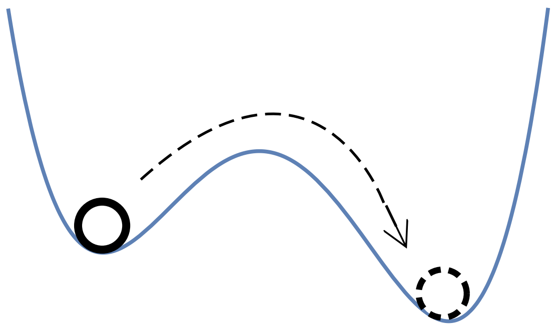
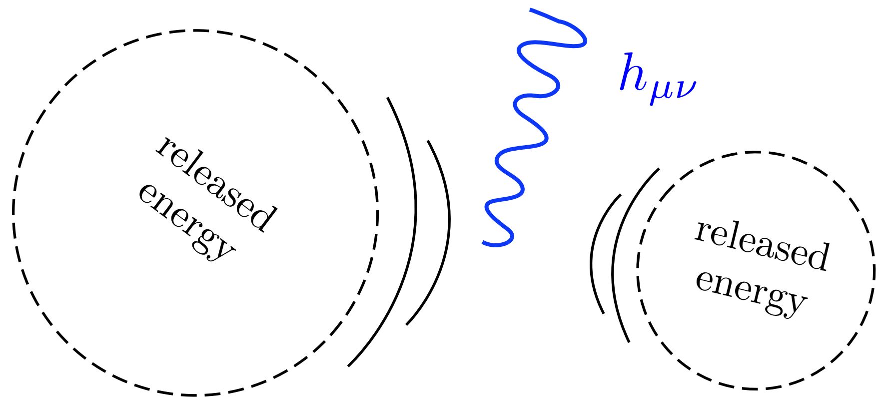
Precise Computation of Thermal Phase Transition Parameters

Mikael Chala

External review of the FTAE Unit of Excellence; April 24, 2026

Motivation





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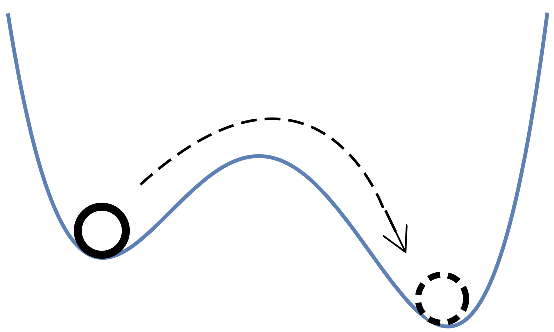

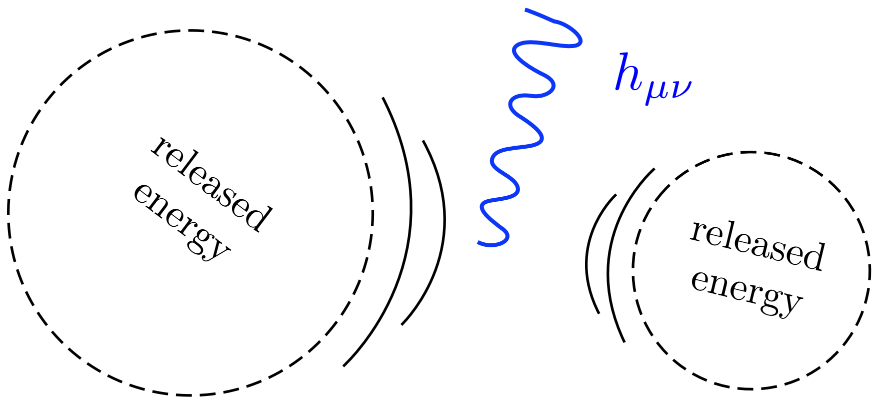

The high-T limit of QFT is captured by a static, bosonic, 3D EFT where Matsubara modes have been integrated out.

This EFT is known only to NLO accuracy, for example to $O(g^4)$ in gauge couplings.

This is insufficient to accurately describe GWs from (strong) PTs [Chala, Criado, Gil, López-Miras; 2406.02667], including current PTA observations.

Precise Computation of Thermal Phase Transition Parameters

Fact Sheet

Objective

Our understanding of field theories at finite temperature (FTQFT) has undergone significant progress in the last decade. It has been boosted in part by the necessity of correctly interpreting the data from current and future detectors of gravitational waves (GWs) produced during Phase Transitions (PT) in the early and hot Universe. By now it is well established that the correct description is provided by a 3-dimensional Euclidean effective field theory (EFT) involving only those degrees of freedom that do not acquire large effective masses in their interaction with the thermal bath; all others being removed from the theory in favor of Lagrangian terms of increasing number of fields and derivatives of light particles.
However, there are strong reasons to push perturbative calculations within this context beyond the current level of accuracy, computing also Lagrangian terms with more than two derivatives, so far mostly ignored. Remarkably, this would also allow us to address the fundamental question, to the best of my knowledge not yet conceived before, of whether there exist degrees of freedom that have gone unnoticed so far. The most important of these are the so-called skyrmions, namely localized field configurations topologically distinct from the vacuum that can be only stable in the presence of four-derivative interactions, which could very well ensue from thermal effects.
The main goal of this project is to improve the accuracy of the EFT description of FTQFT as well as to analyse the impact of higher-order corrections on PT and GW parameters and also, on a different note, on the potential existence of skyrmions inhabiting only at high temperatures.

Project Information

PC-TP2
Grant agreement ID: 101230200

DOI
10.3030/101230200

EC signature date
6 February 2026

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1 March 2026

End date
28 February 2031

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European Research Council (ERC)

Total cost
€ 1 801 250,00

EU contribution
€ 1 801 250,00

Work Packages (WP)

- **WP 1. General computations**
  - Task 1: *Characterisation of the most general 3D EFT*
  - Task 2: *Elimination of unphysical interactions within the 3D EFT*
  - Task 3: *Renormalisation of the 4D theory and of the 3D EFT*
  - Task 4: *Matching the 4D theory onto the 3D EFT*

  - Key challenge: Two- and three-loop sum-integrals.

- **WP 2. Singlet and doublet extensions of the SM**
  - Task 5: *Computation of the bounce solution*
  - Task 6: *Quantum corrections to the effective action*
  - Task 7: *Computation of PT and GW parameters*

  - Key challenge: Perturbative bounce in multi-field case.

- **WP 3. The SM EFT and skyrmions**
  - Task 5 bis: *Computation of the bounce solution*
  - Task 6 bis: *Quantum corrections to the effective action*
  - Task 7 bis: *Computation of PT and GW parameters*
  - Task 8: *Search for skyrmions in the SM*

  - Key challenge: Numerical approach and perturbative validation.

Work Packages	Tasks	Year 1			Year 2			Year 3			Year 4			Year 5		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1.	1															
	2															
	3						**D**									
	4									**D**						
2.	5												**D**			
	6												**D**			
	7															**D**
3.	5 bis															
	6 bis															
	7 bis												**D**			
	8															**D**

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Some preliminary results

1. Running couplings in high-temperature effective field theory [Chala, Dashko, Guedes; 2510.26878]

2. Correction to QCD Debye mass [Bernardo, Chala, Gil, Schicho; 2602.06962]

3. New results about the PT of the SMEFT [Chala, Fiore, Gil; 2507.16905]

4. Public code to solve multi-loop sum-integrals [Gil, López-Miras; wip]

5. Automation of thermal-field theory calculations [Fuentes-Martín, López-Miras, Moreno-Sánchez; wip] [Chala, Fuentes-Martín, Gil, López-Miras, Moreno-Sánchez; wip]

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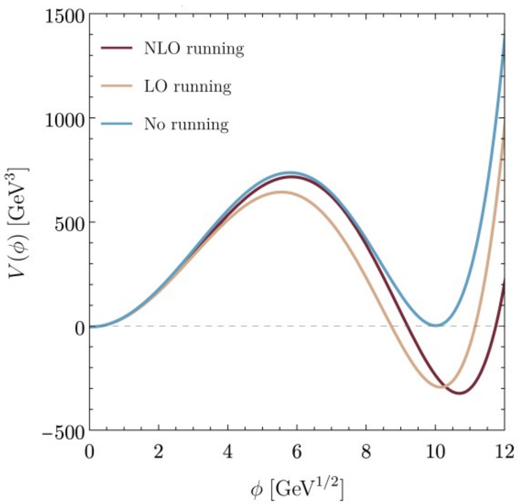
$$\beta_{m_3^2} = \overbrace{-\frac{51}{16}g_3^4 - 9g_3^2\lambda_3 + 12\lambda_3^2}^{\beta^{\mathrm{LO}}_{m_3^2}}$$
$$+ m_3^2\Big[c^{(1)}_{\phi^4D^2}\left(-14g_3^2 + 32\lambda_3\right)$$
$$+ c^{(2)}_{\phi^4D^2}\left(3g_3^2 - 8\lambda_3\right) - 20c_{\phi^2W^2}g_3^2\Big],$$
$$\beta_{\lambda_3} = -c_{\phi^2W^2}\left(\frac{39}{2}g_3^4 + 76g_3^2\lambda_3\right)$$
$$+ c^{(1)}_{\phi^4D^2}\left(128c_{\phi^6}m_3^2 + 3g_3^4 - 52g_3^2\lambda_3 + 256\lambda_3^2\right)$$
$$+ c^{(2)}_{\phi^4D^2}\left(-32c_{\phi^6}m_3^2 - \frac{17}{16}g_3^4 + 9g_3^2\lambda_3 - 60\lambda_3^2\right)$$
$$+ c_{\phi^6}\left(-18g_3^2 + 96\lambda_3\right),$$



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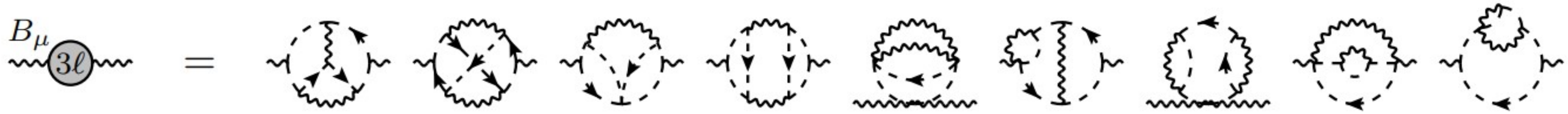

$+$ (240 diagrams)

$$\mathcal{I}_{31111-2} \to \mathcal{I}_{31111-2} + \frac{2T^2}{18(4\pi)^4\epsilon}\left(\frac{1}{4\pi T}\right)^{3\epsilon}$$

$$\mathcal{I}^{\alpha_1\alpha_2\alpha_3}_{s_1s_2s_3s_4s_5s_6} = \sum\!\!\!\!\!\!\int_{K_1K_2K_3} \frac{(k_{1,0})^{\alpha_1}(k_{2,0})^{\alpha_2}(k_{3,0})^{\alpha_3}}{\left[K_1^2\right]^{s_1}\left[K_2^2\right]^{s_2}\left[K_3^2\right]^{s_3}\left[(K_1-K_2)^2\right]^{s_4}\left[(K_1-K_3)^2\right]^{s_5}\left[(K_2-K_3)^2\right]^{s_6}}$$

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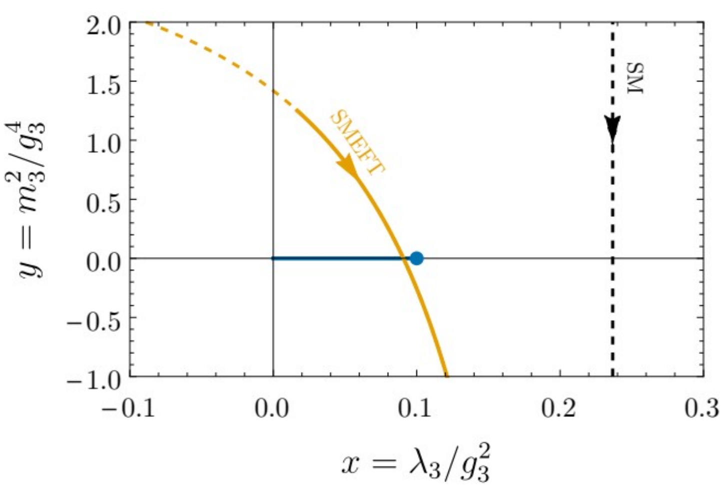

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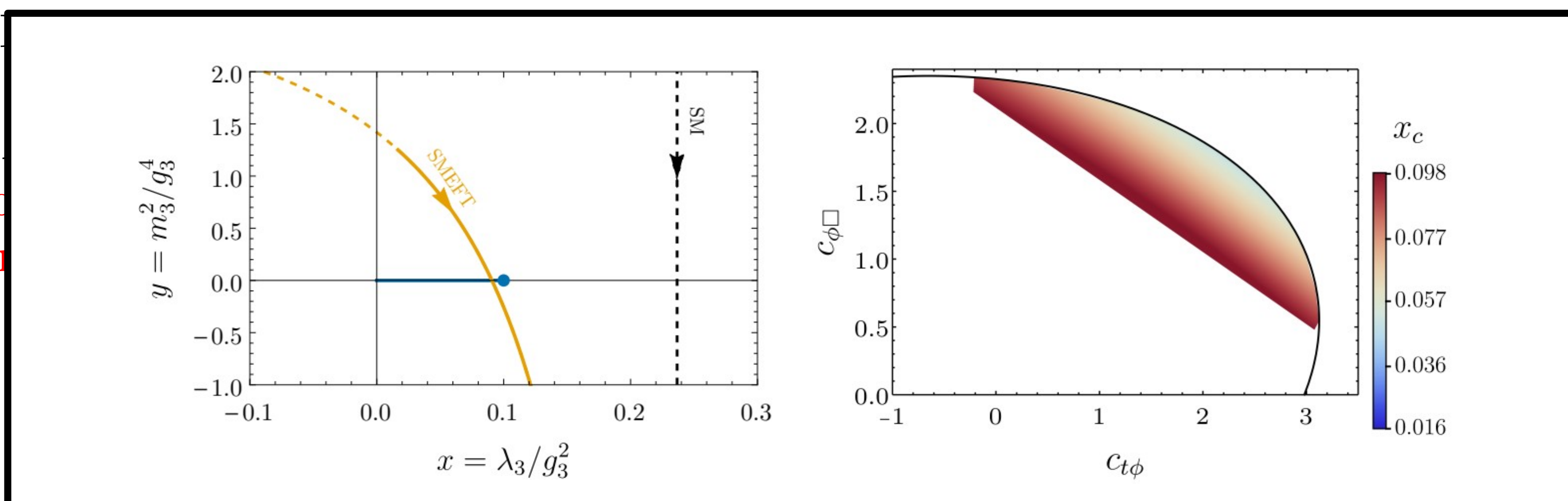

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Backup

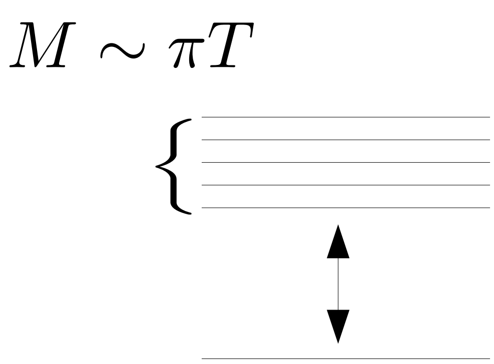
QFT at finite temperature

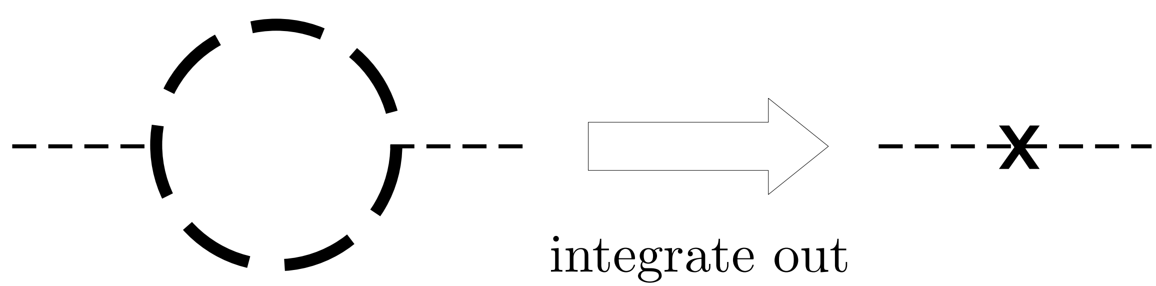
The high-T limit of a given model is described by an Euclidean static 3D EFT involving only bosons

QFT at finite T is equivalent to a regular QFT with periodic time

$$\mathcal{Z} = \mathrm{Tr}(e^{-\beta H}) = \int \mathcal{D}\varphi \langle \varphi | e^{-\beta H} | \varphi \rangle$$

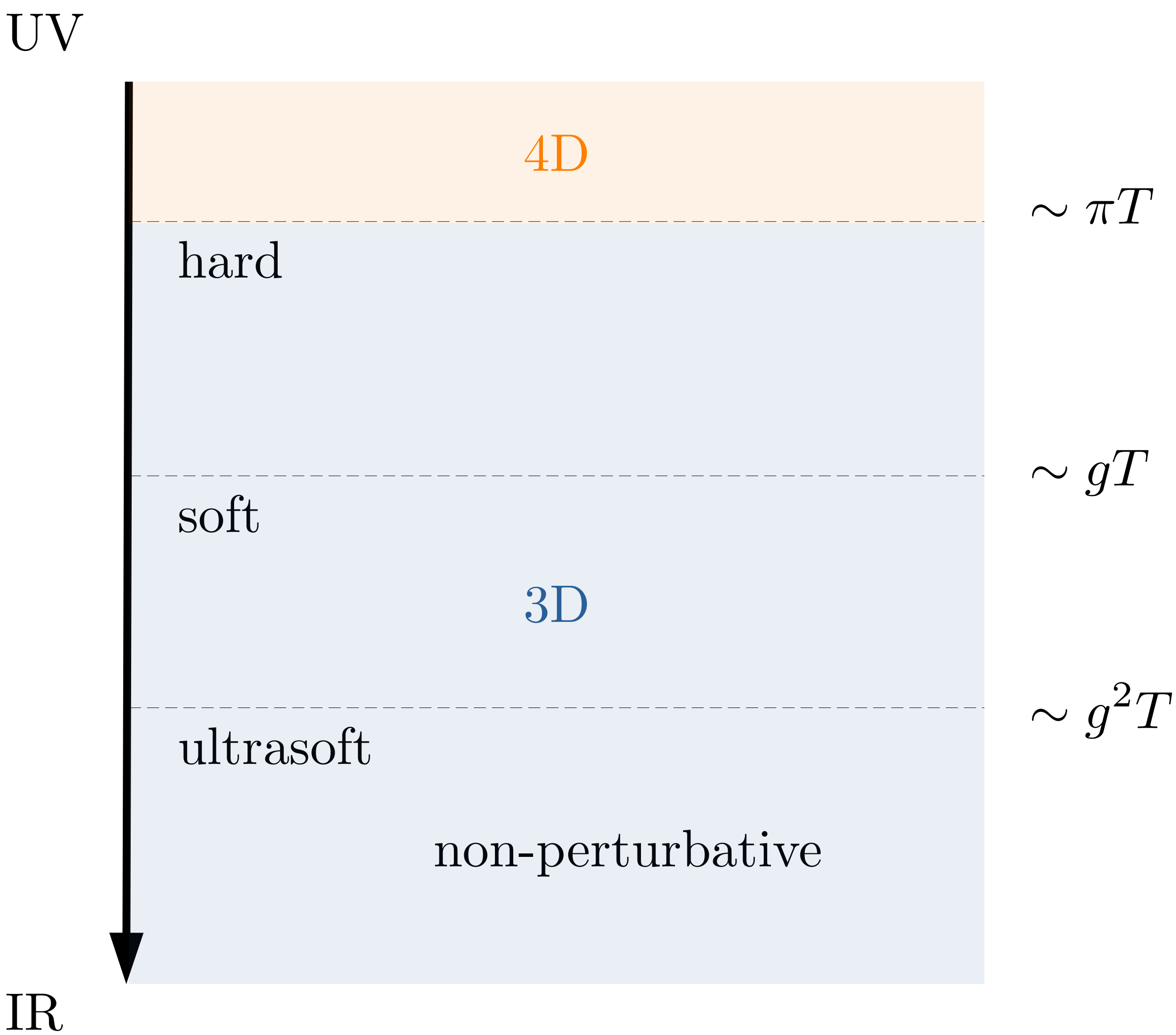
Each field comes with a tower of Matsubara modes (equivalent to Kaluza-Klein excitations in extra-dimensions)





Dimensional reduction

[Kajantie, Laine, Rummukainen, Shaposhnikov; 9508379]



$$\mathcal{L}[\phi, F, \psi]$$

$$\downarrow$$

$$\mathcal{L}[\phi, F, F_0]$$

$$\downarrow$$

$$\mathcal{L}[\phi, F]$$

$$x \sim \frac{g_3^2}{p} \sim \mathcal{O}(1)$$

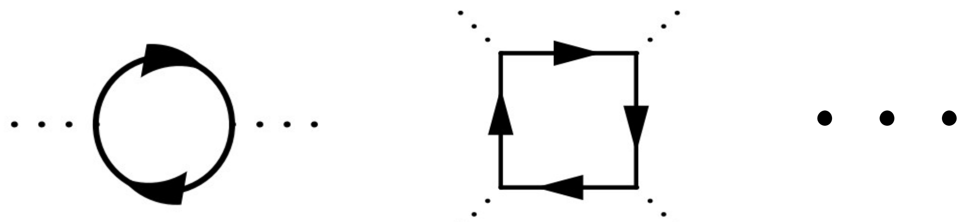
[Essentially, close to a critical point, where log(m/T) grows]

The high-temperature limit of a toy model

Let us consider a model with a real scalar and a fermion

$$L = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \kappa\varphi^3 - \lambda\varphi^4 + i\bar{\psi}D\psi + g\varphi\bar{\psi}\psi$$

We integrate out the Matsubara modes, by performing the hard region expansion of Feynman integrals in



We obtain

$$m_3^2 = m^2 + \frac{g^2T^2}{6}$$

$$\mathscr{L}_3 = \frac{1}{2}K_3(\partial\varphi)^2 + \frac{1}{2}m_3^2\varphi^2 + \kappa_3\varphi^3 + \lambda_3\varphi^4$$

$$+ \alpha_{61}\varphi^6 + \beta_{61}\partial^2\varphi\partial^2\varphi + \beta_{62}\varphi^3\partial^2\varphi + \cdots$$

Perturbative bounce

Use the EFT expansion parameter

$$\varphi_c = \varphi_c^{(0)} + \epsilon\varphi_c^{(1)} + \epsilon^2\varphi_c^{(2)} + \cdots\,, \qquad S_3 = S_3^{(0)} + \epsilon S_3^{(1)} + \epsilon^2 S_3^{(2)} + \cdots$$

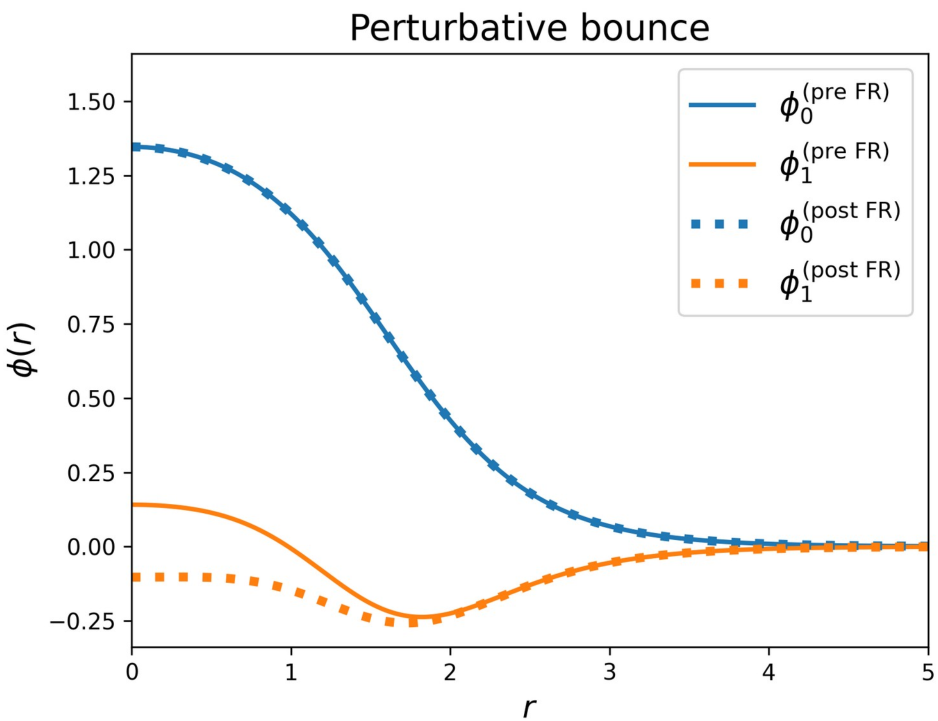
Solve for the minimum of the action order by order in perturbation theory. The leading bounce is the usual one; next corrections solve simple differential equations [Chala, Criado, Gil, López-Miras; 2406.02667]

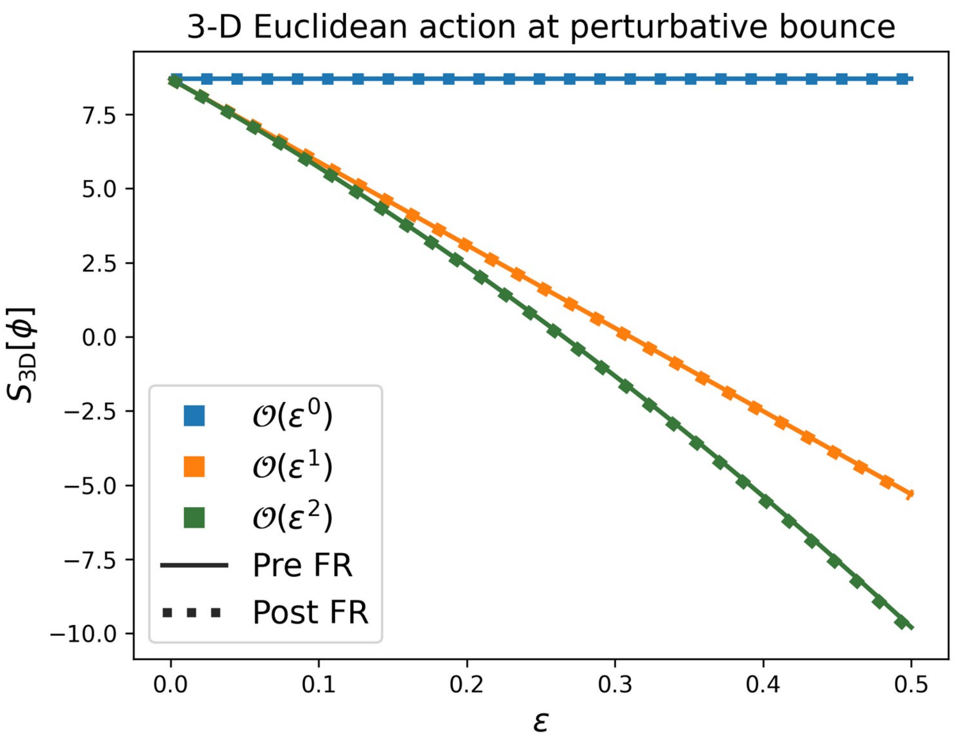
$$\ddot{\varphi}_c^{(1)} + \frac{2}{r}\dot{\varphi}_c^{(1)} - V_3^{(0)\prime\prime}(\varphi_c^{(0)})\,\varphi_c^{(1)} - \frac{1}{4\pi r^2}\frac{\delta S_3^{(1)}}{\delta\varphi}\bigg|_{\varphi_c^{(0)}} = 0$$

$$\dot{\varphi}_c^{(1)}(0) = \lim_{r\to\infty}\varphi_c^{(1)}(r) = 0$$

Perturbative bounce

$S_3[\varphi_c]$ computed this way is physical (i.e. invariant under field redefinitions as well as gauge independent); physical observables independent of how matching is performed [Chala, Criado, Gil, López-Miras; 2406.02667]





PT parameters

$$L = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \kappa\varphi^3 - \lambda\varphi^4 + i\bar{\psi}D\psi + g\varphi\bar{\psi}\psi$$

In dashed, we neglect effective operators

The vertical lines represent the (very conservative) limit of validity of the EFT