QCD factorisation and Super-super-leading logarithms

Jack Holguin

Torino, 2026

Overview

1. Gaps-between-jets
    - I will discuss a brief historical overview of the observable, leading to the discovery of super-leading logs.
    - I will highlight an easily generalisable approach to computing the first super-leading log based on work from Forshaw, JH, 2109.03665.
2. Jettiness
    - I will extend the calculation of the super-leading logs to Jettiness. In doing so, uncovering a super-super-leading log as reported in Banfi, Forshaw, JH, 2511.11799.
    - I'll discuss some physical intuition for the super-super-leading log, from which we can see hints at its all orders structure.

Gapped jet rates

Back to the 90s

Can the Higgs be seen in rapidity gap events at the Tevatron or the LHC?

V.A. Khoze[a], A.D. Martin[a] and M.G. Ryskin[a,b]

[a] Department of Physics, University of Durham, Durham, DH1 3LE
[b] Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188350, Russia

Abstract

Double diffractive Higgs production at pp (or $\bar{p}p$) colliders continues to attract attention as a potential signal in the search for the boson. We present improved perturbative QCD estimates of the event rates for both the exclusive and inclusive double diffractive Higgs processes, paying particular attention to the survival probability of the rapidity gaps. We find that the major uncertainty is in the prediction for the survival probability associated with soft rescattering. We show that an analogous process, the double diffractive production of a pair of jets with large values of E_T, has an event rate which makes it accessible at the Tevatron. Observation of this process can therefore be used as a luminosity monitor for two- gluon exchange processes, such as the production of a Higgs boson with rapidity gaps on either side.

Rapidity gaps in Higgs production

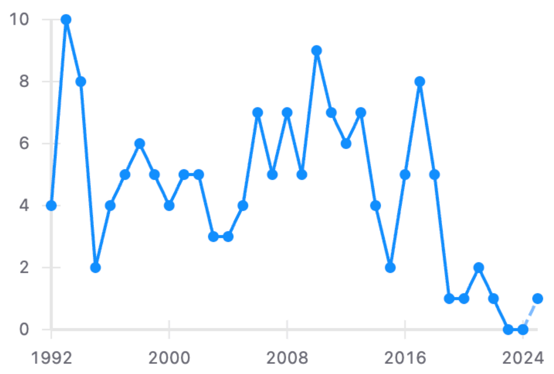
Yu.L. Dokshitzer [1]
Department of Theoretical Physics, University of Lund, Sölvegatan 14A, S-223 62 Lund, Sweden

V.A. Khoze [1]
Centre for Particle Theory, University of Durham, Durham DH1 3LE, UK
and INFN Eloisatron project

and

T. Sjöstrand
CERN, CH-1211 Geneva 23, Switzerland

Received 11 October 1991



The possibility to discriminate different Higgs production mechanisms using a rapidity gap signature is discussed. The results of Monte Carlo calculations are presented, which show that the two processes WW→H and gg→H indeed have different event structures, but also that these differences, without special care, would be masked by other effects.

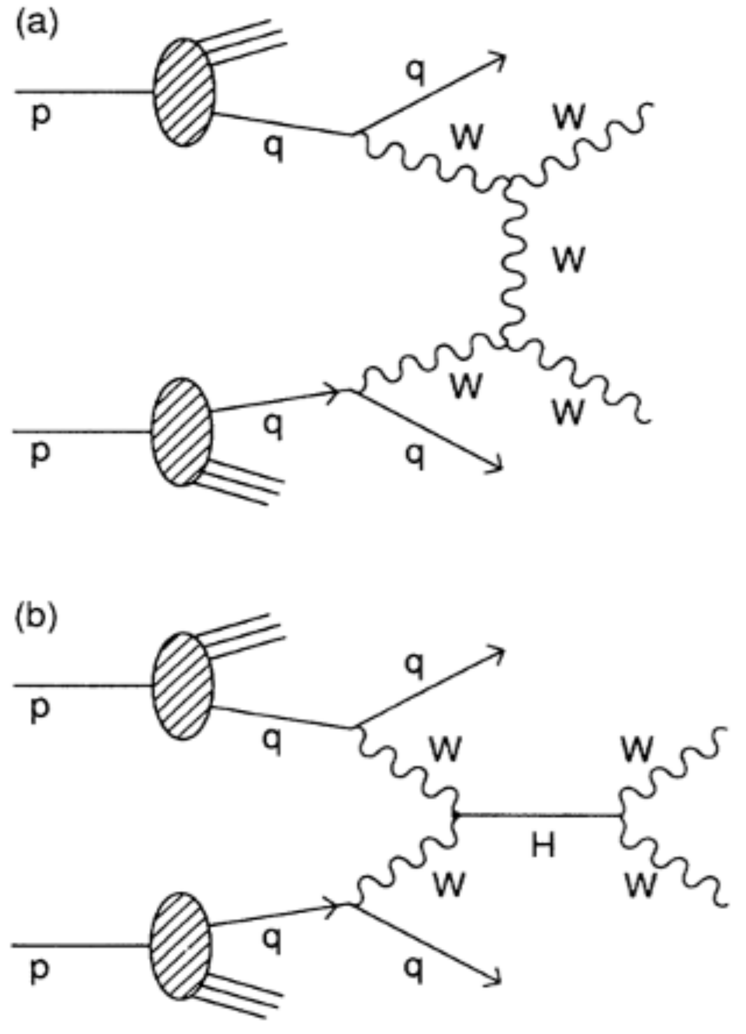
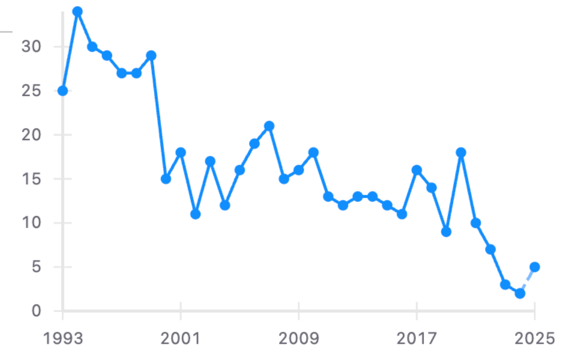


FIG. 1. Basic mechanism for producing W-W interaction processes in high-energy pp collisions, with the presence of a rapidity gap in the final state.

Rapidity gaps and jets as a new-physics signature in very-high-energy hadron-hadron collisions

J. D. Bjorken
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 30 March 1992)

In hadron-hadron collisions, production of Higgs bosons and other color-singlet systems can occur via fusion of electroweak bosons, occasionally leaving a "rapidity gap" in the underlying-event structure. This observation, due to Dokshitzer, Khoze, and Troyan, is studied to see whether it serves as a signature for detection of the Higgs bosons, etc. We find it is a very strong signature at subprocess c.m. energies in excess of a few TeV. The most serious problem with this strategy is the estimation of the fraction of events containing the rapidity gap; most of the time the gap is filled by soft interactions of spectator degrees of freedom. We also study this question and estimate this "survival probability of the rapidity gap" to be of order 5%, with an uncertainty of a factor 3. Ways of testing this estimate and further discussion of the underlying hard-diffraction physics are presented.



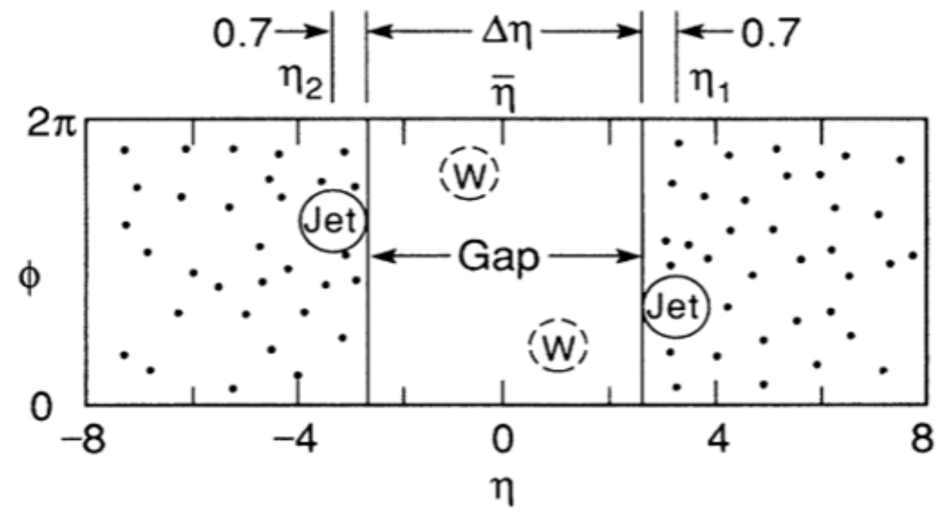
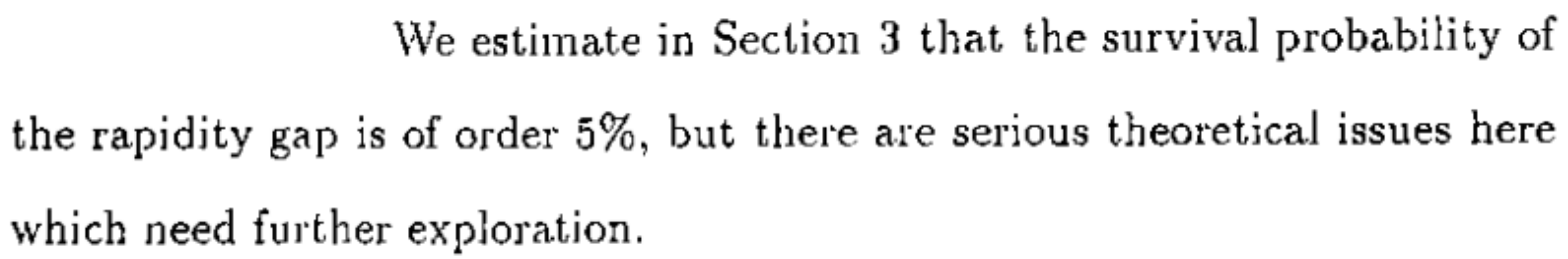


FIG. 2. Event morphology in lego variables for the processes depicted in Fig. 1. The tagging jets are the hadronization products of the quarks, while for large Higgs masses, almost all of the W-decay products lie within the dashed circles. The remaining region, marked gap, contains on average no more than 2 or 3 hadrons.

Gapped jet rates

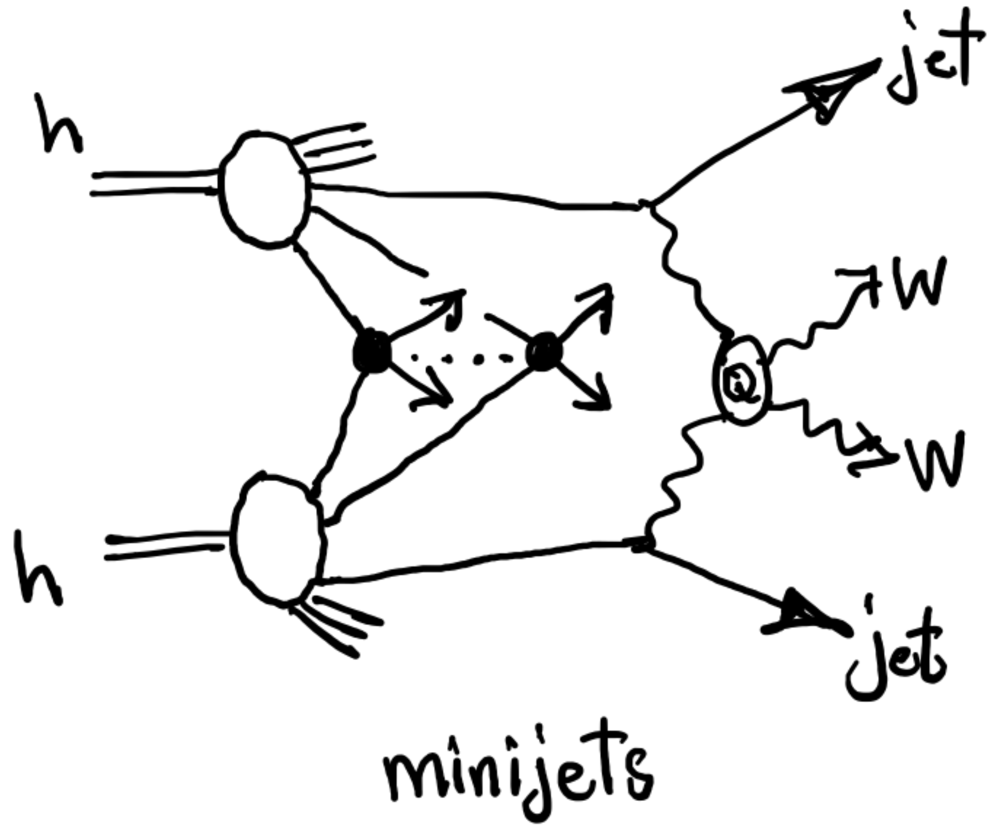
Back to the 90s

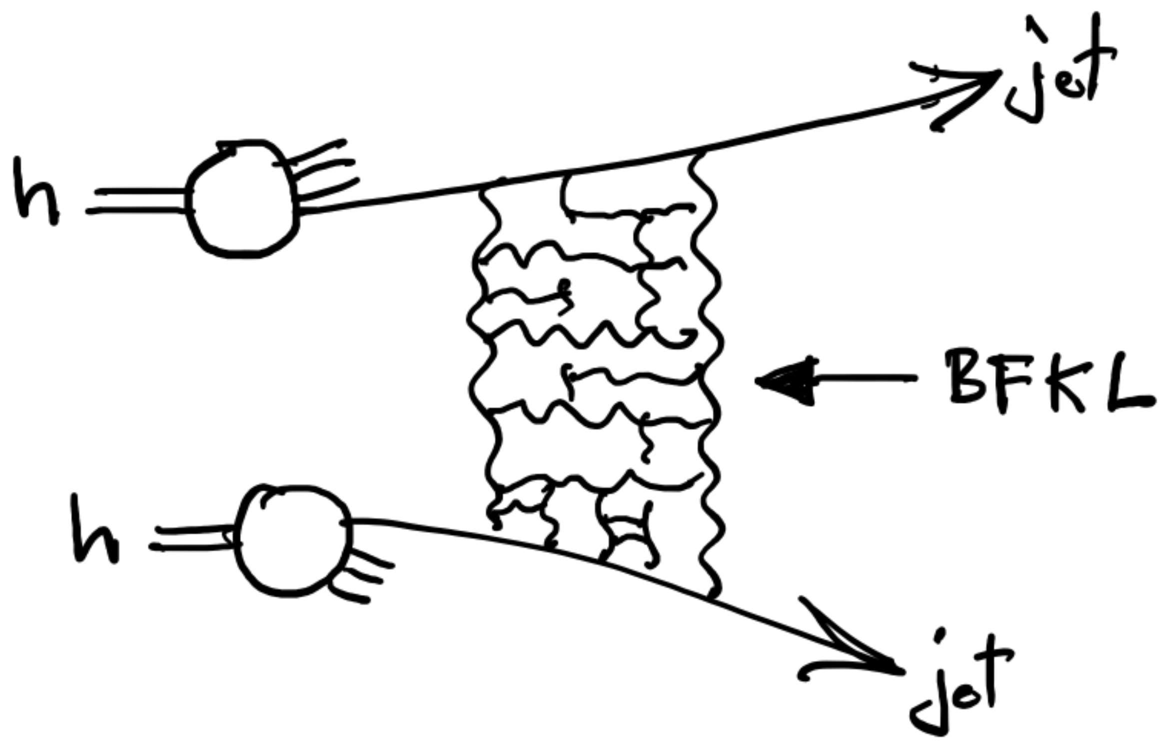
- However, theoretically controlled predictions were lacking. Quoting Bjorken:



> We estimate in Section 3 that the survival probability of the rapidity gap is of order 5%, but there are serious theoretical issues here which need further exploration.

- The problem was two major backgrounds. Gaps could be filled by "mini-jets", i.e. MPIs, and are a signature of diffractive physics Mueller, Navelet '87 which, even now, is not yet under precise control (for recent progress see Lee, Schindler, Stewart 2508.10231).



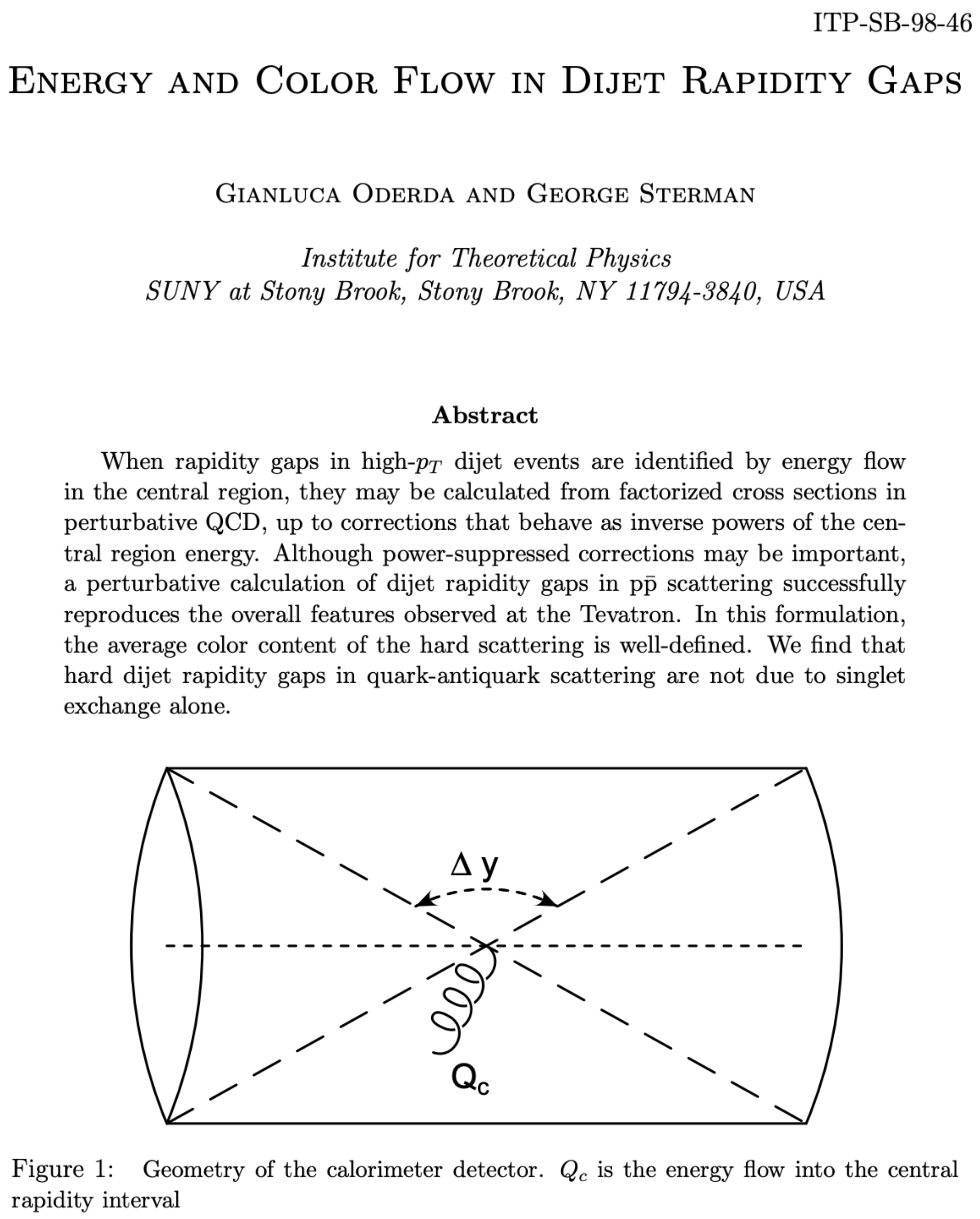


Gaps between jets

Back to the 90s

- A step forward was made by Oderda and Sterman ’98.
- Whilst a gap with nothing in is problematic, a gap with nothing above a veto (Q_c) was perturbatively calculable up to power corrections in $\Lambda_{\mathrm{QCD}}/Q_c$.
- Additionally, they proposed a simple factorisation for this measurement.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}Q_0} = f^a(Q) f^b(Q) \otimes Tr\left[\vec{H}(a+b \to 2\text{jets} : Q, \Delta y) \otimes \vec{S}_{2,2}(\Delta y \ln(Q/Q_c))\right]$$



ITP-SB-98-46

Energy and Color Flow in Dijet Rapidity Gaps

Gianluca Oderda and George Sterman

Institute for Theoretical Physics
SUNY at Stony Brook, Stony Brook, NY 11794-3840, USA

Abstract

When rapidity gaps in high-p_T dijet events are identified by energy flow in the central region, they may be calculated from factorized cross sections in perturbative QCD, up to corrections that behave as inverse powers of the central region energy. Although power-suppressed corrections may be important, a perturbative calculation of dijet rapidity gaps in $p\bar{p}$ scattering successfully reproduces the overall features observed at the Tevatron. In this formulation, the average color content of the hard scattering is well-defined. We find that hard dijet rapidity gaps in quark-antiquark scattering are not due to singlet exchange alone.

Figure 1: Geometry of the calorimeter detector. Q_c is the energy flow into the central rapidity interval

Gaps between jets

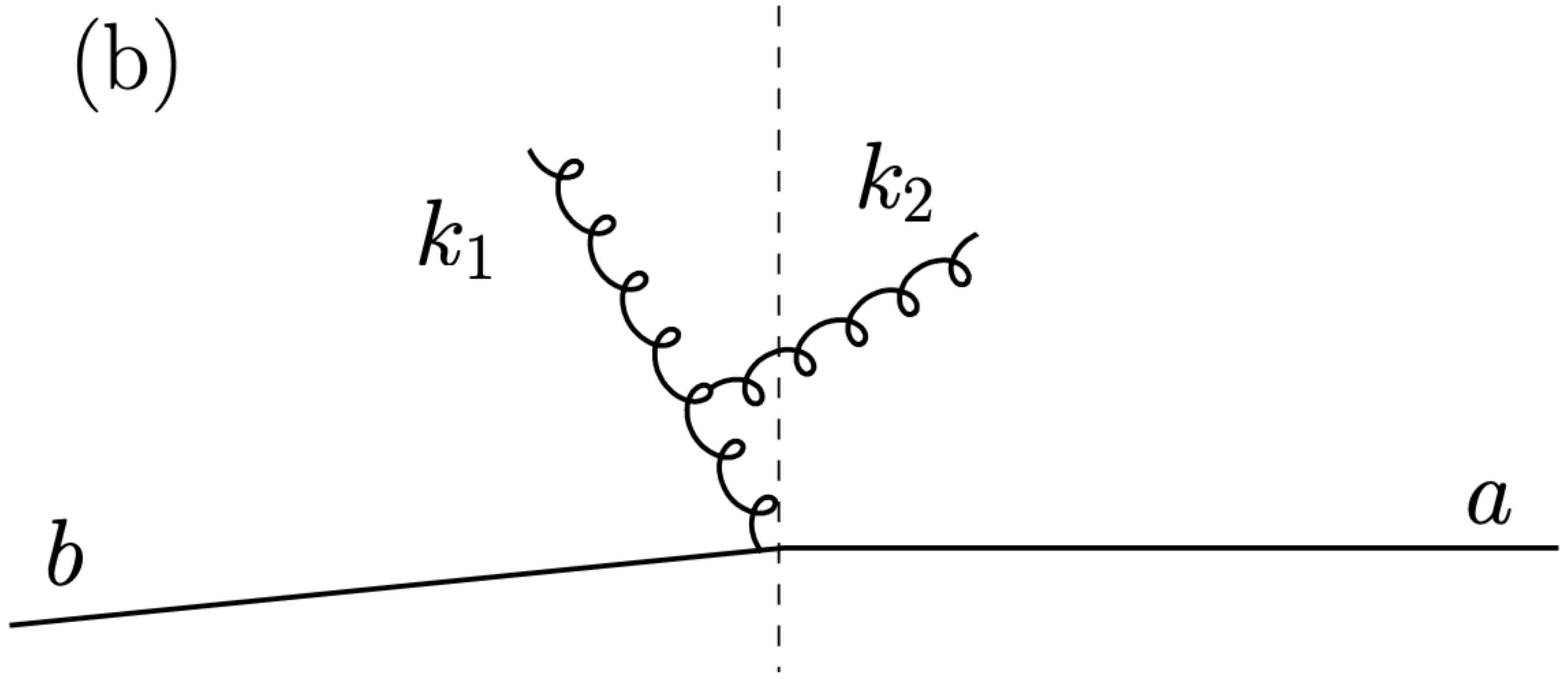
Back to the 00s

- Unfortunately, we now understand that this factorisation misses a lot of important physics.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}Q_0} = f^a(Q)f^b(Q) \otimes Tr\left[\overrightarrow{H}(a+b \to 2\text{jets} : Q, \Delta y) \times \vec{S}_{2,2}(\Delta y \ln(Q/Q_c))\right]$$

- Firstly, $\vec{S}_{2,2}(\Delta y \ln(Q/Q_c))$ does not include the full complexity of the soft logarithms contributing to the gaps-between-jets distribution. It misses non-global logarithms (Dasgupta, Salam '01).

- As it is a single-log observable (at one loop), these NGLs are leading for this observable and the factorisation is invalid at any accuracy.



$$\frac{\mathrm{d}\sigma_{\mathrm{NG}}}{\mathrm{d}Y\mathrm{d}Q_0} \sim \alpha_s^2 \ln^2(Q/Q_c)$$

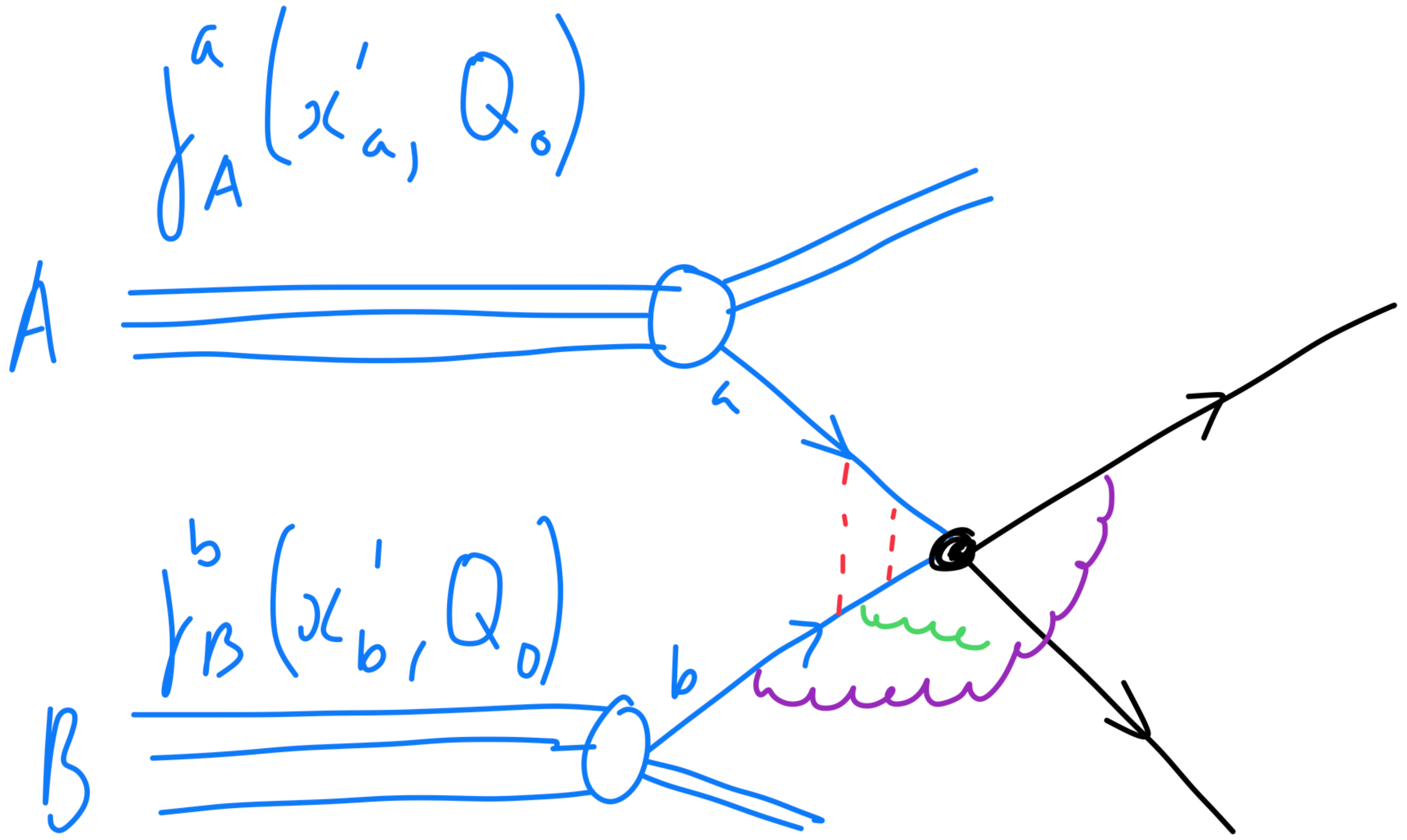
Gaps between jets

Back to the 00s

- Unfortunately, we do now understand that this factorisation misses a lot of important physics.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}Q_0} = f^a(Q)f^b(Q) \otimes Tr\left[\vec{H}(a+b \to 2\text{jets} : Q, \Delta y) \times \vec{S}_{2,2}(\Delta y \ln(Q/Q_c))\right]$$

- Secondly, the full structure of collinear logs mixed with soft logs does not factorise into PDFs and $\vec{S}_{2,2}(\Delta y \ln(Q/Q_c))$ due to the presence of Coulomb/Glauber phases. What's missing are the "super-leading logs" from factorisation breaking (coherence violation).
  Forshaw, Kyrieleis, Seymour '06



$$\frac{\mathrm{d}\sigma_{\mathrm{CVL}}}{\mathrm{d}Y\mathrm{d}Q_0} \sim \alpha_s^4 \ln^5(Q/Q_c)$$

A pause for nomenclature

What is leading log? What is super-leading? Unfortunately nomenclature is based on gaps-between-jets which is single log a la Oderda and Sterman.

- The typical starting point:

$$\ln \Sigma \left(v = e^{-L}\right) = L g_{\mathrm{LL}}(\alpha_{\mathrm{s}} L) + g_{\mathrm{NLL}}(\alpha_{\mathrm{s}} L) + \alpha_{\mathrm{s}} g_{\mathrm{NNLL}}(\alpha_{\mathrm{s}} L) + \cdots$$

$$\Sigma \left(v = e^{-L}\right) = \sum_n C_n^{\mathrm{DL}} \alpha_{\mathrm{s}}^n L^{2n} + \alpha_{\mathrm{s}} L \sum_n C_n^{\mathrm{NDL}} \alpha_{\mathrm{s}}^n L^{2n} + \cdots$$

Factorise and count loops in functions and RG evolution.

- Super-leading (coherence violating) logs do not exponentiate and factorisation is far from simple. Only the second counting works consistently and easily.

A pause for nomenclature

What is leading log? What is super-leading? Unfortunately nomenclature is based on gaps-between-jets which is single log a la Oderda and Sterman.

- So where do super-leading logs fit in? They are N^{3}DL or $\alpha_s^n L^{2n-3}$. But, gaps-between-jets is has no LL piece, the leading piece is NLL ($\alpha_s^n L^n$) and N^{3}DL is super-leading relative to NLL.

- How does this extend to other observables?
  - The $\alpha_s^4 L^5$ logs and factorisation breaking is often referred to as the “super-leading logs” but it rarely is super-leading compared to the whole expansion of an observable.
  - I will use two countings: relative to the double log, and factorisation breaking relative to DGLAP. Relative to DGLAP evolution of the initial state, $\alpha_s^4 L^5$ is always super-leading.

A pause for nomenclature

What is leading log? What is super-leading? Unfortunately nomenclature is based on gaps-between-jets which is single log a la Oderda and Sterman.

- So where do super-leading logs fit in? They are N^{3}DL or $\alpha_s^n L^{2n-3}$. But, gaps-between-jets is has no LL piece, the leading piece is NLL ($\alpha_s^n L^n$) and N^{3}DL is super-leading relative to NLL.

- How does this extend to other observables?
  - The $\alpha_s^4 L^5$ logs and factorisation breaking is often referred to as the “super-leading logs” but it rarely is super-leading compared to the whole expansion of an observable.
  - I will use two countings: relative to the double log, and factorisation breaking relative to DGLAP. Relative to DGLAP evolution of the initial state, $\alpha_s^4 L^5$ is always super-leading.

* I’ll give a 3rd counting at the end.

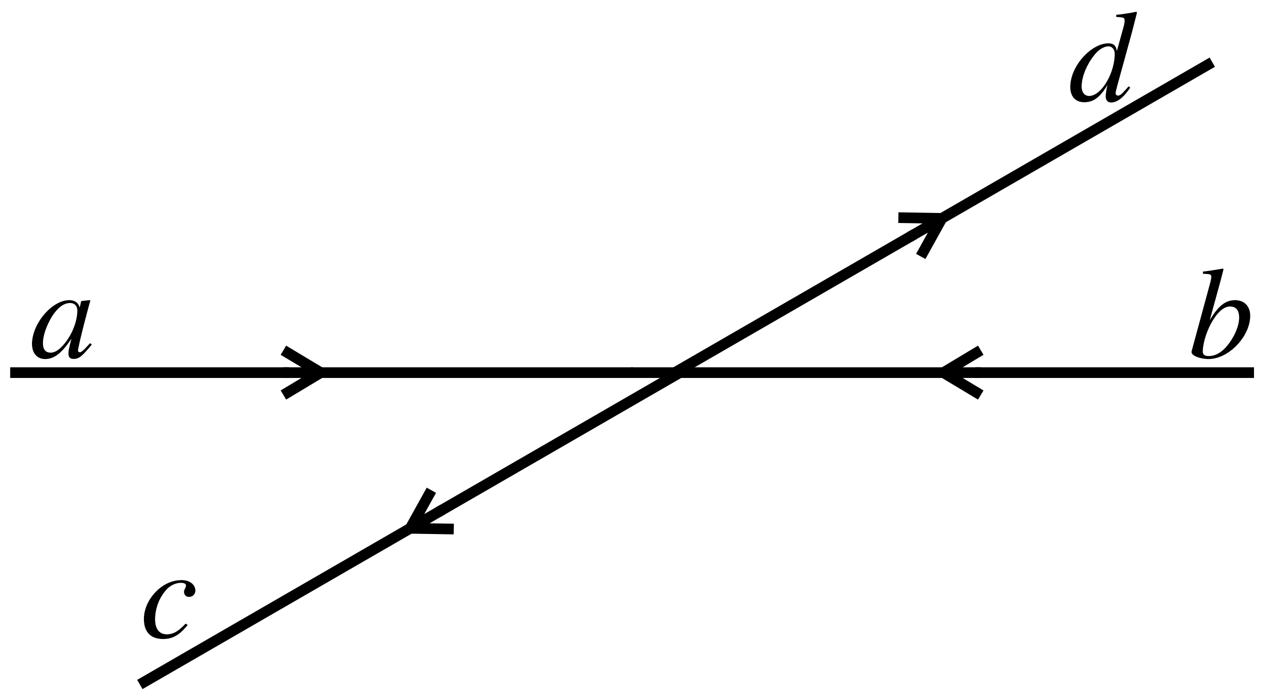
Computing the super-leading Log

Based on Forshaw, JH, 2109.03665.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}Q_0} = f^a(Q)f^b(Q) \otimes Tr\left[\overrightarrow{H}(a+b \to 2\text{jets} : Q, \Delta y) \otimes \vec{S}_{2,2}(\Delta y \ln(Q/Q_c))\right]$$

1-loop evolution
at fixed coupling

$$\frac{\mathrm{d}\sigma_{\mathrm{OS}}}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\mathcal{B}} = f_A(x_a, Q) f_B(x_b, Q) \mathrm{Tr}(\mathbf{V}_{Q_0,Q} \mathbf{H} \mathbf{V}^\dagger_{Q_0,Q})$$



$$\mathbf{V}_{Q_0,Q} \approx \exp\left(-\frac{\alpha_s}{\pi} \ln \frac{Q}{Q_0} (Y\mathbf{T}_t^2 + i\pi \mathbf{T}_s^2)\right)$$

$$\mathbf{T}_t^2 = (\mathbf{T}_a + \mathbf{T}_c)^2 = (\mathbf{T}_b + \mathbf{T}_d)^2$$

$$\mathbf{T}_s^2 = (\mathbf{T}_a + \mathbf{T}_b)^2$$

Computing the super-leading Log

Based on Forshaw, JH, 2109.03665.

- Let's see where the OS result breaks down. First, we manually re-instate the plus-prescription.

$$\frac{\mathrm{d}\sigma_{\mathrm{OS}}}{\mathrm{d}x_a\mathrm{d}x_b\,\mathrm{d}\mathcal{B}} = f_A(x_a,Q)f_B(x_b,Q)\mathrm{Tr}(\mathbf{V}_{Q_0,Q}\mathbf{H}\mathbf{V}^\dagger_{Q_0,Q}) \qquad \mathbf{V}_{Q_0,Q} \approx \exp\left(-\frac{\alpha_s}{\pi}\ln\frac{Q}{Q_0}(Y\mathbf{T}_t^2 + i\pi\mathbf{T}_s^2)\right)$$

Expand DGLAP evolution of hadron A to leading order

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a\mathrm{d}x_b\,\mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{\mathrm{d}k_T}{k_T}\int_0^{1-\frac{k_T}{Q}e^{Y/2}}\frac{\mathrm{d}z}{z}\,P_{qq}(z)\,f_B(x_b,Q)$$

$$\times\left[\Theta(z-x_a)\,f_A(x_a/z,Q_0)\frac{1}{\mathbf{T}_a^2}\mathrm{Tr}(\mathbf{V}_{Q_0,k_T}\mathbf{T}_a\mathbf{V}_{k_T,Q}\mathbf{H}\mathbf{V}^\dagger_{k_T,Q}\mathbf{T}_a^\dagger\mathbf{V}^\dagger_{Q_0,k_T}) - z f_A(x_a,Q_0)\mathrm{Tr}(\mathbf{V}_{Q_0,Q}\mathbf{H}\mathbf{V}^\dagger_{Q_0,Q})\right]$$

We have not assumed factorisation. The collinear is interleaved with the soft, the scale determined by the k_T of the collinear gluon.

Computing the super-leading Log

Based on Forshaw, JH, 2109.03665.

- To illustrate the point...

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{\mathrm{d}k_T}{k_T} \int_0^{1-\frac{k_T}{Q}e^{Y/2}} \frac{\mathrm{d}z}{z}\, P_{qq}(z)\, f_B(x_b, Q)$$

$$\times \left[\Theta(z - x_a)\, f_A(x_a/z, Q_0) \frac{1}{\mathbf{T}_a^2} \mathrm{Tr}(\mathbf{V}_{Q_0,k_T} \mathbf{T}_a \mathbf{V}_{k_T,Q} \mathbf{H} \mathbf{V}^\dagger_{k_T,Q} \mathbf{T}_a^\dagger \mathbf{V}^\dagger_{Q_0,k_T}) - z f_A(x_a, Q_0) \mathrm{Tr}(\mathbf{V}_{Q_0,Q} \mathbf{H} \mathbf{V}^\dagger_{Q_0,Q}) \right]$$

No jet veto, we get the DGLAP plus-prescription back

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{\mathrm{d}k_T}{k_T} \int_0^{1-\frac{k_T}{Q}} \frac{\mathrm{d}z}{z}\, P_{qq}(z) \Big[\Theta(z - x_a)\, f_A(x_a/z, Q_0) \;- z f_A(x_a, Q_0) \Big] \mathrm{Tr}(\mathbf{H}) f_B(x_b, Q)$$

$$= \frac{\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{\mathrm{d}k_T}{k_T} \int_{x_a}^{1} \frac{\mathrm{d}z}{z}\, C_F \left(\frac{1+z^2}{1-z} \right)_+ f_A(x_a/z, Q_0) f_B(x_b, Q) \mathrm{Tr}(\mathbf{H})$$

Computing the super-leading Log

Based on Forshaw, JH, 2109.03665.

- So, what happens when we compute this expression?

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{\mathrm{d}k_T}{k_T} \int_0^{1-\frac{k_T}{Q}e^{Y/2}} \frac{\mathrm{d}z}{z}\, P_{qq}(z)\, f_B(x_b, Q)$$

$$\times \left[\Theta(z - x_a)\, f_A(x_a/z, Q_0) \frac{1}{\mathbf{T}_a^2} \mathrm{Tr}(\mathbf{V}_{Q_0,k_T} \mathbf{T}_a \mathbf{V}_{k_T,Q} \mathbf{H} \mathbf{V}^\dagger_{k_T,Q} \mathbf{T}_a^\dagger \mathbf{V}^\dagger_{Q_0,k_T}) - z f_A(x_a, Q_0) \mathrm{Tr}(\mathbf{V}_{Q_0,Q} \mathbf{H} \mathbf{V}^\dagger_{Q_0,Q}) \right]$$

- $[\mathbf{T}_t^2, \mathbf{T}_s^2] \neq 0 \text{ and } [\mathbf{T}_a, \mathbf{T}_s^2] \neq 0$ and so the real emission term is “trapped”.

- Since we can’t pull out a Casimir, the plus-prescription is broken. Expanding in α_s, we find

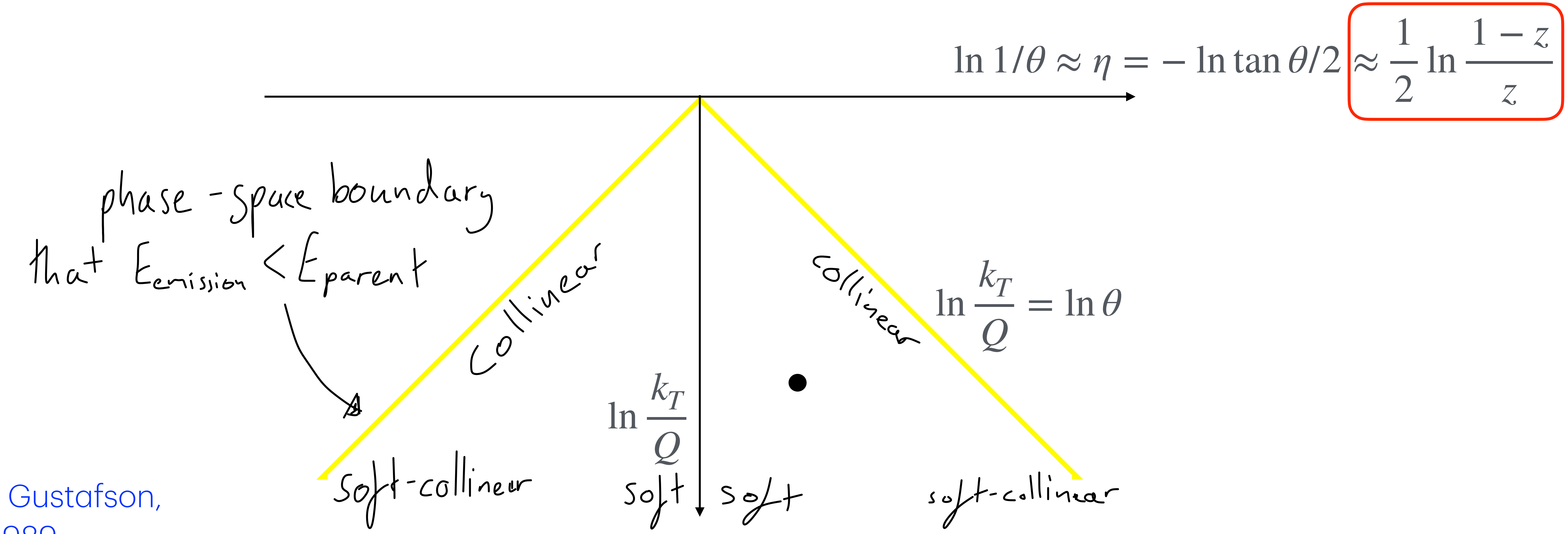
$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\mathcal{B}} \sim \pi^2 N_c^2 Y \alpha_s^4 \log^5(Q/Q_0)$$

Computing the super-leading Log

Diagrammatically

- It is instructive to view this on the Lund plane.

Dipole kinematics



$$\ln 1/\theta \approx \eta = -\ln\tan\theta/2 \boxed{\approx \frac{1}{2}\ln\frac{1-z}{z}}$$

$$\ln\frac{k_T}{Q} = \ln\theta$$

$$\ln\frac{k_T}{Q}$$

Andersson, Gustafson, Lonnblad, 1989

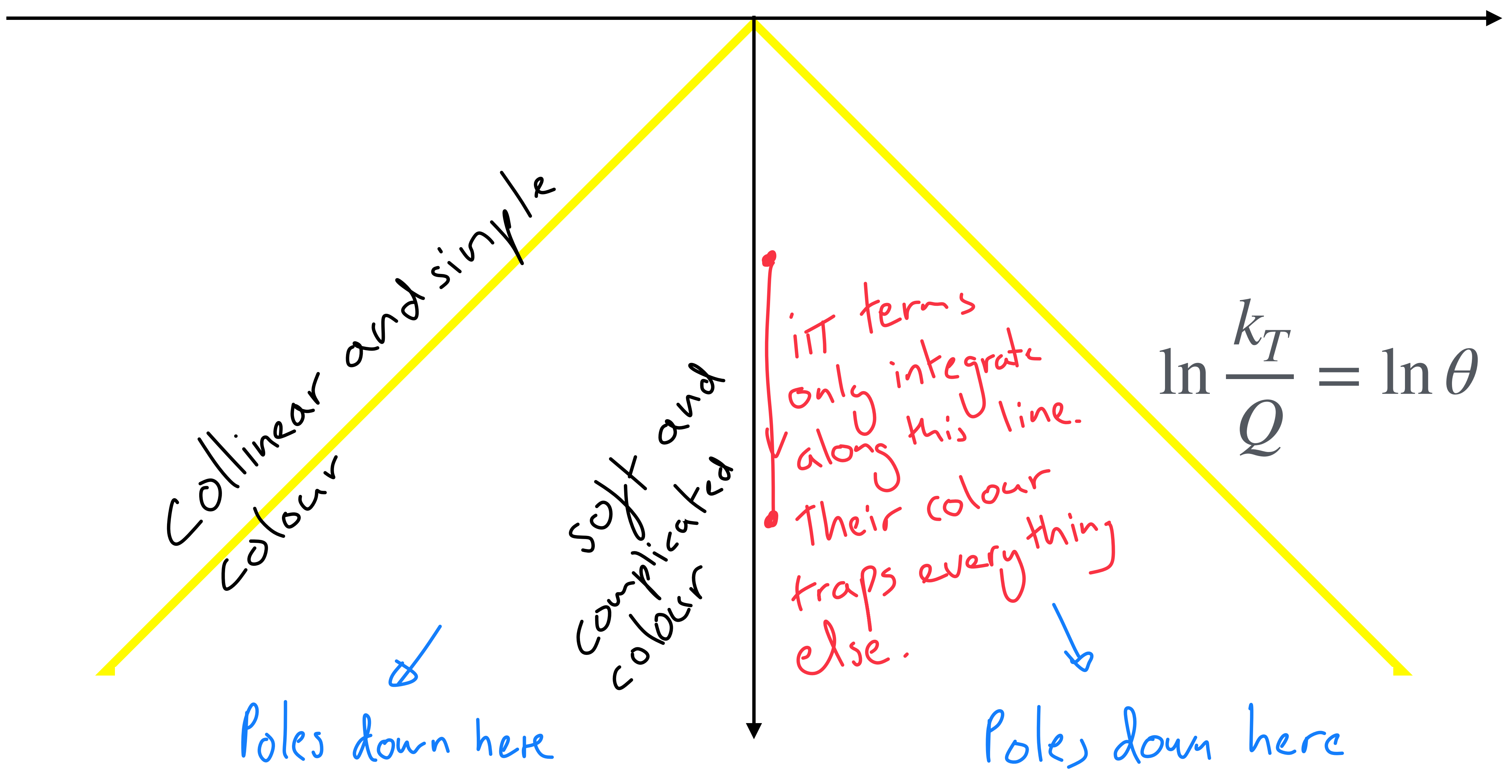
Computing the super-leading Log

Diagrammatically

- It is instructive to view this on the Lund plane.

Dipole kinematics

$$\ln 1/\theta \approx \eta = -\ln\tan\theta/2 \boxed{\approx \frac{1}{2}\ln\frac{1-z}{z}}$$



iπ terms only integrate along this line. Their colour traps everything else.

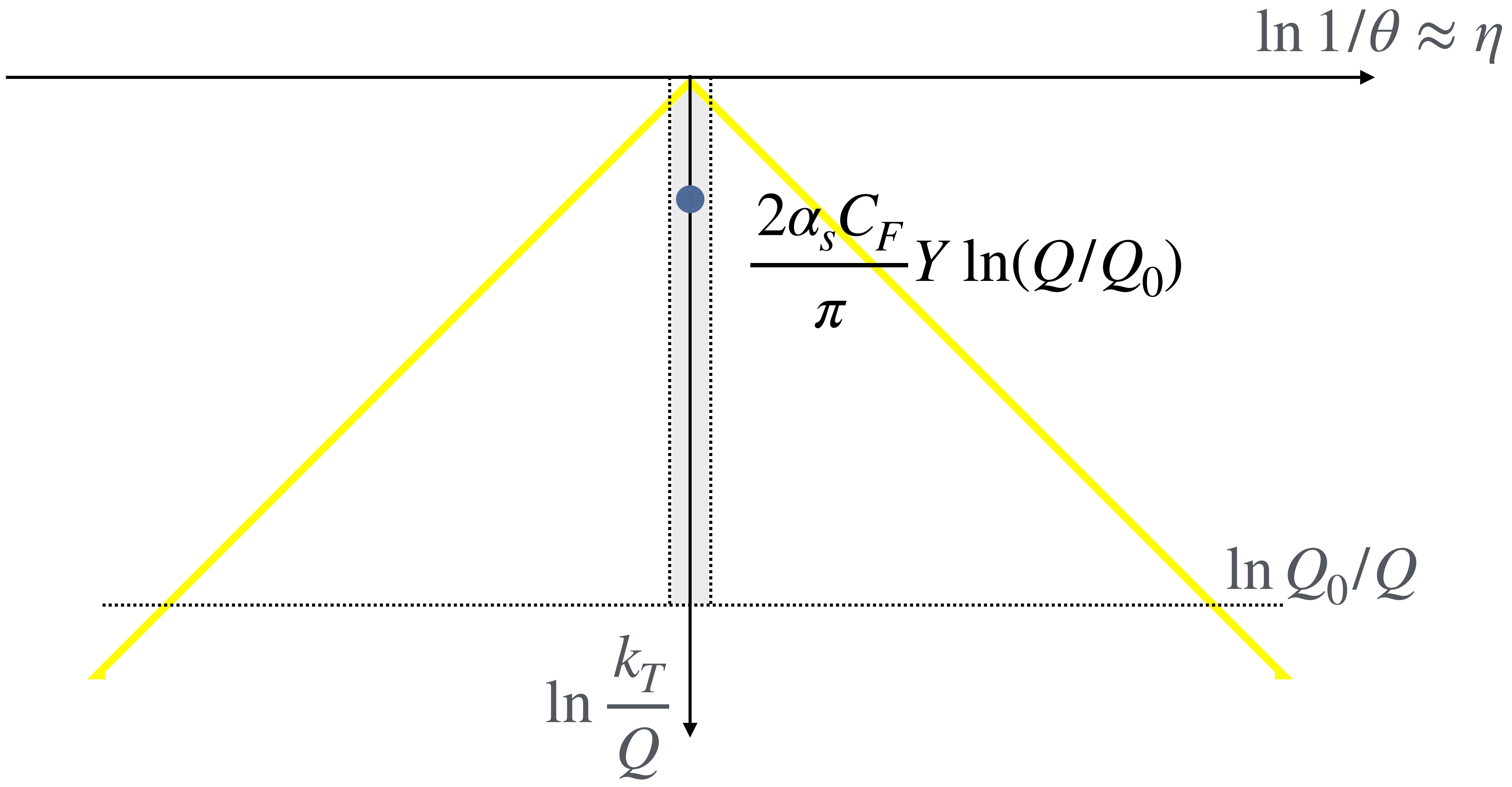
$$\ln\frac{k_T}{Q} = \ln\theta$$

Andersson, Gustafson, Lonnblad, 1989

Computing the super-leading Log

Diagrammatically

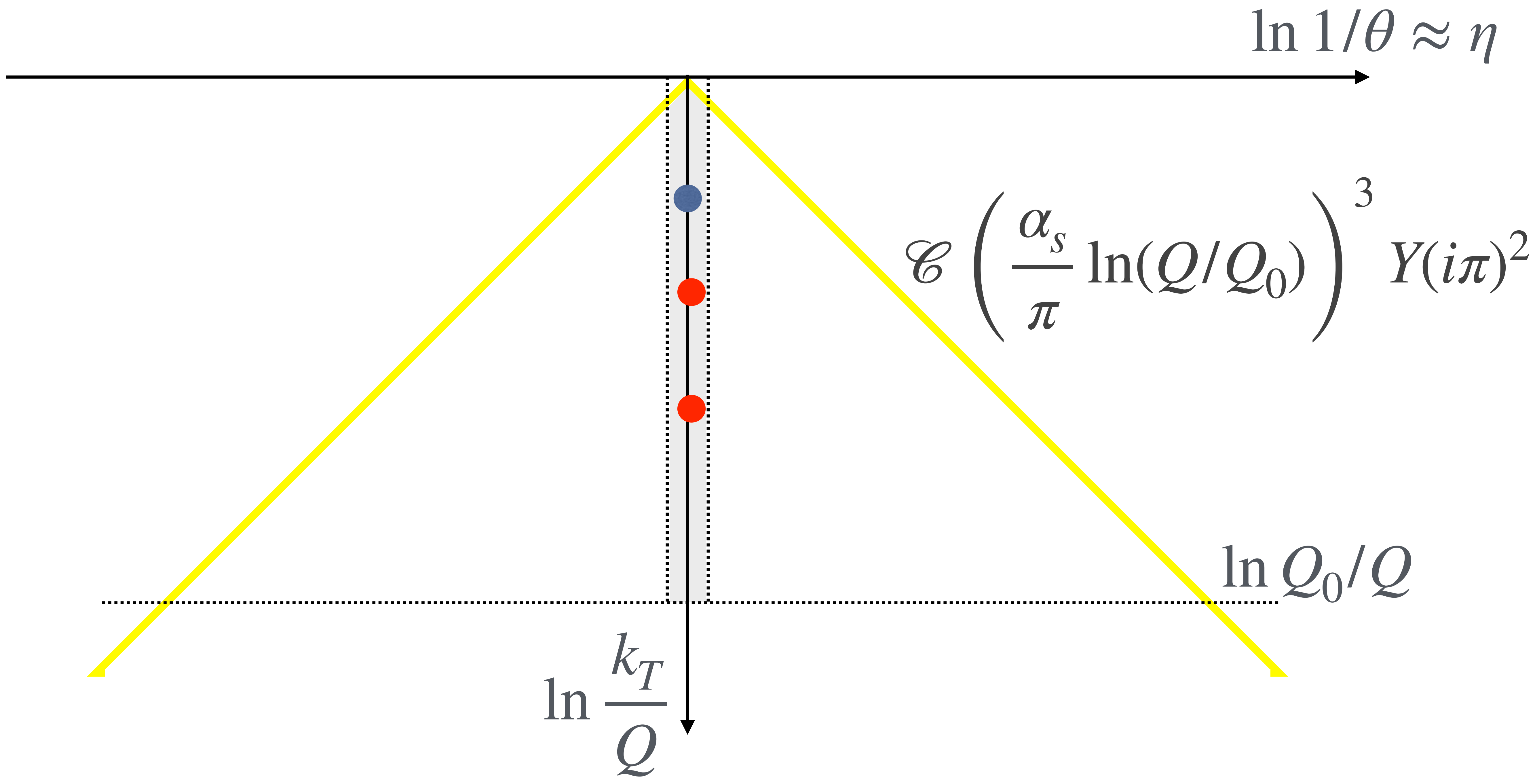
- It is instructive to view this on the Lund plane.



Computing the super-leading Log

Diagrammatically

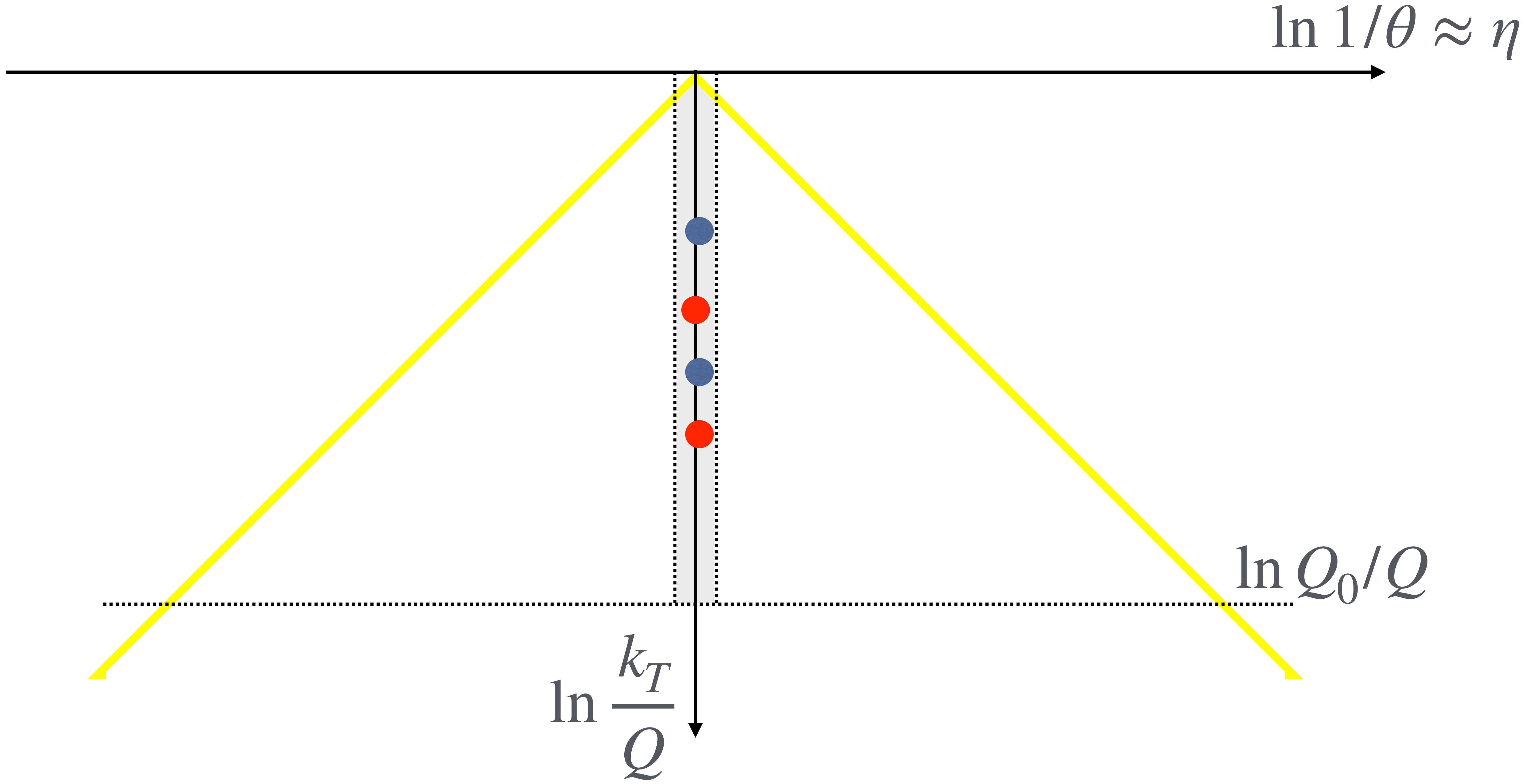
- It is instructive to view this on the Lund plane.



Computing the super-leading Log

Diagrammatically

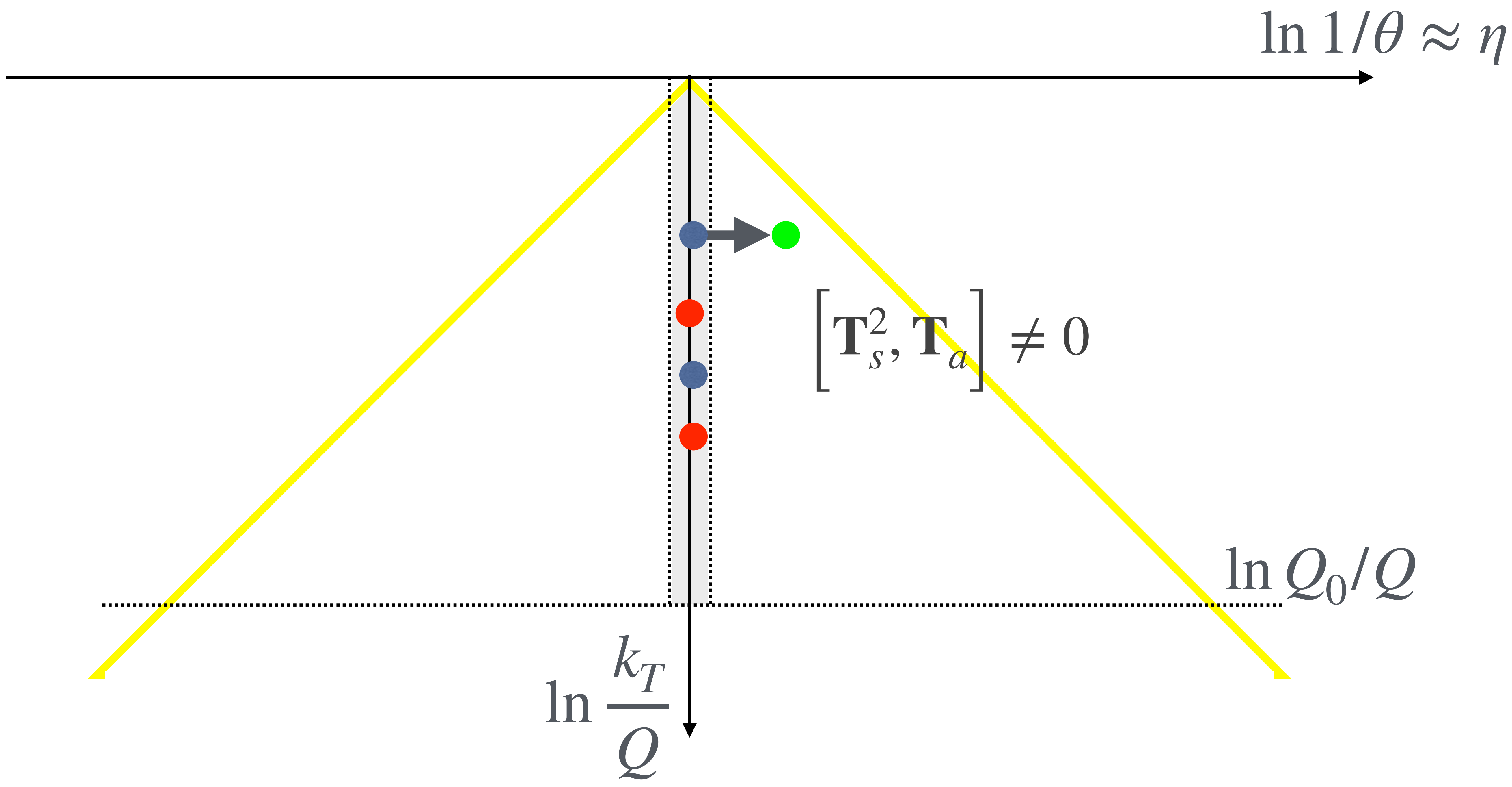
- It is instructive to view this on the Lund plane.



Computing the super-leading Log

Diagrammatically

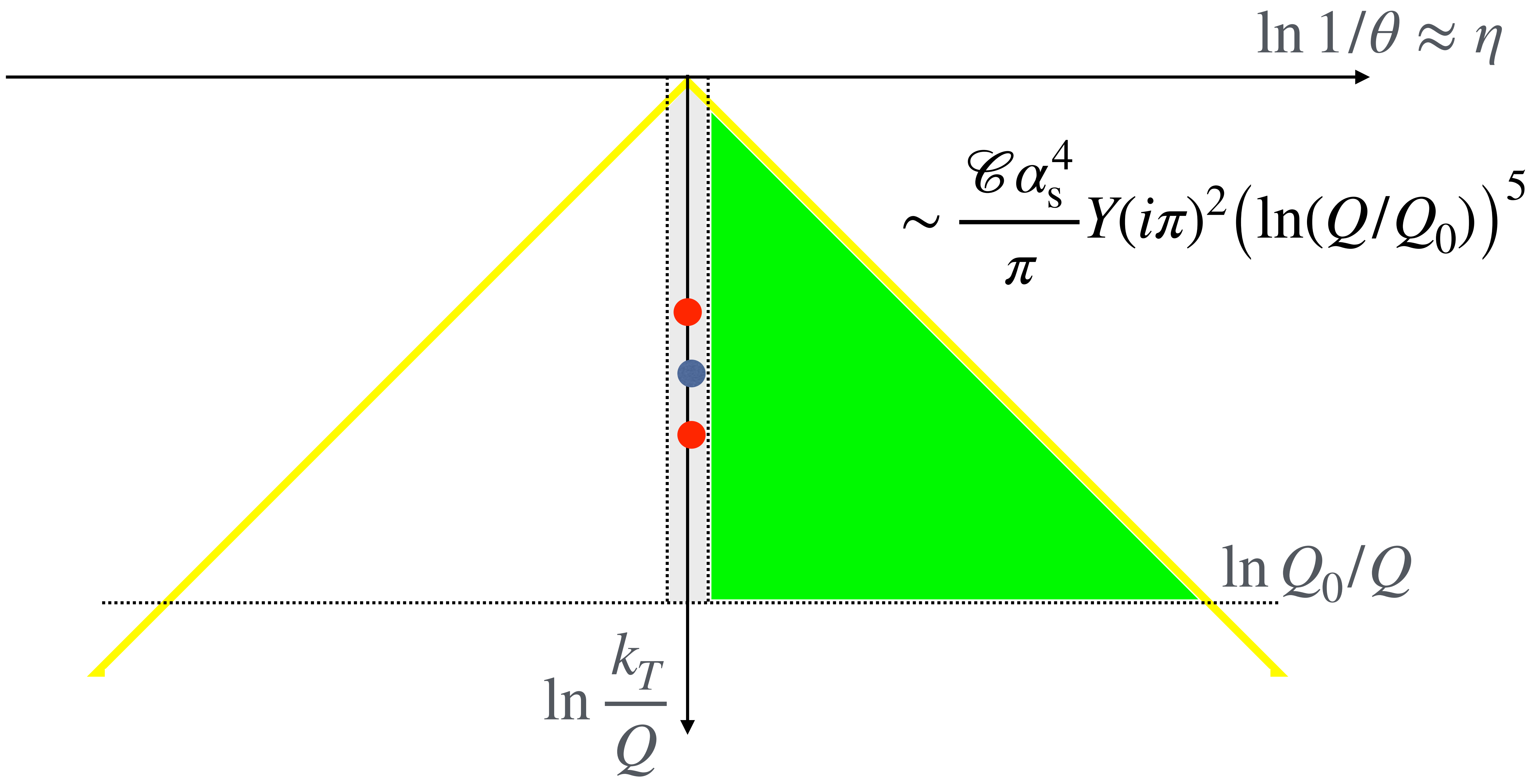
- It is instructive to view this on the Lund plane.



Computing the super-leading Log

Diagrammatically

- It is instructive to view this on the Lund plane.



$$\mathscr{C} \sim \begin{cases} \mathrm{Tr}\left([\mathbf{T}_s^2, \mathbf{T}_i \cdot \mathbf{T}_j] \, (\mathbf{T}_a [\mathbf{T}_s^2, \mathbf{H}] \mathbf{T}_a^\dagger - \mathbf{T}_a^2 [\mathbf{T}_s^2, \mathbf{H}]) \right) \\ \mathrm{Tr}\left([\mathbf{T}_s^2, [\mathbf{T}_s^2, \mathbf{T}_i \cdot \mathbf{T}_j]] \, (\mathbf{T}_a \mathbf{H} \mathbf{T}_a^\dagger - \mathbf{T}_a^2 \mathbf{H}) \right) \end{cases}$$

Gaps between jets

Recent progress

- The “full colour” computation of exclusive jet rates with coloured initial states has received a lot of attention in recent years, driven by a few collaborations:
  - Bern-Mainz: Becher, Hager, Jaskiewicz, Martinelli, Neubert, Schwienbacher and others*

    This is the main group working on the resummation of SLLs.
  - Graz-Manchester: DeAngelis, Forshaw, JH, Martínez, Plätzer, Ruffa, Seymour, Torre González

    Developing “amplitude evolution”. The central goal is a numerical code which is capable of resumming SLLs and non-global logs.
  - Also work by: Hagiwara, Hatta, Ueda, Mueller, Triantafyllopoulos and others*

    Other work on the numerical resummation of non-global logs but not applied to SLLs.

*sorry to anyone whose name I’ve missed.

Super-leading logarithms

Recent progress

- The super-leading logarithms have non-trivial colour, the space of which grows with α_s.

- Using an approach based in SCET and some very fiddly colour algebra, the Bern-Mainz collaboration demonstrated that the super-leading logarithms can be resummed analytically with a new iterative factorisation.

$$\sigma(Q_0) = \sum_{a_1,a_2=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=4}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

Becher, Neubert, Shao 2107.01212, Philipp Böer, Hager, Neubert, Stillger, Xu 2307.11089, 2311.18811, 2405.05305, 2407.01691 Becher, Hager, Jaskiewicz, Neubert, Schwienbacher 2408.10308, Becher, Hager, Martinelli, Neubert, Schwienbacher 2411.12742

- The key point is that $\boldsymbol{\mathcal{W}}_m$ contains both initial and final state low scale physics, including PDFs which must be evaluated at Q_0, not the hard scale!

Super-leading logarithms

Recent progress

- Recently Becher, Hager, Martinelli, Neubert, Schwienbacher, Stillger 2411.12742 performed an initial pheno study:

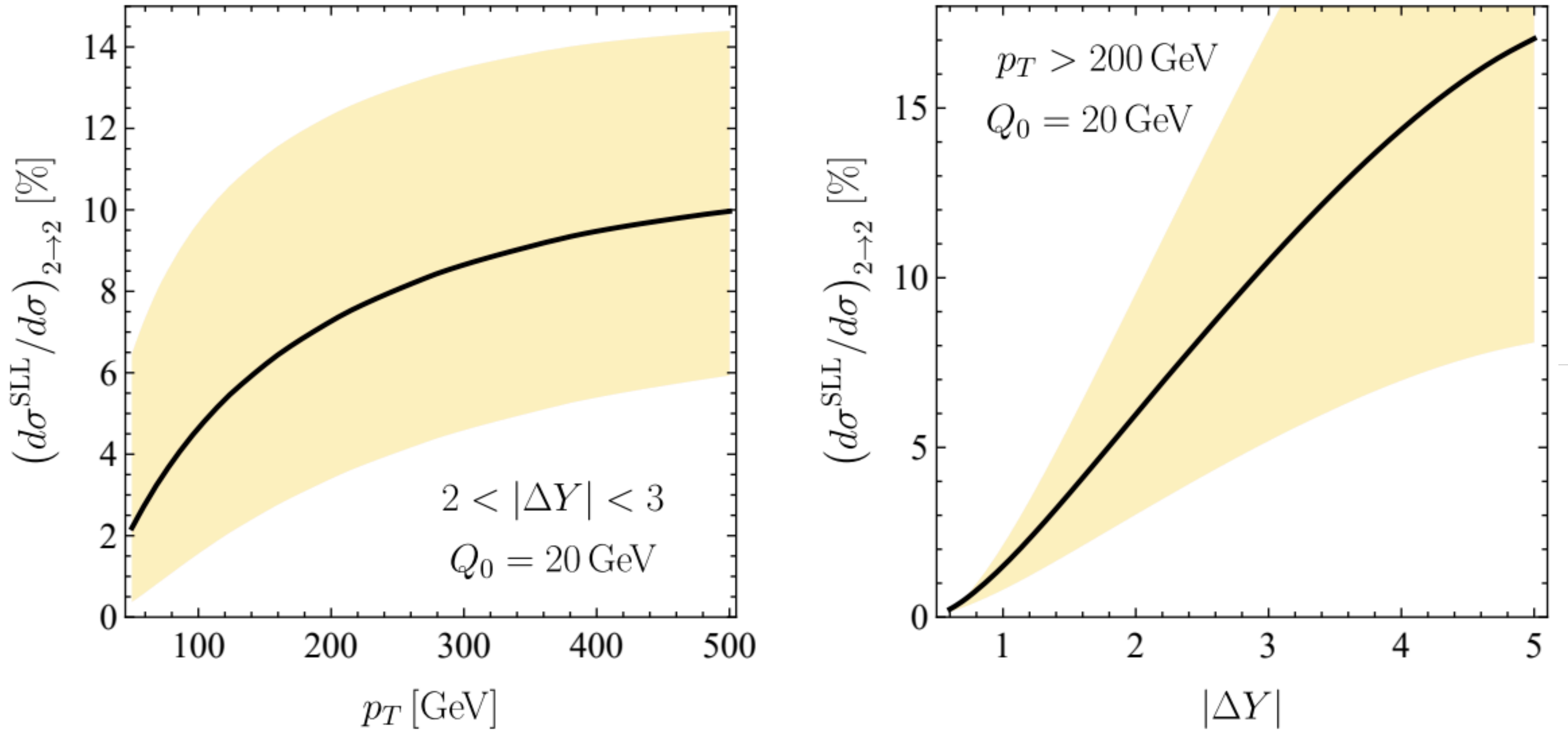


Figure 3: SLL contribution to the differential $pp \to 2$ jets cross section as function of transverse momentum p_T (left) and gap size $|\Delta Y|$ (right). The veto scale is fixed to $Q_0 = 20\,\mathrm{GeV}$.

$$\hat{\sigma}^{\mathrm{SLL}}_{2\to M}(Q_0) = \hat{\sigma}_{2\to M}\,\frac{\alpha_s L}{\pi N_c}\left(\frac{N_c\alpha_s}{\pi}\,i\pi L\right)^2 \sum_{n=0}^{\infty} c_n \left(\frac{N_c\alpha_s}{\pi}\,L^2\right)^n$$

- Despite appearing first at 4-loops, effects can be as large as 10% to 15% for large gaps!

Part 2 - jettiness

What is jettiness?

Background

- Jettiness is a class of observables introduced by Stewart, Tackmann, Waalewijn 1004.2489.
- Broadly, the n-jettiness (τ_n) is defined so that in the limit $\tau_n \to 0$ an event contains just n collimated coloured jets.
- They're recursively IRC safe and can be computed with exceptional precision (absent Coulomb/Glauber modes). Alioli et al, 2312.06496
- Jettiness is used for precision QCD but, due to its simplicity, it also is used for:
  - Selection cuts to isolate processes with n jets. This is particularly important for "missing neutrals" searches (i.e. BSM, dark matter). Lindert et al. 1705.04664
  - Phase-space slicing for NNLO matching (Geneva collaboration)

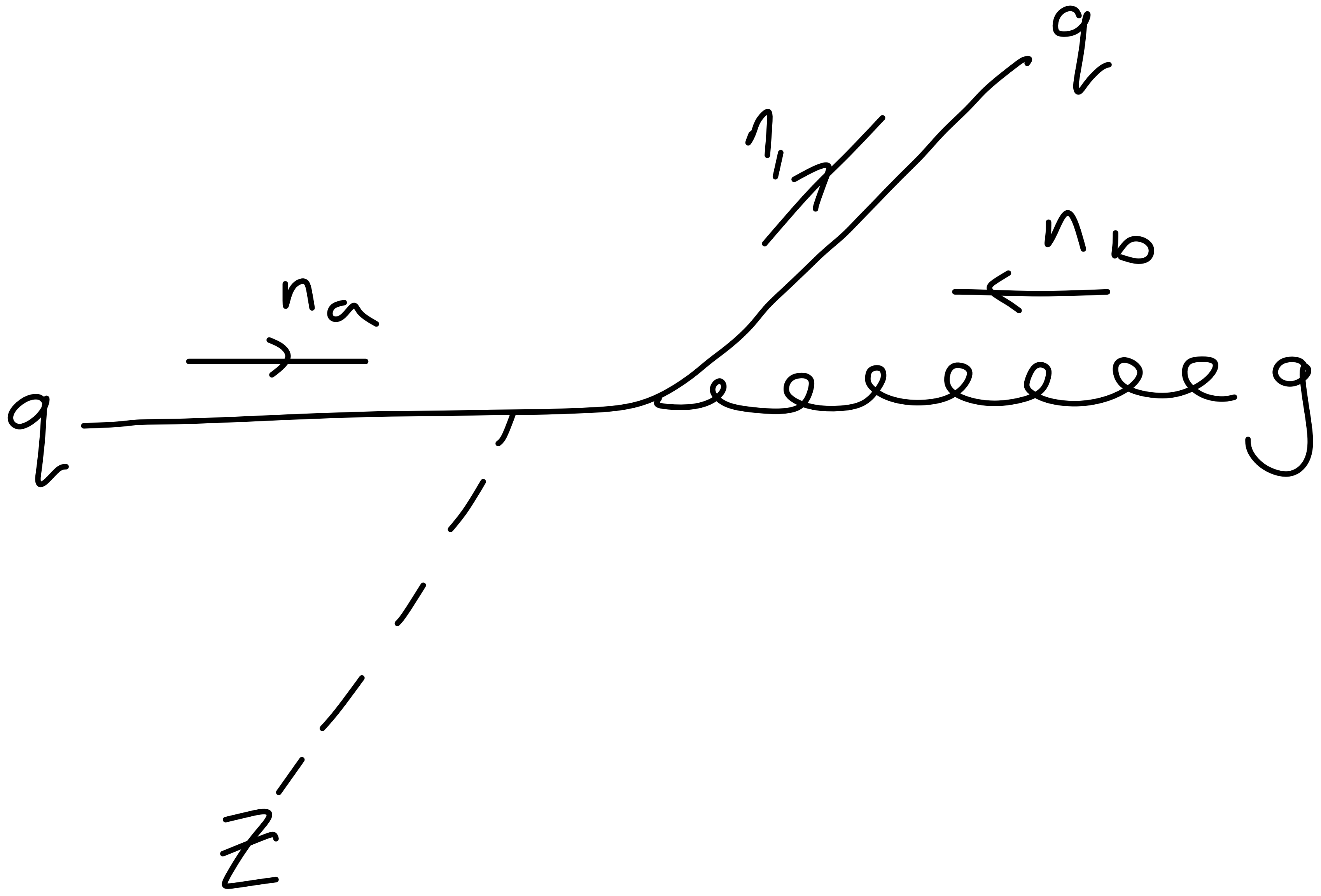
What is jettiness?

The definition of one-jettiness

The one-jettiness at the LHC:

$$\tau_1^X = \sum_{i \in X} \frac{1}{Q} \min(n_a \cdot p_i, n_b \cdot p_i, n_1 \cdot p_i)$$

$$\frac{d\sigma}{d\tau_1} = \sum_X \int d\sigma_{pp \to X}\, \delta\left(\tau_1 - \tau_1^X\right)$$

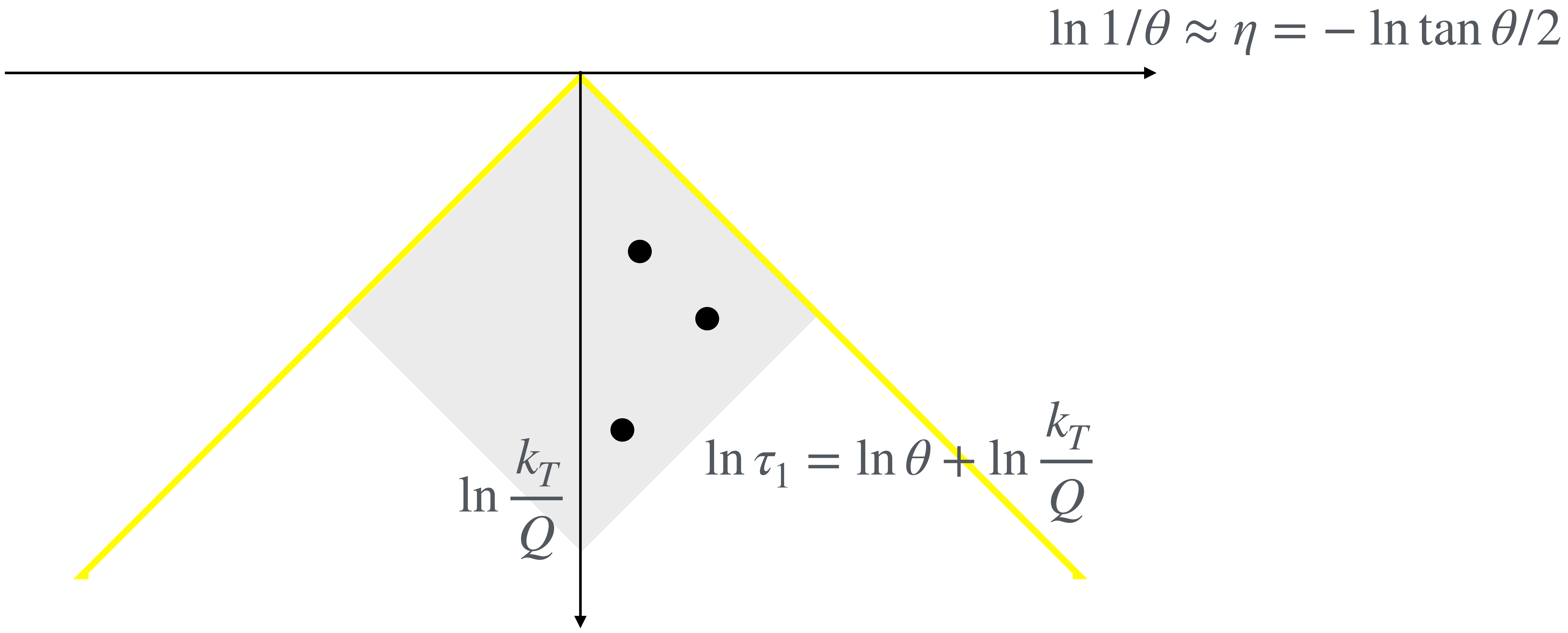


There are a few topologies that can contribute but, for concreteness, I will focus on just

$$p(q)p(g) \to Z + q$$

The Lund plane for jettiness

Leading-log resummation



$$\frac{\mathrm{d}\sigma(\tau_1)}{\mathrm{d}x_a\mathrm{d}x_b\mathrm{d}B} = \sum_n \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}x_a\mathrm{d}x_b\mathrm{d}B}\,\frac{1}{n!}\left(-\frac{\alpha_s}{\pi}(2C_F+C_A)\frac{1}{2}L^2+\cdots\right)^n = \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}x_a\mathrm{d}x_b\mathrm{d}B}\exp\left(-\frac{\alpha_s}{\pi}(2C_F+C_A)\frac{1}{2}L^2+\cdots\right)$$

The Lund plane for jettiness

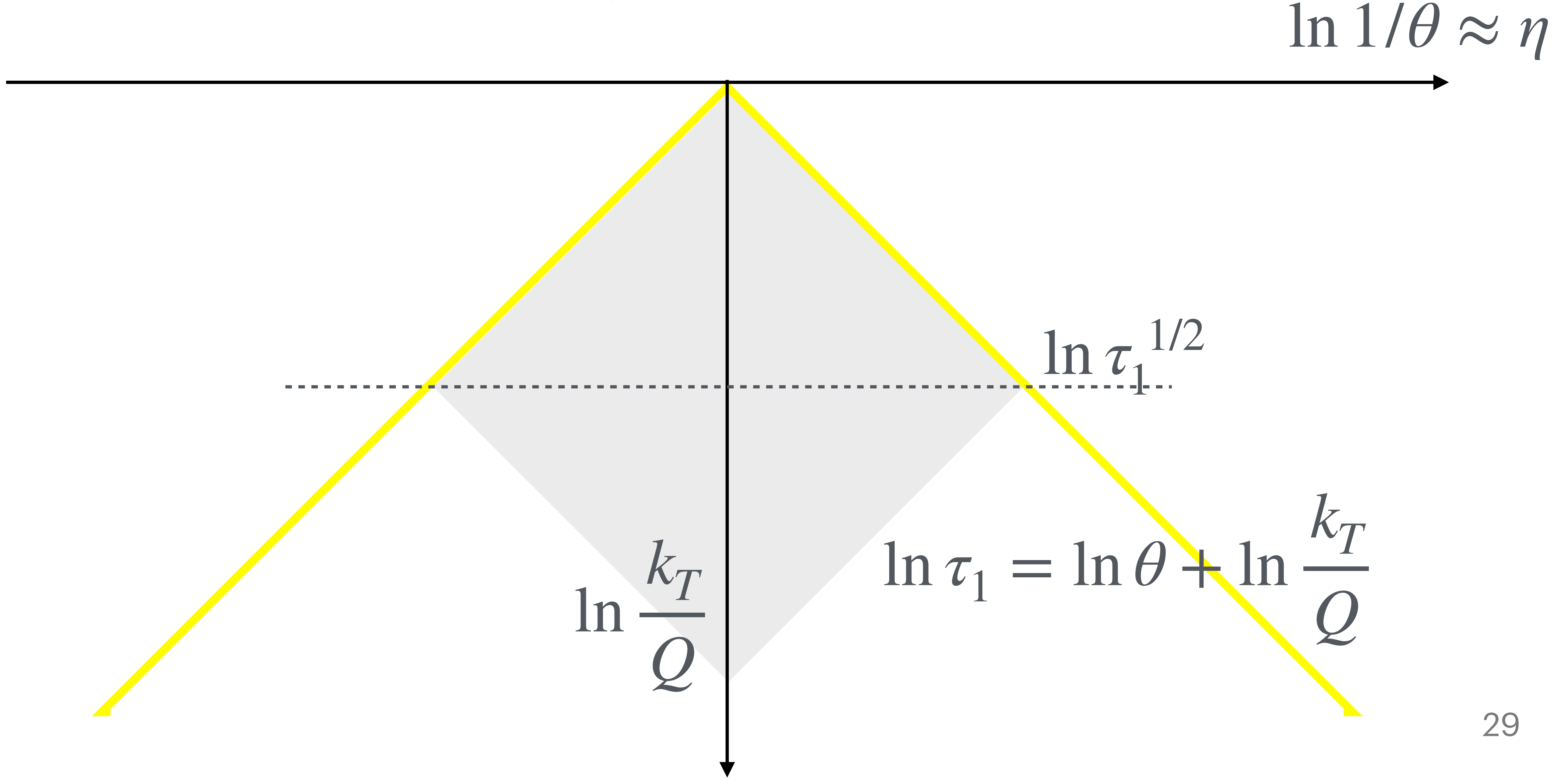
Collinear logs without Coulomb/Glauber phases

The partonic result should be convoluted against PDFs

$$\frac{\mathrm{d}\,\sigma}{\mathrm{d}\tau_1 \mathrm{d}B} = \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b\ f_A^a(x_a, \mu_F)\, f_B^b(x_b, \mu_F)\, \frac{\mathrm{d}\,\hat{\sigma}_{ab}(\mu_F)}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\tau_1\, \mathrm{d}B}$$

From the Lund plane, we should fix $\mu_F = \tau^{1/2} Q$ so that the DGLAP evolution of the PDFs captures all the logs from the initial state.

The collinear scale is the geometric mean between the hard and soft scale.



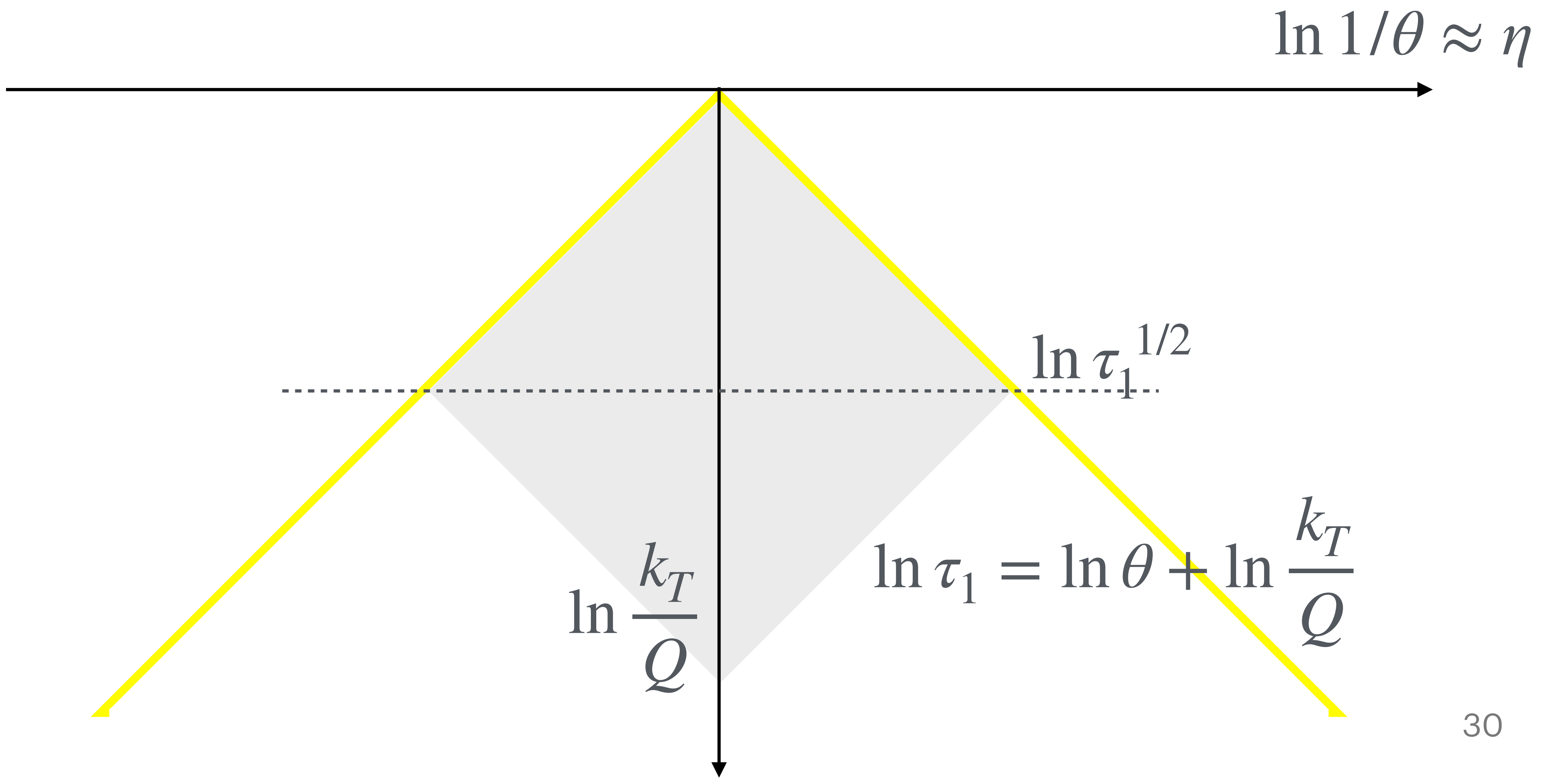
The jettiness factorised

The full result…

The complete leading power result Stewart, Tackmann, Waalewijn 2010:

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_N} = & \int \mathrm{d}x_a \mathrm{d}x_b \int \mathrm{d}^4 q \, \mathrm{d}\Phi_L(q) \int \mathrm{d}\Phi_N(\{q_J\}) \\
& \times F_N(\{q_m\}, L) \, (2\pi)^4 \delta^4\Big(q_a + q_b - \sum_J q_J - q\Big) \\
& \times \sum_{ij,\kappa} \mathrm{tr}\, \widehat{H}_{ij\to\kappa}(\{q_m\}, L, \mu) \prod_J \int \mathrm{d}s_J \, J_{\kappa_J}(s_J, \mu) \\
& \times \int \mathrm{d}t_a \, B_i(t_a, x_a, \mu) \int \mathrm{d}t_b \, B_j(t_b, x_b, \mu) \\
& \times \widehat{S}_N^{ij\to\kappa}\Big(\tau_N - \frac{t_a + t_b + \sum_J s_J}{Q^2}, \{q_m\}, \mu\Big) . \quad (14)
\end{aligned}
$$

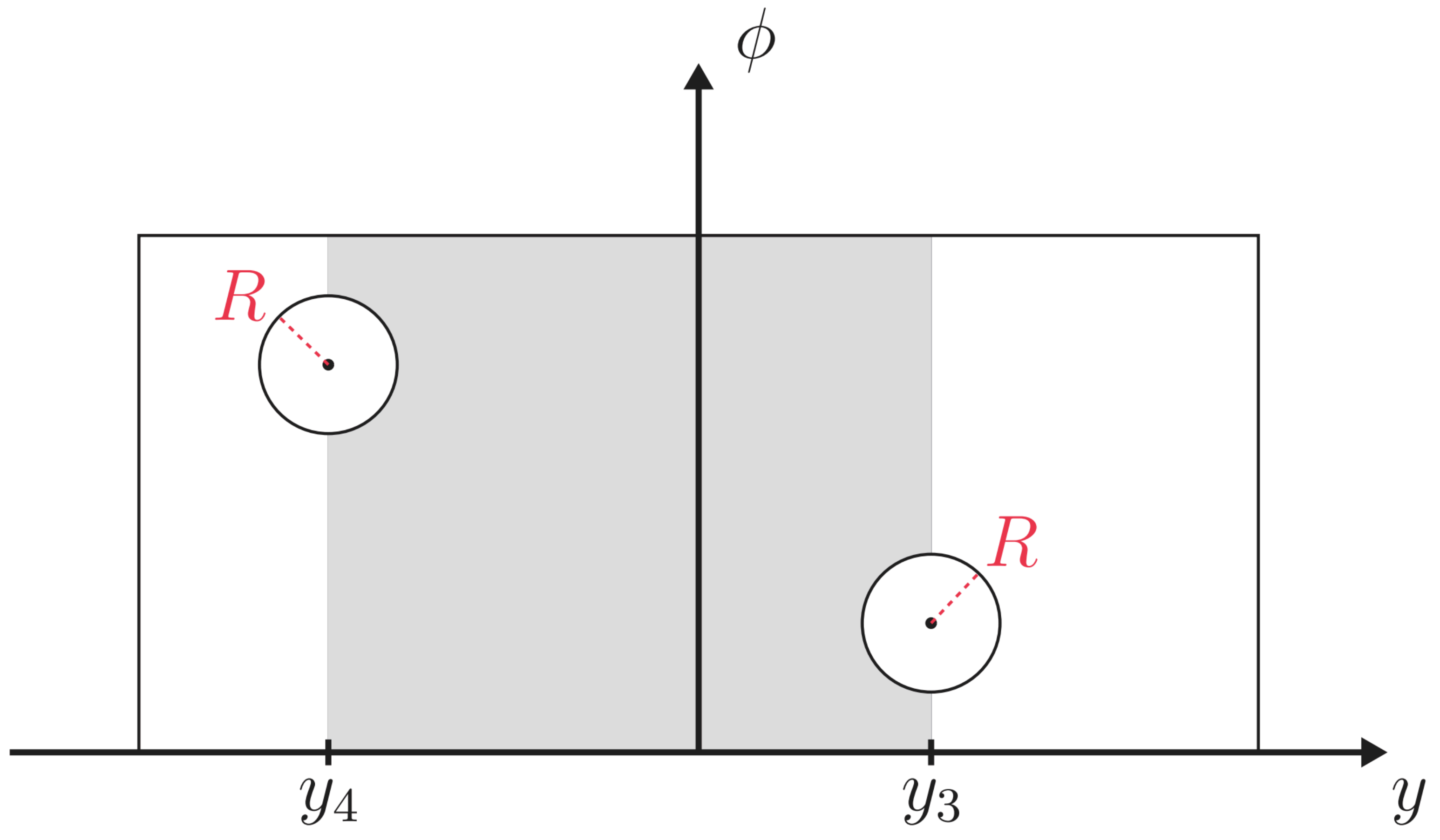
$$
B_i(t, x, \mu) = \sum_{i'} \int_x^1 \frac{\mathrm{d}\xi}{\xi} \, \mathcal{I}_{ii'}\Big(t, \frac{x}{\xi}, \mu\Big) f_{i'}(\xi, \mu)
$$

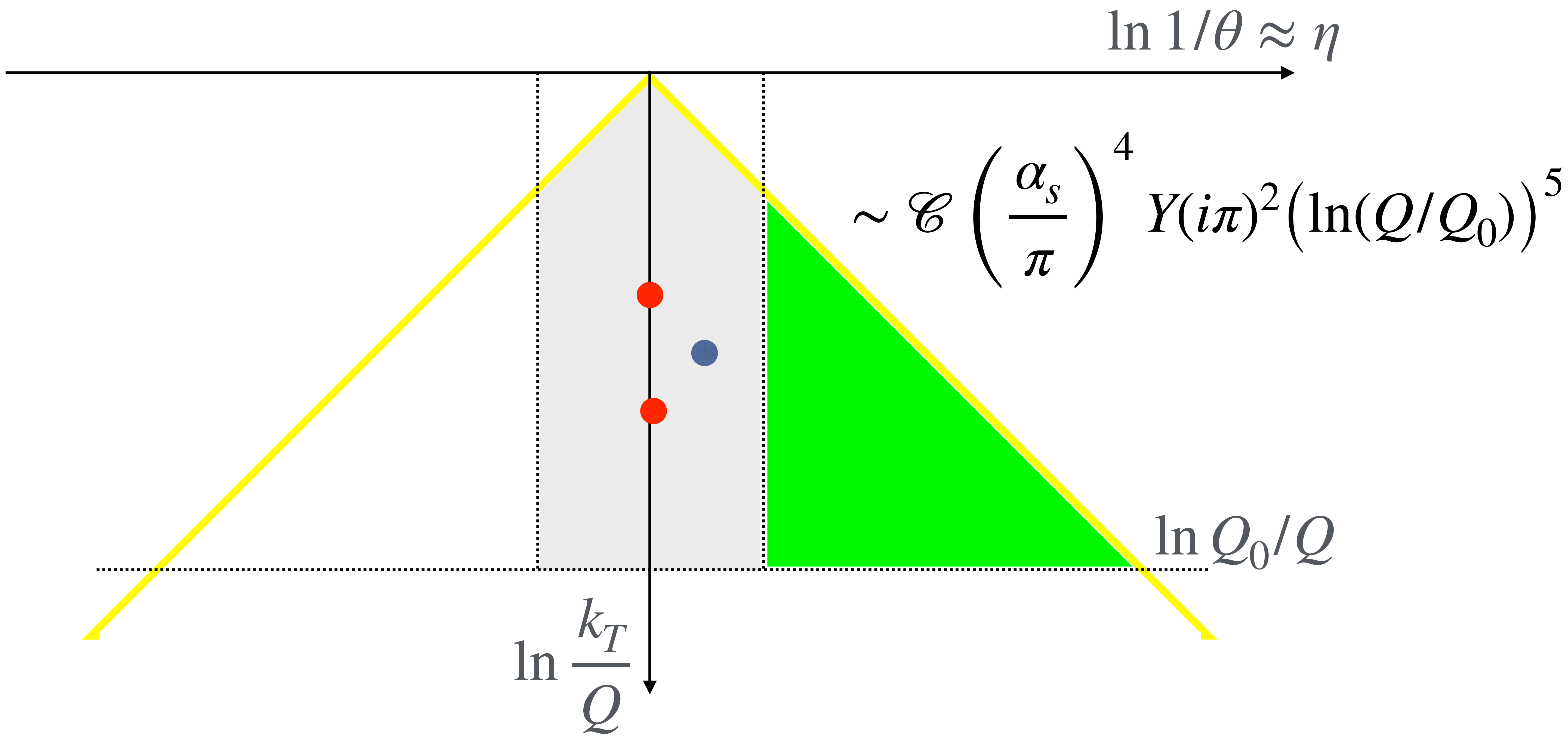


From gaps-between-jets to jettiness

Could we get a similar scale hierarchy in gaps between jets?

Grow the gap to induce a collinear scale...





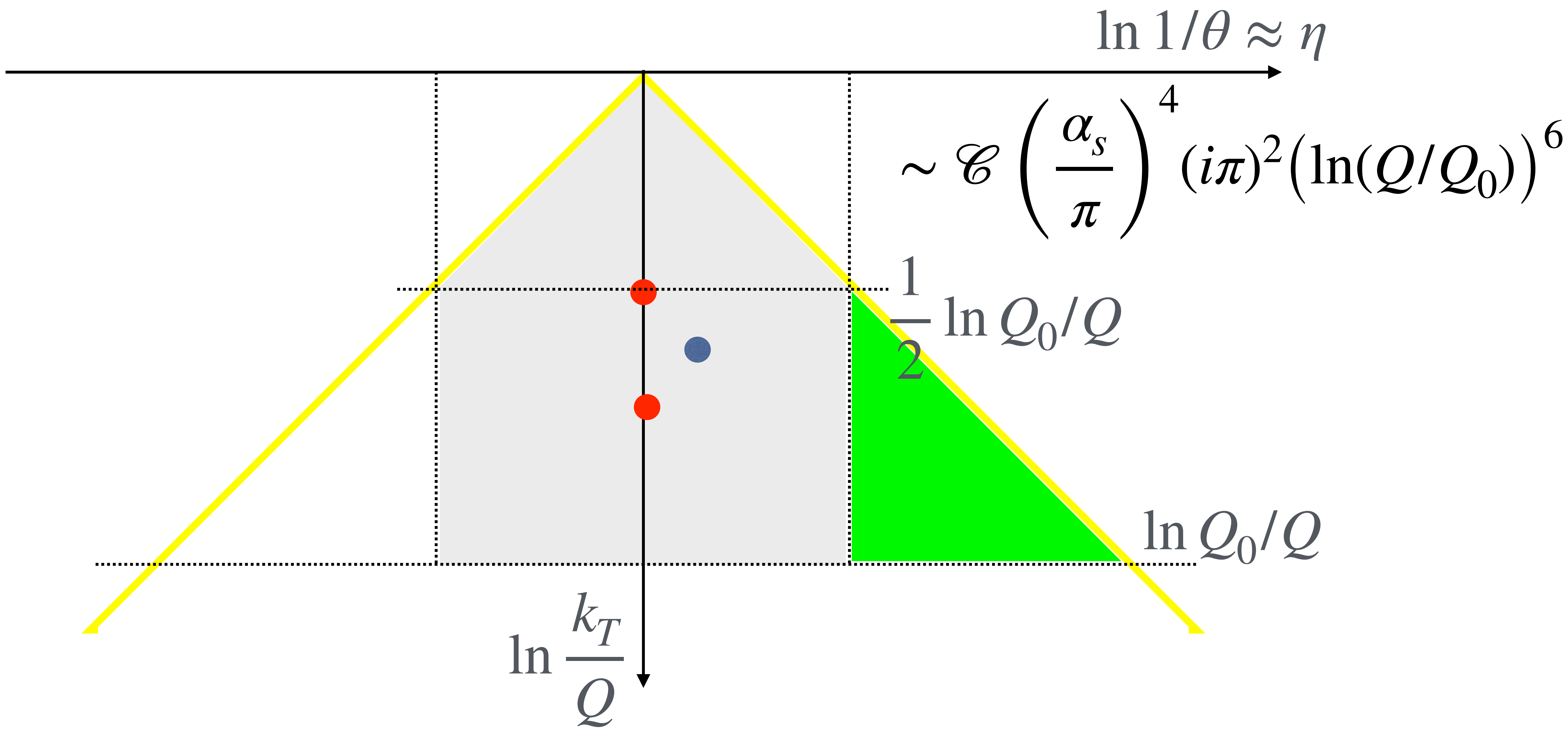
From gaps-between-jets to jettiness

Could we get a similar scale hierarchy in gaps between jets?

Grow the gap… until it is a log

Consequently, factorisation breaking is now $\alpha_s^4 L^6$!

Crucially, nothing changes in the colour structure to protect GBJs from this log.

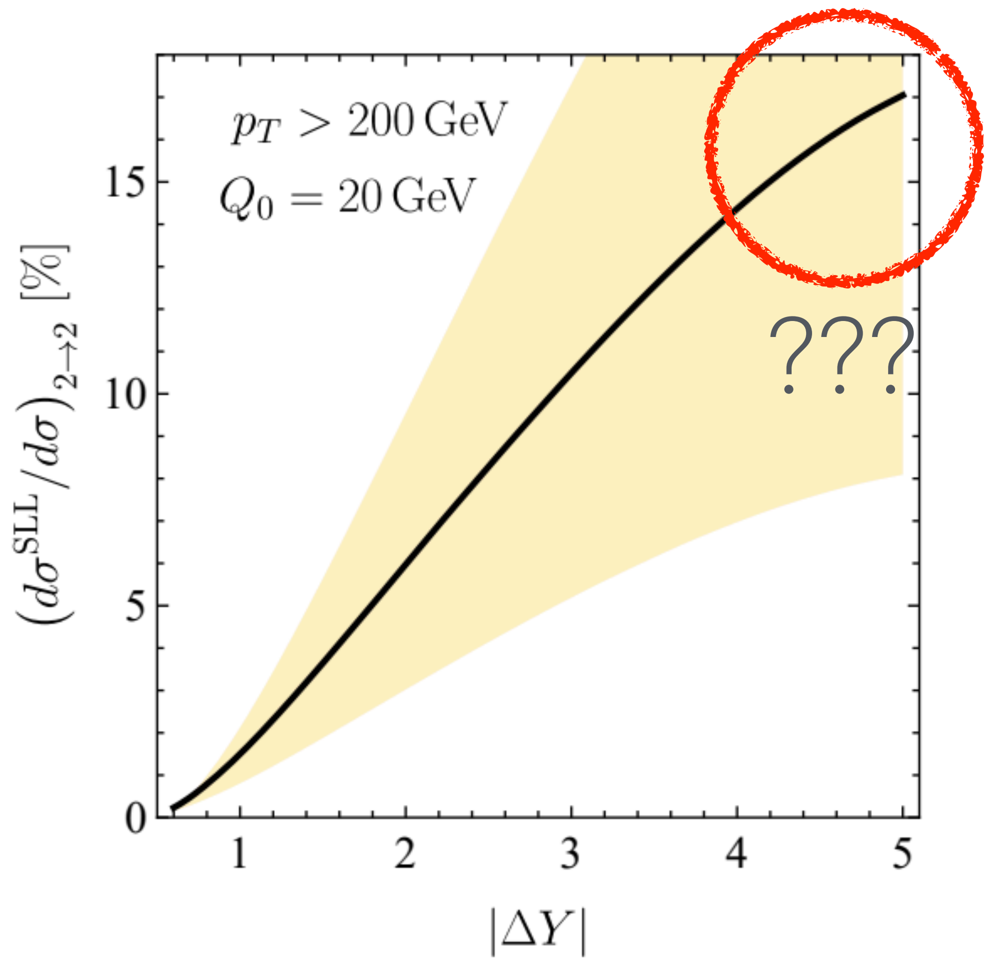


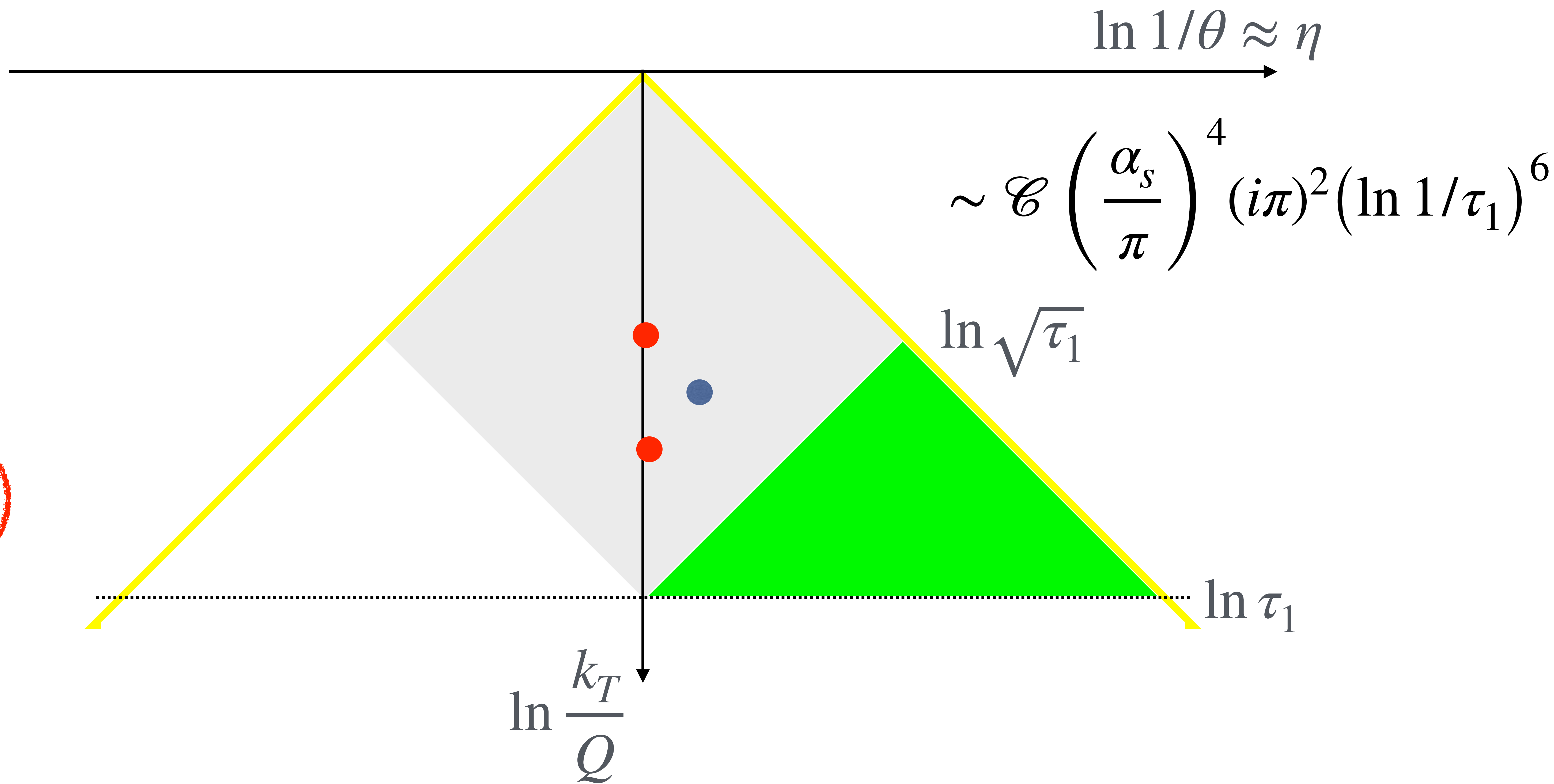
From gaps-between-jets to jettiness

Based on Forshaw, JH, 2109.03665 and Banfi, Forshaw, JH, 2511.11799

This looks just like our distorted version of gaps-between-jets but with a diagonal boundary.

An $\alpha_s^4 L^6$ will be present unless the colour factor vanishes…





$$\sim \mathscr{C} \left(\frac{\alpha_s}{\pi}\right)^4 (i\pi)^2 \left(\ln 1/\tau_1\right)^6$$

Banfi et al, 1001.4082

Generalising our method to jettiness

Based on Forshaw, JH, 2109.03665 and Banfi, Forshaw, JH, 2511.11799

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a\mathrm{d}x_b\,\mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{\mathrm{d}k_T}{k_T}\int_0^{1-\frac{k_T}{Q}e^{Y/2}}\frac{\mathrm{d}z}{z}\,P_{qq}(z)\,f_B(x_b,Q)$$
$$\times\left[\Theta(z-x_a)\,f_A(x_a/z,Q_0)\frac{1}{\mathbf{T}_a^2}\mathrm{Tr}(\mathbf{V}_{Q_0,k_T}\mathbf{T}_a\mathbf{V}_{k_T,Q}\mathbf{H}\mathbf{V}^\dagger_{k_T,Q}\mathbf{T}_a^\dagger\mathbf{V}^\dagger_{Q_0,k_T}) - z f_A(x_a,Q_0)\mathrm{Tr}(\mathbf{V}_{Q_0,Q}\mathbf{H}\mathbf{V}^\dagger_{Q_0,Q})\right]$$

Generalise the observable

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a\mathrm{d}x_b\,\mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi}\int_{\mu_\mathrm{F}}^{Q}\frac{\mathrm{d}k_\perp}{k_\perp}\int_0^{1-k_\perp/Q}\frac{\mathrm{d}z}{z}\,P_{qq}(z)\,u_1(k)\,f_B(x_b,Q)$$
$$\times\left[\Theta(z-x_a)\,f_A(x_a/z,\mu_\mathrm{F})\frac{1}{\mathbf{T}_a^2}\mathrm{Tr}(\mathbf{V}_{\mu_\mathrm{F},k_\perp}\mathbf{T}_a\mathbf{V}_{k_\perp,Q}\mathbf{H}\mathbf{V}^\dagger_{k_\perp,Q}\mathbf{T}_a^\dagger\mathbf{V}^\dagger_{\mu_\mathrm{F},k_\perp}) - z f_A(x_a,\mu_\mathrm{F})\mathrm{Tr}(\mathbf{V}_{\mu_\mathrm{F},Q}\mathbf{H}\mathbf{V}^\dagger_{\mu_\mathrm{F},Q})\right]$$

$$\mathbf{V}_{\alpha,\beta} \approx \mathrm{Pexp}\left(\frac{\alpha_\mathrm{s}}{\pi}\left[\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\int_\alpha^\beta\frac{\mathrm{d}q_\perp^{(ij)}}{q_\perp^{(ij)}}\int_{-\ln Q/q_\perp^{(ij)}}^{\ln Q/q_\perp^{(ij)}}\mathrm{d}y^{(ij)}\int_0^{2\pi}\frac{\mathrm{d}\phi^{(ij)}}{4\pi}\left(1-u_n(q,\{k\}_{n-1})\right) - i\pi\mathbf{T}_s^2\ln\frac{b}{a}\right]\right)$$

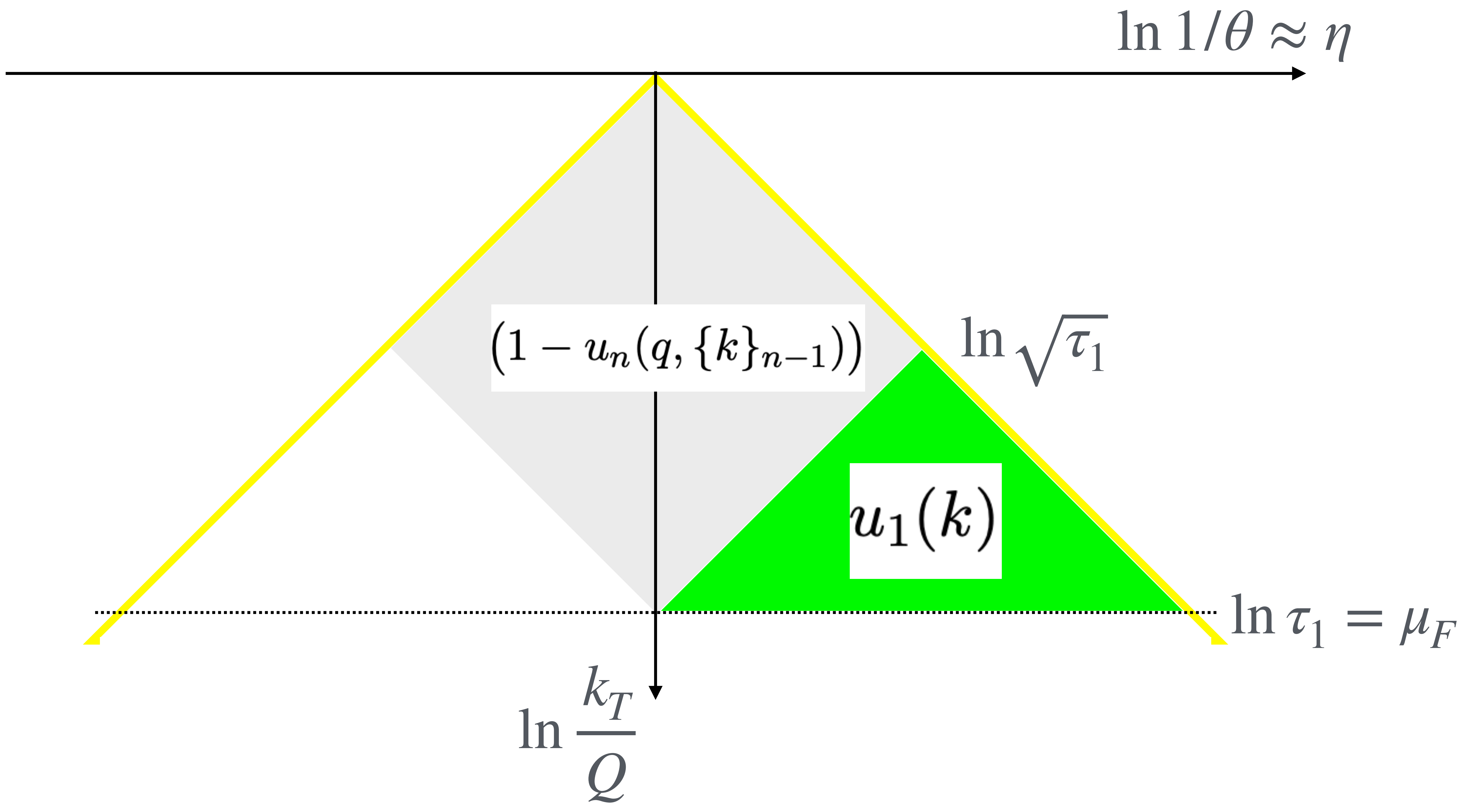
From gaps-between-jets to jettiness

Could we get a $\alpha_s^4 L^6$ in jettiness?

On the Lund plane:

- We integrate loops over the vetoed region.
- We integrate real emissions over the accepted regions above the factorisation scale.

Corrections to $\frac{d\sigma_1}{dx_a dx_b\, d\mathcal{B}}$ are suppressed by α_s since it requires two emissions into the green region.



Generalising our method to jettiness

Based on Forshaw, JH, 2109.03665 and Banfi, Forshaw, JH, 2511.11799

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_a\mathrm{d}x_b\,\mathrm{d}\mathcal{B}} = \frac{\alpha_s}{\pi}\int_{\mu_\mathrm{F}}^{Q}\frac{\mathrm{d}k_\perp}{k_\perp}\int_0^{1-k_\perp/Q}\frac{\mathrm{d}z}{z}\,P_{qq}(z)\,u_1(k)\,f_B(x_b,Q)$$
$$\times\left[\Theta(z-x_a)\,f_A(x_a/z,\mu_\mathrm{F})\frac{1}{\mathbf{T}_a^2}\mathrm{Tr}(\mathbf{V}_{\mu_\mathrm{F},k_\perp}\mathbf{T}_a\mathbf{V}_{k_\perp,Q}\mathbf{H}\mathbf{V}^\dagger_{k_\perp,Q}\mathbf{T}_a^\dagger\mathbf{V}^\dagger_{\mu_\mathrm{F},k_\perp}) - z f_A(x_a,\mu_\mathrm{F})\mathrm{Tr}(\mathbf{V}_{\mu_\mathrm{F},Q}\mathbf{H}\mathbf{V}^\dagger_{\mu_\mathrm{F},Q})\right]$$

Put in one-jettiness

$$\frac{\mathrm{d}\sigma_1^{\mathrm{CVL}}}{\mathrm{d}x_a\mathrm{d}x_b\,\mathrm{d}\mathcal{B}} = \sum_{a=q,g}\frac{\alpha_\mathrm{s}}{\pi}f_A^a(x_a,\mu_\mathrm{F})f_B^b(x_b,\mu_\mathrm{F})\int_{\mu_\mathrm{F}}^{Q}\frac{\mathrm{d}k_T}{k_T}\int_{x_a}^{1-\frac{k_T}{Q}}\mathrm{d}z\,\frac{2}{1-z}\,u(k)\,W_{k_T,Q}\,\mathrm{Tr}(\mathbf{V}^\dagger_{\mu_\mathrm{F},k_T}\mathbf{V}_{\mu_\mathrm{F},k_T}\mathbf{t}_a\mathbf{H}_{ab}(\mathcal{B})\mathbf{t}_a^\dagger),$$

I'm just interested in the CVL so I've dropped the virtual term which does not contribute.

Lower case colour charges are on the 3-parton state. Capital on the 4 parton state.

Generalising our method to jettiness

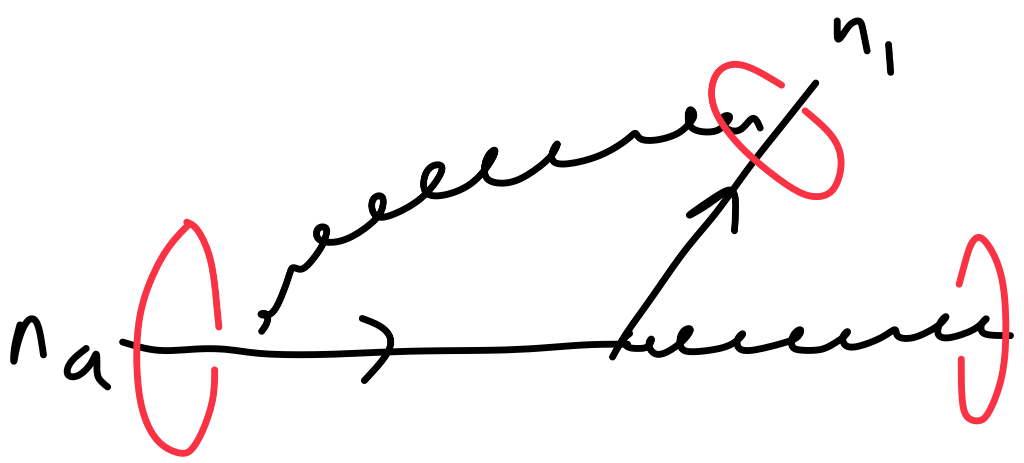
Based on Forshaw, JH, 2109.03665 and Banfi, Forshaw, JH, 2511.11799

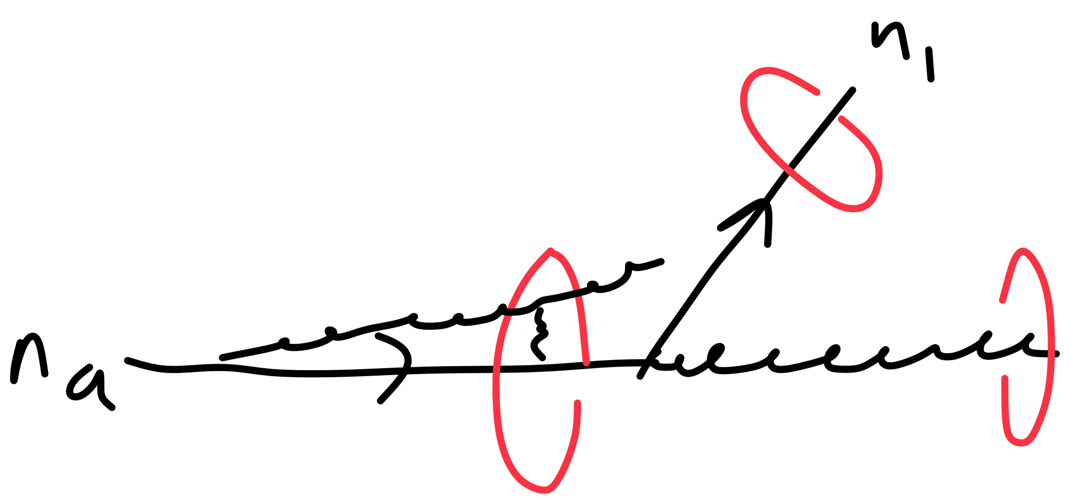
Let's look to the Sudakov operator:

$$\mathbf{V}_{\alpha,\beta} \approx \mathrm{Pexp}\left(\frac{\alpha_\mathrm{s}}{\pi}\left[\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\int_\alpha^\beta\frac{\mathrm{d}q_\perp^{(ij)}}{q_\perp^{(ij)}}\int_{-\ln Q/q_\perp^{(ij)}}^{\ln Q/q_\perp^{(ij)}}\mathrm{d}y^{(ij)}\int_0^{2\pi}\frac{\mathrm{d}\phi^{(ij)}}{4\pi}\left(1-u_n(q,\{k\}_{n-1})\right)-i\pi\mathbf{T}_s^2\ln\frac{b}{a}\right]\right)$$

Put in one-jettiness, two core integrals

$$\int_{q.n_{i,j}>\tau_1 Q}\mathrm{d}y^{(ij)}\frac{\mathrm{d}\phi^{(ij)}}{2\pi} = \ln\left(\frac{n_i\cdot n_j}{1-\frac{q_T^2-\tau_1^2Q^2}{q_T^2+\tau_1^2Q^2}}-1\right)$$
$$\approx \ln\left(\frac{q_T^2}{\tau_1^2Q^2}\frac{n_i\cdot n_j}{2}\right)$$





$$\int_{q.n_{a,b}>\tau_1 Q}\mathrm{d}y^{(ad)}\frac{\mathrm{d}\phi^{(ad)}}{2\pi} = \frac{1}{2}\ln\left(\frac{\tau_1^2Q^2}{q_T^2}\frac{\cos\theta+\frac{q_T^2-\tau_1^2Q^2}{q_T^2+\tau_1^2Q^2}}{\cos\theta-\frac{q_T^2-\tau_1^2Q^2}{q_T^2+\tau_1^2Q^2}}\right)$$

Simplifies the Sudakov greatly

$$\mathbf{V}_{\mu_\mathrm{F},k_T} \approx \exp\left(\frac{\alpha_\mathrm{s}}{\pi}\left[\mathbf{T}_{ad}^2 L_2(\mu_\mathrm{F},k_T)-i\pi\mathbf{T}_s^2\ln\frac{k_T}{\mu_\mathrm{F}}\right]\right)$$

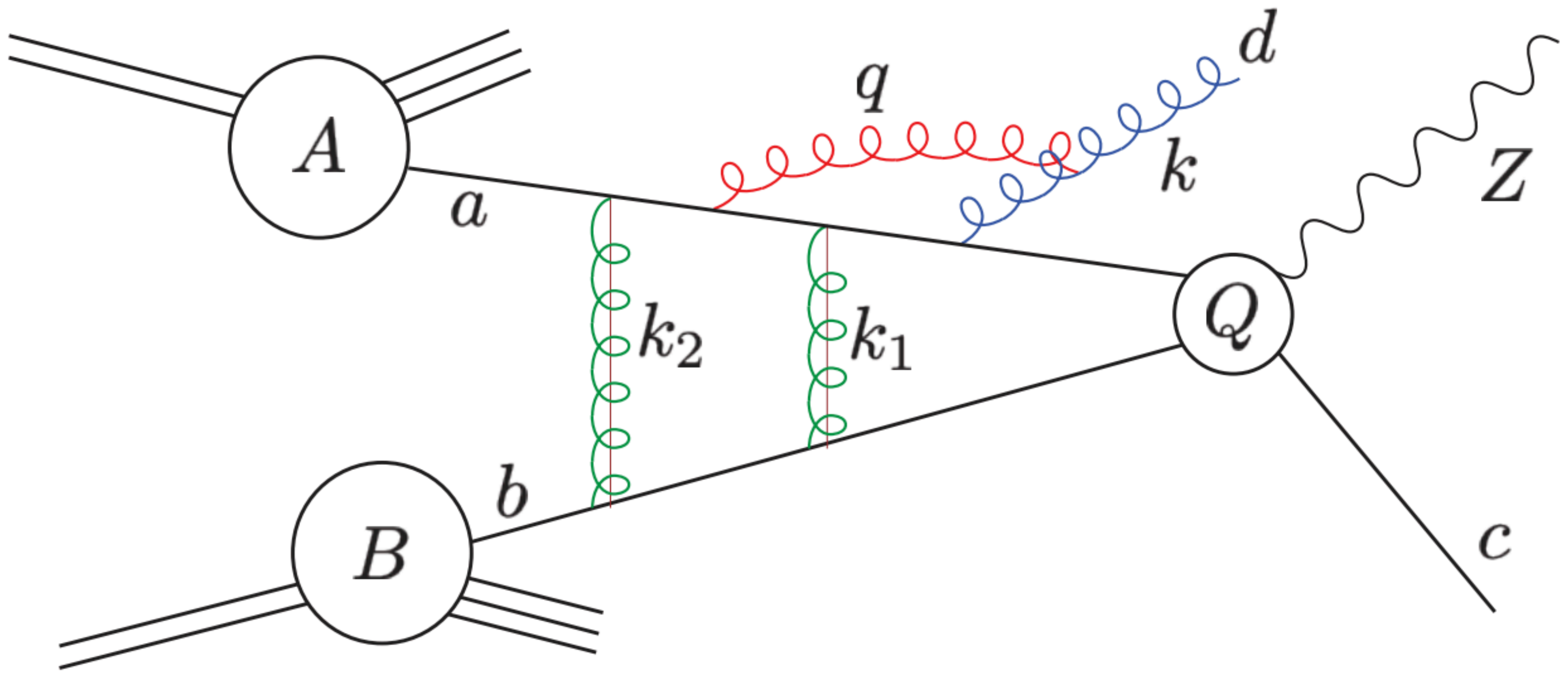
$$\mathbf{T}_{ad}^2 = \sum_{\substack{i<j\\ i\nparallel j}}\mathbf{T}_i\cdot\mathbf{T}_j = -\frac{1}{2}\sum_i\mathbf{T}_i^2-\mathbf{T}_a\cdot\mathbf{T}_d,$$

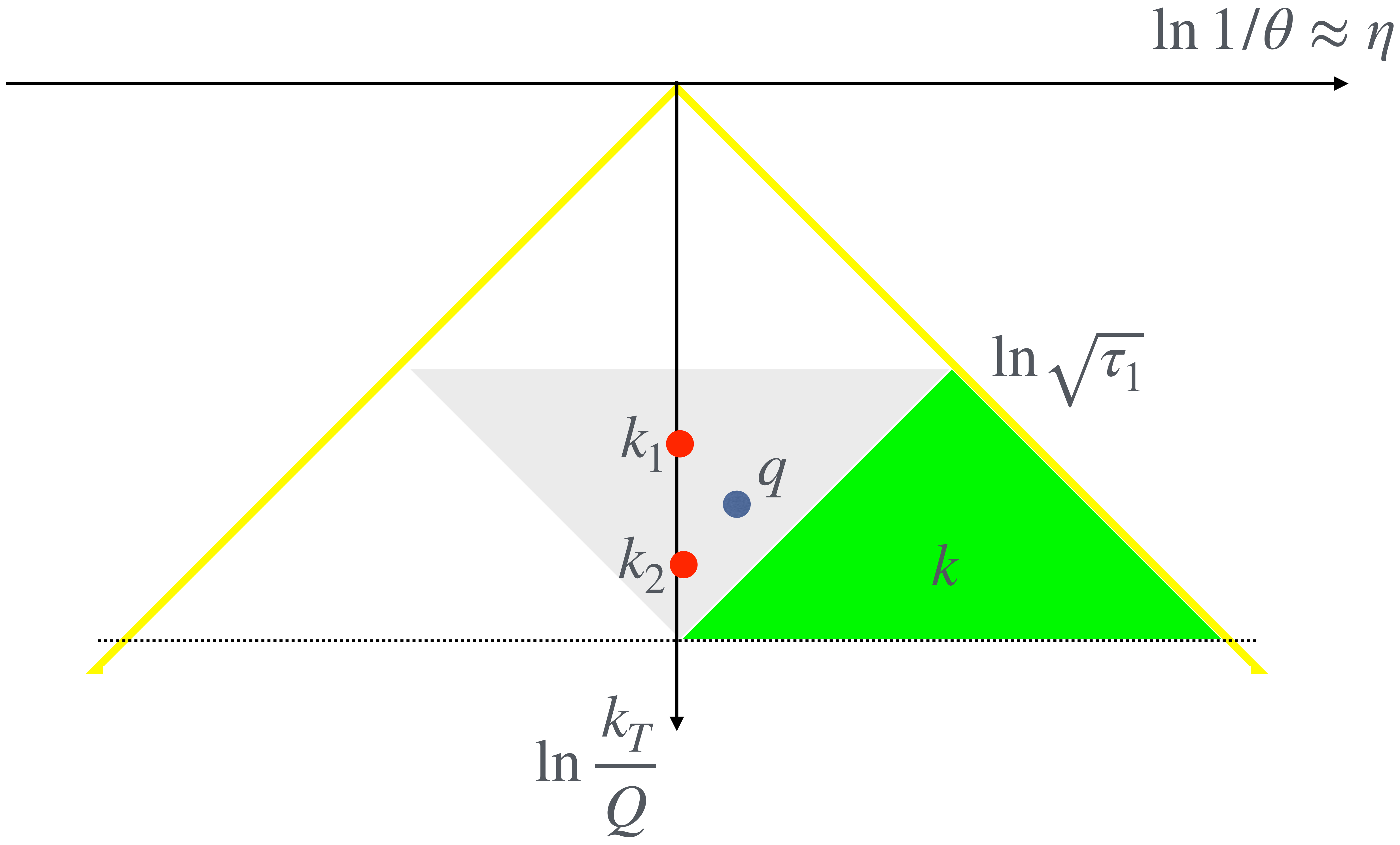
$$L_2(\mu_\mathrm{F},k_T) = \ln\left(\frac{k_T}{\tau_1 Q}\right)^2-\ln\left(\frac{\mu_\mathrm{F}}{\tau_1 Q}\right)^2$$

Generalising our method to jettiness

Based on Forshaw, JH, 2109.03665 and Banfi, Forshaw, JH, 2511.11799

Physical picture of the colour structure:





Generalising our method to jettiness

Based on Forshaw, JH, 2109.03665 and Banfi, Forshaw, JH, 2511.11799

Expand and get the bottom line result:

$$\mathbf{V}^{\dagger}_{\mu_{\mathrm{F}},k_T}\mathbf{V}_{\mu_{\mathrm{F}},k_T} = \exp\left(\frac{2\alpha_{\mathrm{s}}}{\pi}\mathbf{T}^2_{ad}L_2(\mu_{\mathrm{F}},k_T) - \left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^2 L_2(\mu_{\mathrm{F}},k_T)(-i\pi)\ln\frac{k_T}{\mu_{\mathrm{F}}}[\mathbf{T}^2_{ad},\mathbf{T}^2_s] + \left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^3 L_2(\mu_{\mathrm{F}},k_T)\left((-i\pi)\ln\frac{k_T}{\mu_{\mathrm{F}}}\right)^2\frac{1}{3}[\mathbf{T}^2_s,[\mathbf{T}^2_s,\mathbf{T}^2_{ad}] + \ldots\right)$$

$$\frac{\mathrm{d}\sigma_1^{\mathrm{CVL}}}{\mathrm{d}x_a\mathrm{d}x_b\mathrm{d}\mathcal{B}} \approx \sum_{a=q,g} A_a\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^4\int_{\tau_1 Q}^{Q}\frac{\mathrm{d}k_T}{k_T}\int_{k_T/Q}^{1}\frac{\mathrm{d}\theta}{\theta}\,2u(k) \times L_2(\tau_1 Q,k_T)\left((-i\pi)\ln\frac{k_T}{\tau_1 Q}\right)^2$$

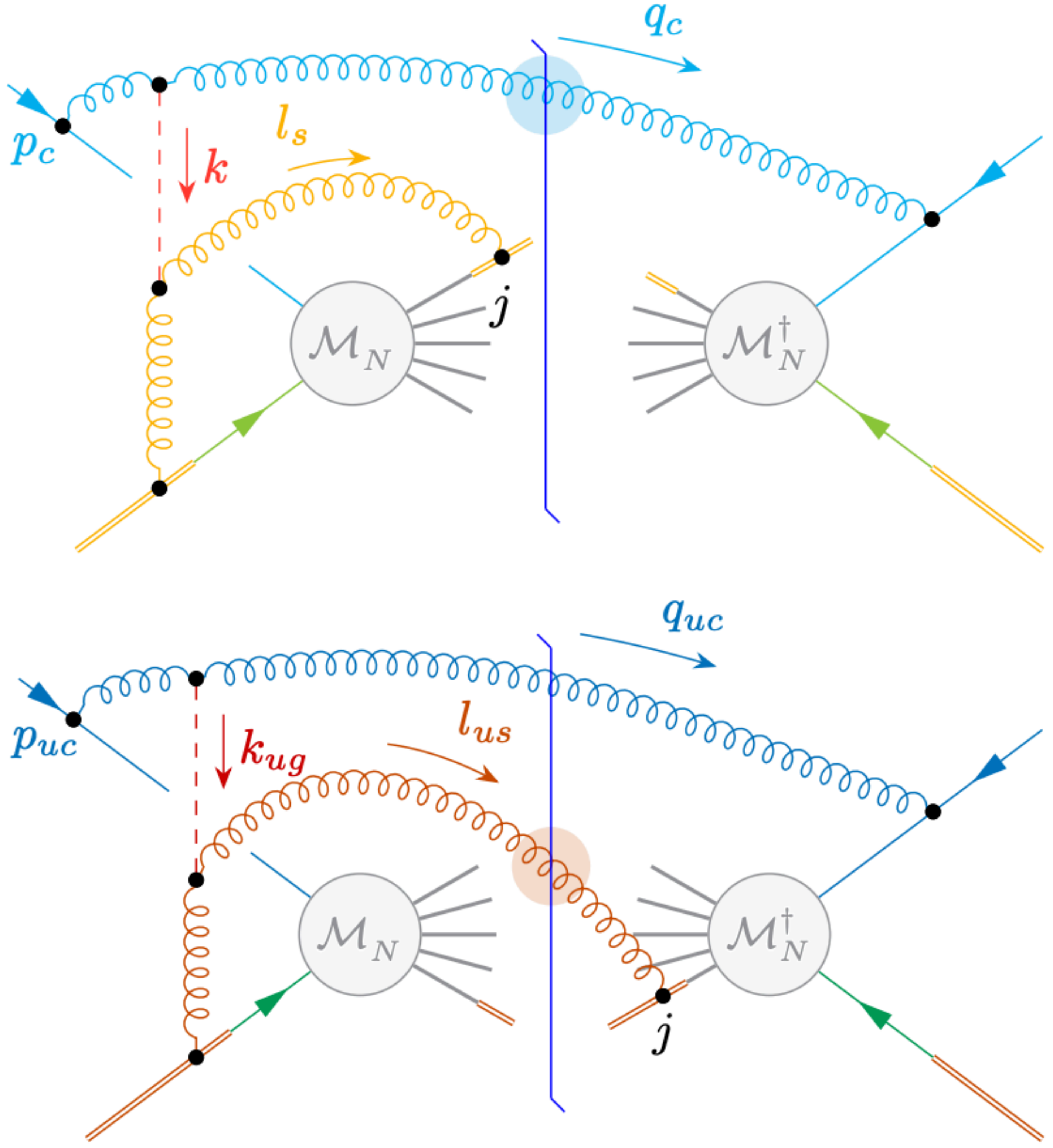
$$\boxed{\frac{\mathrm{d}\sigma_1^{\mathrm{CVL}}}{\mathrm{d}x_a\mathrm{d}x_b\mathrm{d}\mathcal{B}} \approx \sum_{a=q,g} A_a\left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^4(-i\pi)^2\frac{1}{480}\left(\ln\frac{1}{\tau_1}\right)^6}$$

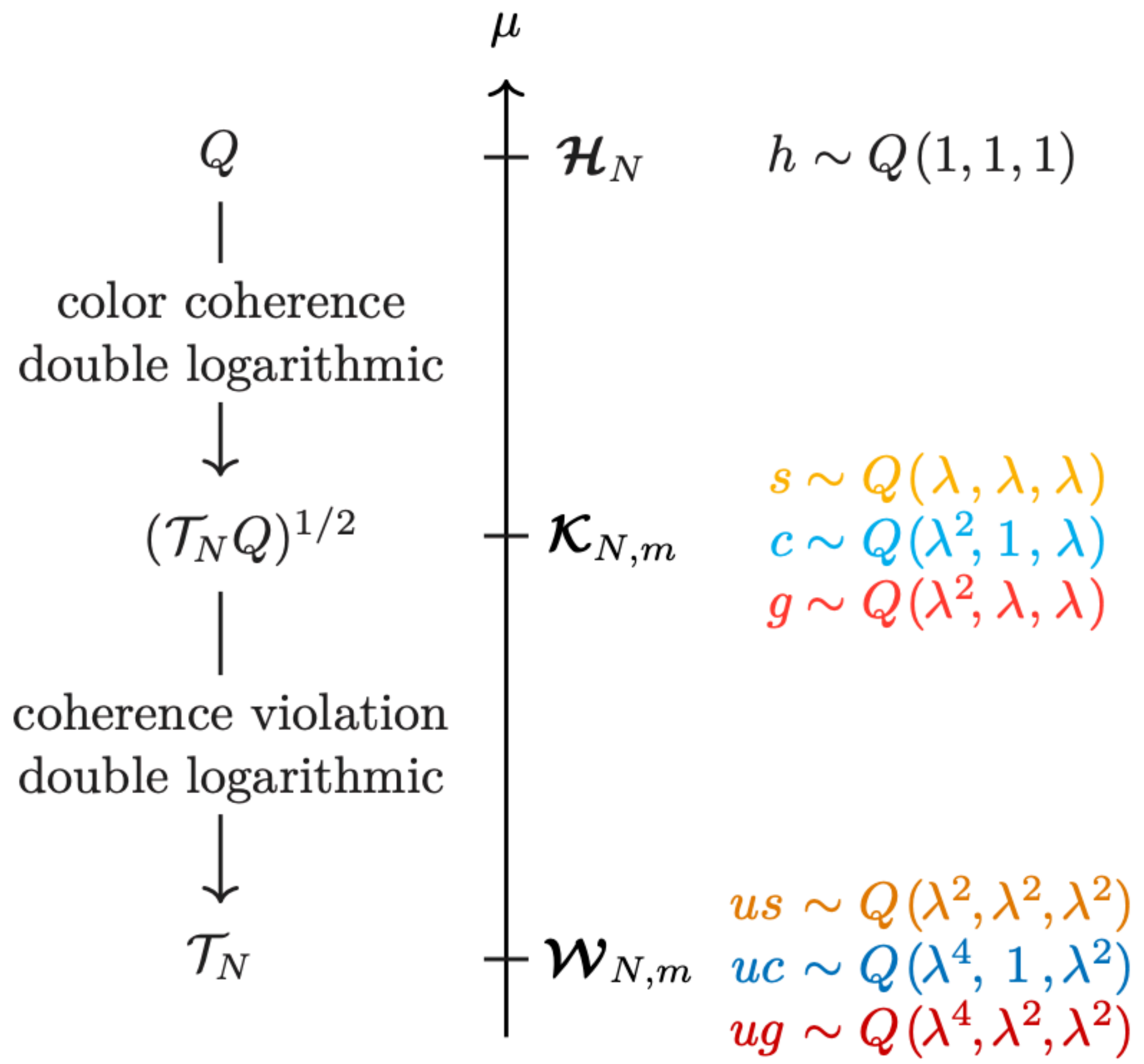
$$A_q = f^q_A f^g_B\,\mathrm{Tr}\left[\left(\frac{1}{3}[\mathbf{T}^2_s,[\mathbf{T}^2_s,\mathbf{T}^2_{ad}]\right)\mathbf{t}_a\mathbf{H}_{ab}(\mathcal{B})\mathbf{t}^{\dagger}_a\right]_{a=\mathrm{quark}} = \frac{8}{3}N_{\mathrm{c}}^2T_R^4\; f^q_A f^g_B\;\sigma^{(0)}_{qg}(\mathcal{B}),$$

$$A_g = f^g_A f^q_B\,\mathrm{Tr}\left[\left(\frac{1}{3}[\mathbf{T}^2_s,[\mathbf{T}^2_s,\mathbf{T}^2_{ad}]\right)\mathbf{t}_a\mathbf{H}_{ab}(\mathcal{B})\mathbf{t}^{\dagger}_a\right]_{a=\mathrm{gluon}} = -\frac{32}{3}N_{\mathrm{c}}^2T_R^4\; f^g_A f^q_B\;\sigma^{(0)}_{gq}(\mathcal{B}).$$

Independently verified

T. Becher, P. Hager, M. Neubert, D. Schwienbacher, arXiv:2603.12383





μ

Q

color coherence double logarithmic

$(\mathcal{T}_N Q)^{1/2}$

coherence violation double logarithmic

$\mathcal{T}_N$

$\boldsymbol{\mathcal{H}}_N$ $h \sim Q(1,1,1)$

$\boldsymbol{\mathcal{K}}_{N,m}$ $s \sim Q(\lambda,\lambda,\lambda)$ $c \sim Q(\lambda^2, 1, \lambda)$ $g \sim Q(\lambda^2, \lambda, \lambda)$

$\boldsymbol{\mathcal{W}}_{N,m}$ $us \sim Q(\lambda^2,\lambda^2,\lambda^2)$ $uc \sim Q(\lambda^4, 1, \lambda^2)$ $ug \sim Q(\lambda^4,\lambda^2,\lambda^2)$

$$\cancel{\frac{d\sigma}{d\mathcal{T}_N} = \Big\langle \boldsymbol{\mathcal{H}}_N \otimes B_a \otimes B_b \otimes \Big[\prod_{i=1}^{N} J_i\Big] \otimes \boldsymbol{S}_N \Big\rangle}$$

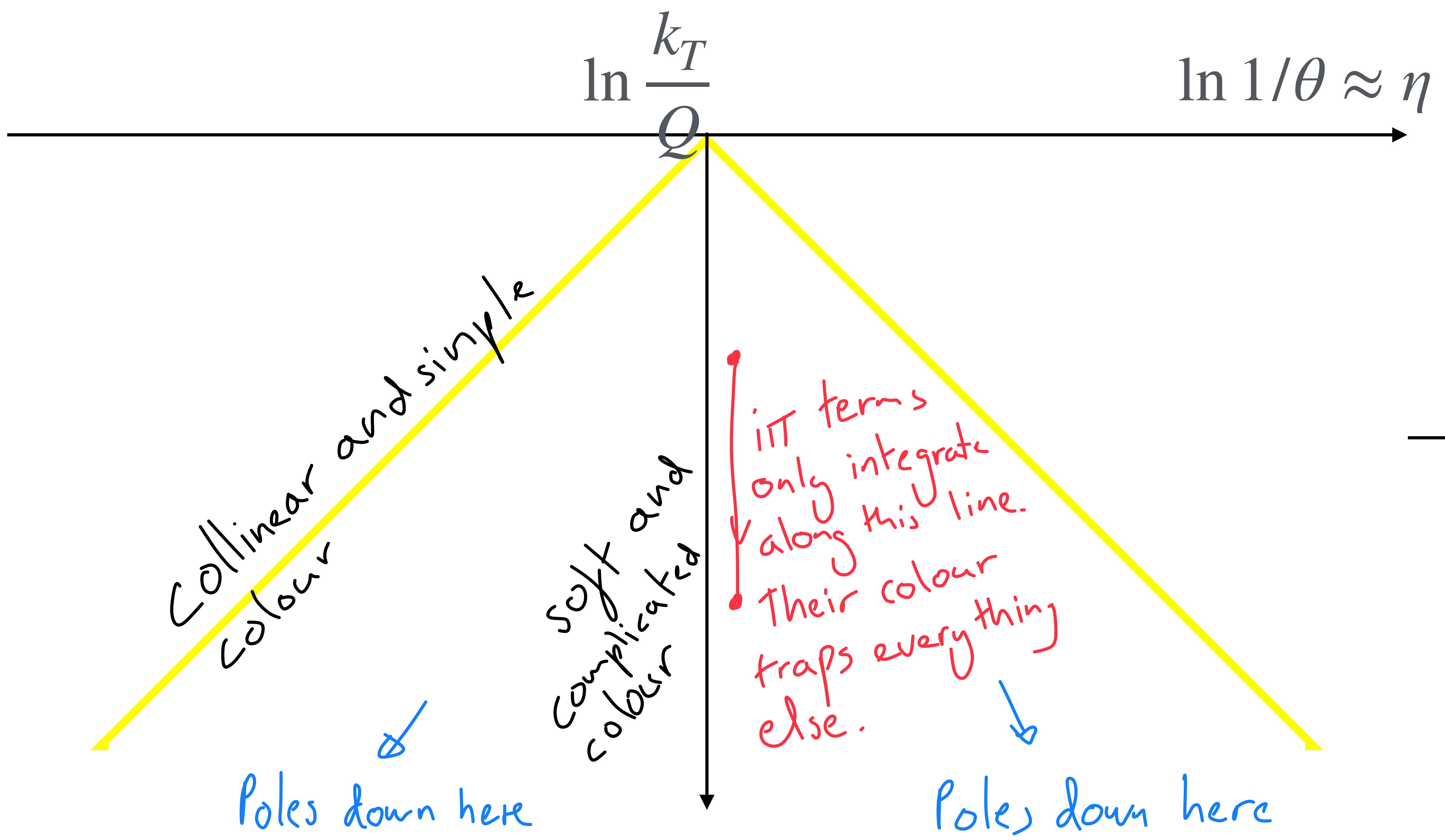
$$\frac{d\sigma}{d\omega} = \sum_m \big\langle \boldsymbol{\mathcal{H}}_N(Q,\mu)\, \widetilde{\boldsymbol{\mathcal{K}}}_{N,m}(\sqrt{Q\omega},\mu)\, \widetilde{\boldsymbol{\mathcal{W}}}_{N,m}(\omega,\mu) \big\rangle$$

$$\int_\omega^{\omega_c} \frac{d\mu_1}{\mu_1} \ln\frac{\omega_c}{\mu_1} \int_\omega^{\mu_1} \frac{d\mu_2}{\mu_2} \int_\omega^{\mu_2} \frac{d\mu_3}{\mu_3} \int_\omega^{\mu_3} \frac{d\mu_4}{\mu_4} \ln\frac{\mu_4}{\omega} = \frac{1}{46080} \ln^6\left(\frac{Q}{\omega}\right). \tag{11}$$

For the $qg \to q$ channel, expression (10) confirms the corresponding result obtained in [21]. Our formalism also makes it clear that the leading CVLs scale as $d\sigma_{\text{CVL}}/d\omega \sim \alpha_s^{2+n} \ln^{2+2n}(Q/\omega)$ with $n \geq 2$. For 0-jettiness, the CVL series starts at five-loop order, because an additional insertion of $\boldsymbol{\Gamma}^{\mathcal{W}}_{\text{real}}$ is needed for a non-trivial color structure.

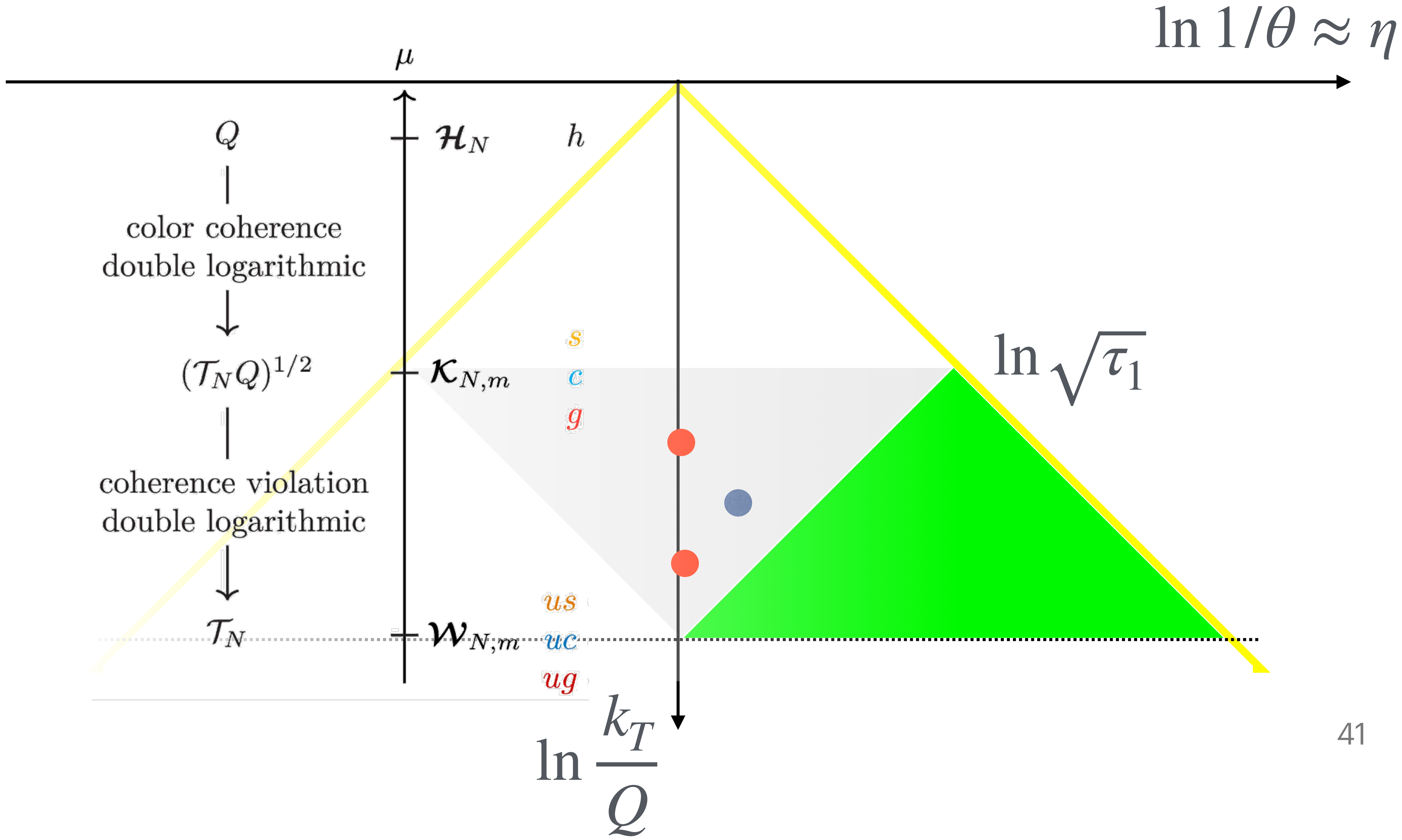
What does this mean?

How does this not cancel?



Whilst there is a double log, it (grey region) is not associated with a collinear pole/boundary. It is a log enhanced soft phase-space volume. It can therefore have complicated colour.

Compare the Lund plane for the super-super leading log against our intuition.



What does this mean?

Log counting

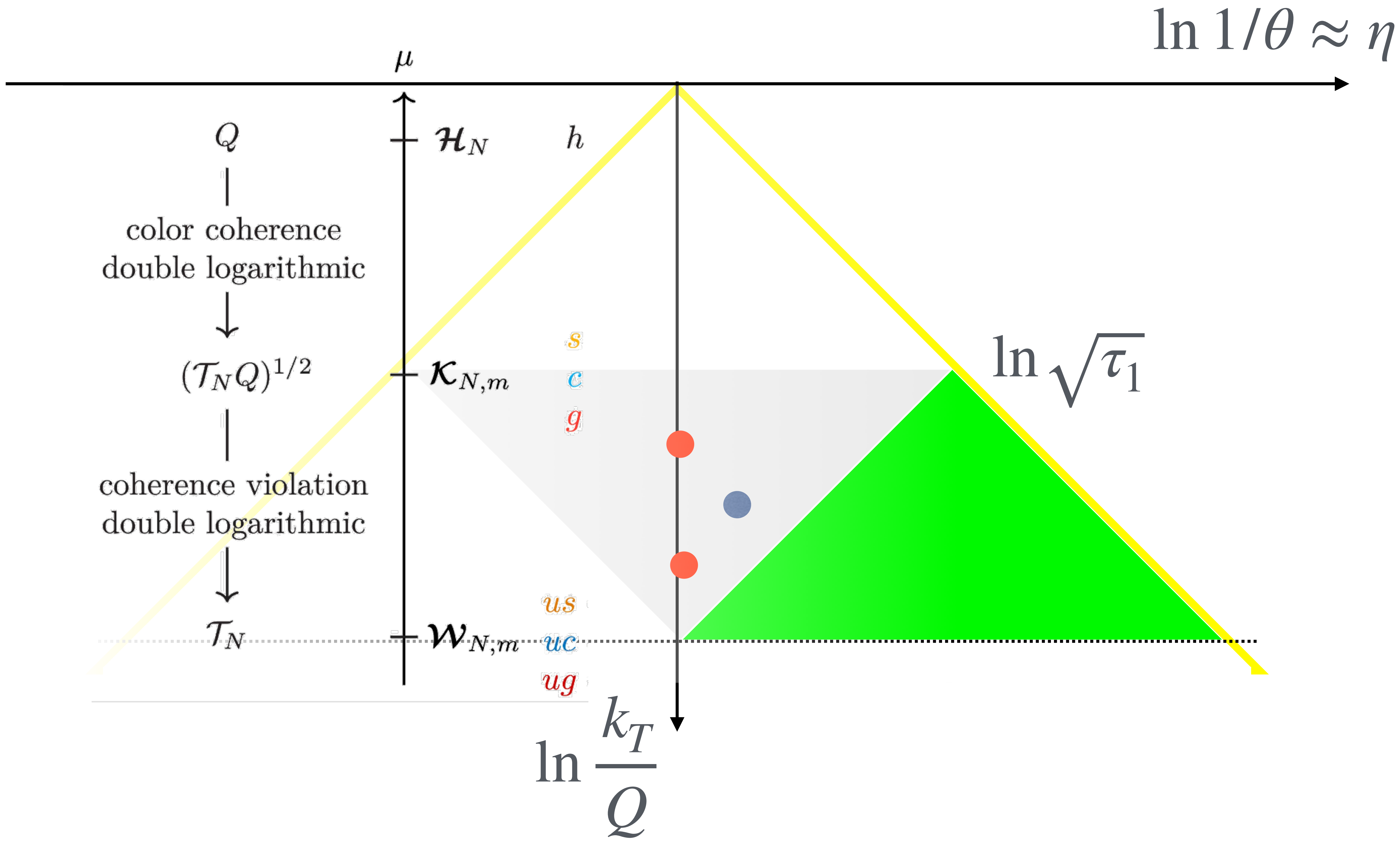
We're talking about areas on the Lund plane -> these are double logs.

They are computable at fixed coupling with only the soft-collinear singularities of one-loop kernels. This is just DL resummation?

$$\ln s_{ij} - i\pi \equiv \ln(-s_{ij})$$

$$\alpha_s^4 \ln(\tau_1)^6 (i\pi)^2 \mapsto \alpha_s^4 \ln(\tau_1)^6 \ln(-\tau_1)^2 \mapsto \alpha_s^4 L_\pm^8$$

All orders this is $\alpha_s^n L_\pm^{2n}$. **It is double log resummation with two large logs!**

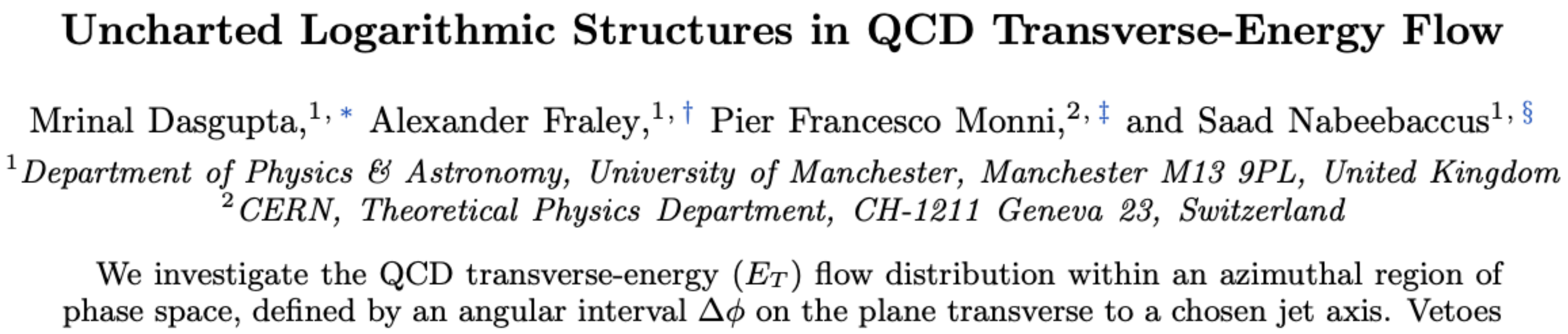


A similar result

QCD Transverse-Energy Flow

$$A_q = f_A^q f_B^g \,\mathrm{Tr}\left[\left(\frac{1}{3}[\mathbf{T}_s^2,[\mathbf{T}_s^2,\mathbf{T}_{ad}^2]]\right)\mathbf{t}_a \mathbf{H}_{ab}(\mathcal{B})\mathbf{t}_a^\dagger\right]_{a=\text{quark}}$$
$$= \frac{8}{3} N_\mathrm{c}^2 T_R^4 \; f_A^q f_B^g \; \sigma_{qg}^{(0)}(\mathcal{B}), \qquad (16)$$

$$\frac{\mathrm{d}\sigma_1^{\mathrm{CVL}}}{\mathrm{d}x_a \mathrm{d}x_b \mathrm{d}\mathcal{B}} \approx \sum_{a=q,g} A_a \left(\frac{\alpha_\mathrm{s}}{\pi}\right)^4 (-i\pi)^2 \frac{1}{480}\left(\ln\frac{1}{\tau_1}\right)^6$$



Uncharted Logarithmic Structures in QCD Transverse-Energy Flow

Mrinal Dasgupta,[1,*] Alexander Fraley,[1,†] Pier Francesco Monni,[2,‡] and Saad Nabeebaccus[1,§]
[1] *Department of Physics & Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom*
[2] *CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland*

We investigate the QCD transverse-energy (E_T) flow distribution within an azimuthal region of phase space, defined by an angular interval $\Delta\phi$ on the plane transverse to a chosen jet axis. Vetoes

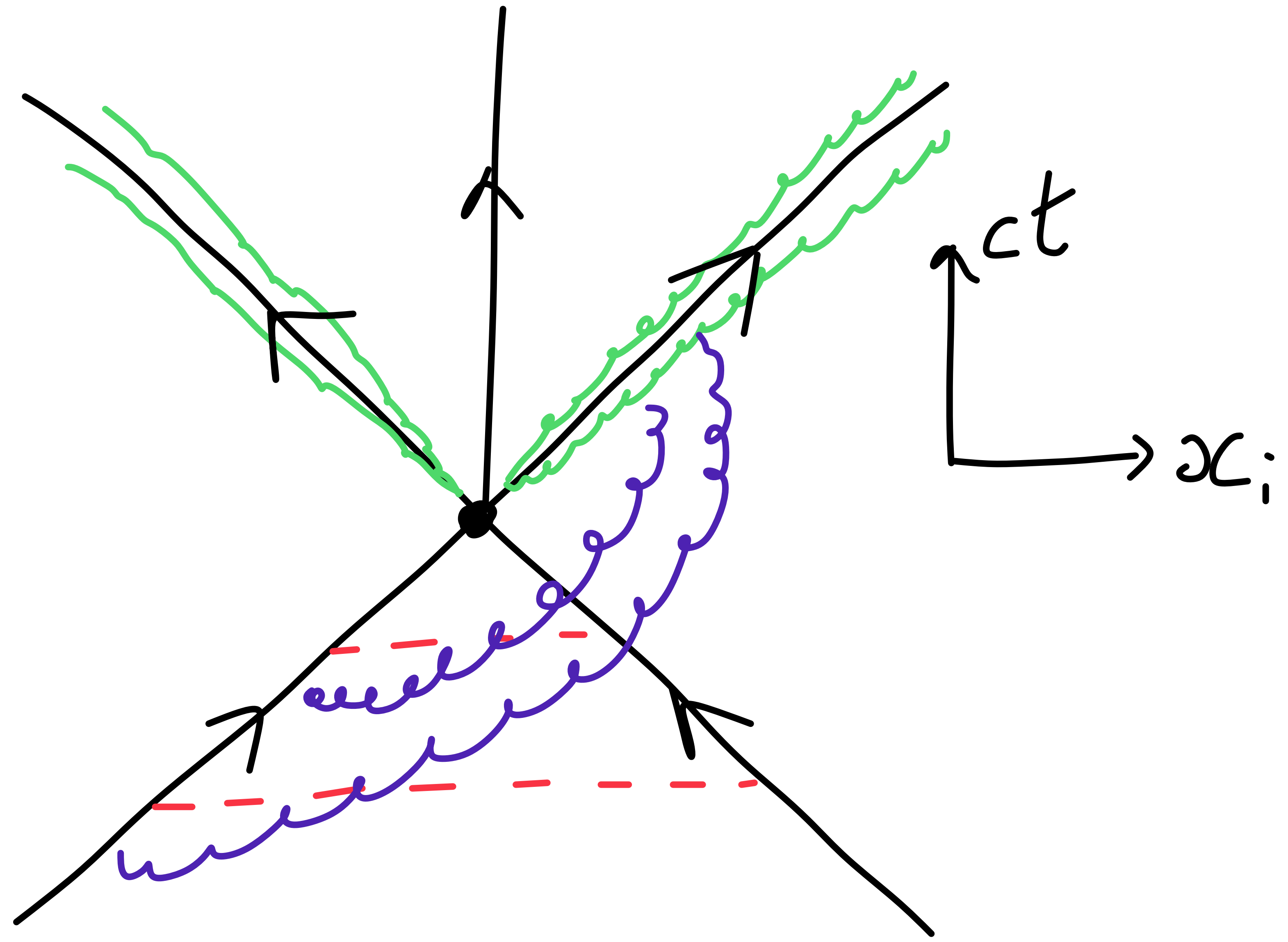
$$\hat{\sigma}_{\mathrm{CVL}} = \left(-\frac{2\alpha_s}{\pi}\right)^5 (1-f)^2 \int_{E_T}^{Q} \frac{dk_{t,3}}{k_{t,3}} \int_{-\eta_3^{\max}}^{\eta_3^{\max}} d\eta_3 \int_{E_T}^{k_{t,3}} \frac{dk_{t,4}}{k_{t,4}} \int_{-\eta_4^{\max}}^{\eta_4^{\max}} d\eta_4 \int_{E_T}^{k_{t,4}} \frac{dk_t}{k_t}$$
$$\left[\frac{1}{3}\ln^2\left(\frac{k_{t,4}}{E_T}\right) \langle m_0 | \boldsymbol{D}_{a_3,(2)}^{\mu_3\,\dagger} \boldsymbol{D}_{a_4,(3)}^{\mu_4\,\dagger} \left[\boldsymbol{\Gamma}_{(4)}^{\mathrm{I}}, \left[\boldsymbol{\Gamma}_{(4)}^{\mathrm{I}}, \boldsymbol{\Gamma}_{(4)}^{\mathrm{R}}\right]\right] \boldsymbol{D}_{a_4,(3)}^{\mu_4} \boldsymbol{D}_{a_3,(2)}^{\mu_3} | m_0 \rangle\right]$$

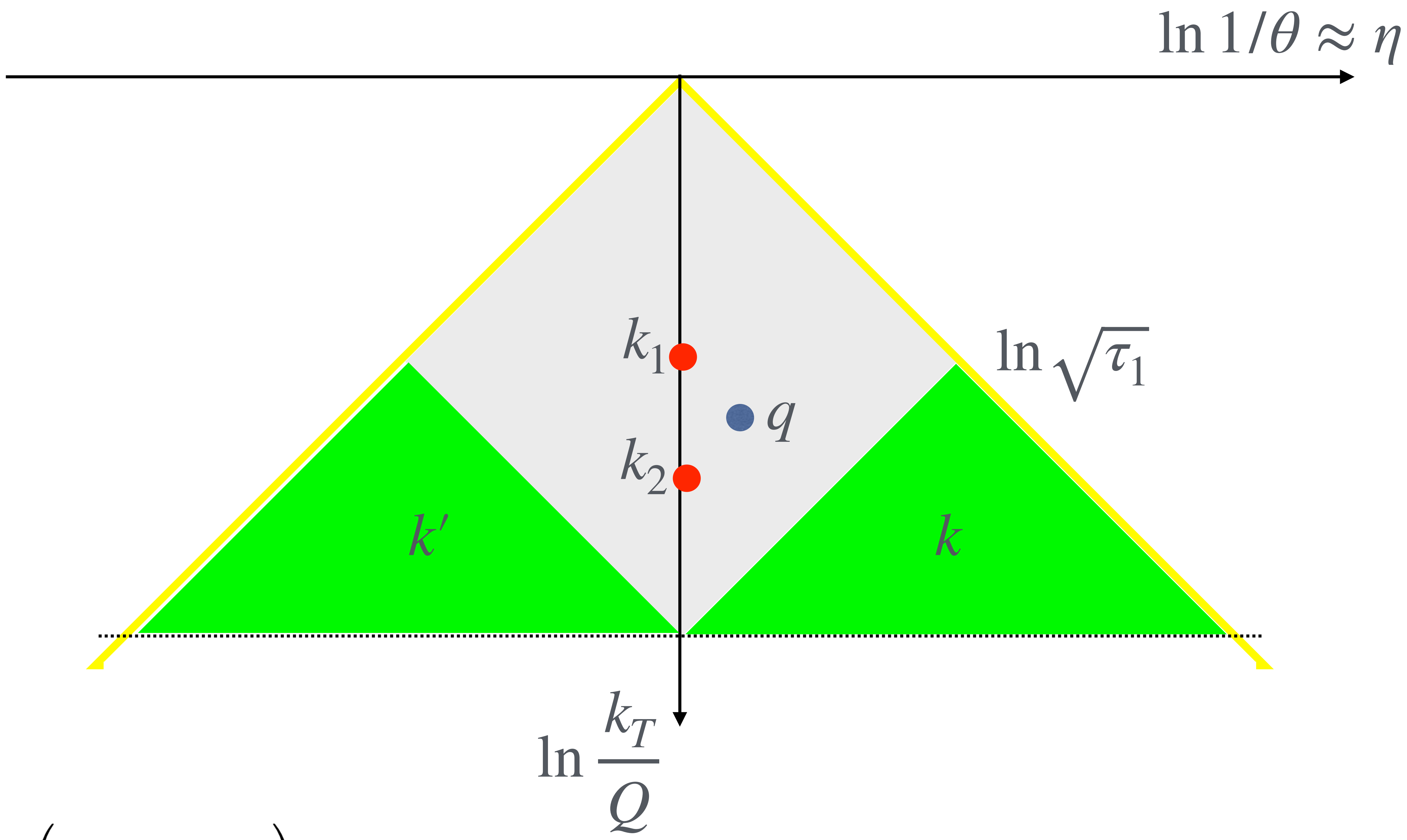
$$\Sigma_{\mathrm{DY}}^{(\mathrm{CVL})} = -\left(\frac{\alpha_s}{\pi}\right)^5 \pi^2 C_F N_c^2 f \;(1-f)^2 \frac{L^8}{180}$$

The all-orders structure of jettiness?

Based on speculation (and work to come)

Physical picture of the colour structure:





$$\int_{q.n_{a,b}>\tau_1 Q} \mathrm{d}y^{(ad)}\frac{\mathrm{d}\phi^{(ad)}}{2\pi} = \frac{1}{2}\ln\left(\frac{\tau_1^2 Q^2}{q_T^2}\frac{\cos\theta + \frac{q_T^2-\tau_1^2Q^2}{q_T^2+\tau_1^2Q^2}}{\cos\theta - \frac{q_T^2-\tau_1^2Q^2}{q_T^2+\tau_1^2Q^2}}\right) \qquad \int_{q.n_{i,j}>\tau_1 Q} \mathrm{d}y^{(ij)}\frac{\mathrm{d}\phi^{(ij)}}{2\pi} = \ln\left(\frac{n_i\cdot n_j}{1-\frac{q_T^2-\tau_1^2Q^2}{q_T^2+\tau_1^2Q^2}} - 1\right) \approx \ln\left(\frac{q_T^2}{\tau_1^2Q^2}\frac{n_i\cdot n_j}{2}\right)$$

$$\mathbf{V}_{\mu_\mathrm{F},k_T} \approx \exp\left(\frac{\alpha_\mathrm{s}}{\pi}\left[\mathbf{T}_{ad}^2 L_2(\mu_\mathrm{F},k_T) - i\pi\mathbf{T}_s^2\ln\frac{k_T}{\mu_\mathrm{F}}\right]\right) \quad \mathbf{T}_{ad}^2 = \sum_{\substack{i<j\\ i\nparallel j}}\mathbf{T}_i\cdot\mathbf{T}_j = -\frac{1}{2}\sum_i \mathbf{T}_i^2 - \mathbf{T}_a\cdot\mathbf{T}_d,$$

The all-orders structure of jettiness?

Based on speculation (and work to come)

A hierarchy of challenge to do the CVL leading resummation?

GBJs	Collinear must be resummed	Colour space grows indefinitely with soft multiplicity	Single soft is enough	Two Coloumb/Glauber needed (resummation achieved anyway)	SLL	NDL$_{\pm}$
n-Jettiness	Collinear must be resummed	Colour space is n+4 parton-like	Soft must be resummed	Two Coloumb/Glauber needed	SSLL	DL$_{\pm}$
rIRC safe	Collinear must be resummed	Colour space is finite	Soft only must be resummed if forward suppressed	Two Coloumb/Glauber needed	SSLL if forward suppressed	DL$_{\pm}$ if forward suppressed
QCD Transverse-Energy Flow	Collinear must be resummed	Colour space grows indefinitely with soft multiplicity	Soft must be resummed	Two Coloumb/Glauber needed	SSLL	DL$_{\pm}$

Conclusions

- One-jettiness contains a super-super leading log.

$$\frac{\mathrm{d}\sigma(\tau_1)^{\mathrm{CVL}}}{\mathrm{d}x_a\mathrm{d}x_b\mathrm{d}B} \sim N_c^2\left(\frac{\alpha_s}{\pi}\right)^4 (i\pi)^2\left(\ln\tau_1\right)^6 f^a(x_a,\tau_1 Q)f^b(x_b,\tau_1 Q)$$

- The log doesn’t touch the low scale pole structure, so PDFs can still be defined but must be at the soft scale, not the collinear scale.
- Resummation of these logs might be tough but possible. Unlike GBJs, where only a single soft needs to be considered, here arbitrary softs are required. However, the colour space remains finite.
- Numerical codes may also provide a route out, CVolver - Forshaw, Plätzer et al, Deductor - Nagy, Soper…