Chaos or Noise?
Identifying the Origin of Irregular Dynamics from Time Series Using Machine Learning

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Asia Pacific Center for Theoretical Physics

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KOREA INSTITUTE FOR ADVANCED STUDY

Jaesung Choi (CAINS, KIAS)
Athokpam Langlen Chanu (APCTP)

3rd INPP Demokritos-APCTP workshop @ Athens, 29 Jun. 2026

Statistical Physics in APCTP Group (S-PA Group)

Junior Research Group

Thermodynamics of microscopic nonequilibrium systems

Jong-Min Park

Athokpam Langlen Chanu

Yuanfei Huang

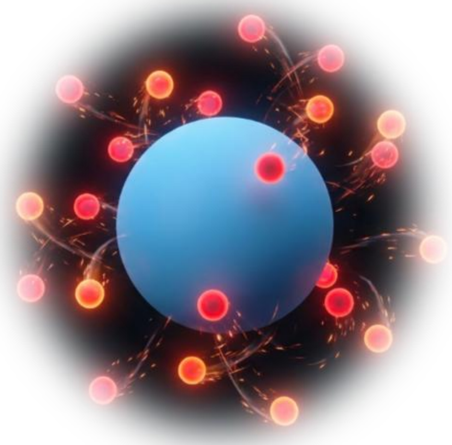
Kiran Vijayakumar

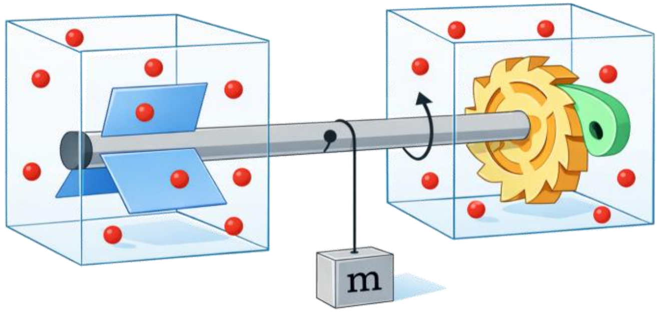
Kirandeep Kaur

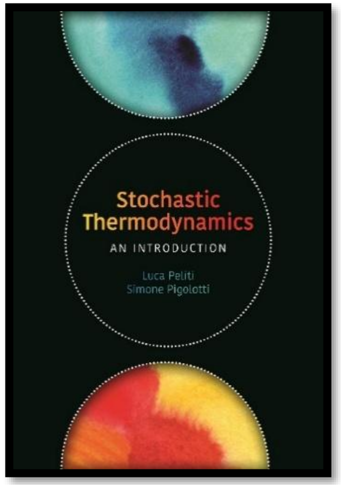
Young Scientist Training program

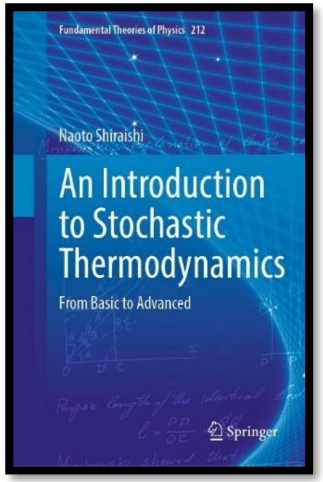
Roni Muslim

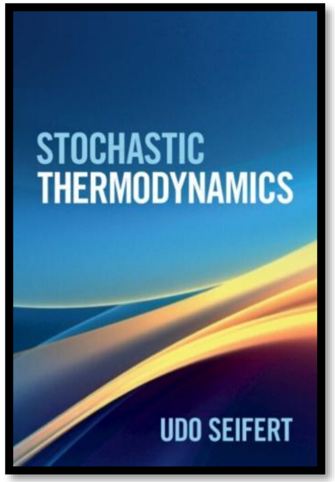
Research scope
Stochastic thermodynamics



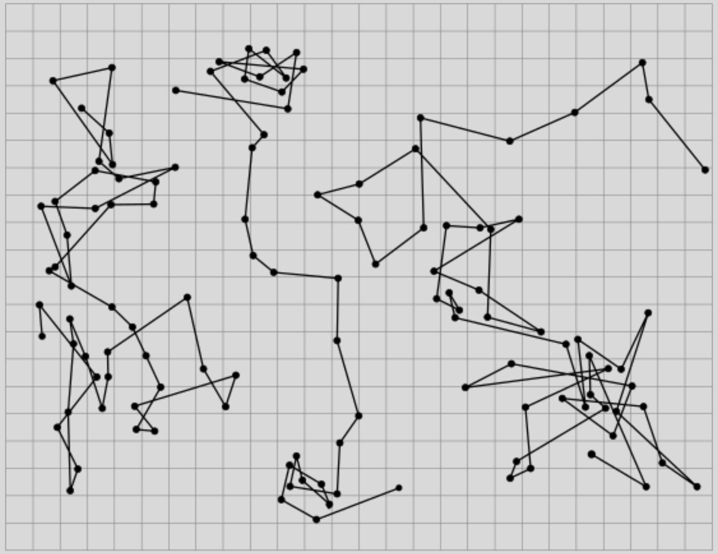




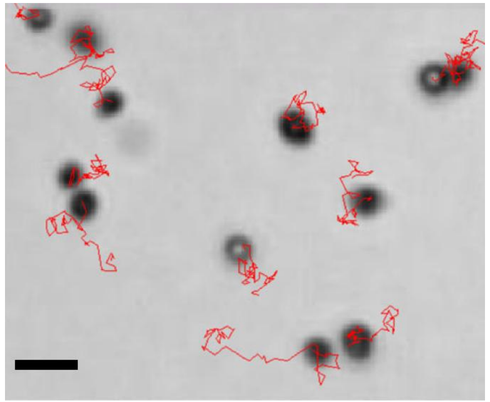




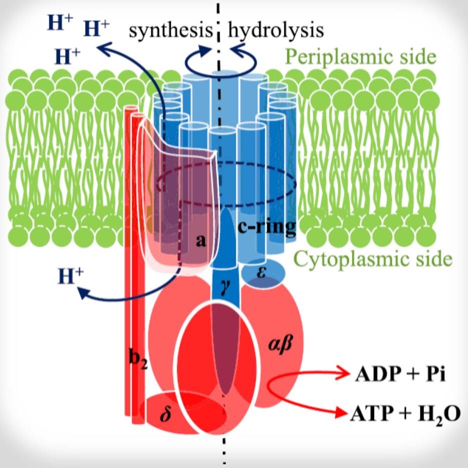
Brownian motion (1900s)



J. B. Perrins' drawing ©Wikipedia



colloidal particles
Singh *et al.*, Nat. Commun. (2020)



molecular motor
Hou and Wang, BBA-Bioenergetics (2021)

Research scope

Stochastic thermodynamics

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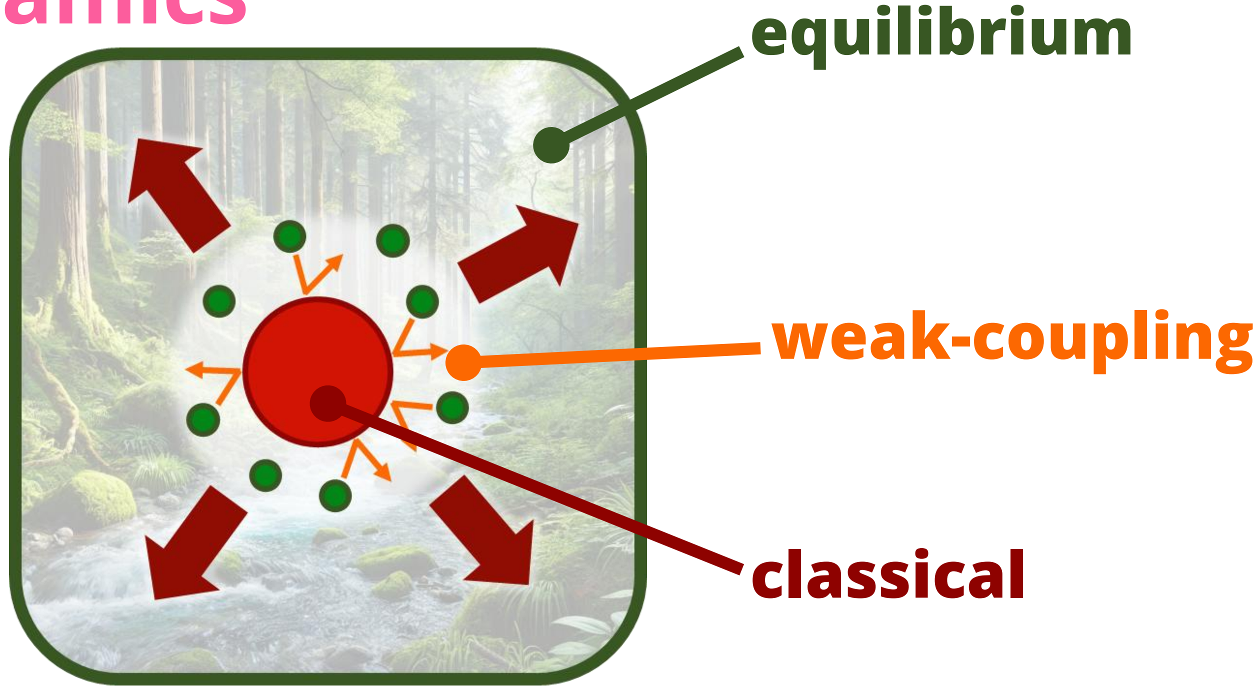

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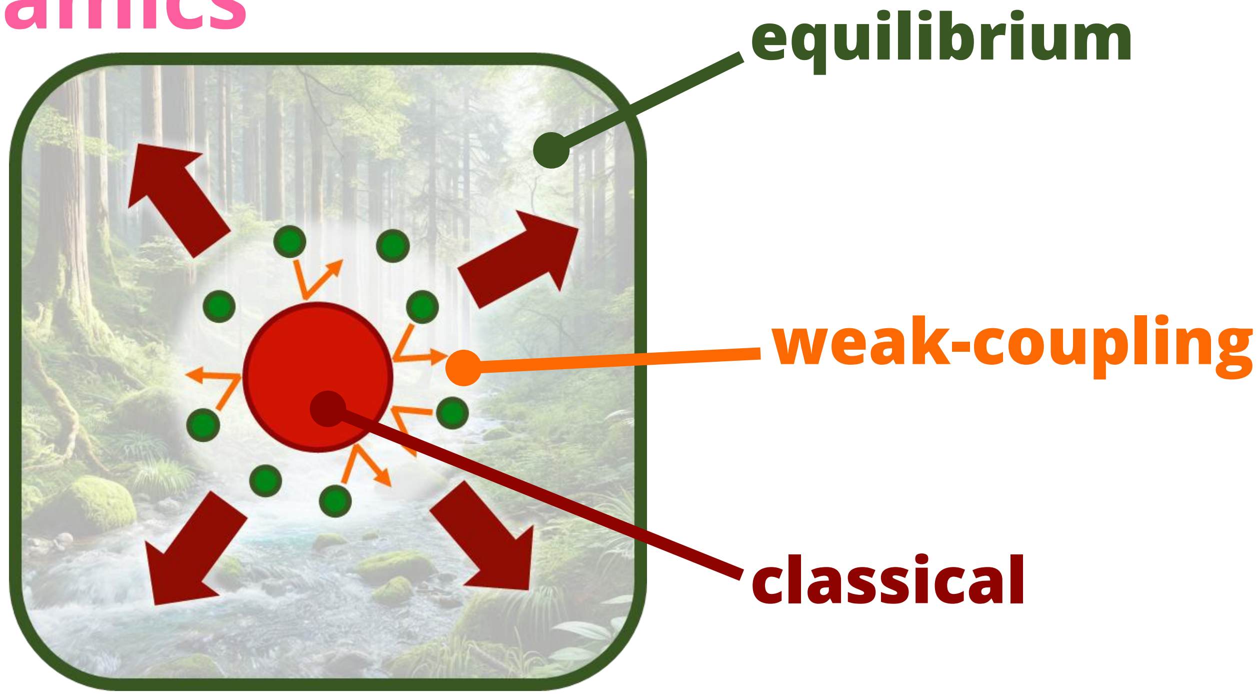
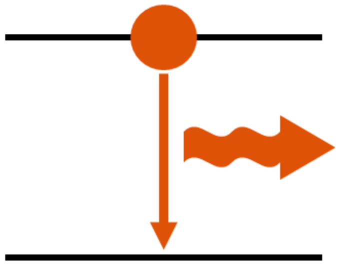
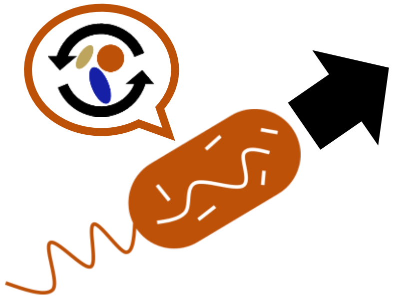
Stochastic thermodynamics



quantum system



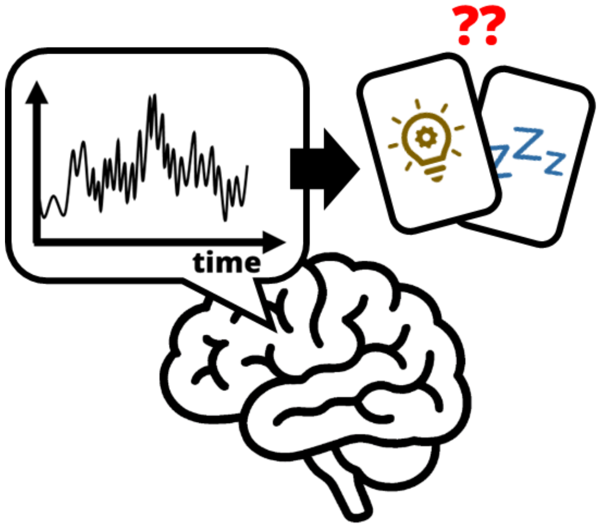
active system

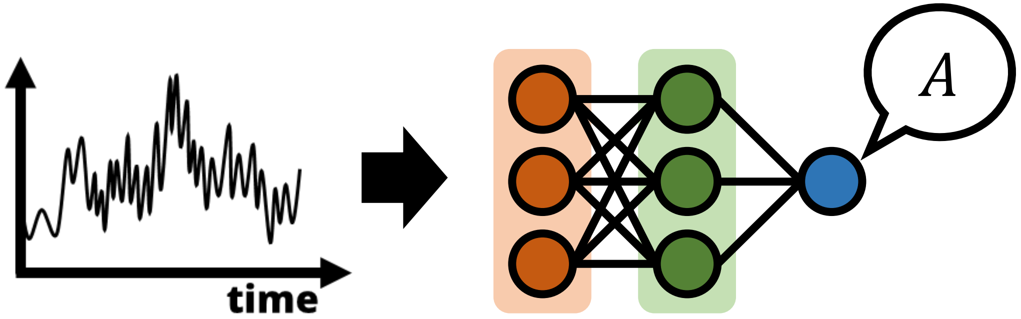


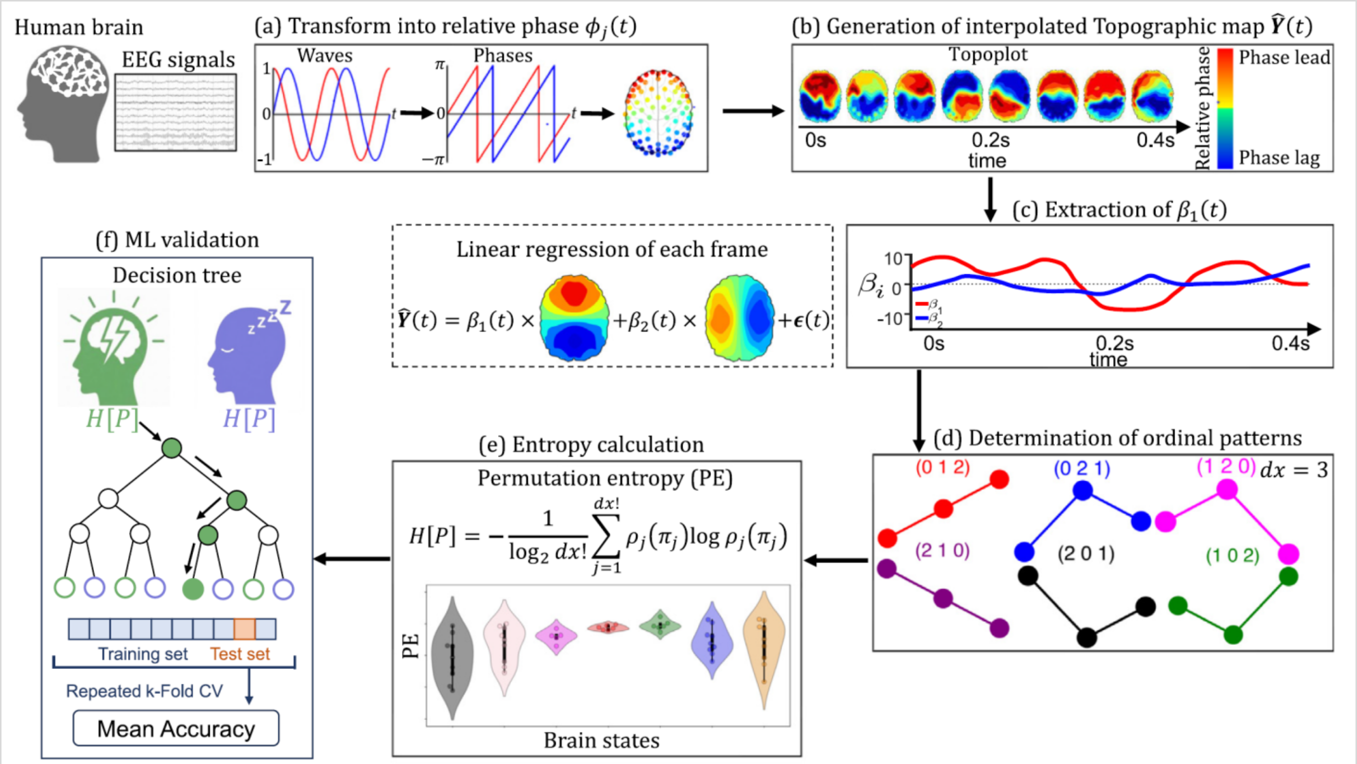
strongly-coupled
systems

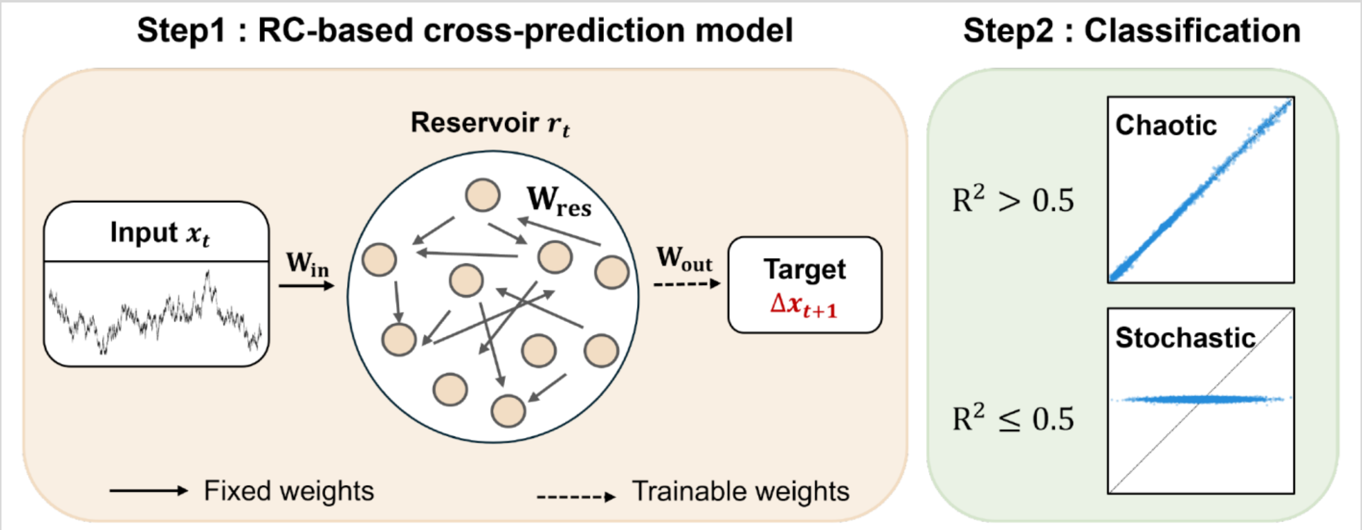
Research scope

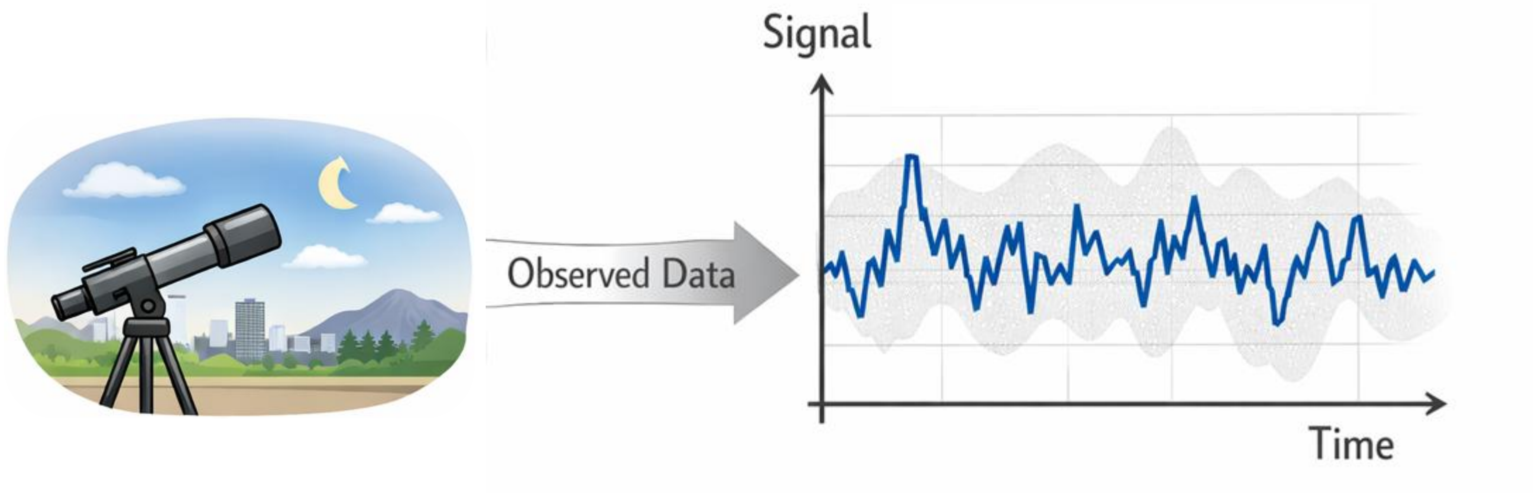
Time series analysis



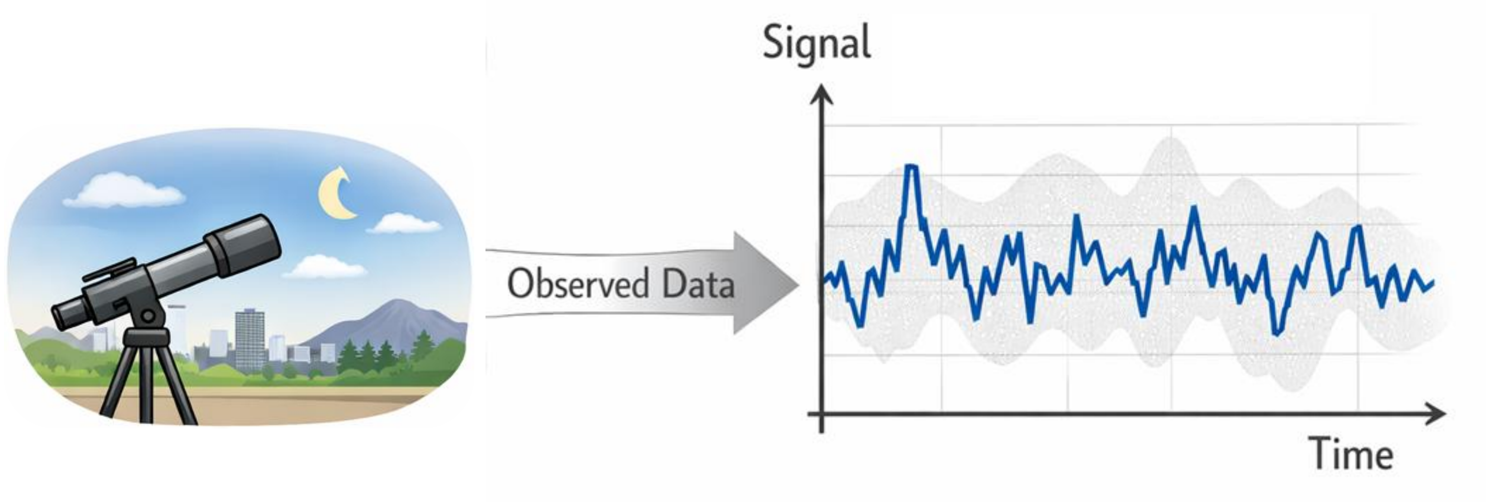








Where does **irregularity** in empirical time series come from?



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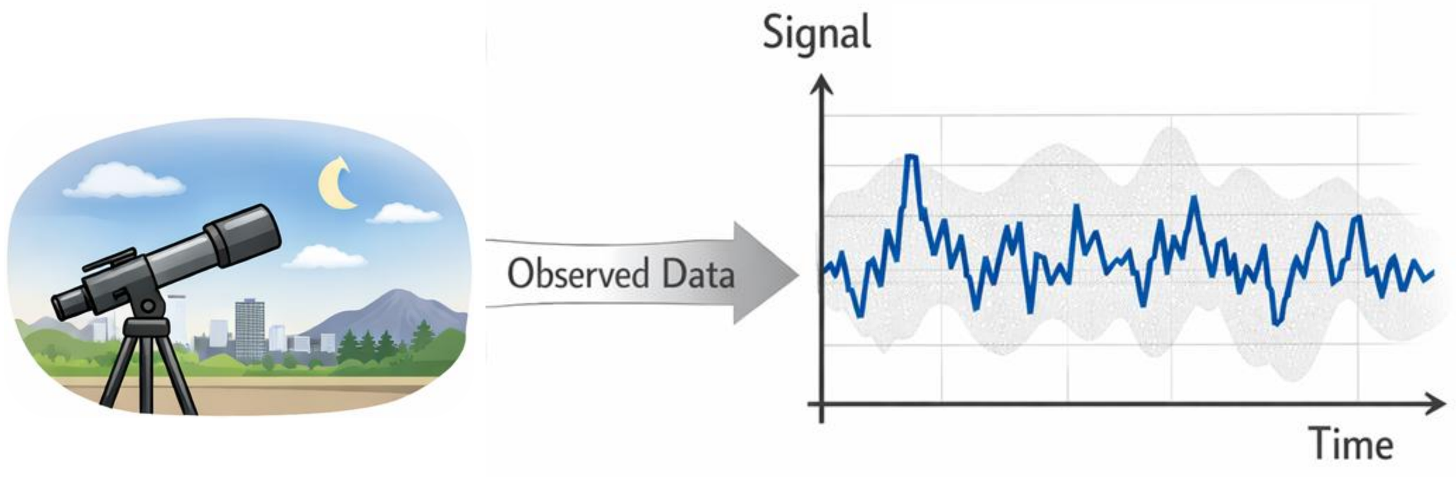
Noise



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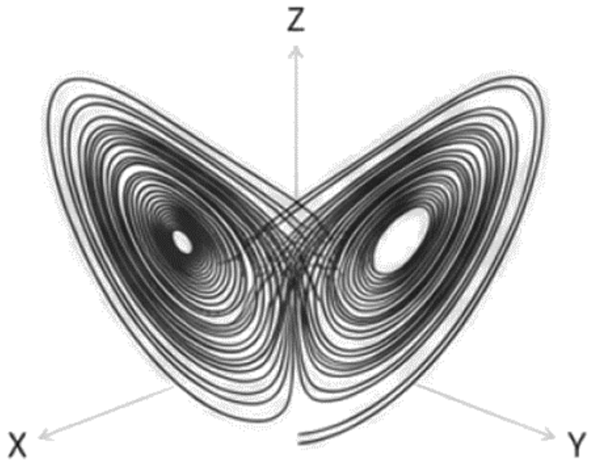
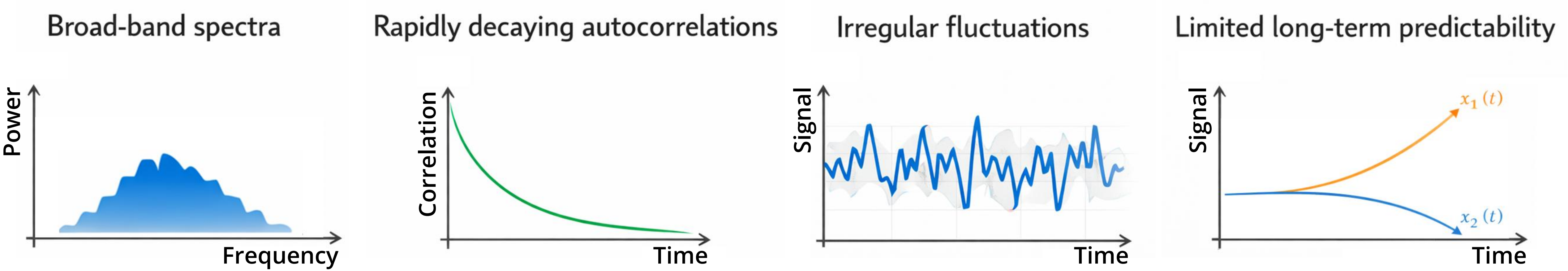
Chaos



Noise

Chaos and noise are often difficult to distinguish due to their **similar behavior**.



Some stochastic processes can mimic chaotic processes

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- spurious positive Lyapunov exponent

[A. R. Osborne and A. Provenzale, Physica D **35**, 357 (1989)]
[J. Theiler, Physics letters A **155**, 480 (1991)]

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- spurious positive Lyapunov exponent

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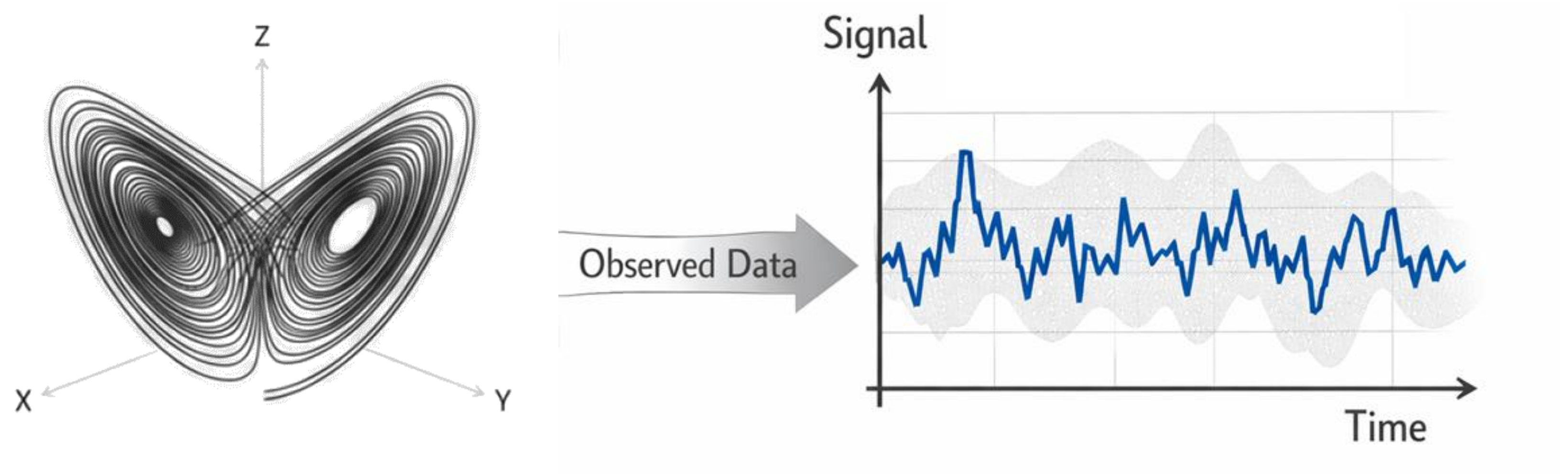
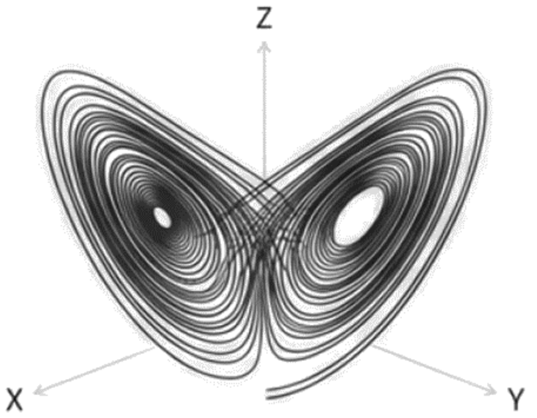
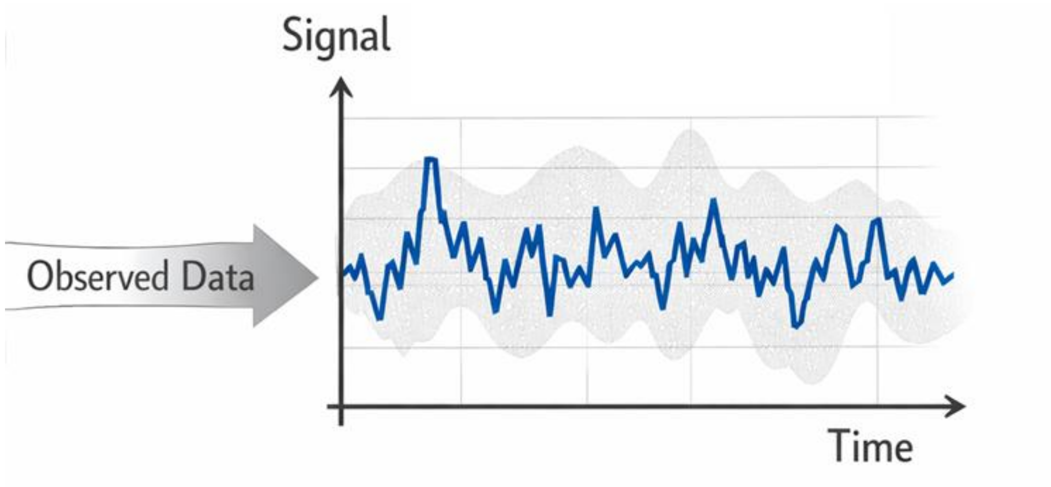
- finite correlation dimension

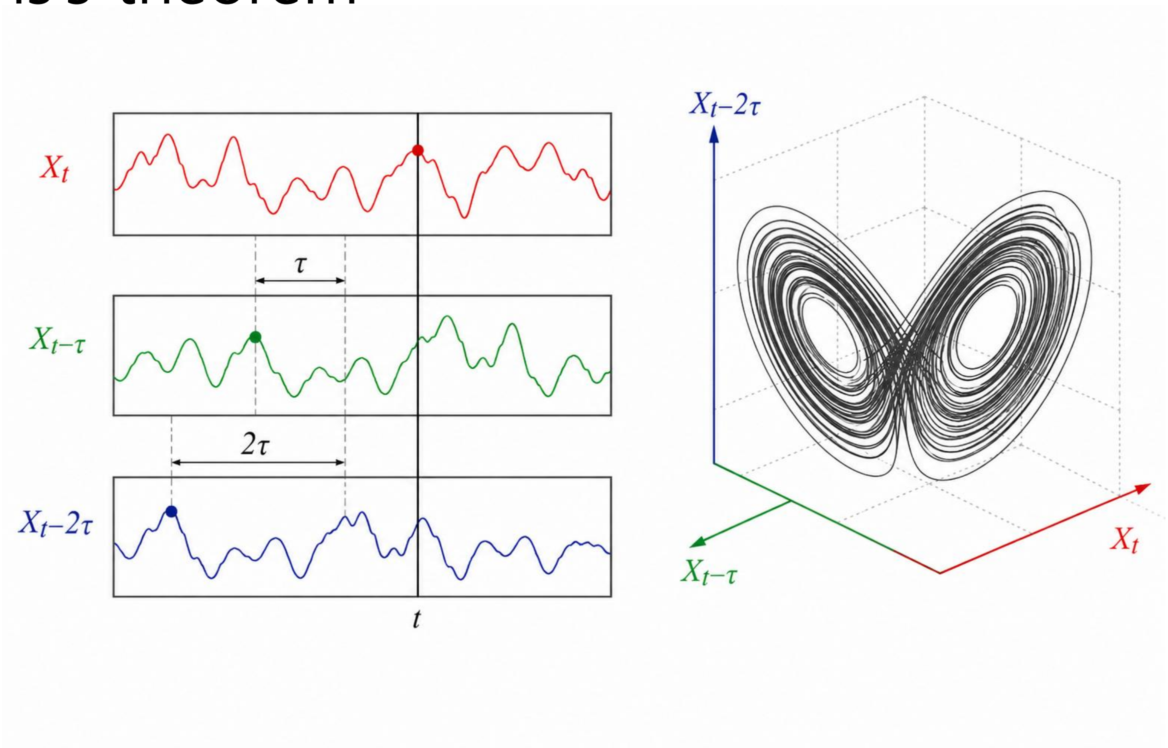
  [T. Tanaka, K. Aihara, and M. Taki, Physical Review E **54**, 2122 (1996)]
  [T. Ikeguchi and K. Aihara, International Journal of Bifurcation and Chaos **7**, 1267 (1997)]

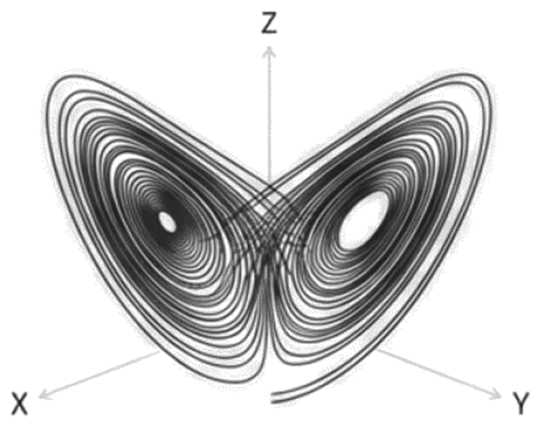

Z
X
Y
Observed Data
Signal
Time

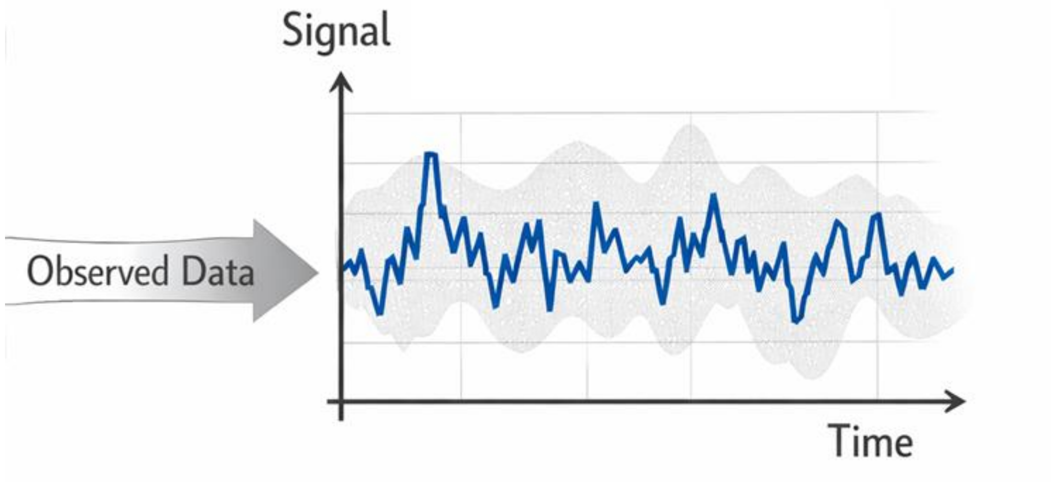




Takens's theorem







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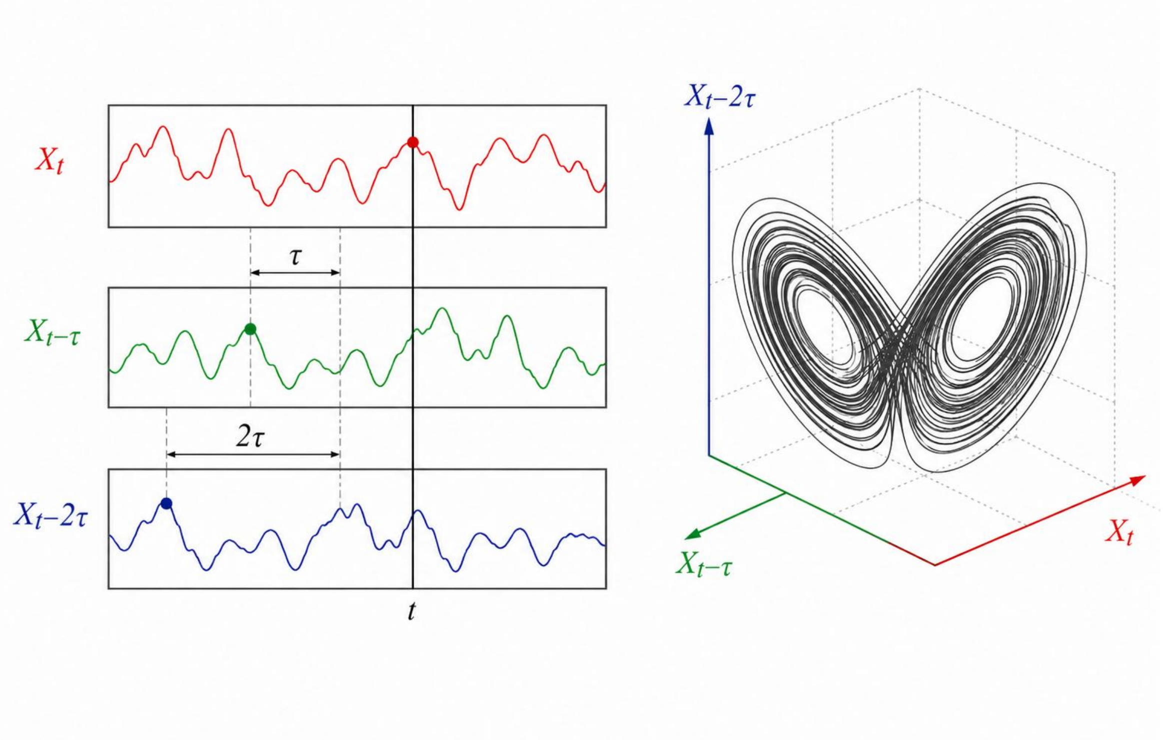

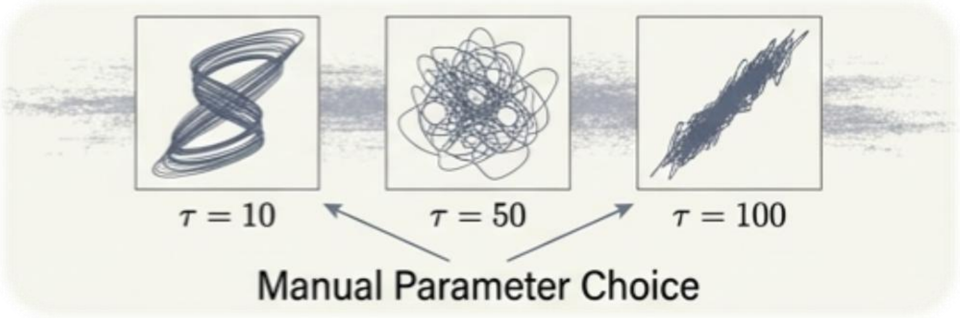

Detecting chaos from empirical time series is difficult due to **measurement noise**, **finite resolution** and **limited data length**.

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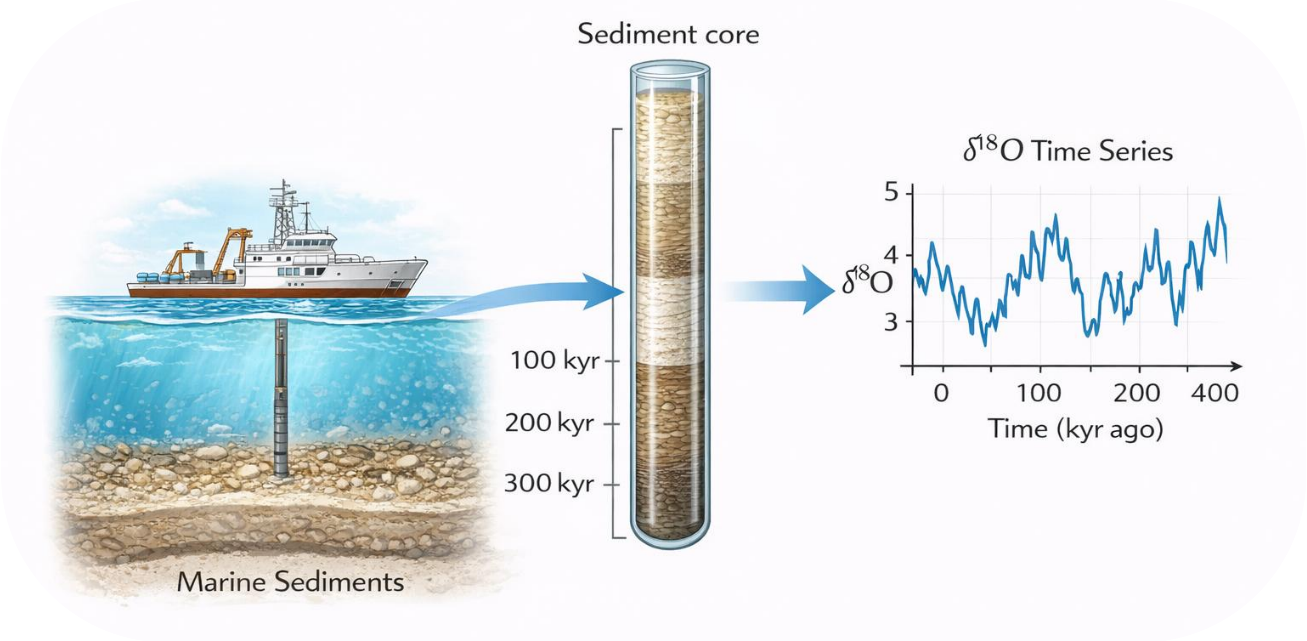

Dose climate dynamics lie on a low-dimensional strange attractor?

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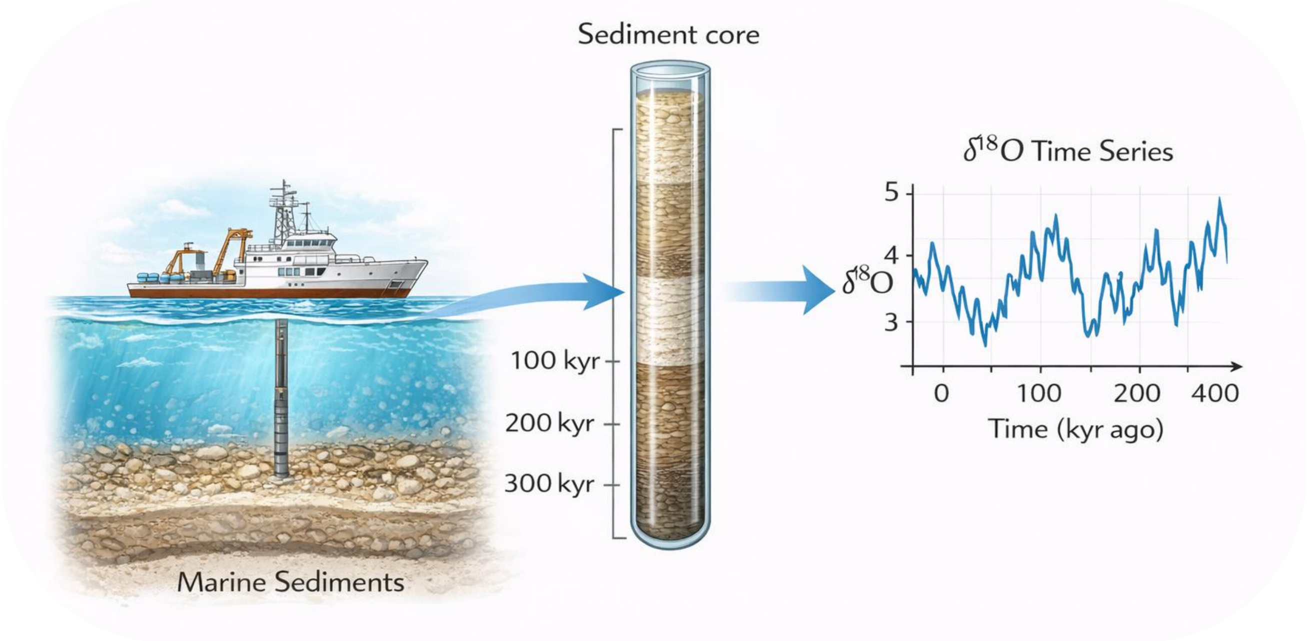


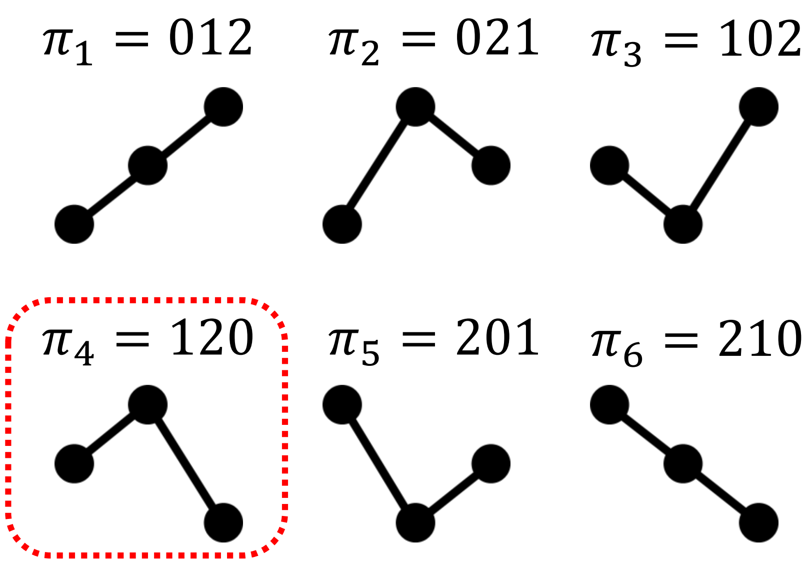
Dose climate dynamics lie on a low-dimensional strange attractor?

1984 Nicolis	1986 Grassberger	1987 Nicolis	1987 Grassberger
Yes	No	Yes	No
[Nature **311**, 529]	[Nature **323**, 609]	[Nature **326**, 523]	[Nature **326**, 524]

Ordinal pattern analysis

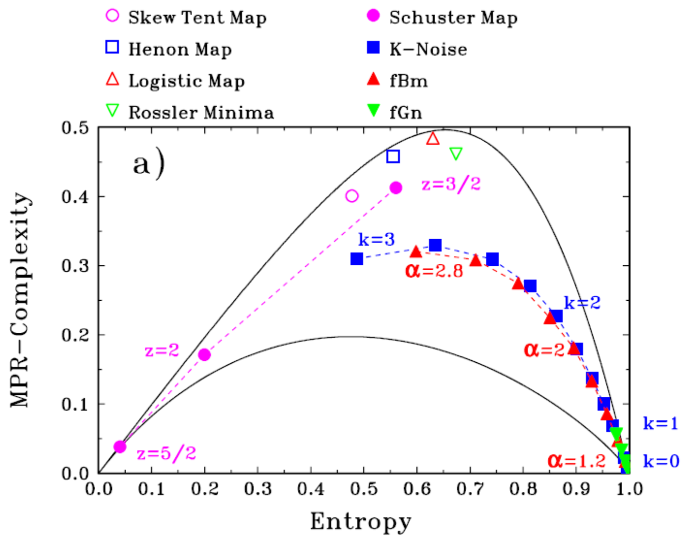
$$x(t) = \{0.5, 0.4, \underbrace{\overset{1}{-0.7}, \overset{2}{-0.6}, \overset{0}{-0.9}}_{dx=3}, \dots\}$$

$$\pi(t) = \{\pi_6, \pi_5, \pi_4, \dots\}$$



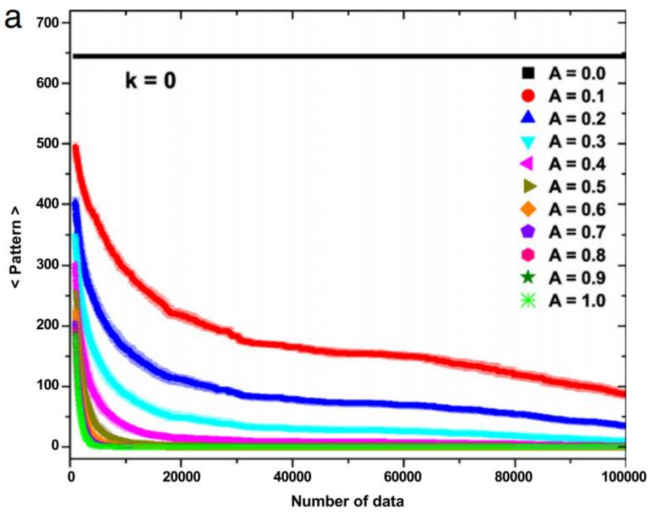
Entropy-complexity plain

[PRL **99**, 154102 (2007)]



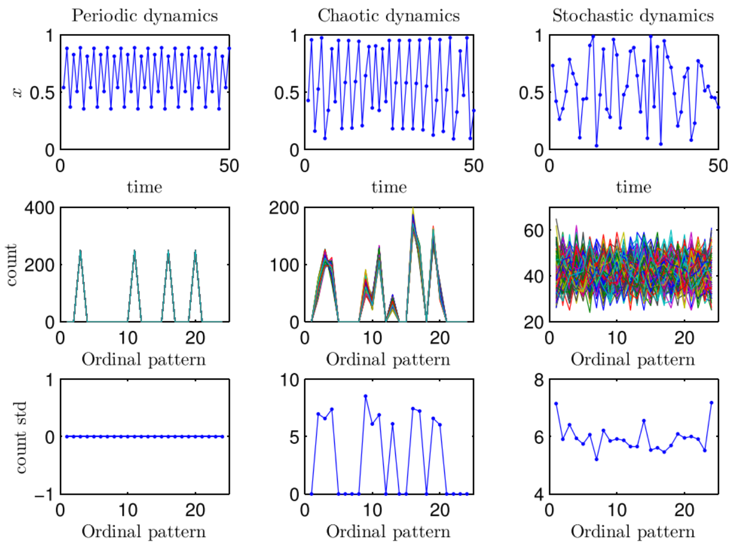
Forbidden pattern

[Physica A **391**, 42 (2012)]



Permutation spectrum

[Chaos **24**, 033116 (2014)]



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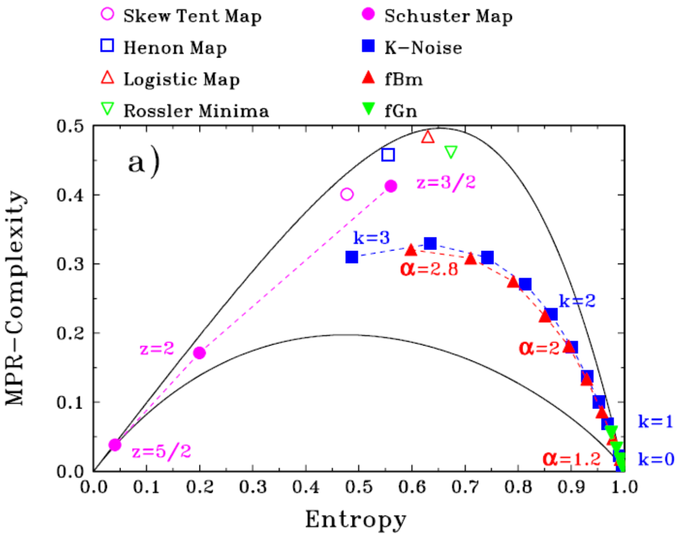

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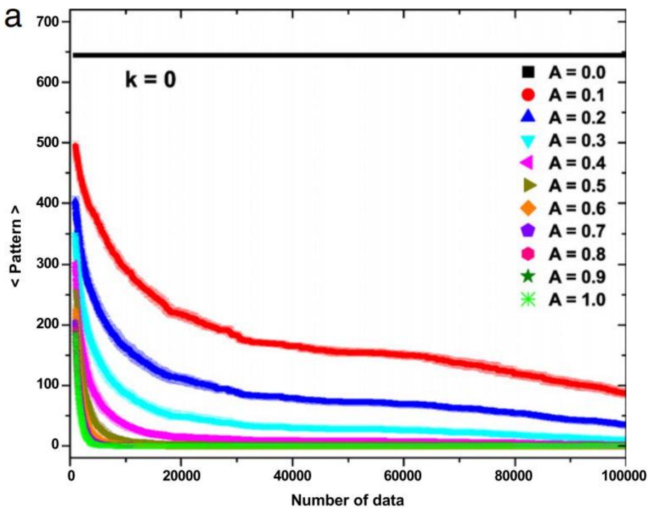

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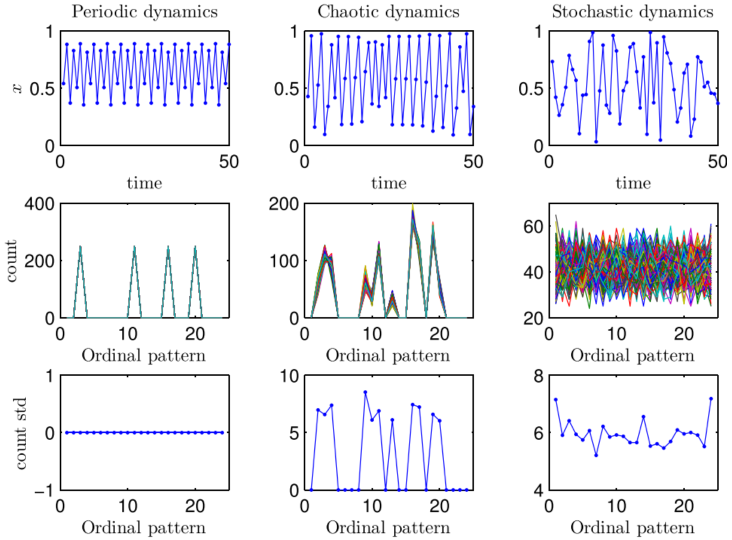
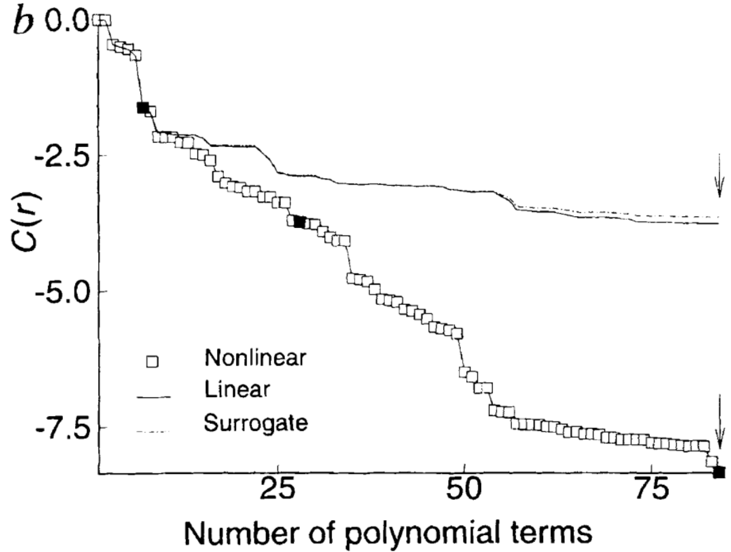


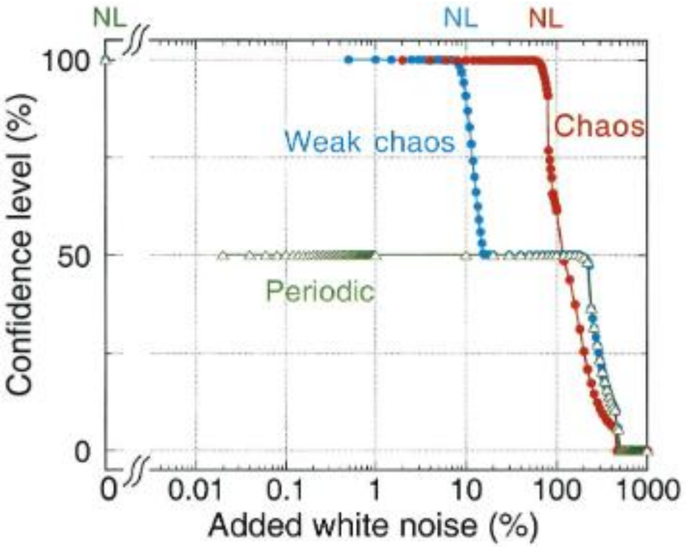
Nonlinear prediction test

[Nature **381**, 215 (1996)]



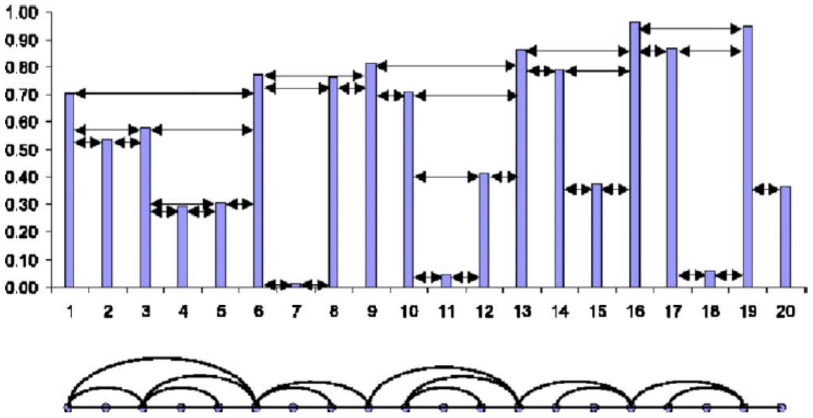
Noise titration

[PNAS **98**, 7107 (2001)]



Visibility graph

[PRE **82**, 036120 (2010)]



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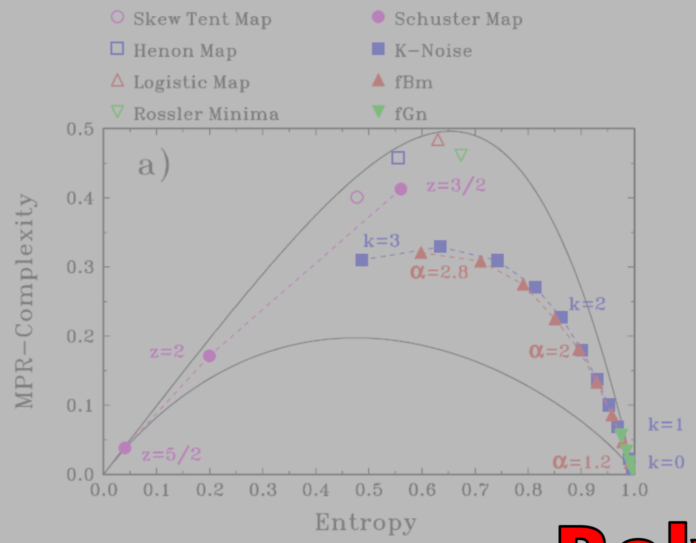

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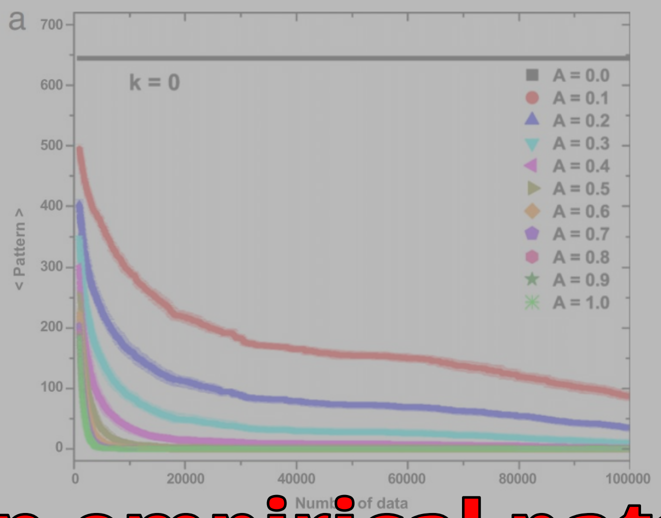

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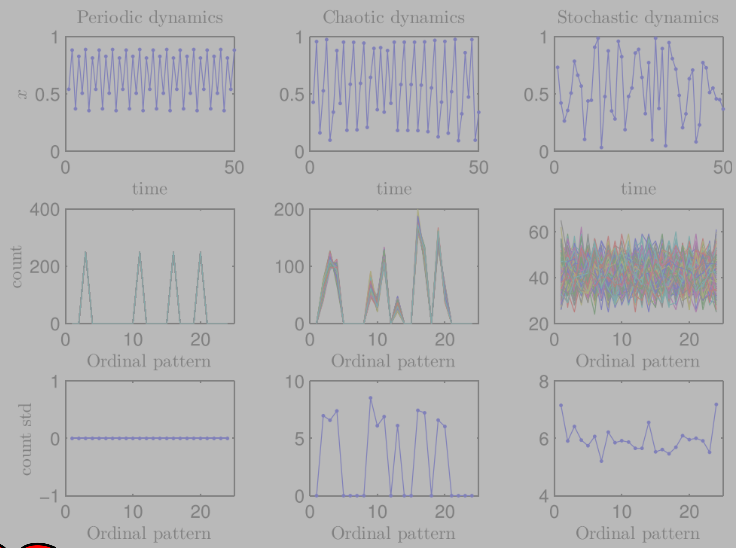

Rely on empirical patterns

Nonlinear prediction test

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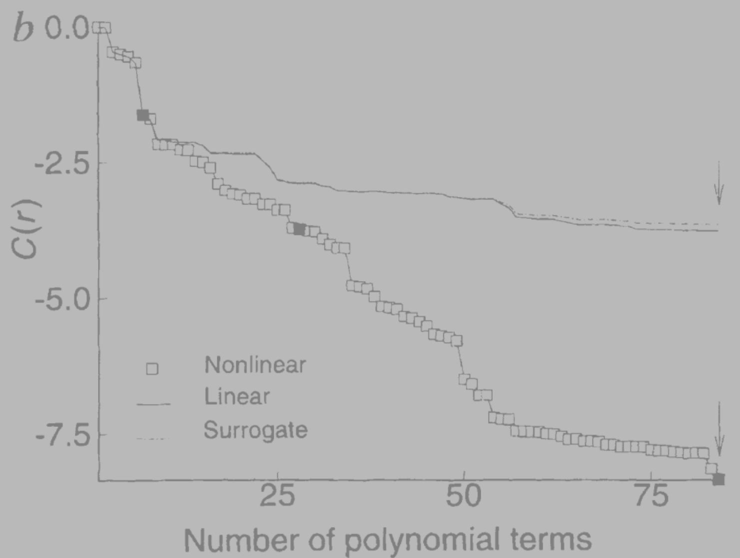

Noise titration

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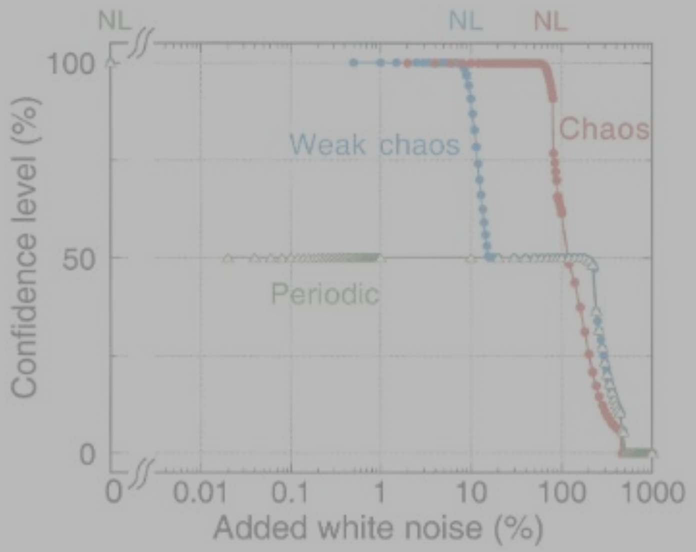
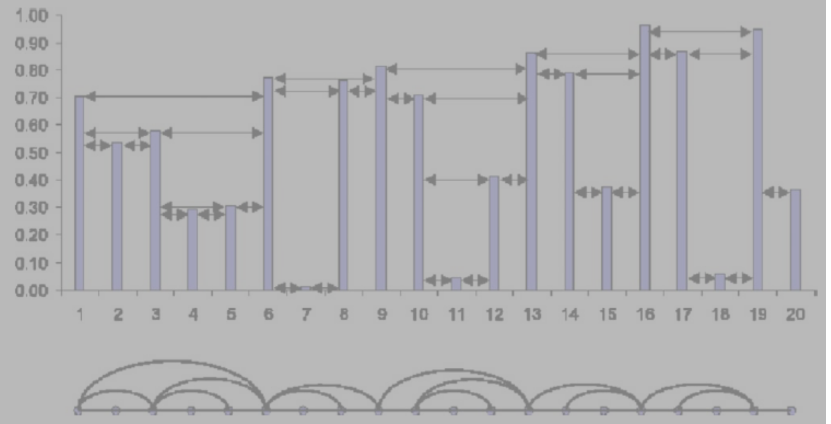


Visibility graph

[PRE **82**, 036120 (2010)]

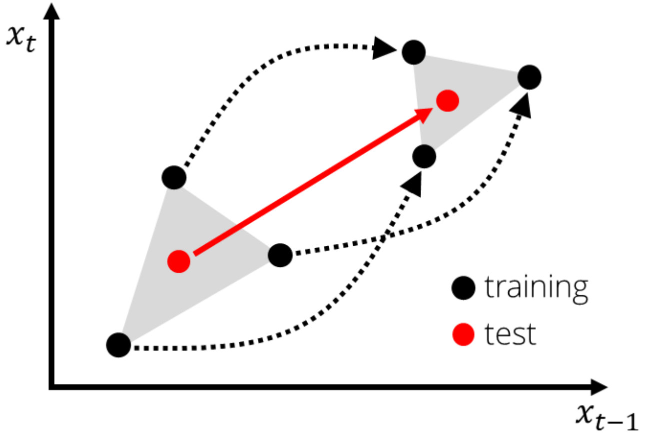


Short-term predictability

Nonlinear forecasting

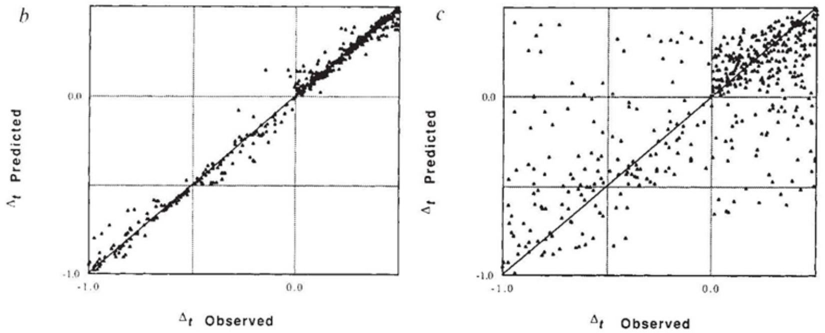
[Suglihara and May, Nature (1990)]

Simplex projection



Chaos

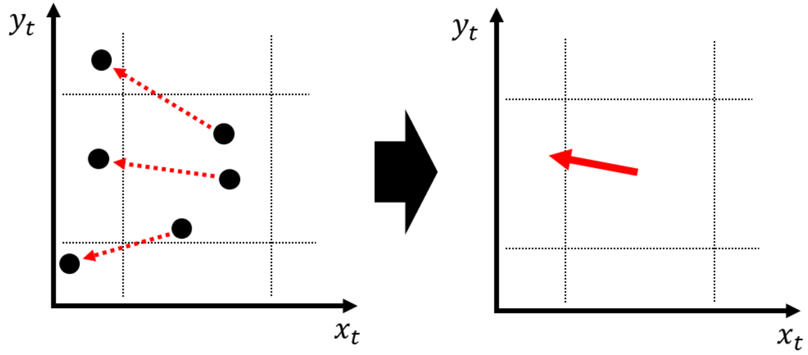
Noise

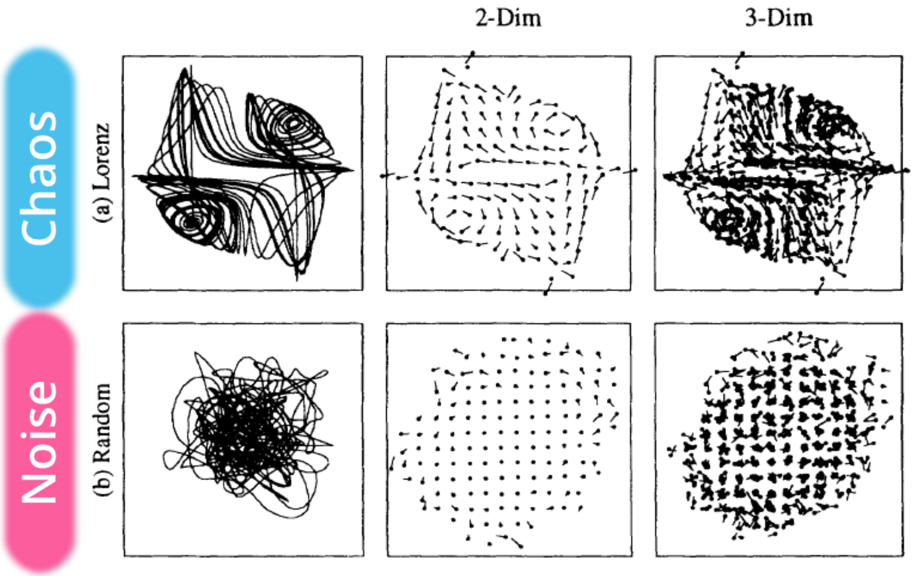


Deterministic dynamical flow

Text for determinism

[Kaplan and Glass, Phys. Rev. Lett. (1992)]





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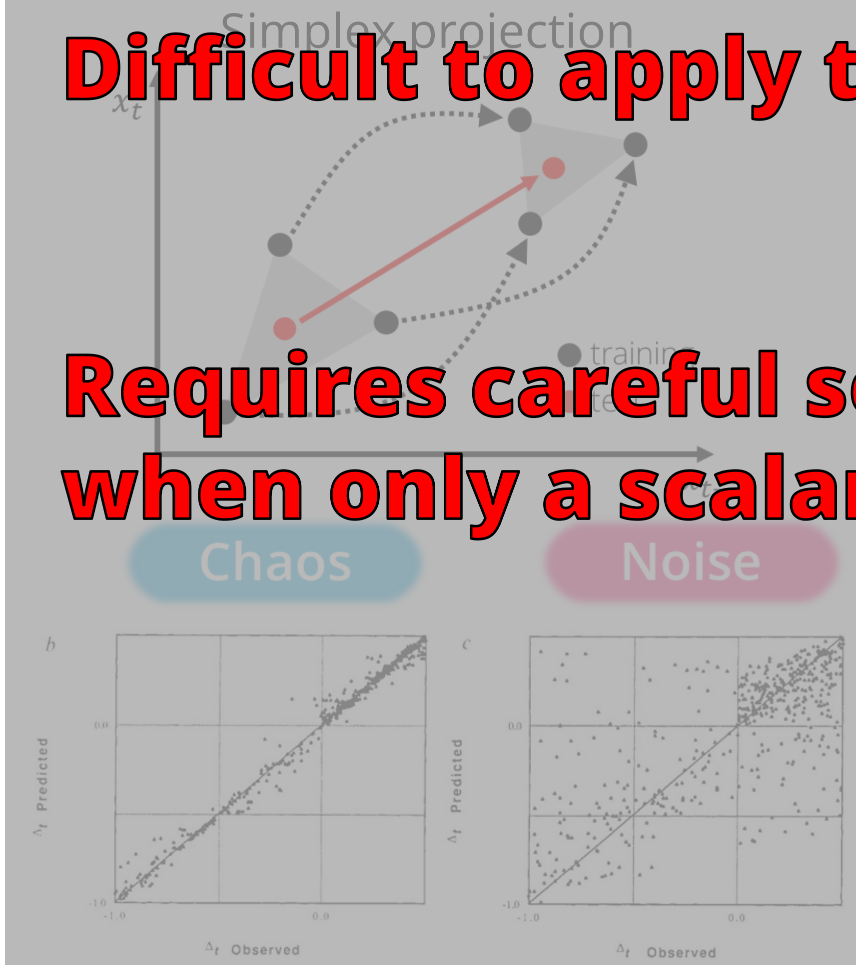
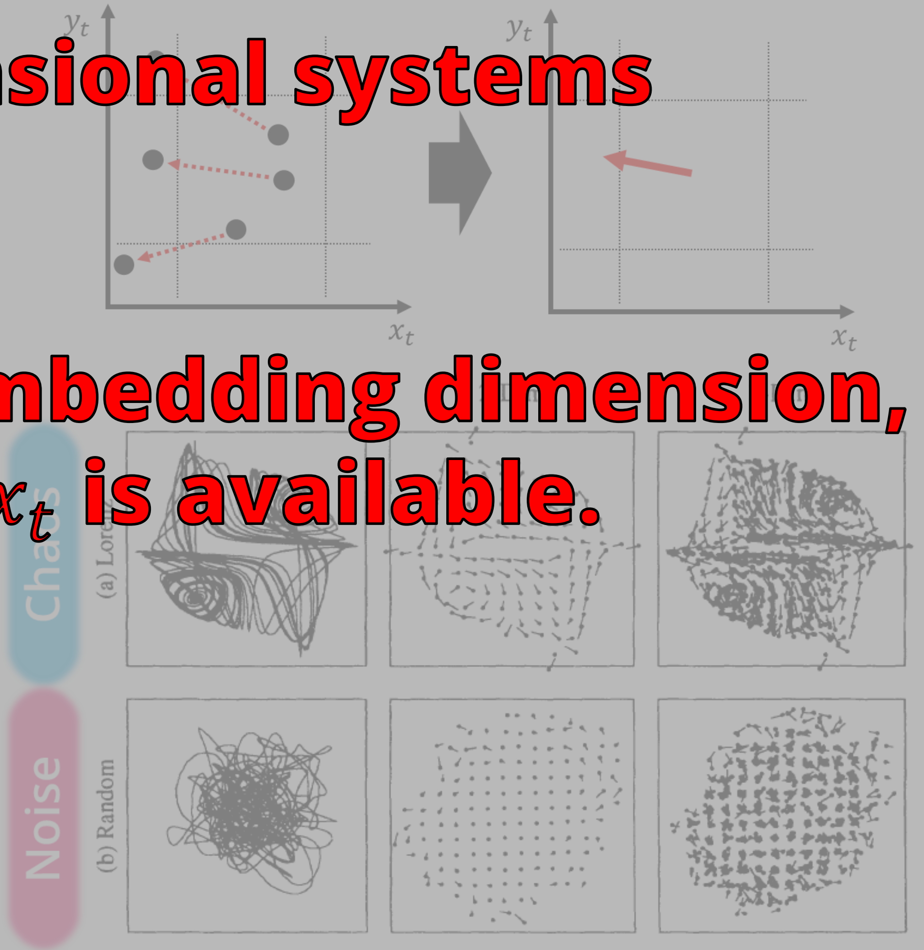
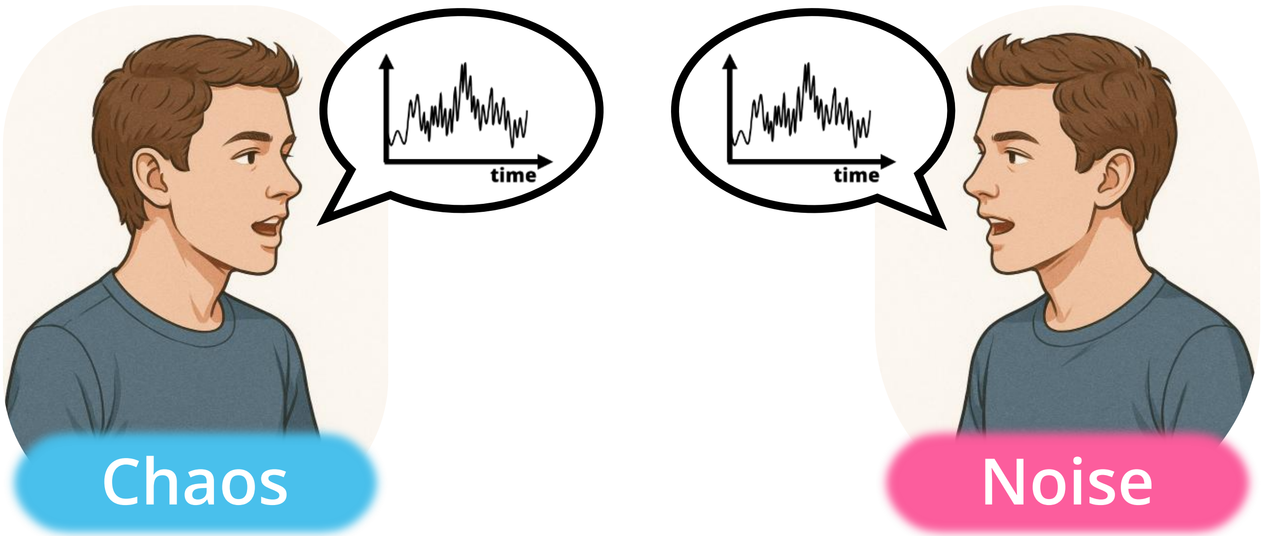




Difficult to apply to high-dimensional systems

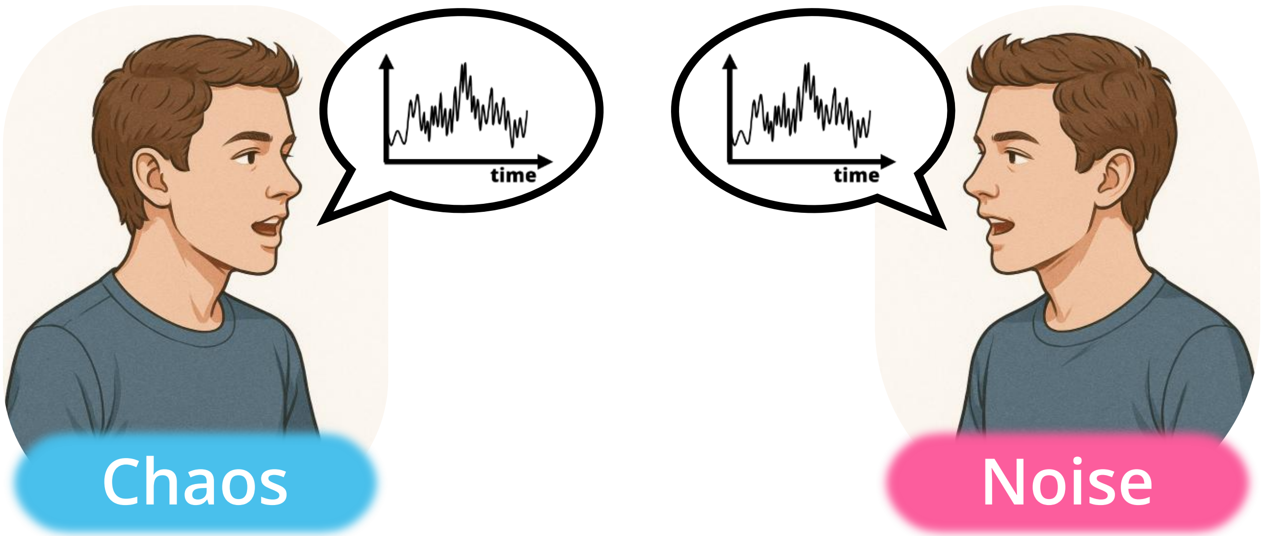
Requires careful selection of embedding dimension, when only a scalar time series x_t is available.

Voice of twins



It is challenging to distinguish chaos and noise.

Voice of twins



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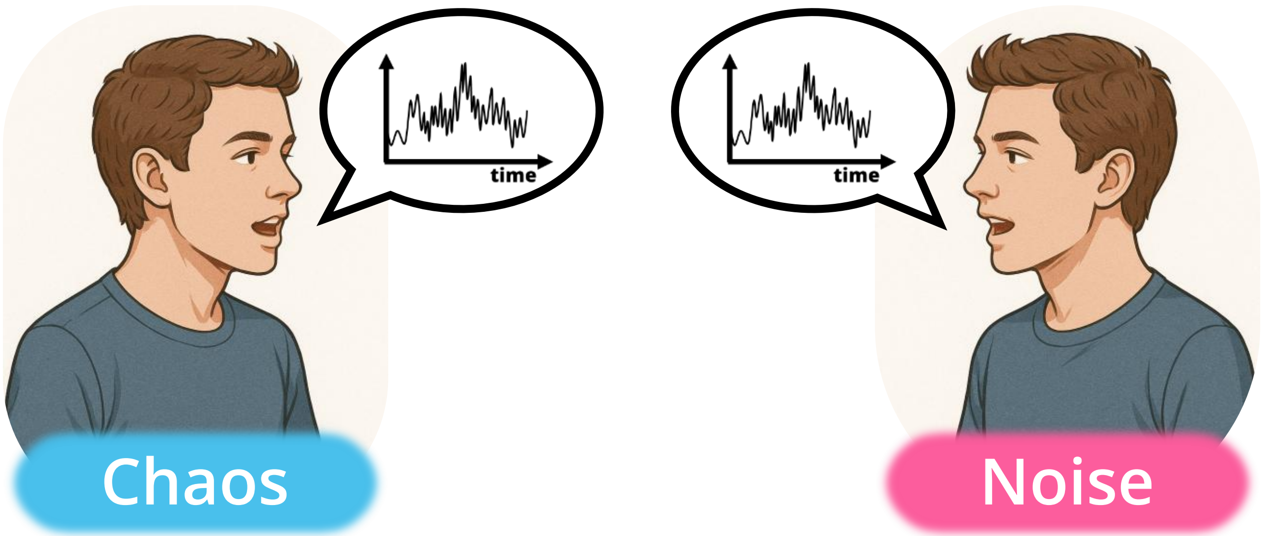

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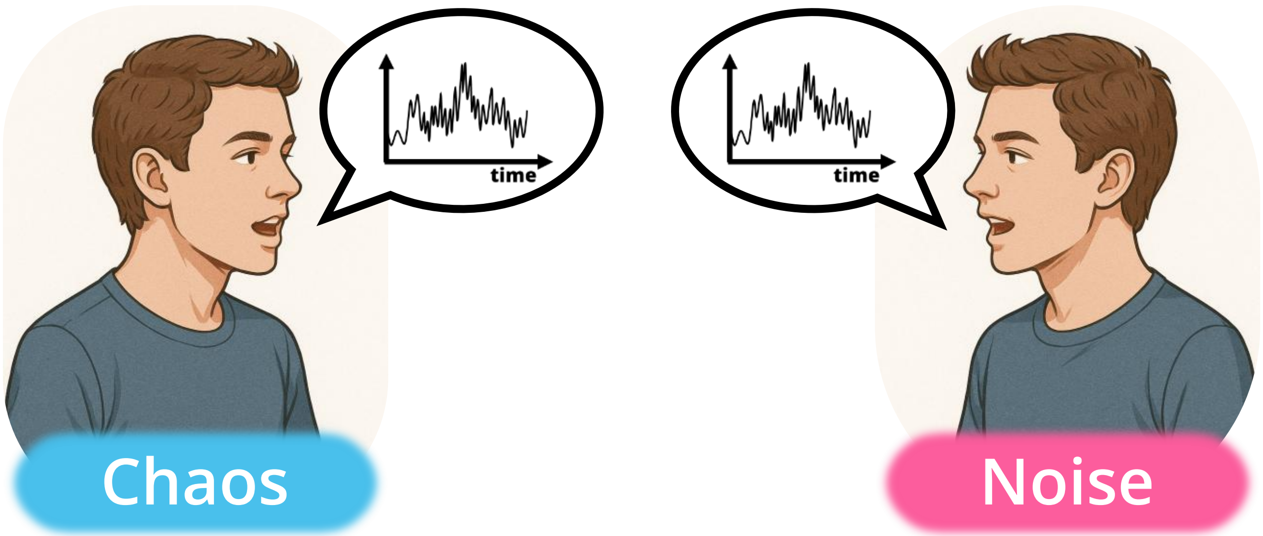
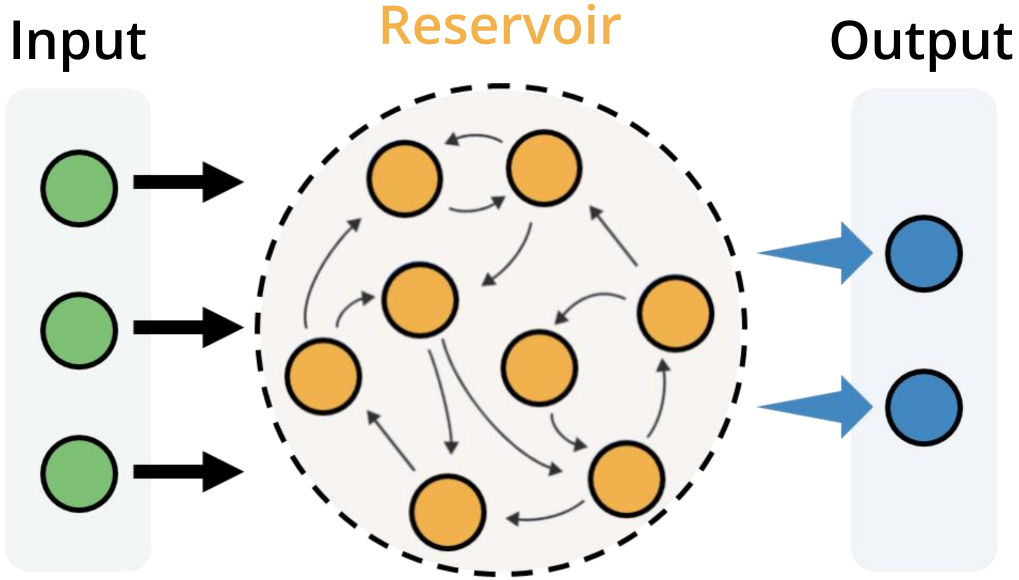


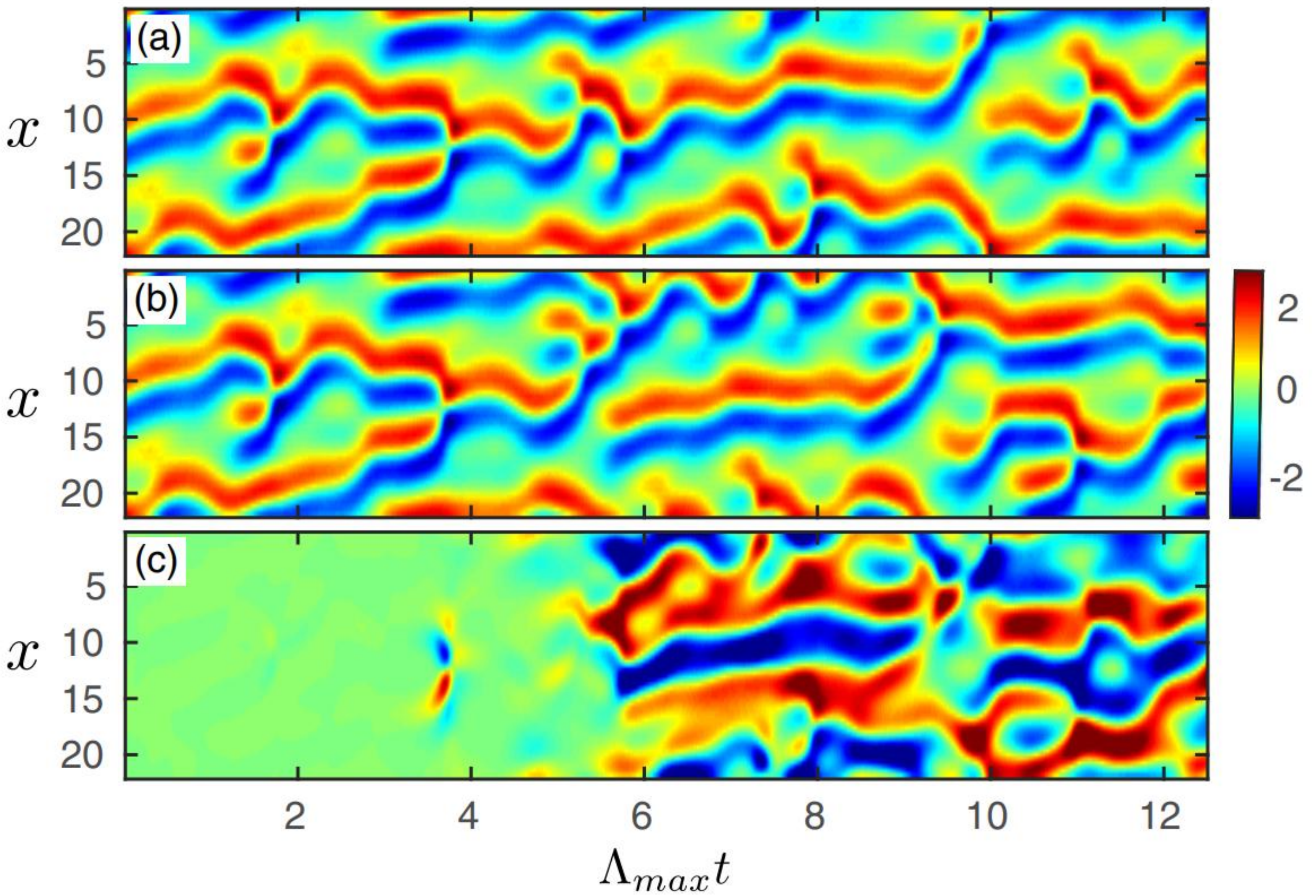
It is challenging to distinguish chaos and noise.

Reservoir computing

[Pathak *et al.*, PRL **120**, 024102 (2018)]



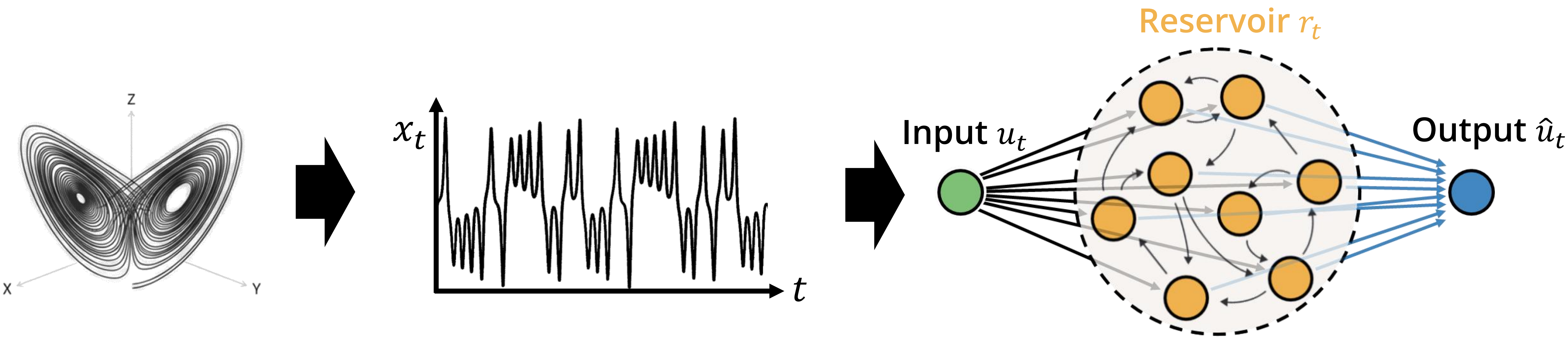
(a) actual data, (b) prediction, (c) error



Modified Kuramoto-Sivashinsky equation

$$\dot{y} = -yy' - y'' - y'''' + \mu \cos\left(\frac{2\pi x}{\lambda}\right)$$

Method



Update rule $\quad \boldsymbol{r}_{t+1} = (1 - a_{\text{leak}})\boldsymbol{r}_t + a_{\text{leak}} \tanh(\mathbf{W}_{\text{res}}\boldsymbol{r}_t + \mathbf{W}_{\text{in}} u_t)$

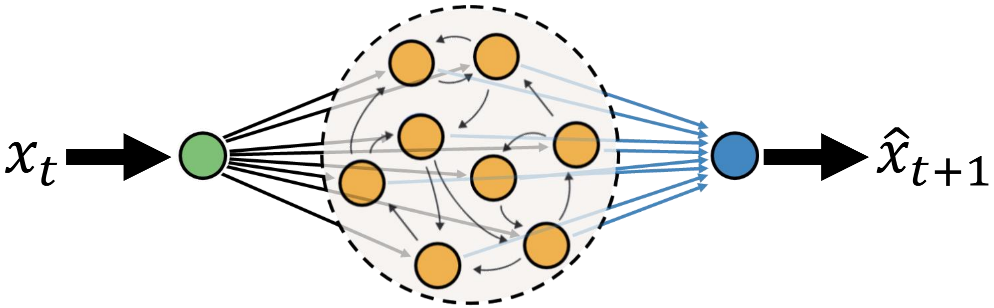
Loss function $\quad L \equiv \sum_{t=0}^{L-1} (u_{t+1} - \hat{u}_{t+1})^2 , \qquad \hat{u}_{t+1} = \mathbf{W}_{\text{out}} \boldsymbol{r}_{t+1}$

Preprocessing $\quad x_t \mapsto x_t - \langle x_t \rangle \quad x_t \mapsto x_t / \max_t \{|x_t|\}$

Method

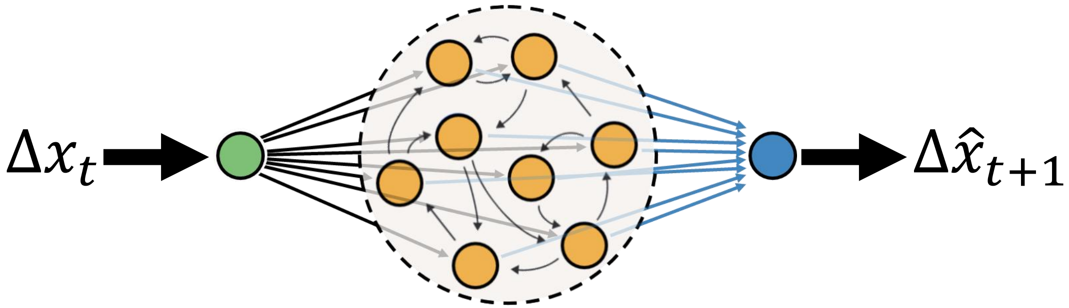
Short-term prediction with reservoir computing

Scheme 1
Value-to-value



Scheme 2
Difference-to-difference

$$\Delta x_t \equiv x_{t+1} - x_t$$



Stochastic noise

$$x(t) \sim \mathcal{N}$$

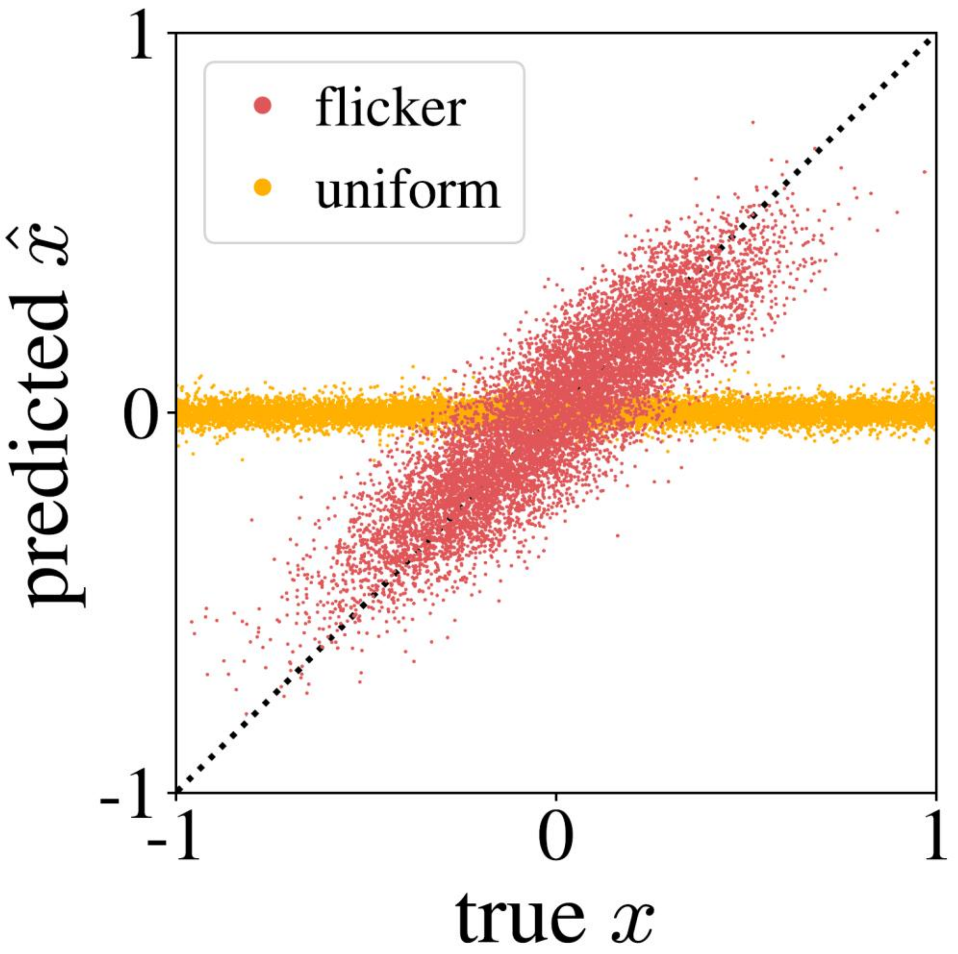
flicker noise $S(f) \sim 1/f$

uniform noise $x(t) \in (-1,1)$

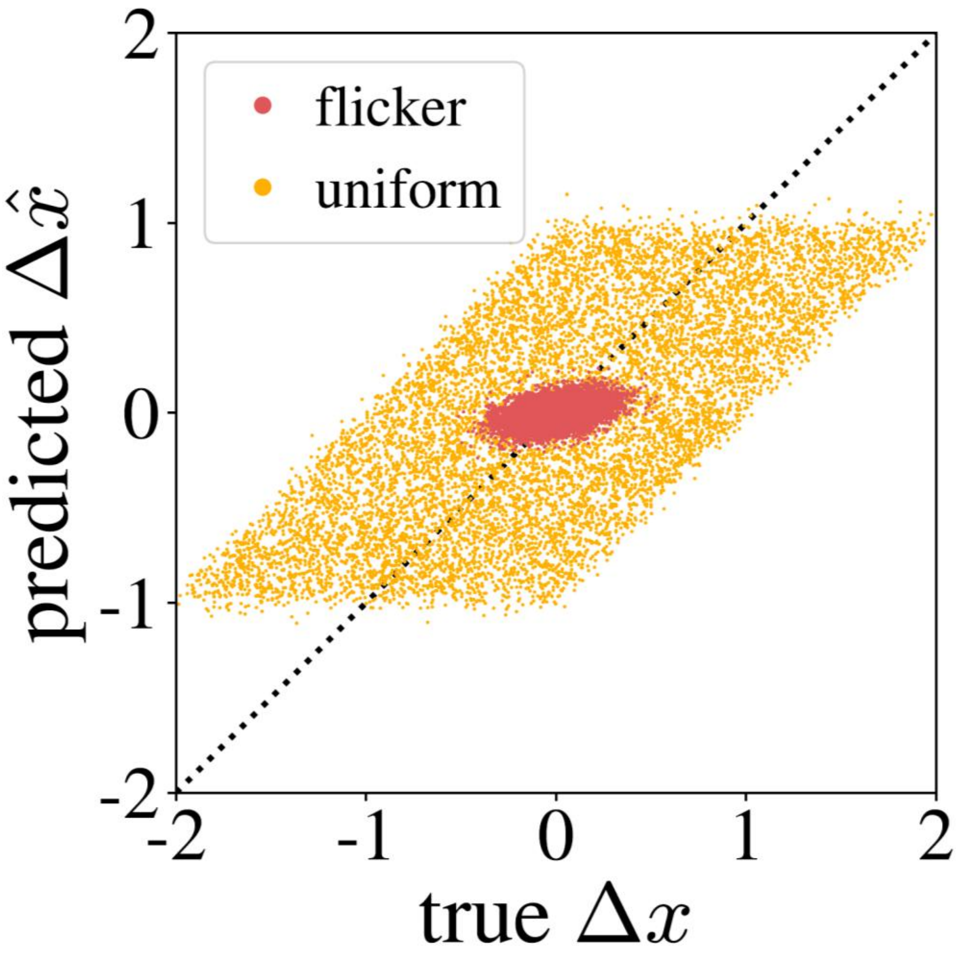
Method

Testing on synthetic dynamics

Scheme 1
Value-to-value



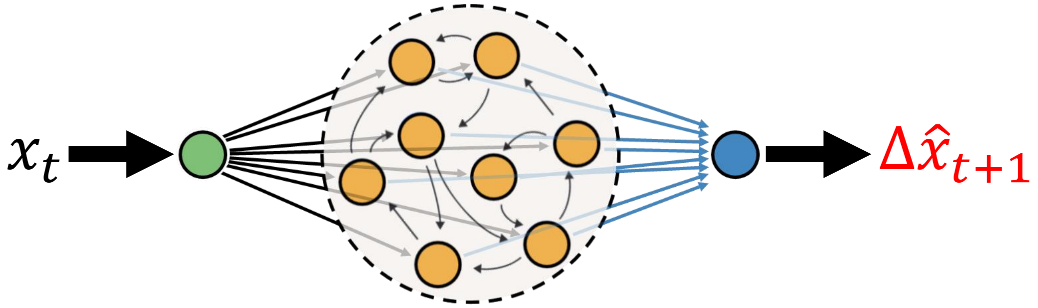
Scheme 2
Difference-to-difference



Method

cross-prediction scheme

Scheme 3
Value-to-difference

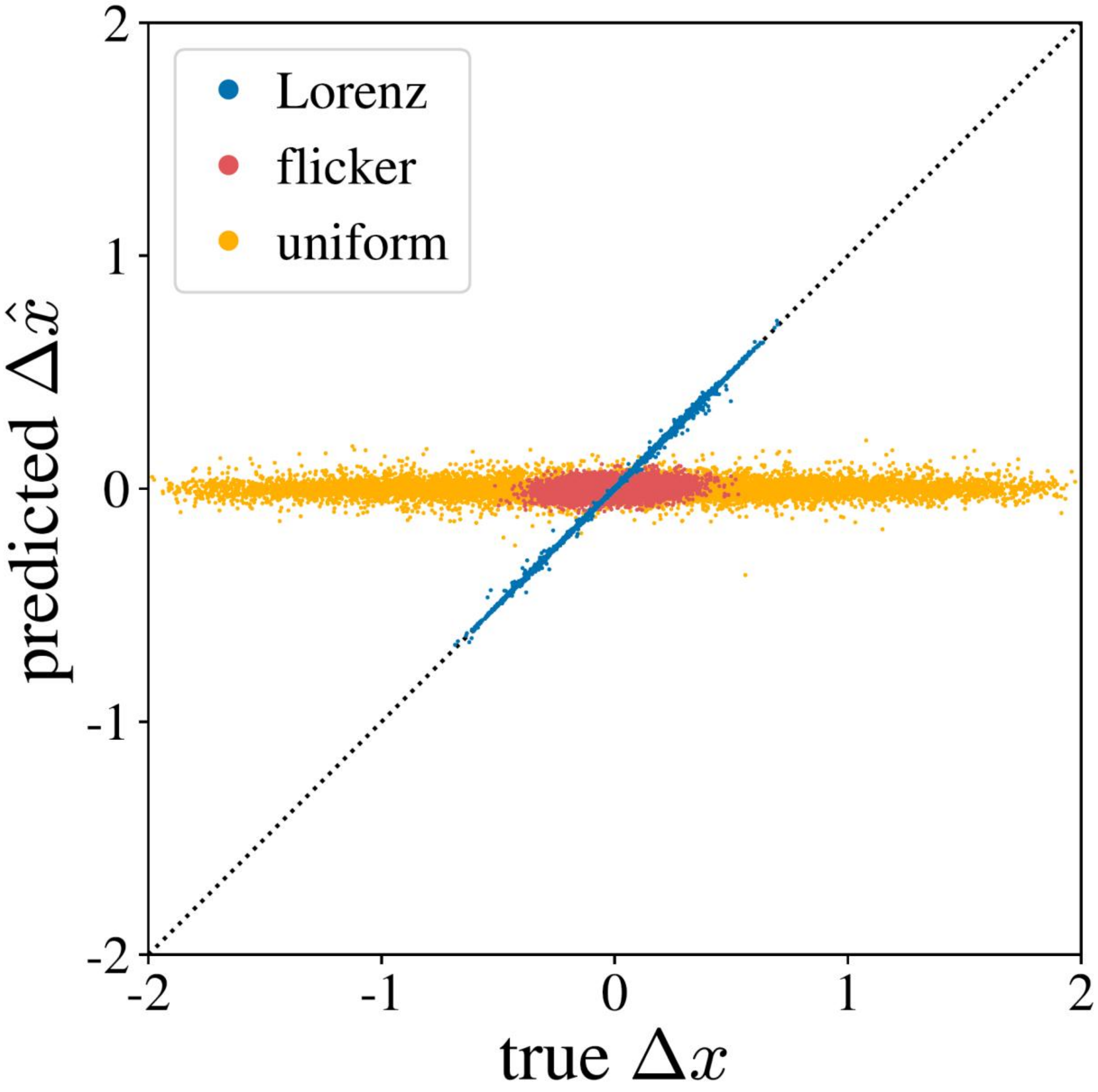


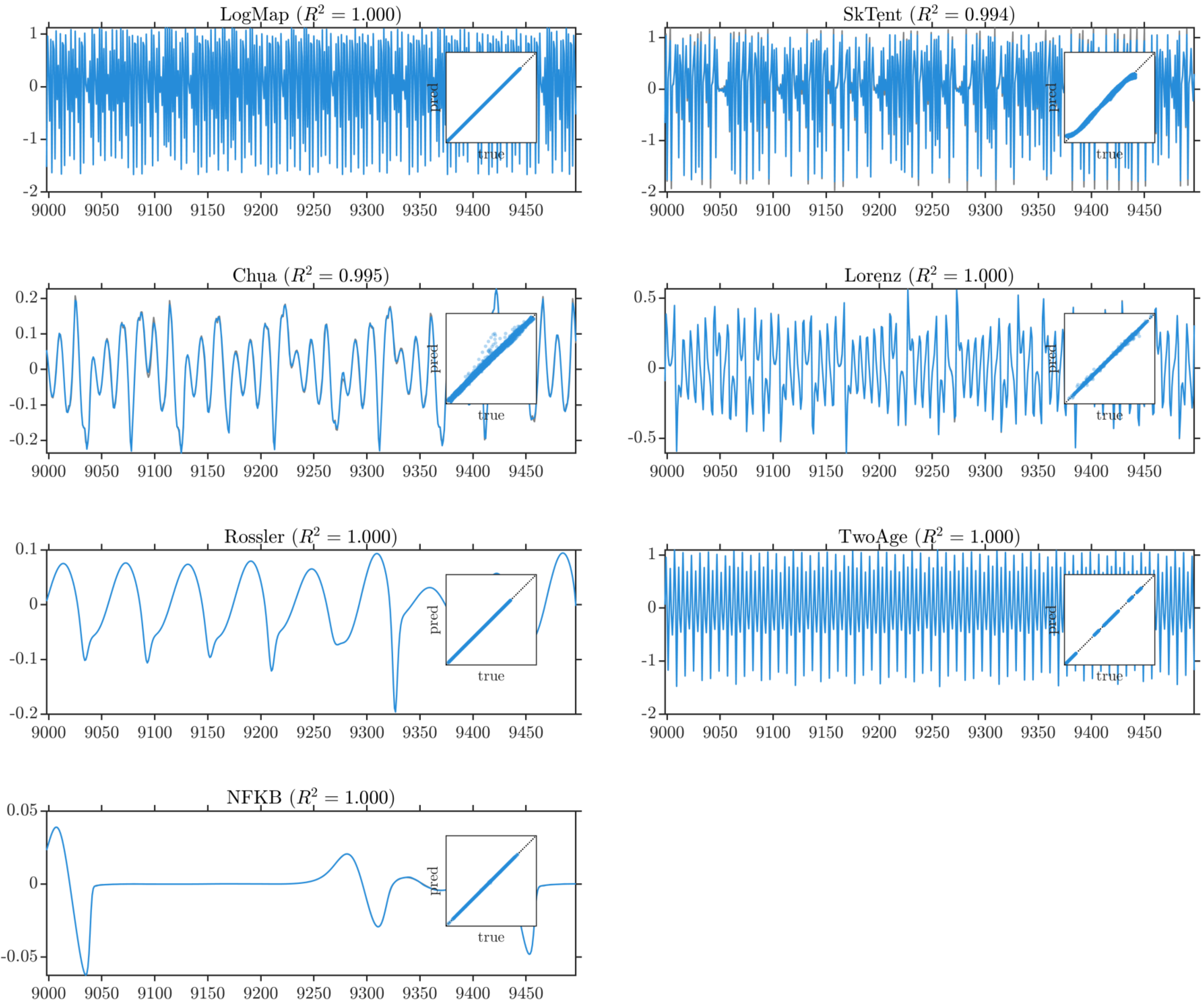
Lorenz system

$$\dot{x} = \sigma(y - x)$$

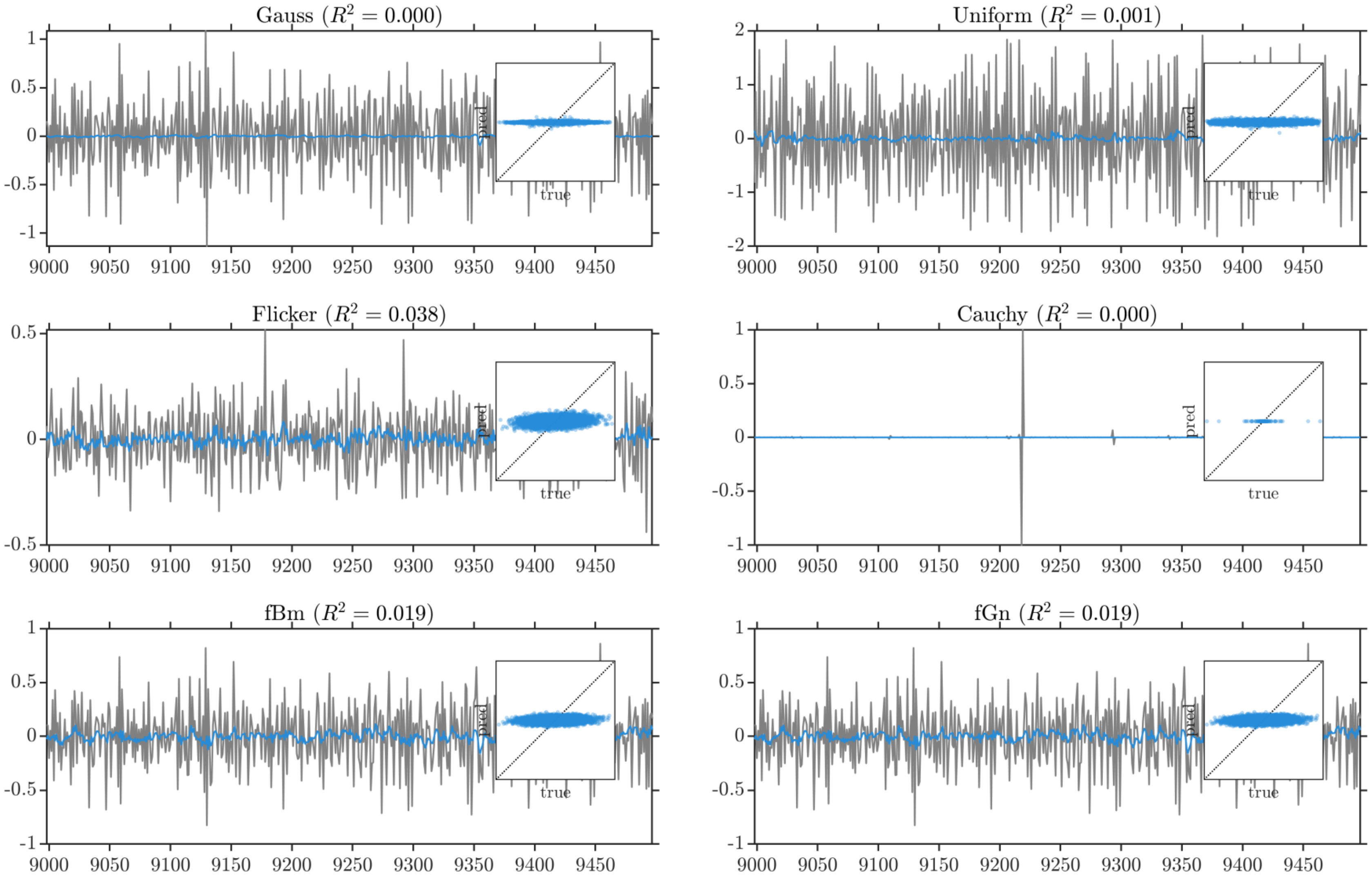
$$\dot{y} = x(\rho - z) - y$$

$$\dot{z} = xy - \beta z$$





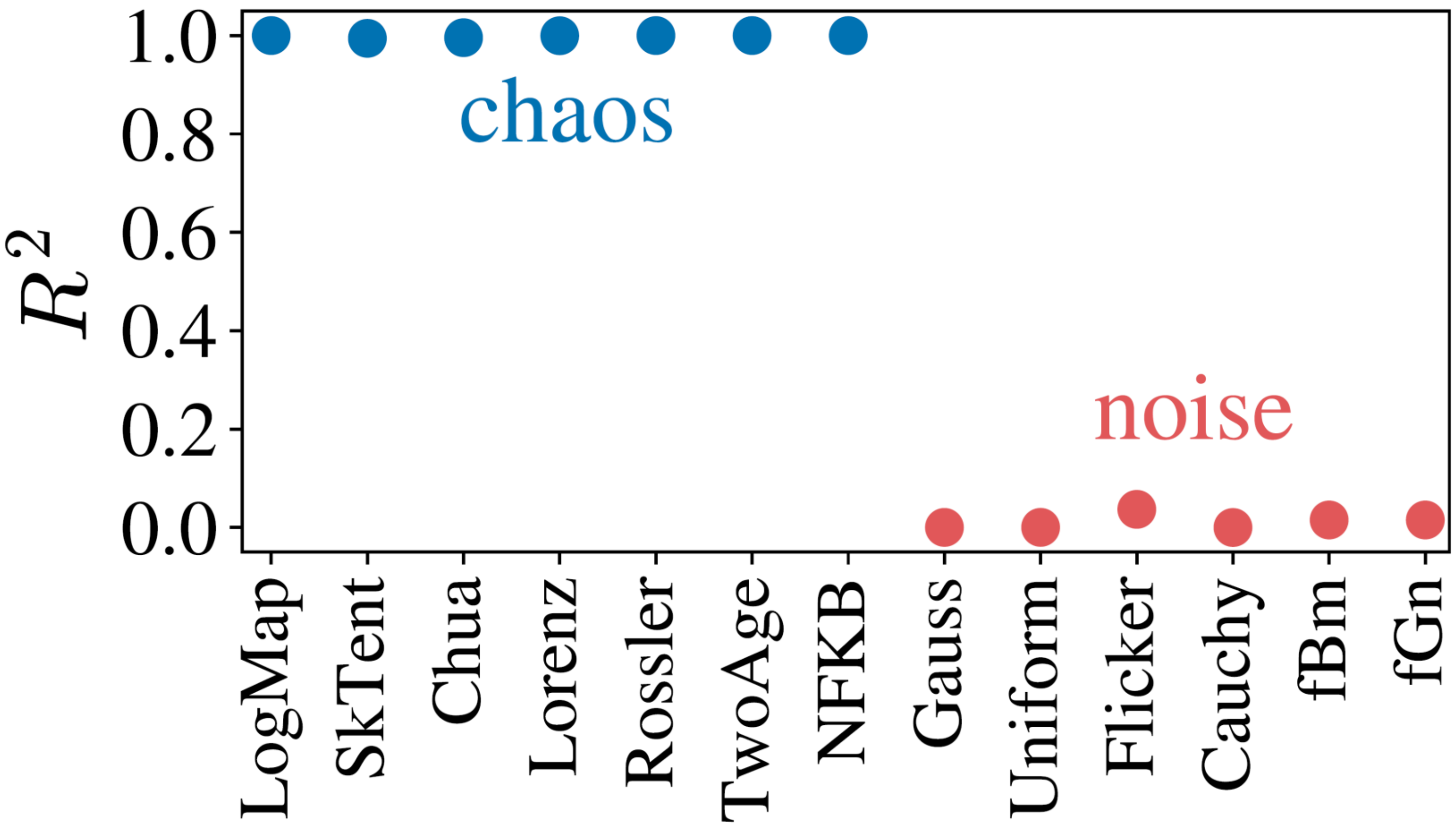
LogMap – logistic map
SkTent – skew tent map
Chua – Chua's circuit
Lorenz – Lorenz system
Rossler – Rossler system
TwoAge – two-age-class model
NFKB – NF-κB model ($d = 6$)



Gauss – white Gaussian noise
Uniform – white uniform noise
Flicker – flicker noise
Cauchy – Cauchy noise
fBm – fractional Brownian motion
fGn – fractional Gaussan noise

Results for simulated synthetic dynamics

coefficient of determination R^2



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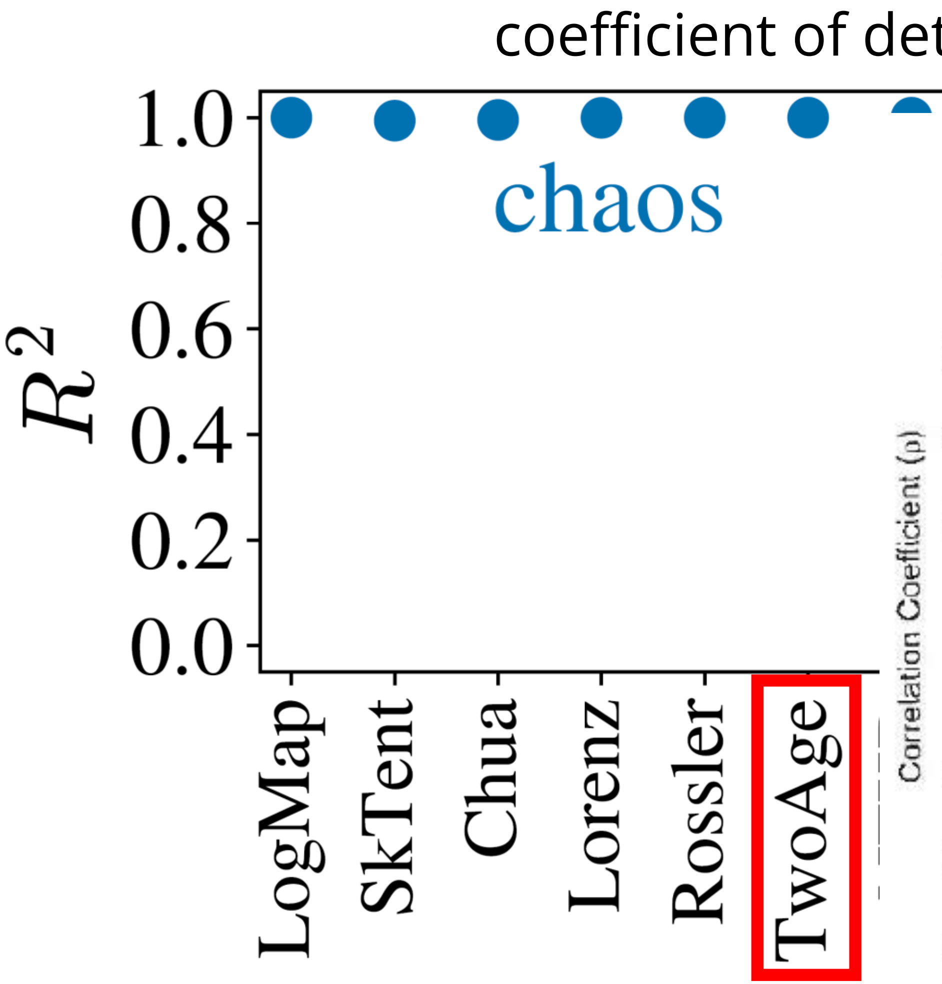

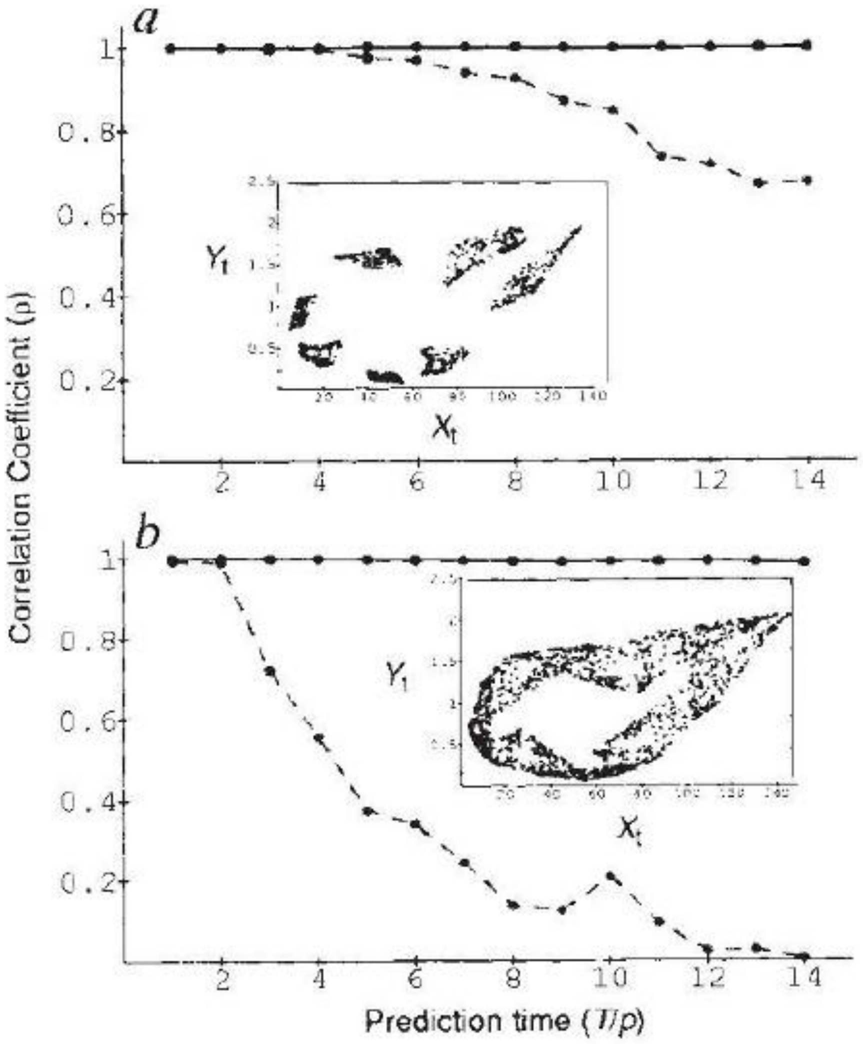

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er – Rossler system
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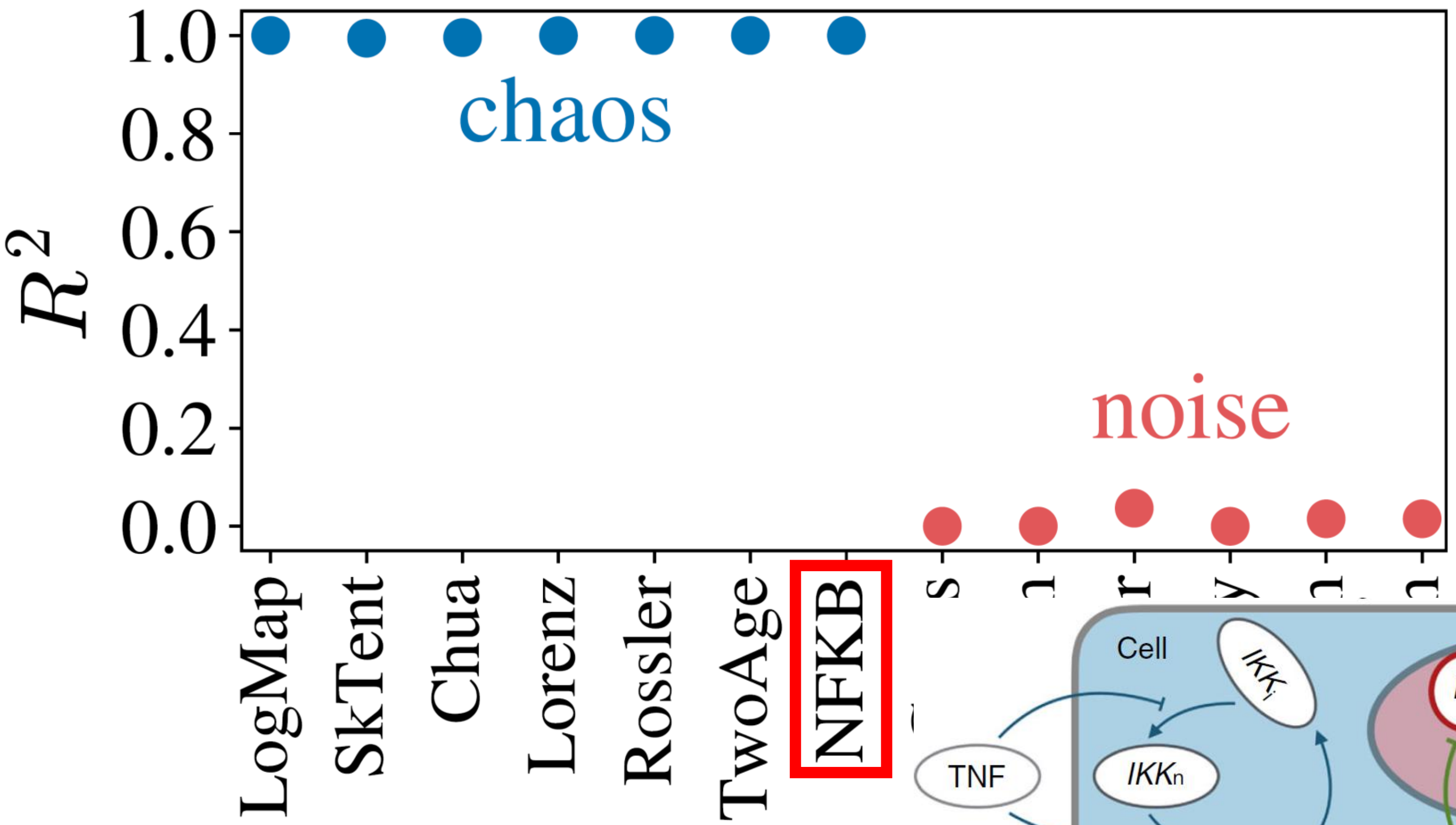
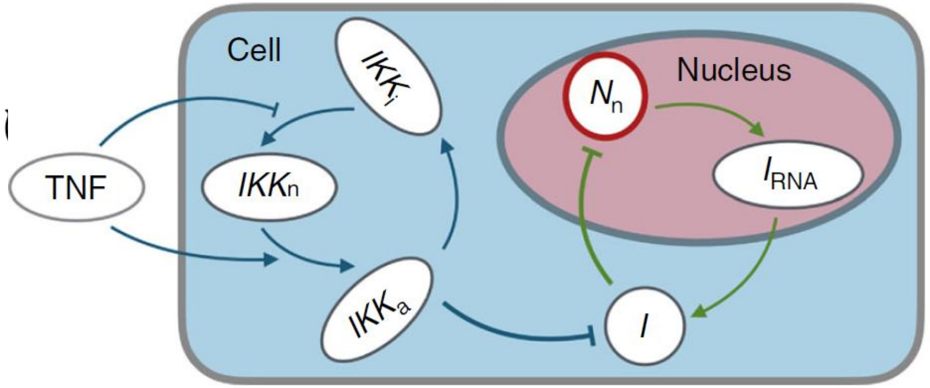
er – flicker noise
hy – Cauchy noise
- fractional Brownian motion
fractional Gaussan noise

Results for simulated synthetic dynamics

coefficient of determination R^2





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white uniform noise
cker noise
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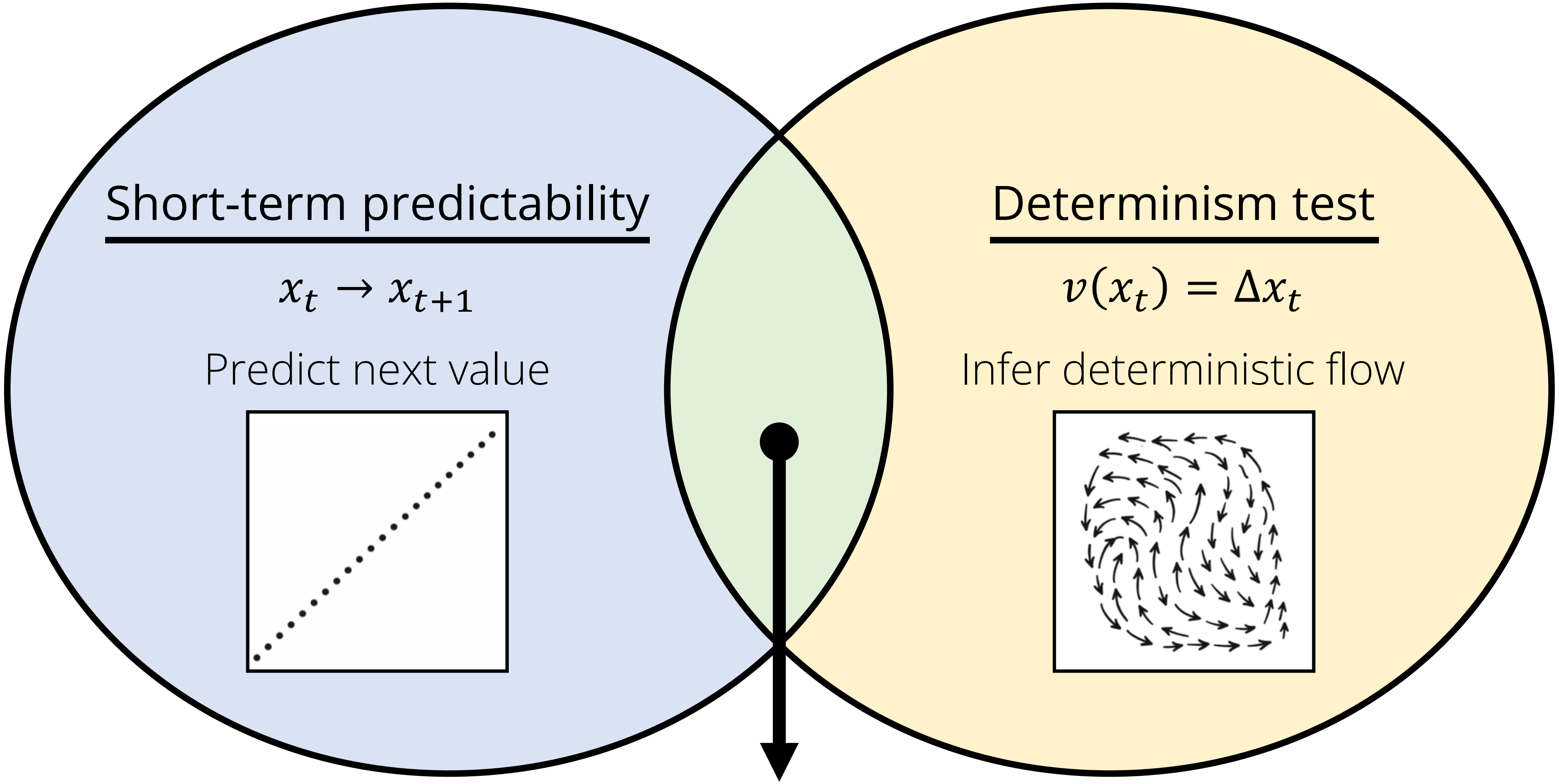
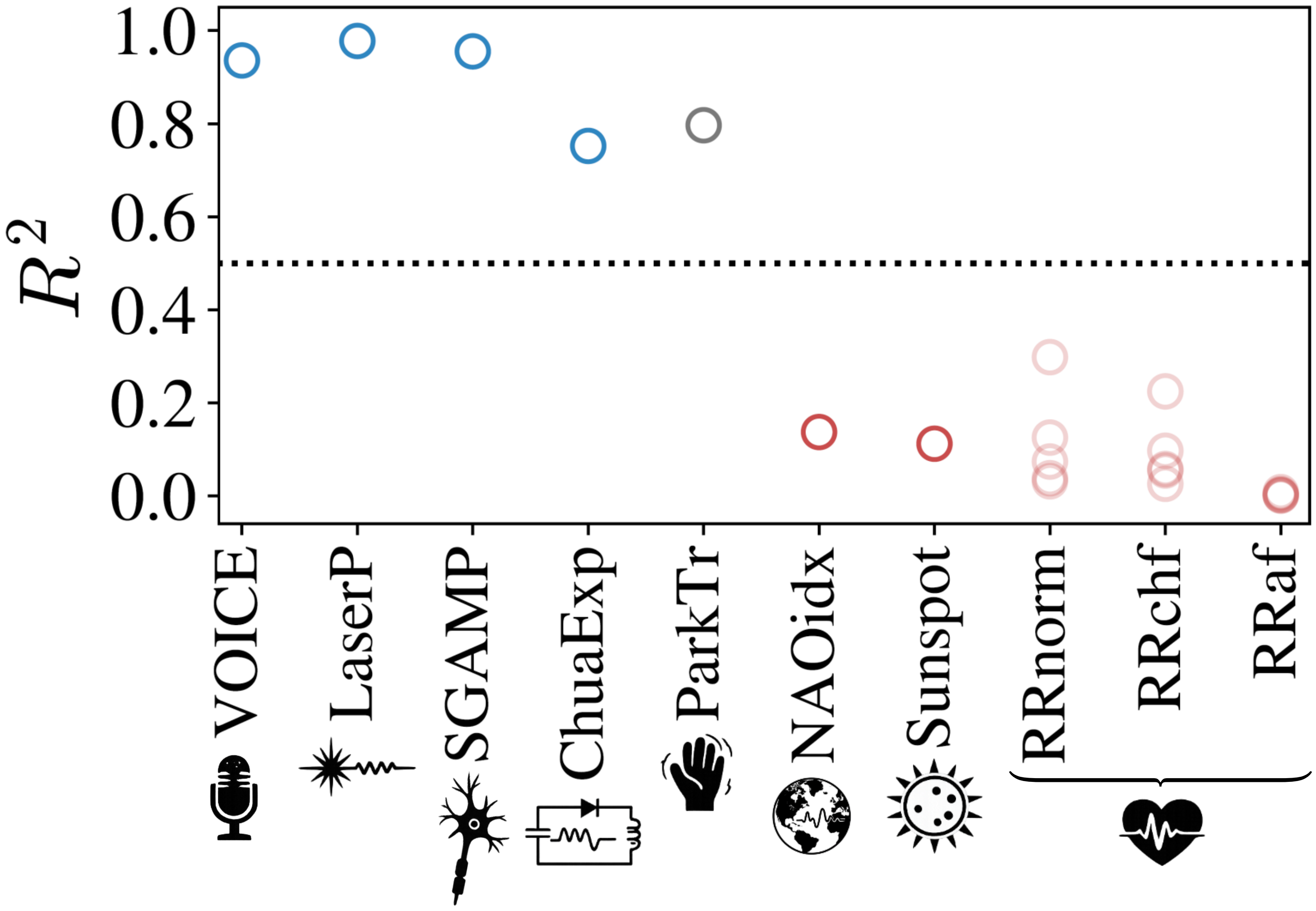
Intuition for the mechanism



Cross-prediction

$$x_t \rightarrow \Delta x_{t+1} = v(x_{t+1})$$

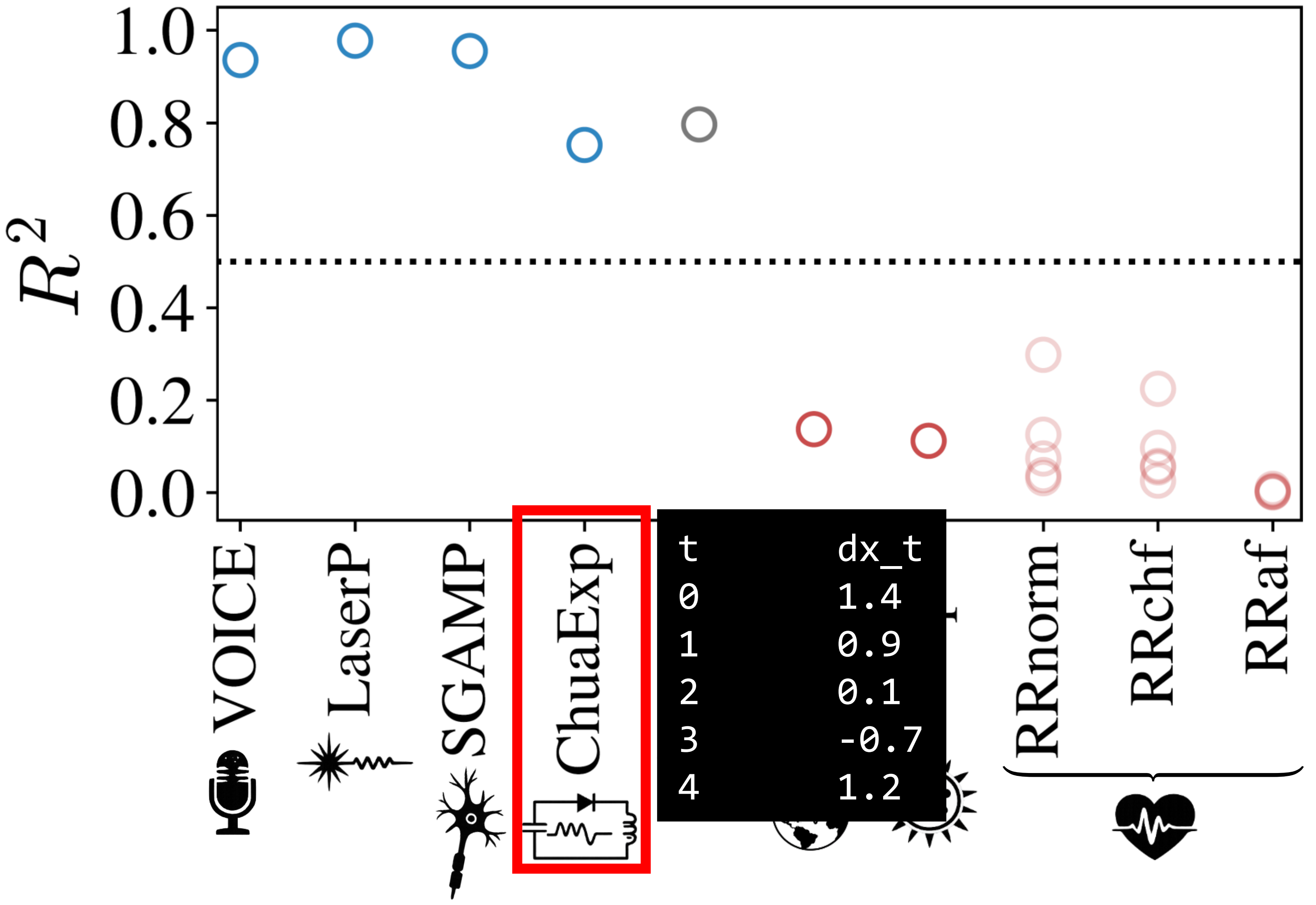
Results for empirical data



VOICE – voice recoding
LaserP – laser intensity pulsing
SGAMP – squid giant axon membrane potential
ChuaExp – Chua's circuit experiment
ParkTr – Parkinsonian tremor
NAOidx – mean North Atlantic Oscillation index
Sunspot – sunspot number

RR-interval data from...
RRnorm – healthy subjects
RRchf – congestive heart failure
RRaf – atrial fibrillation

Results for empirical data



t	dx_t
0	1.4
1	0.9
2	0.1
3	-0.7
4	1.2

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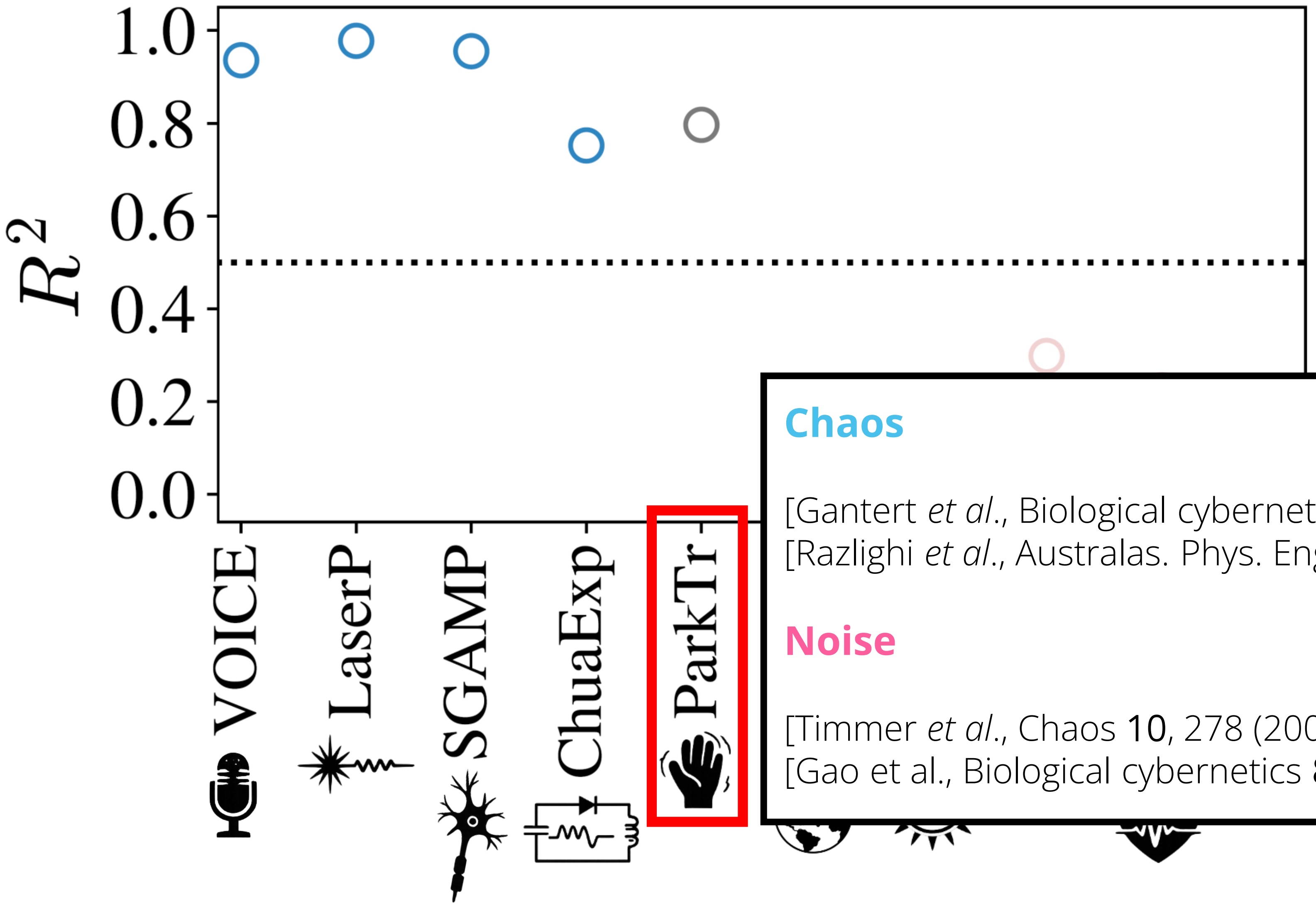

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Chaos

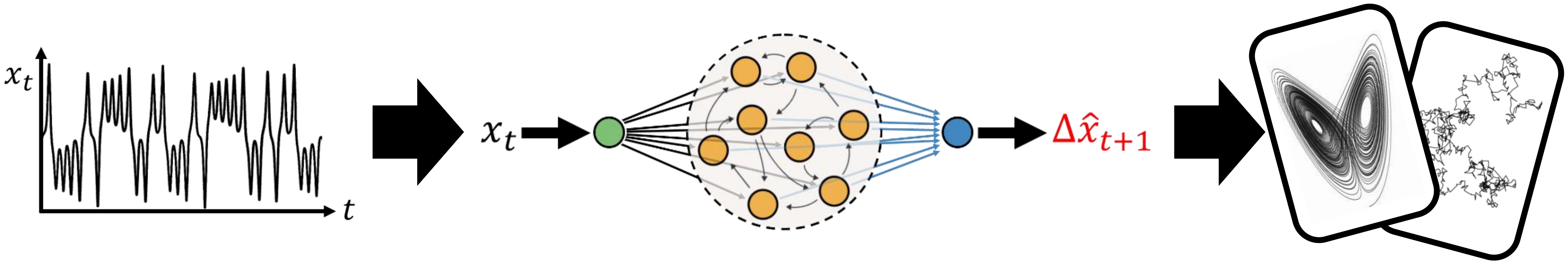
[Gantert *et al*., Biological cybernetics **66**, 479 (1992)]
[Razlighi *et al*., Australas. Phys. Eng. Sci. Med. **35**, 25 (2012)]

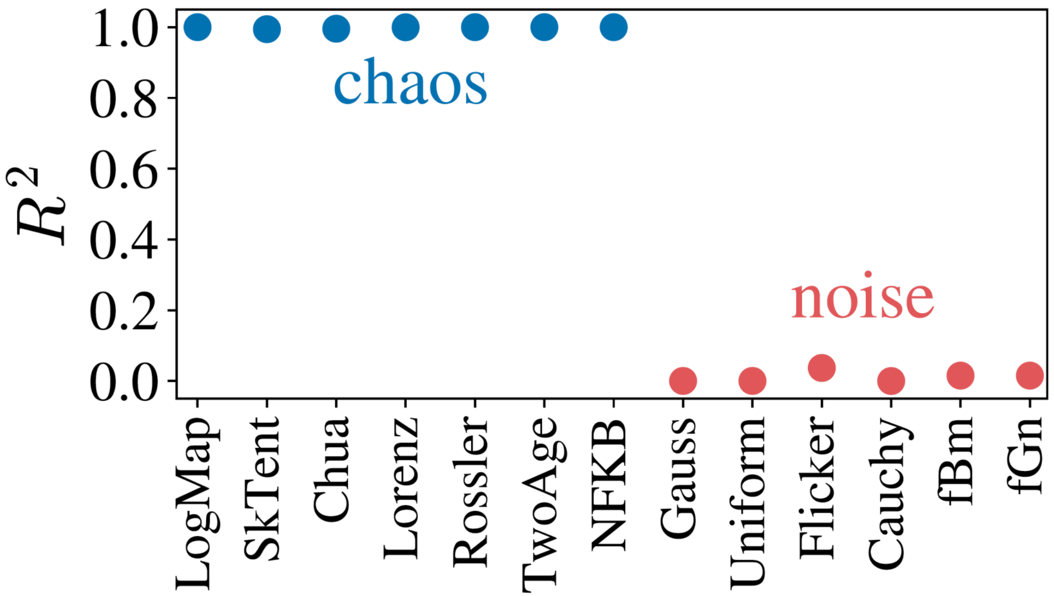
Noise

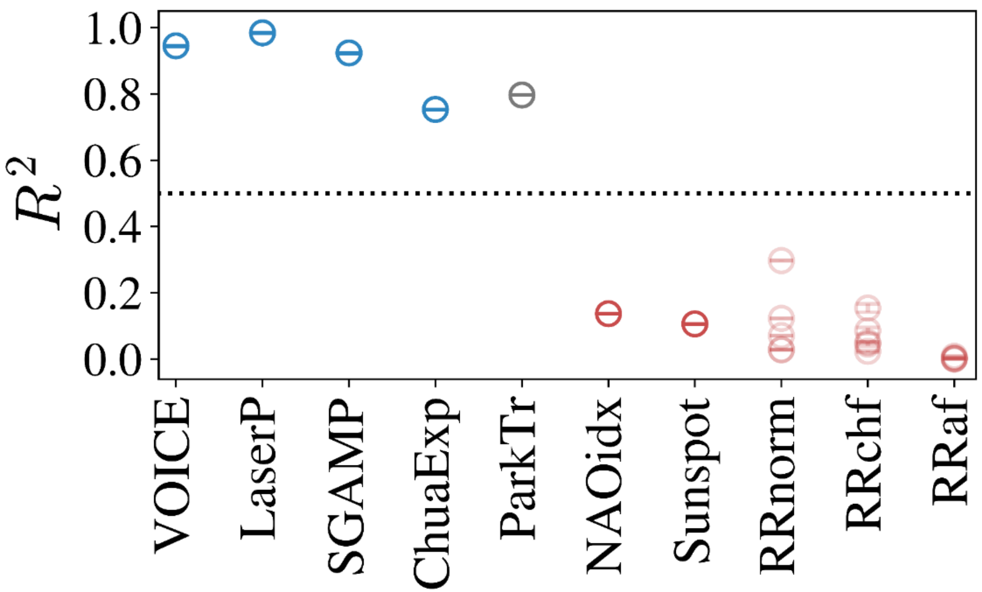
[Timmer *et al*., Chaos **10**, 278 (2000)]
[Gao et al., Biological cybernetics **86**, 263 (2002)]

Summary

Pure data-driven method to distinguish chaos and noise
using cross-prediction with reservoir computing







Thank you for your kind attention!

✉ jongmin.park@apctp.org

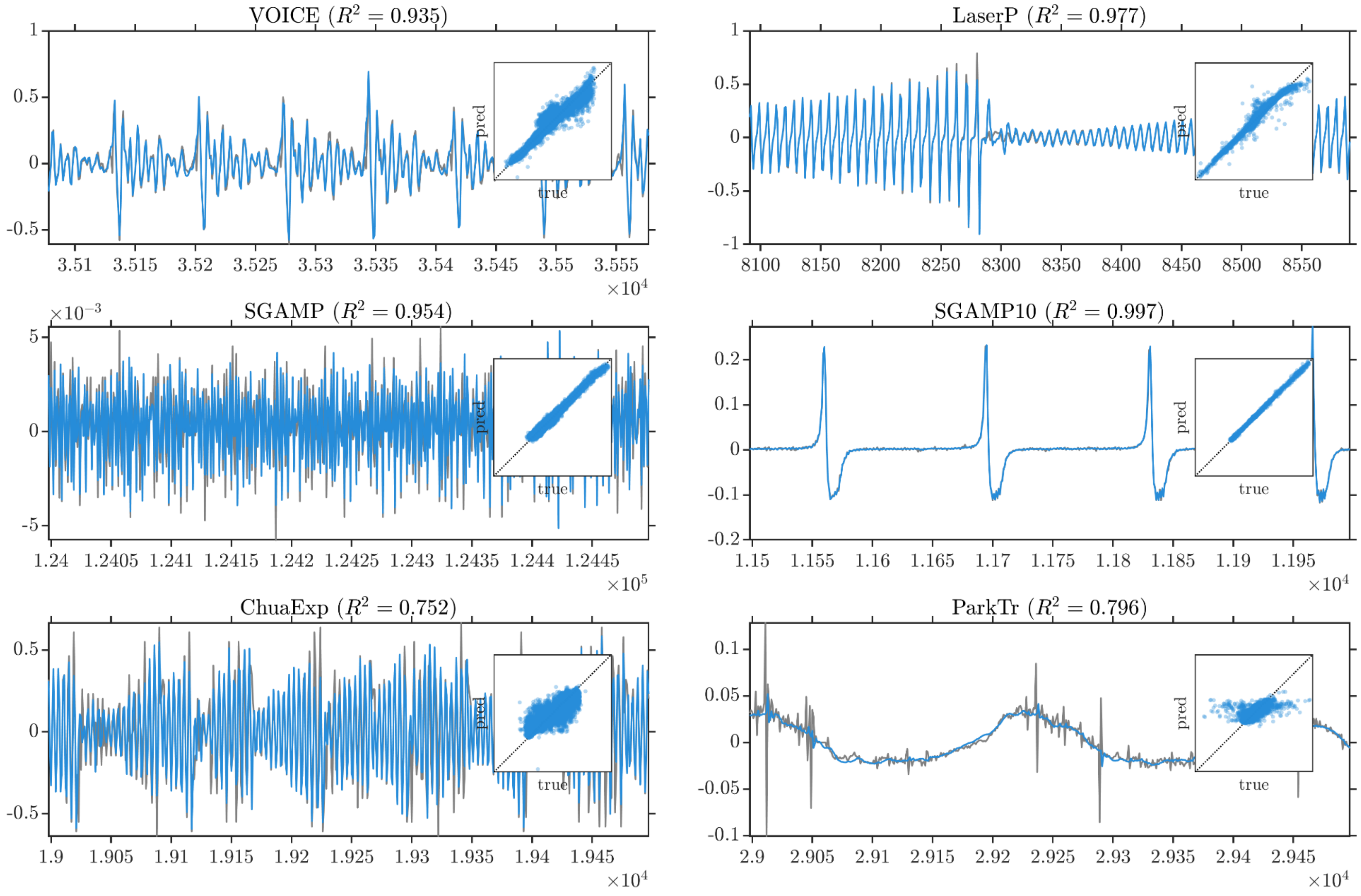
KIAS KOREA INSTITUTE FOR ADVANCED STUDY

Jaesung Choi

apctp asia pacific center for theoretical physics

Athokpam Langlen Chanu

Appendix 1 – Results for empirical data



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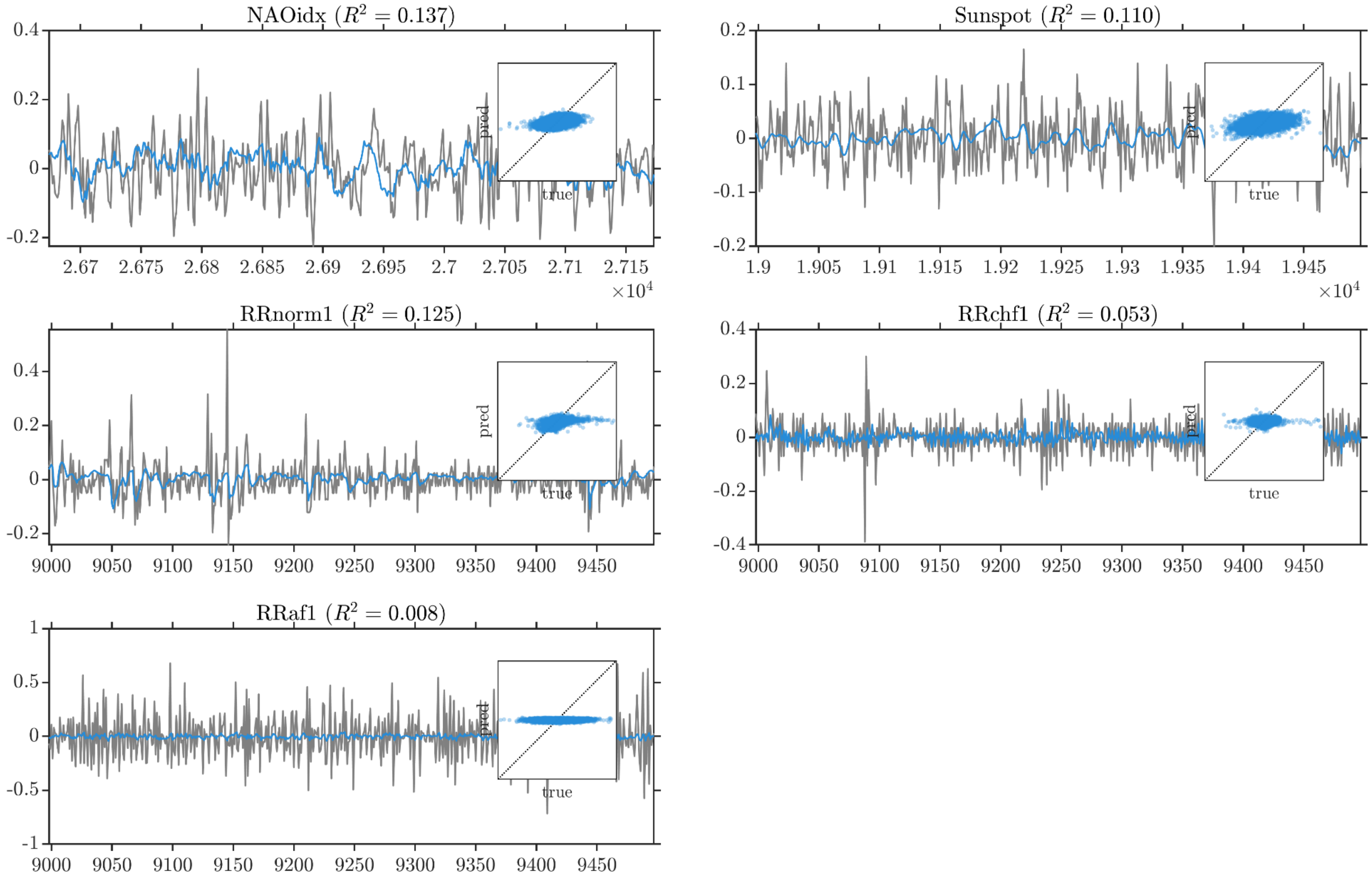

NAOidx – mean North Atlantic Oscillation index
Sunspot – sunspot number

RR-interval data from…
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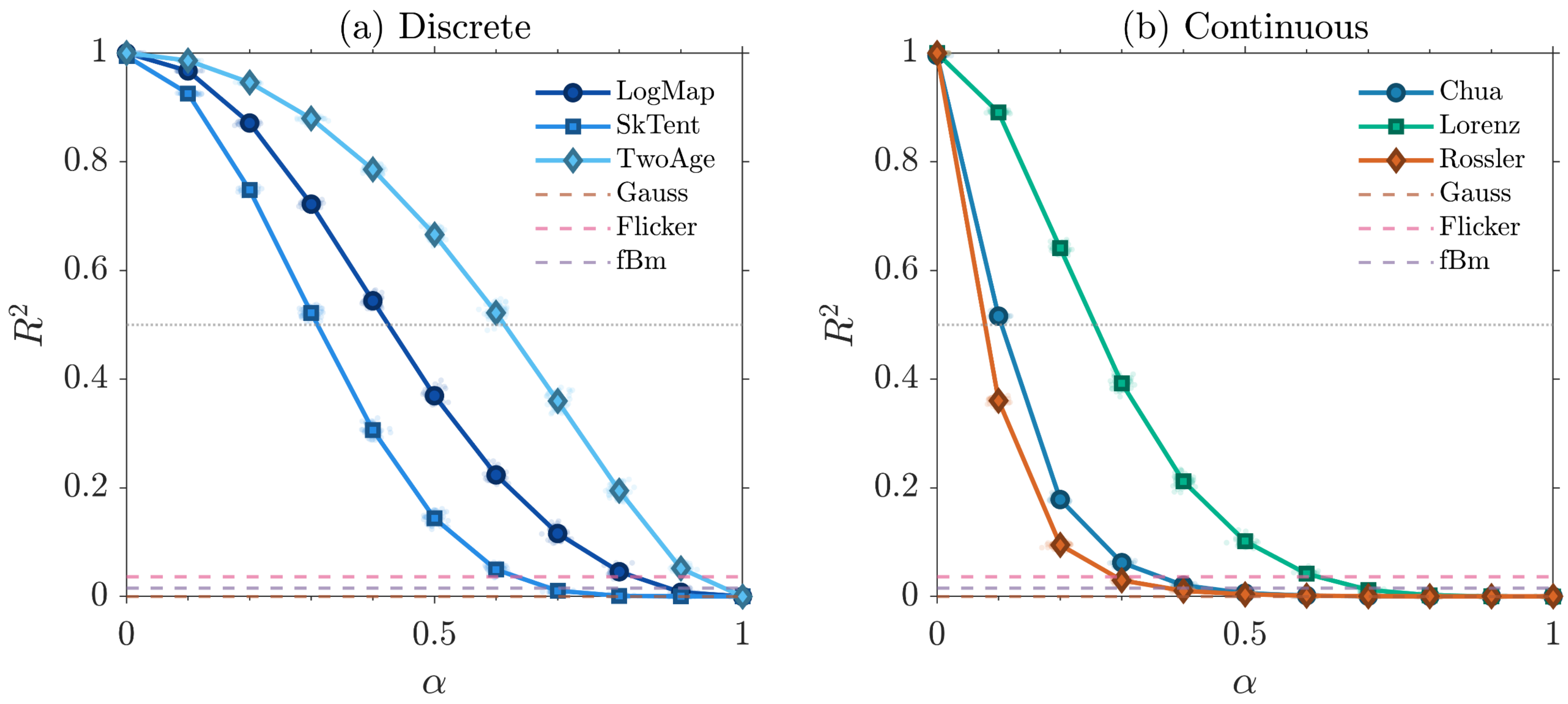
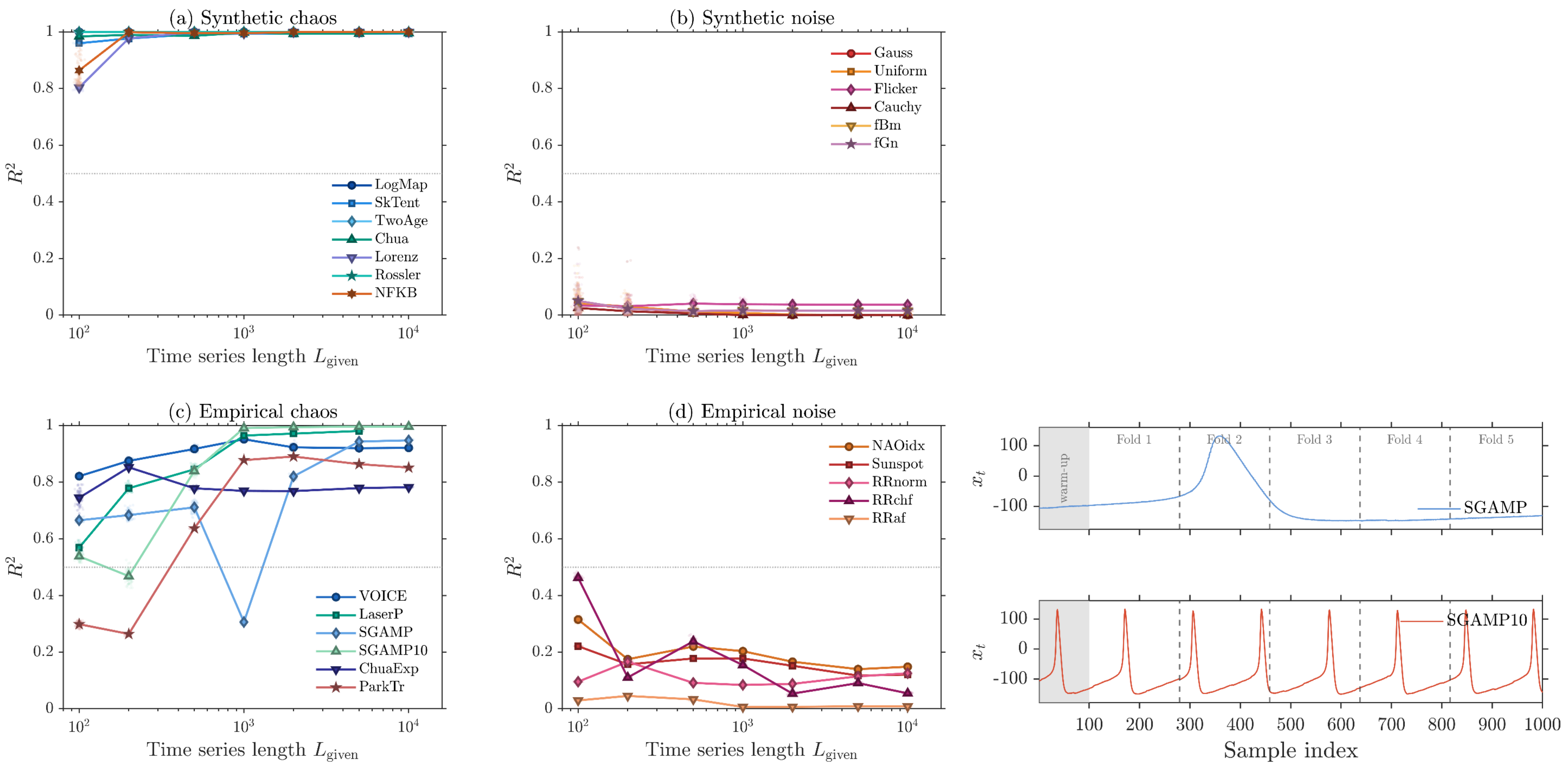
Appendix 2 – Noise mixing

$$z_t = \sqrt{1-\alpha}x_t, +\sqrt{\alpha}\xi_t$$

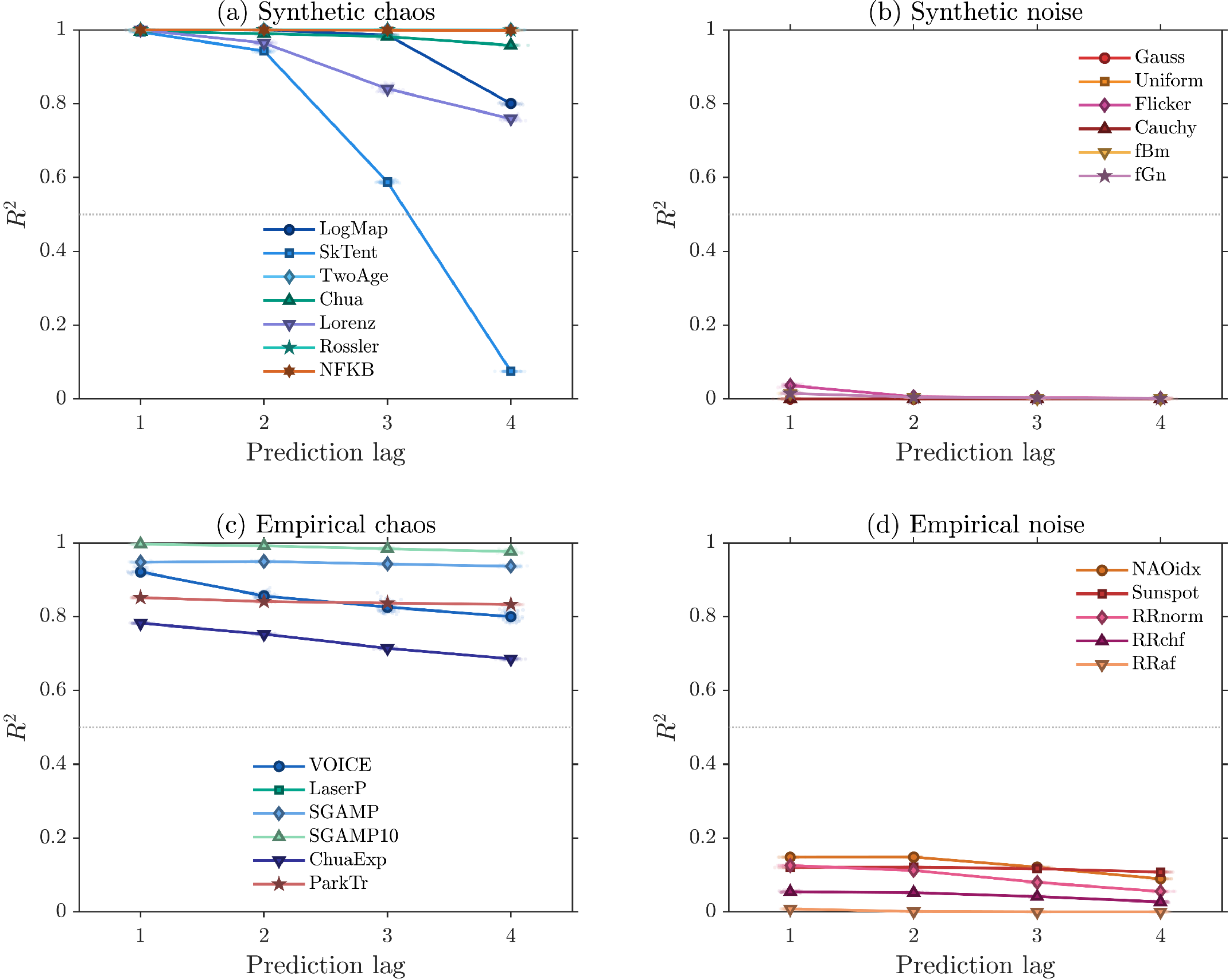


Appendix 3 – Data length



(a) Synthetic chaos
R^2
Time series length L_{given}
LogMap
SkTent
TwoAge
Chua
Lorenz
Rossler
NFKB
(b) Synthetic noise
Gauss
Uniform
Flicker
Cauchy
fBm
fGn
(c) Empirical chaos
VOICE
LaserP
SGAMP
SGAMP10
ChuaExp
ParkTr
(d) Empirical noise
NAOidx
Sunspot
RRnorm
RRchf
RRaf
warm-up
Fold 1
Fold 2
Fold 3
Fold 4
Fold 5
x_t
SGAMP
SGAMP10
Sample index

Appendix 4 – Prediction horizon



(a) Synthetic chaos
(b) Synthetic noise
(c) Empirical chaos
(d) Empirical noise
R^2
Prediction lag
LogMap
SkTent
TwoAge
Chua
Lorenz
Rossler
NFKB
Gauss
Uniform
Flicker
Cauchy
fBm
fGn
VOICE
LaserP
SGAMP
SGAMP10
ChuaExp
ParkTr
NAOidx
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