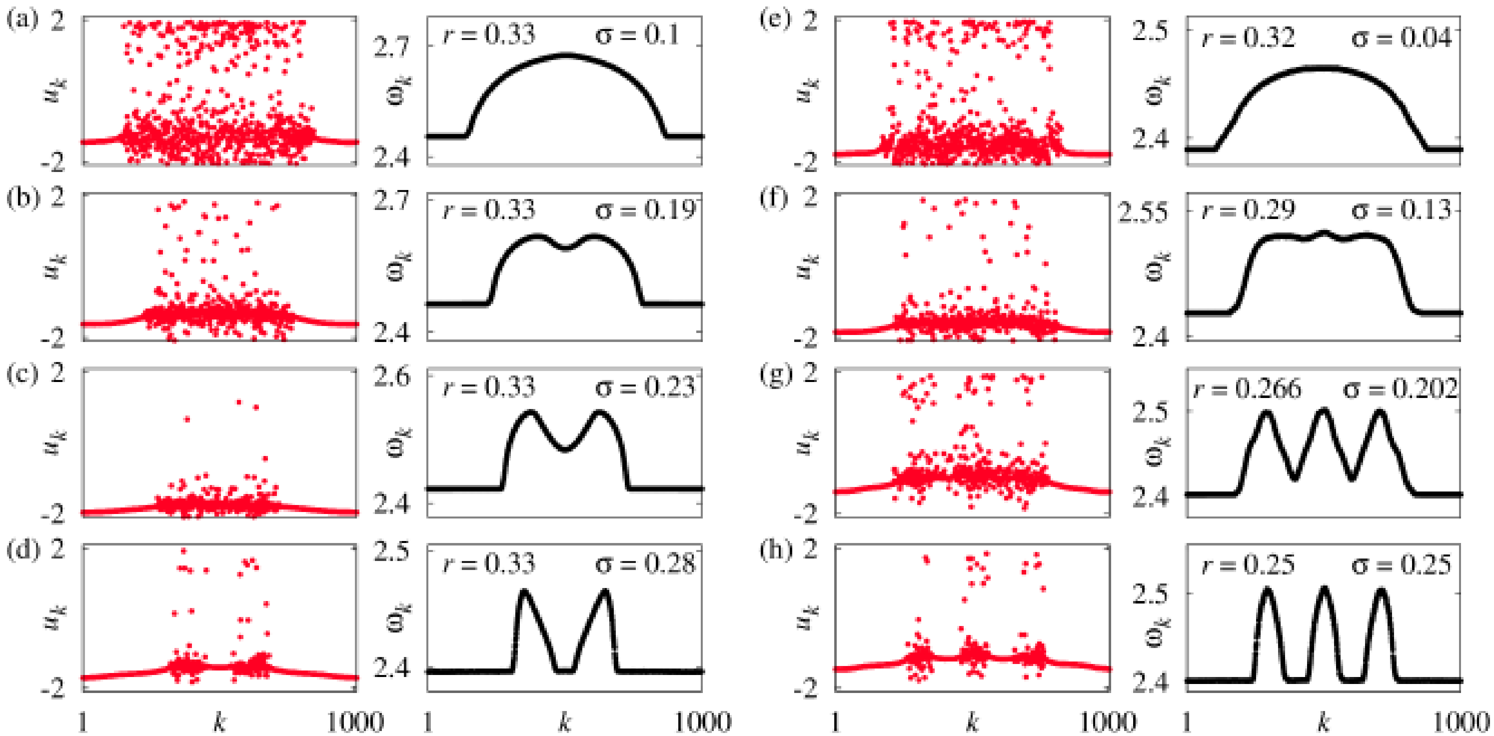
$$\sigma_{ij} = \begin{cases} \sigma & \text{if } i - R \le j \le i + R \\ 0 & \text{otherwise} \end{cases}$$

$$\epsilon \frac{du_i(t)}{dt} = u_i(t) - \frac{u_i^3(t)}{3} - v_i(t) + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} \left[b_{uu}\left[u_j(t) - u_i(t)\right] + b_{uv}\left[v_j(t) - v_i(t)\right] \right]$$

$$\frac{dv_i(t)}{dt} = u_i(t) + \gamma + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} \left[b_{vu}\left[u_j(t) - u_i(t)\right] + b_{vv}\left[v_j(t) - v_i(t)\right] \right]$$

Watts and Strogatz, Nature, 393 (6684): 440–442, 1998
Omelchenko et al, PRL, 110(22), 224101, 2013
Omelchenko et al, PRE, 91(2), 022917, 2015

5.1 Coupled FHN oscillators with nonlocal connectivity in 1D

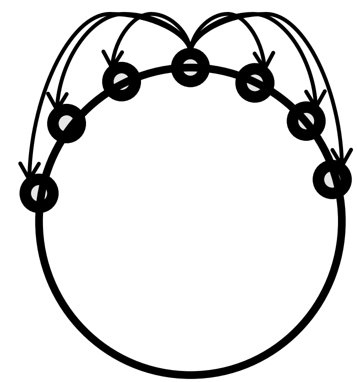


Chimera states for parameters:

N=1000, r=R/N

ε=0.05, γ=0.5, φ=π/2-0.1

buu=cos φ, buv= sin φ, bvu=-sin φ, bvv=cos φ



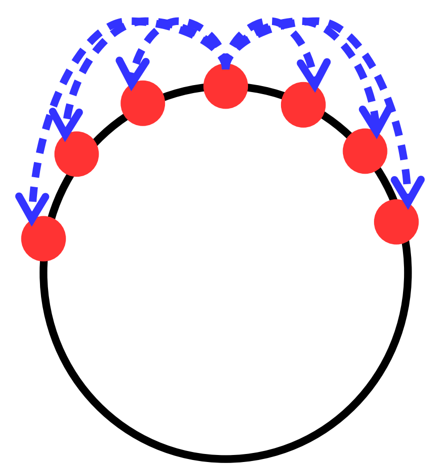
Omelchenko et al, PRL, 110(22), 224101, 2013

6.0 FitzHugh-Nagumo networks with adaptive coupling

$$\epsilon \frac{du_i(t)}{dt} = u_i(t) - \frac{u_i^3(t)}{3} - v_i(t) + \frac{\sigma_c}{2R} \sum_{j=i-R}^{i+R} \sigma_{jk} \left[b_{uu}[u_j(t) - u_i(t)] + b_{uv}[v_j(t) - v_i(t)] \right]$$

$$\frac{dv_i(t)}{dt} = u_i(t) + \gamma + \frac{\sigma_c}{2R} \sum_{j=i-R}^{i+R} \sigma_{jk} \left[b_{vu}[u_j(t) - u_i(t)] + b_{vv}[v_j(t) - v_i(t)] \right]$$

$$\tau_\sigma \frac{d\sigma_{jk}(t)}{dt} = u_j u_k - \alpha\, u_j u_j \sigma_{jk}$$



FHN adaptive network:

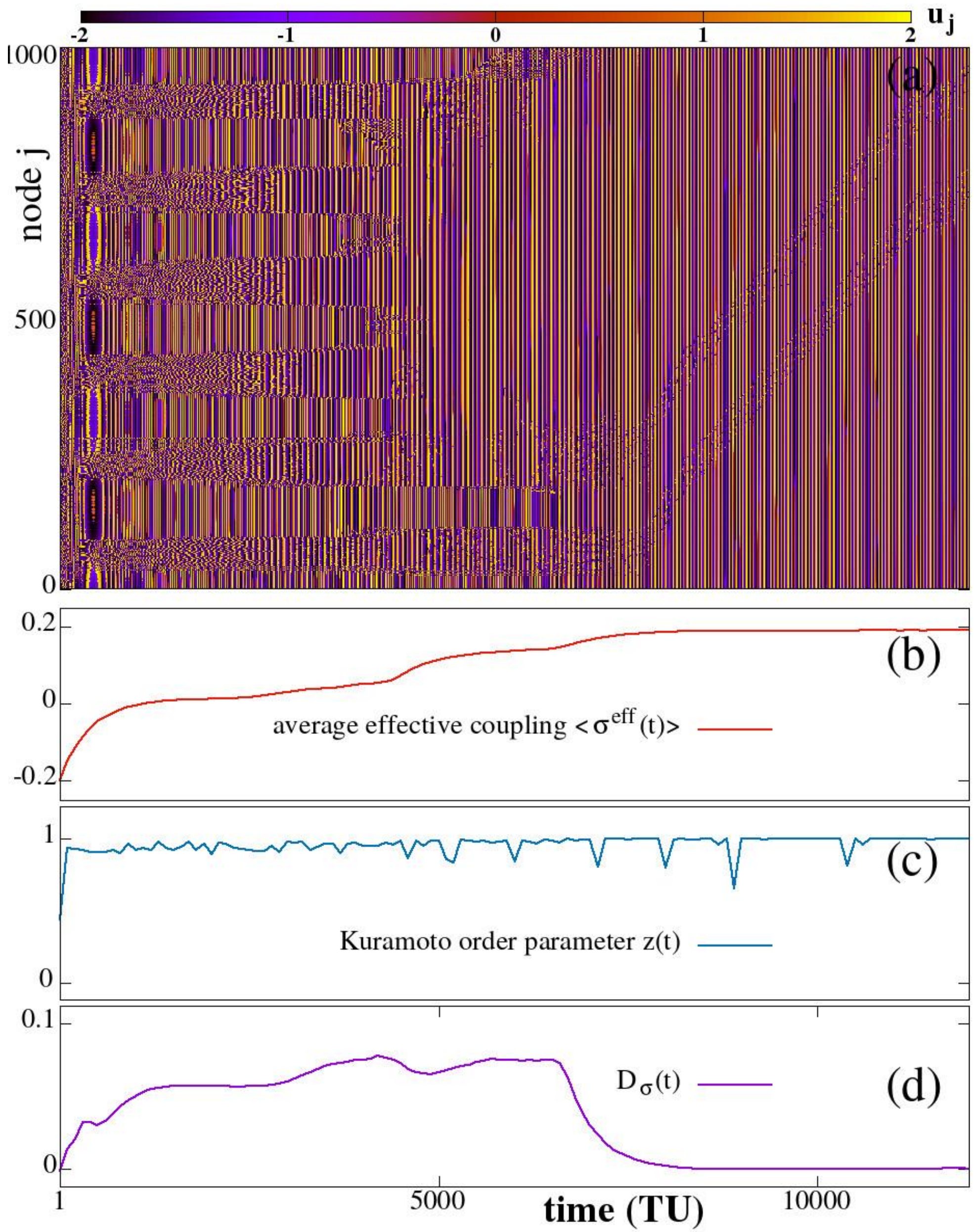
$\varepsilon=0.01$, $\gamma=0.5$

$N=1000$, $R=260$, $\varphi=\pi/2-0.1$, $\sigma_c=0.2$,

$\tau_\sigma=10000$, α(Oja)$=1.0$

…Not possible to form bump states in FHN networks…

6.1 FitzHugh-Nagumo networks with adaptive coupling



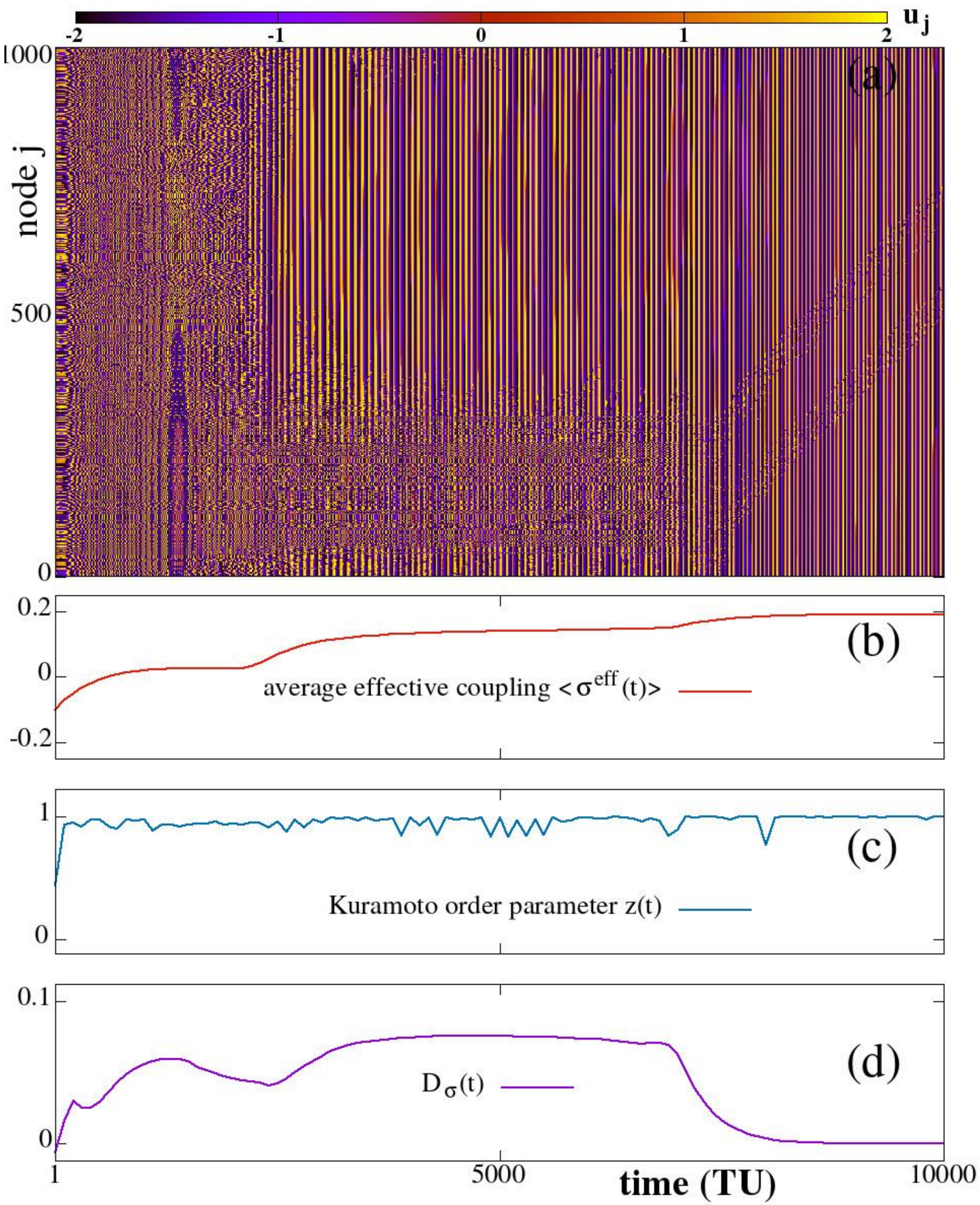
u(t=0) = random
v(t=0) = random

< σ_{ij}(t=0)> = -0.2
(randomly distributed around -0.2)

=>

Note abrupt transitions …

6.2 FitzHugh-Nagumo networks with adaptive coupling



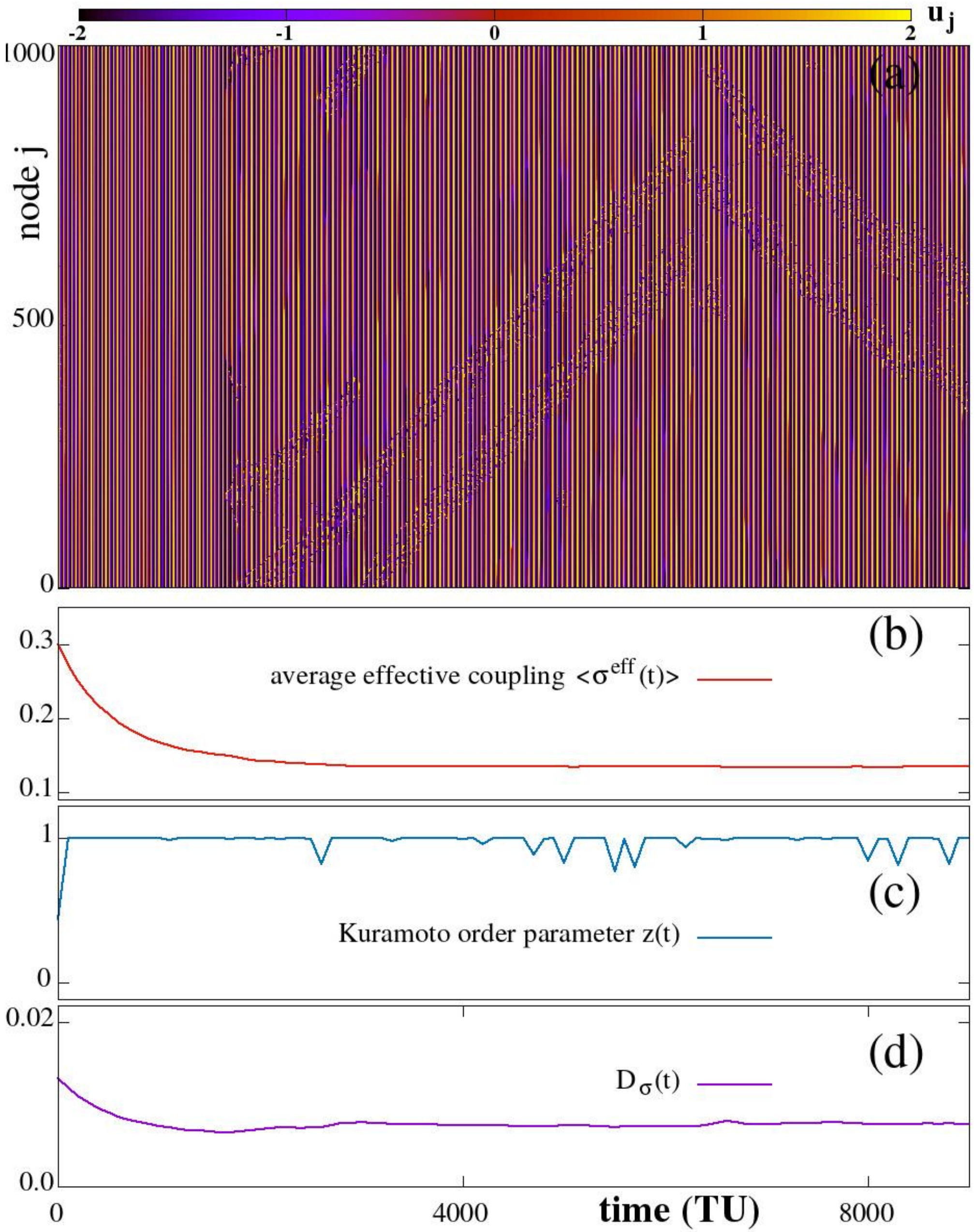
$u(t=0)$ = random
(same as before)

$v(t=0)$ = random
(same as before)

$<\sigma_{ij}(t=0)> = -0.2$
(different random realization around -0.2)

=> different realizations lead to different routes to same final state

6.3 FitzHugh-Nagumo networks with adaptive coupling



u(t=0) = random (same as before)

v(t=0) = random (same as before)

$<\sigma_{ij}(t=0)> = +0.3$
(different random realization around -0.2)

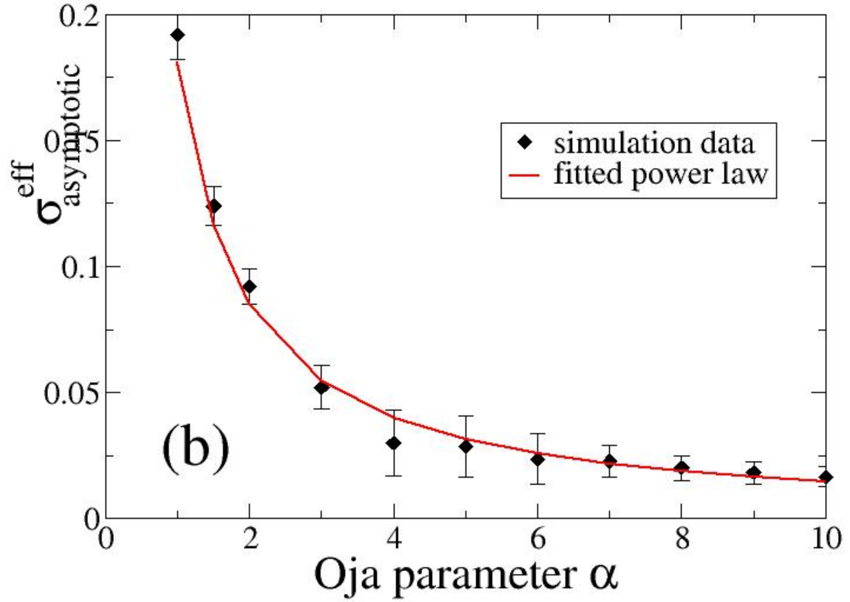
$\Rightarrow$ ends at
$<\sigma_{ij}> = 0.14$

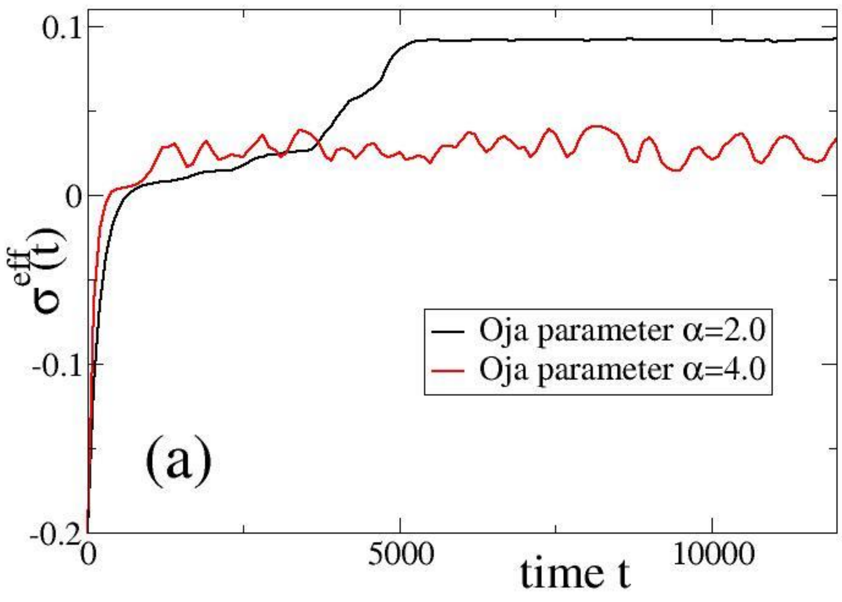
As $t \to \infty$ $\langle\sigma\rangle \to 1/\alpha$

or $\langle\sigma^{eff}\rangle \to \sigma_c/\alpha$

for $\sigma_c = 0.2$

$y = 0.18\ x^{-1.073}$





for $\sigma_c = 0.2$ & $\alpha = 2$

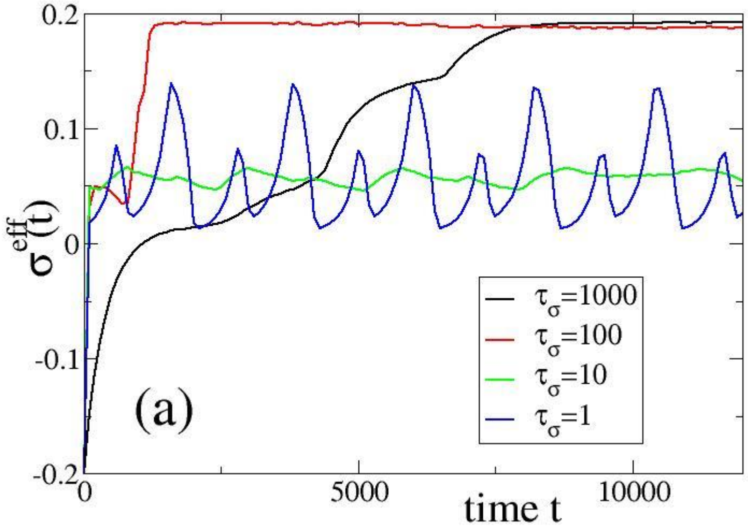
$\langle\sigma^{eff}\rangle \to 0.2/2 \to 0.1$

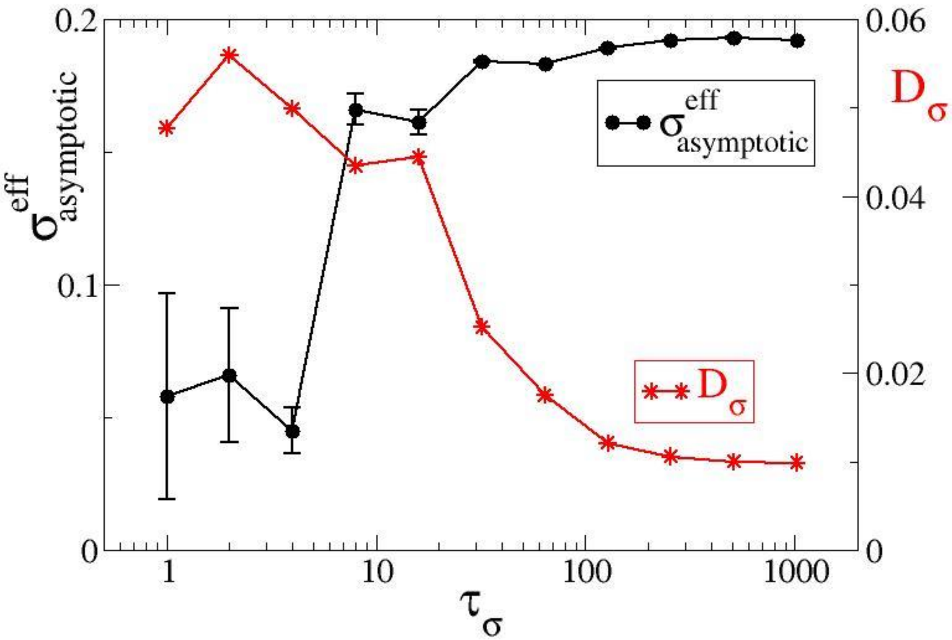
for $\sigma_c = 0.2$ & $\alpha = 4$

$\langle\sigma^{eff}\rangle \to 0.2/4 \to 0.05$

As $t \to \infty$ $\langle\sigma^{eff}\rangle \to \sigma_c/\alpha$

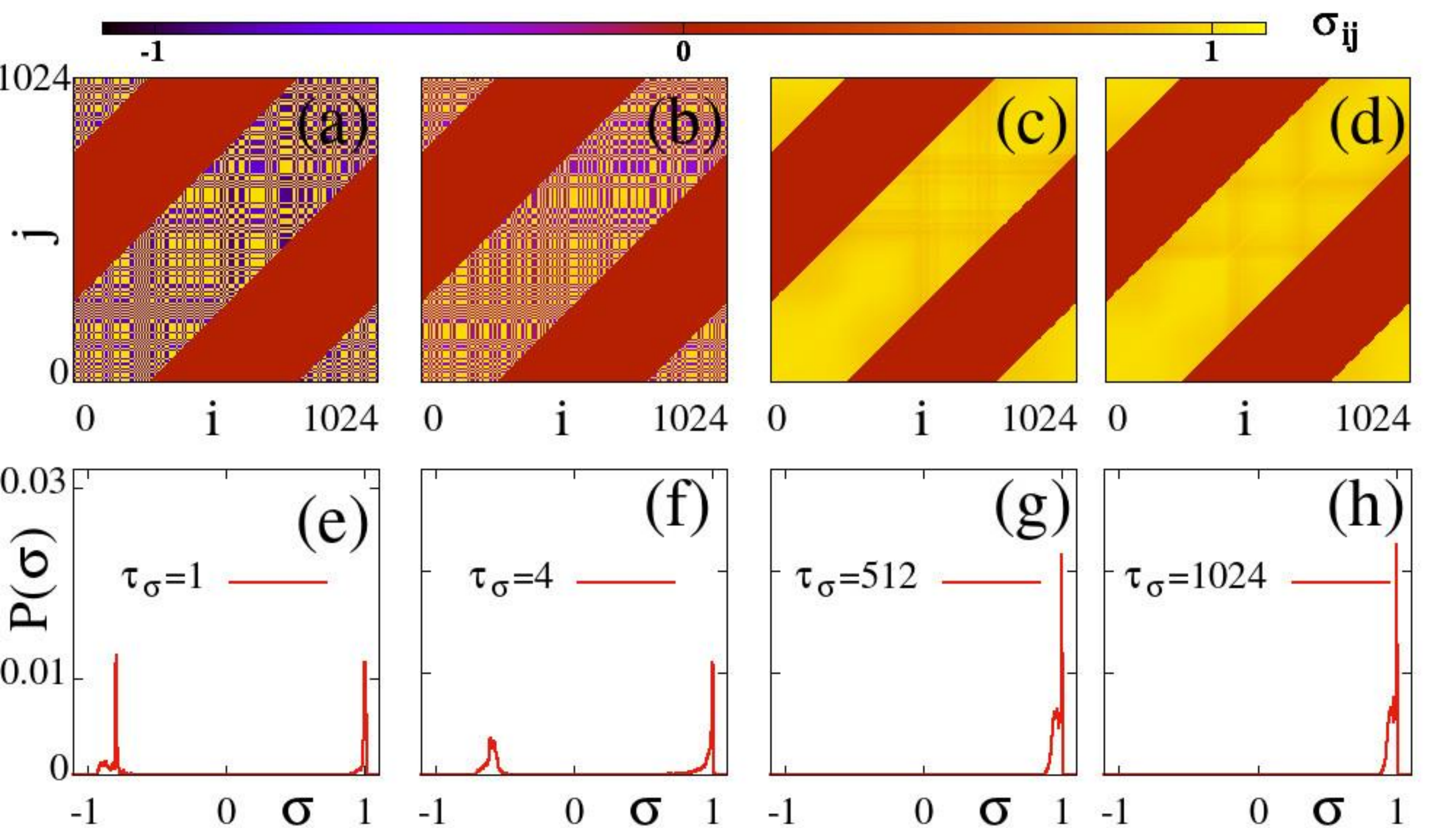
Only for large τ_σ the system reaches the asymptotics





Note the sharp transition at intermediate τ_σ values: $\langle\sigma\rangle$ abruptly increases & fluctuations abruptly decrease.

Distribution of σ_{ij} at the asymptotic state for different of τ_σ



7. Conclusions

- Chimera and bump states in LIF & FHN limit cycle dynamics
- Nonlocal connectivity with constant weights produces hybrid synchronization states
- Adaptivity of the link weights leads to interesting transition phenomena crossing domains of different synchronization

Open Problems

- Not all multiplicities “show” as the average weights cross from negative to positive values (and back)….
- Still the problem of the spatial symmetry breaking leading to hybrid synchronization remains open ...
- Explore further the sharp transition in σ_{ij} at intermediate τ_σ
- Study further the effects of plasticity (parameter domains)
- Influence of initial conditions...
- Other learning rules (Bienenstock-Cooper-Murno, etc)…

Collaborations & Thanks

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!!Thank you very much for your attention!!