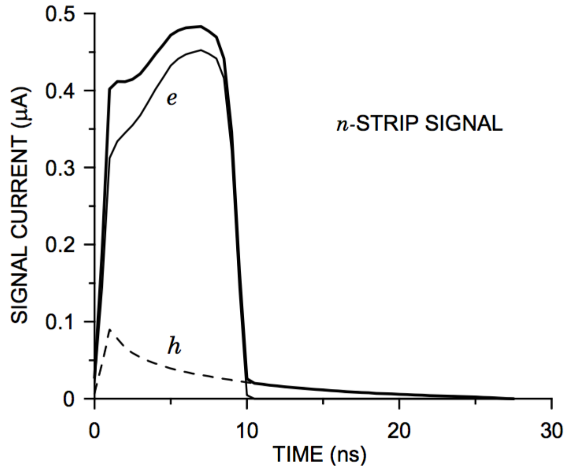
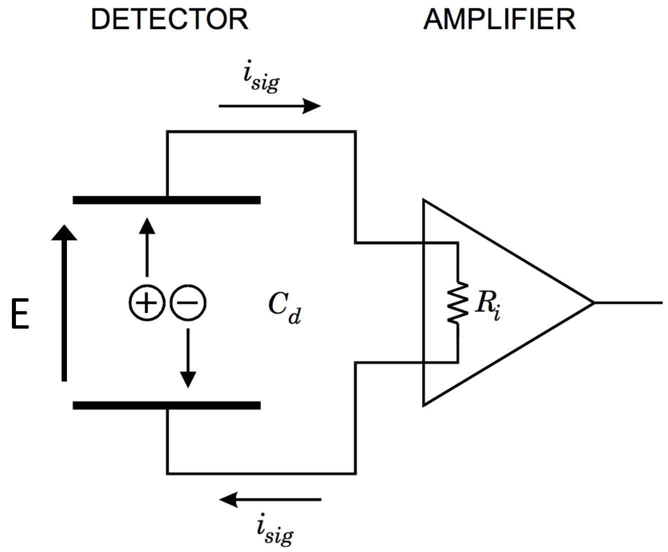
Ionisation chamber model

- Free charge is generated in a volume between two electrodes
  - Volume is the detecting medium, e.g. silicon or a gas
  - Charges: ions + electrons or e-h pairs
  - Apply an electric field to drift the charge to the electrodes
- What is generating the signal?
  - The movement of charges induces the current $i_{sig}(t)$
  - The current depends on the geometry

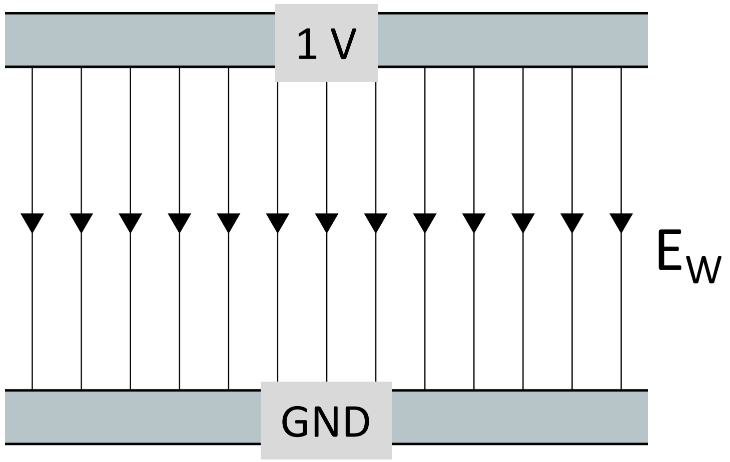




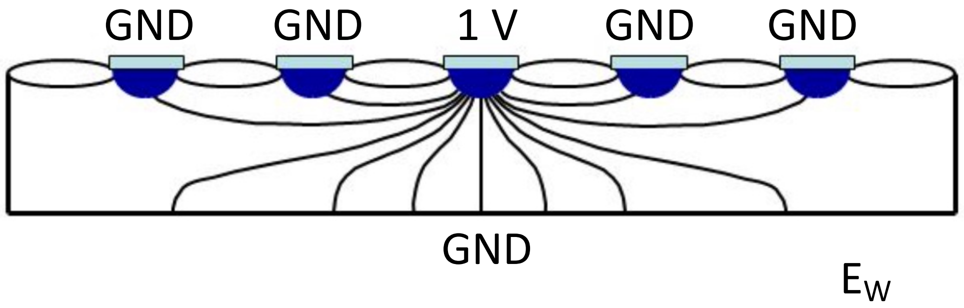
Shockley-Ramo Theorem

- Determines $i_{sig}(t)$ induced on an electrode A
- Construct a hypothetical field, the *weighting field*: E_W
  - Set electrode A to unit potential
  - Set all other potentials to ground

Parallel plate capacitor



Pixelated detector volume



For a full derivation and discussion, see e.g. [H. Spieler's lectures](http://www-physics.lbl.gov/~spieler/Heidelberg_Notes_2005/index.html)

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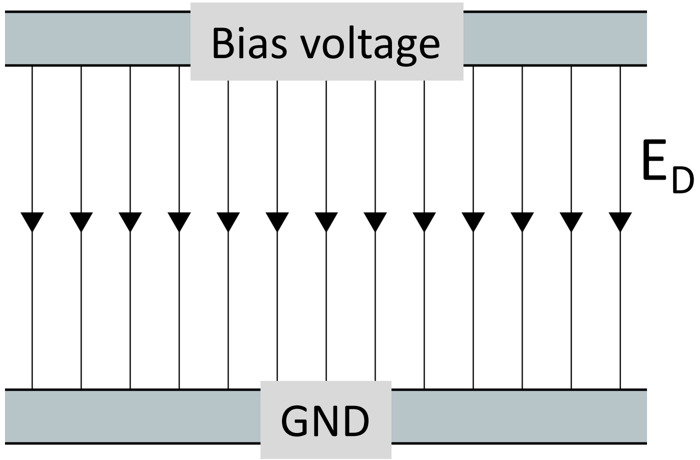
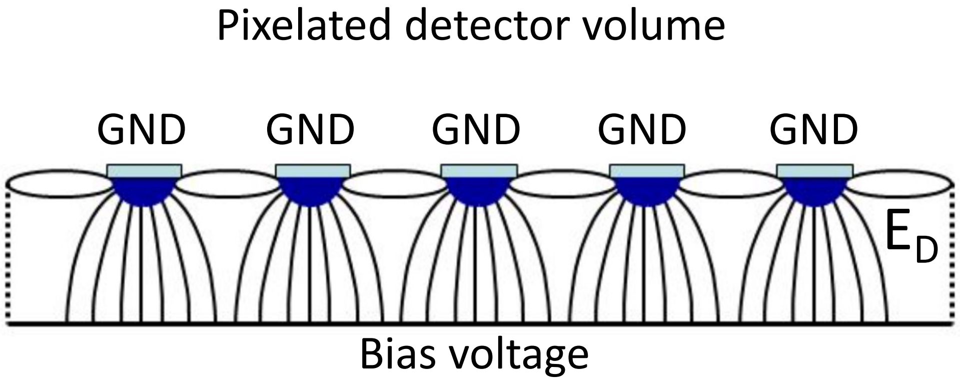
Shockley-Ramo Theorem

- Calculate the actual field that the charges will drift in: E_D
  - The electrode configuration as it is in the detector

Parallel plate capacitor





- The drift field and weighting field will look very different
  - The drift filed determines the trajectory of the particles
  - Special case: they are identical for a parallel plate capacitor

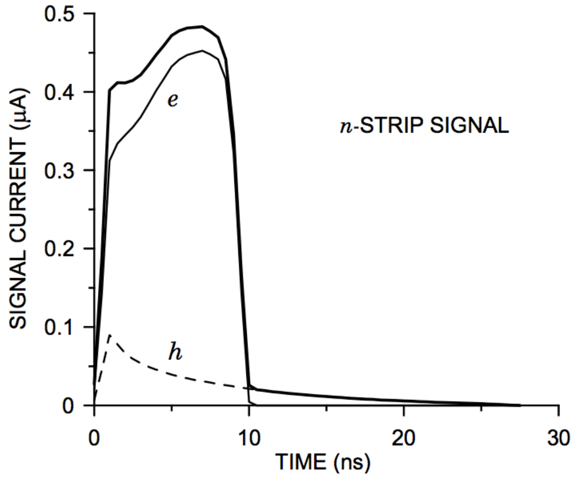
Shockley-Ramo Theorem

- Use these two fields to calculate the induced current
  - Velocity vector of the charge drifting along the field lines

$$\vec{v}_q(t) = \mu \cdot \vec{E}_D\big(\vec{x}(t)\big), \qquad \text{where} \begin{cases} \mu \text{ is the mobility} \\ E_D \text{ is the drift field} \end{cases}$$

- Shockley-Ramo Theorem
  - The time-dependent induced current is given by

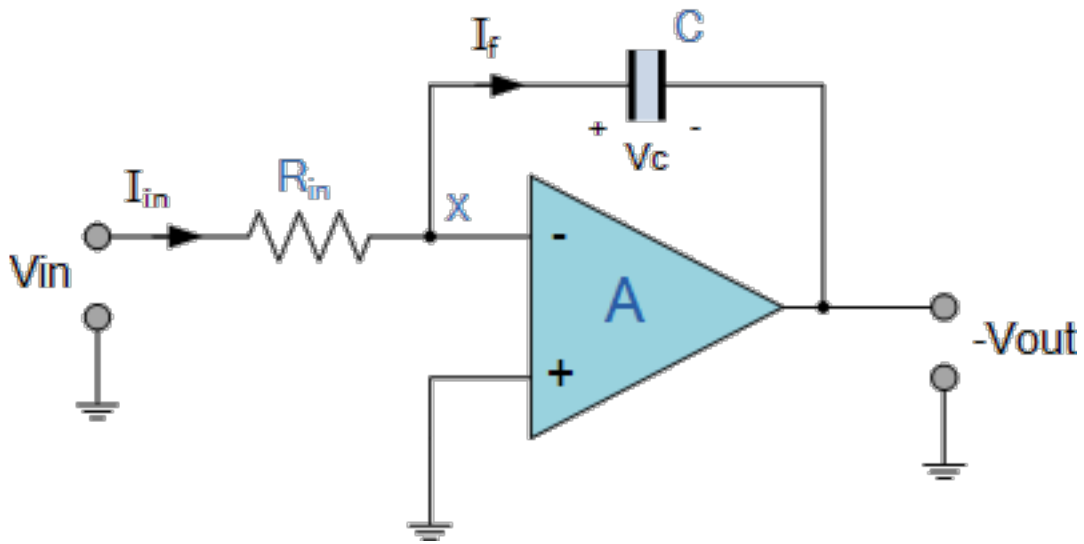
$$i_{sig}(t) = \mathrm{q} \cdot \vec{v}(t) \cdot \vec{E}_W\big(\vec{x}(t)\big), \quad \text{where } E_W \text{ is the weighting field}$$



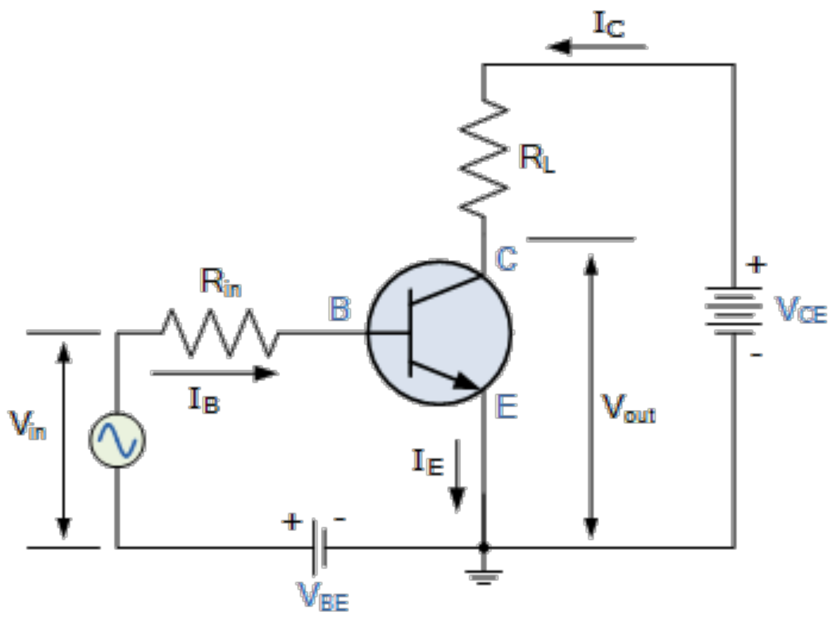
Example of a calculated current from electrons and holes in a silicon detector
(note the difference in mobility)

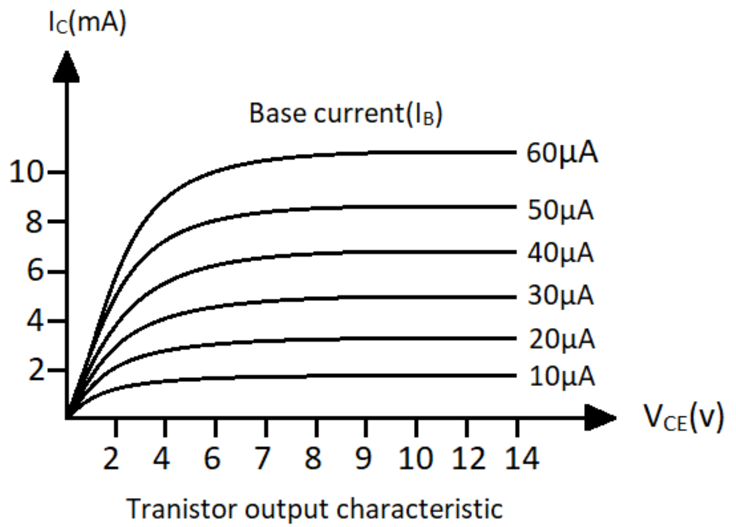
Charge integrating amplifier

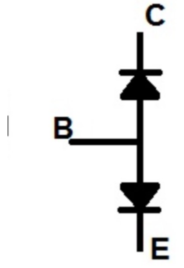
- Integrating amplifier is commonly used to integrate the induced current
  - Gives a signal proportional to the ionised charge
  - Shaping required to bring the signal back to the baseline



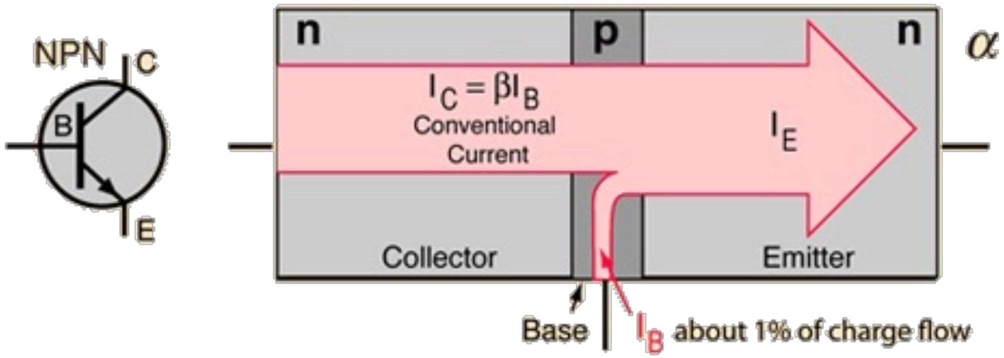
Simple common emitter stage







NPN Simplyfy Diagram

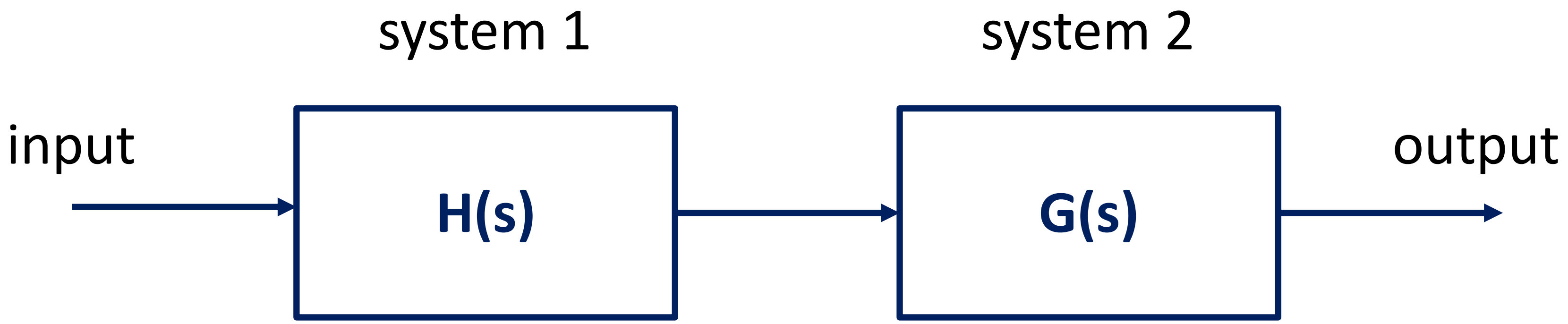


$$\alpha = \frac{I_C}{I_E} \approx 0.99$$

$$I_C = \beta I_B$$

$$\beta = \frac{\alpha}{\alpha - 1}$$

Transfer functions



Laplace transform

$$F(s) = \int_0^\infty f(t)e^{-st}dt$$

where $s = \sigma + i\omega$

Fourier transform

$$F(\omega) = \int_{-\infty}^\infty f(t)e^{-i\omega t}dt$$

Properties of the Laplace transformation

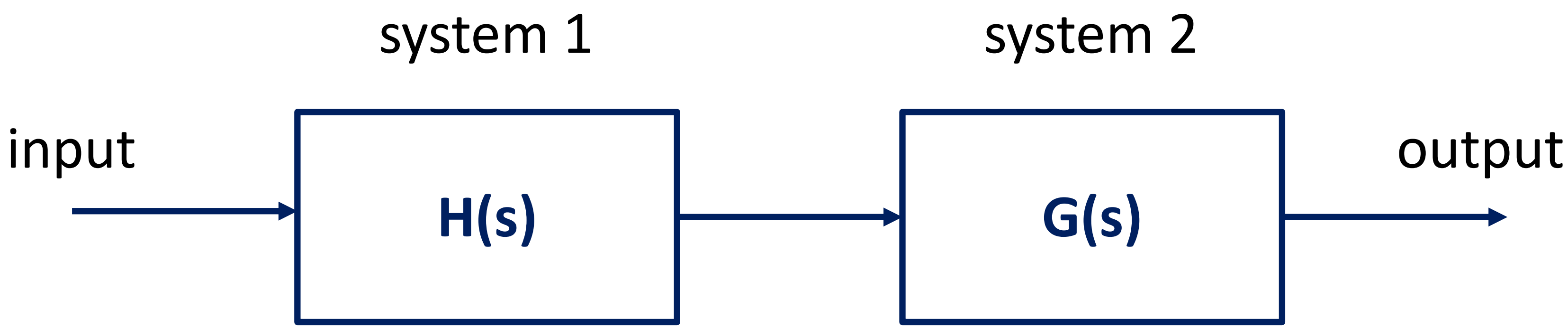
- Laplace transforms of derivatives:

$f'(t)$	$sF(s) - f(0)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) \cdots - s f^{(n-2)}(0) - f^{(n-1)}(0)$

- Transforms (linear) differential equations to polynomial equations
- Solve the equations and find poles and zeroes
  - Defines the dynamical behaviour of the system
- Solution becomes combinations of exponential and sinus functions

e^{at}	$\dfrac{1}{s-a}$	$\sin(at)$	$\dfrac{a}{s^2+a^2}$
		$t\sin(at)$	$\dfrac{2as}{\left(s^2+a^2\right)^2}$

Combining the transfer functions of two systems



- Time domain: convolution integral
  - $f(t) = \int_{-\infty}^{\infty} h(\tau) \cdot g(t - \tau) d\tau$
- Laplace (also frequency) domain: multiplication
  - $F(s) = H(s) \cdot G(s)$
  - Deconvolute: $H(s) = F(s)/G(s)$

Transfer functions – time and frequency domain

Time domain

$$
\begin{tikzcd}
x(t) \arrow[r] \arrow[d, "\text{Laplace}"] & \boxed{h(t)} \arrow[r] \arrow[d, "\text{Laplace}"] & y(t) = h(t) * x(t) \\
X(s) \arrow[r] & \boxed{H(s)} \arrow[r] & Y(s) = H(s) \cdot X(s) \arrow[u, "\text{Inverse Laplace}"']
\end{tikzcd}
$$

Frequency domain

Fourier transformation

Table of Common Functions and their Fourier Transforms

Function name	Function in the time domain	Fourier Transform (in the frequency domain)
	$w(t)$	$\widehat{W}(f)$
Dirac delta	$\delta(t)$	1
Constant	1	$\delta(f)$
Cosine	$\cos(2\pi f_0 t)$	$\dfrac{\delta(f-f_0)+\delta(f+f_0)}{2}$
Sine	$\sin(2\pi f_0 t)$	$\dfrac{\delta(f-f_0)-\delta(f+f_0)}{2j}$
Unit step function	$u(t) = \begin{cases} 0, \text{ if } t < 0 \\ 1, \text{ if } t \geq 0 \end{cases}$	$\frac{1}{j\omega}$ (for $\omega = 2\pi f$)
Decaying exponential (for t > 0)	$e^{-\alpha t} u(t),$	$\frac{1}{\alpha + j2\pi f}$, $\alpha > 0$
Box or rectangle function	$\mathrm{rect}(at) = \begin{cases} 0, \text{ if } \lvert at \rvert > \frac{1}{2} \\ 1, \text{ if } \lvert at \rvert \leq \frac{1}{2} \end{cases}$	$\dfrac{1}{\lvert a \rvert}\mathrm{sinc}\left(\dfrac{f}{a}\right) = \dfrac{\sin(\pi f/a)}{\pi f/a}$
Sinc function	$\mathrm{sinc}(at) = \dfrac{\sin(\pi at)}{\pi at}$	$\dfrac{1}{\lvert a \rvert}\mathrm{rect}\left(\dfrac{f}{a}\right)$
Comb function	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\dfrac{1}{T}\sum_{k=-\infty}^{\infty} \delta\left(f - \dfrac{k}{T}\right)$
Gaussian	$e^{-\alpha t^2}$	$\sqrt{\dfrac{\pi}{\alpha}} e^{-\frac{(\pi f)^2}{\alpha}}$