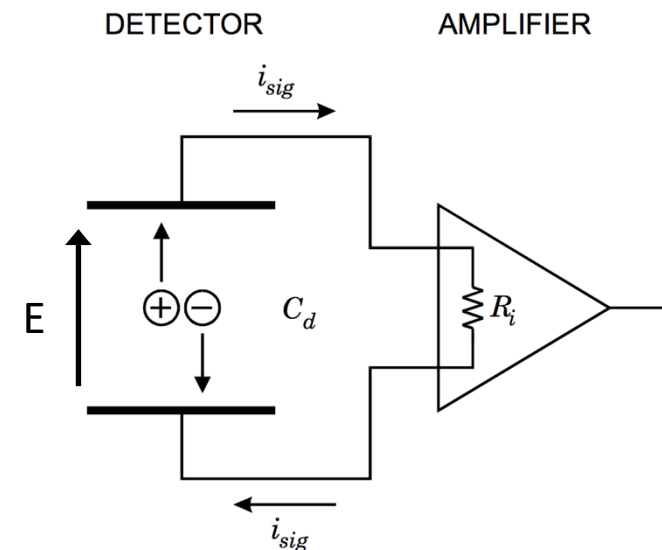
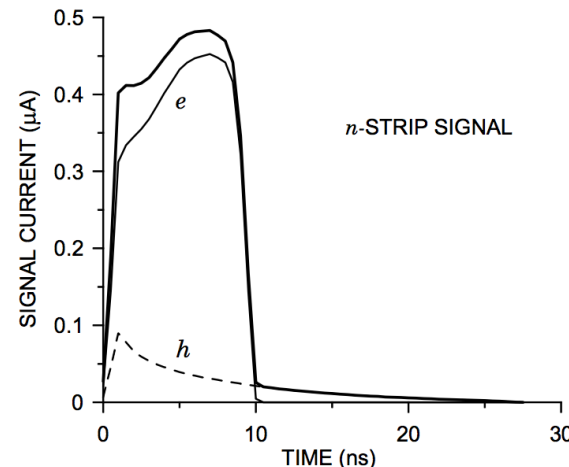


Ionisation chamber model

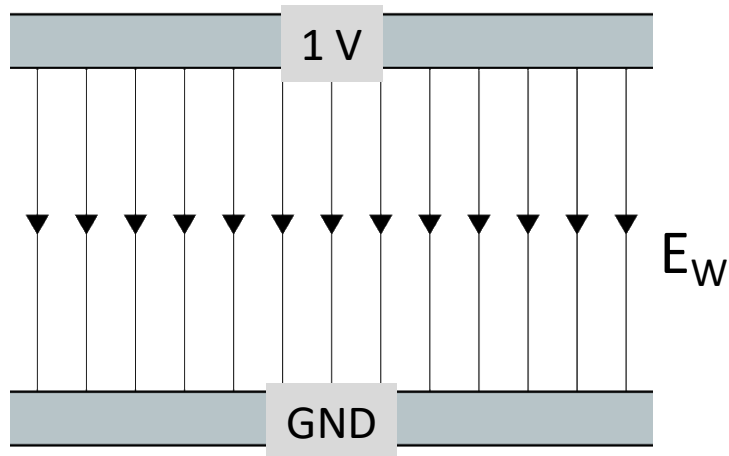
- Free charge is generated in a volume between two electrodes
 - Volume is the detecting medium, e.g. silicon or a gas
 - Charges: ions + electrons or e-h pairs
 - Apply an electric field to drift the charge to the electrodes
- What is generating the signal?
 - The movement of charges induces the current $i_{sig}(t)$
 - The current depends on the geometry



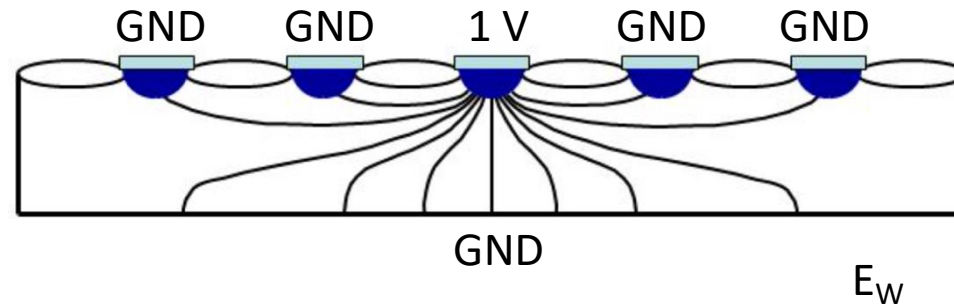
Shockley-Ramo Theorem

- Determines $i_{\text{sig}}(t)$ induced on an electrode A
- Construct a hypothetical field, the *weighting field*: E_w
 - Set electrode A to unit potential
 - Set all other potentials to ground

Parallel plate capacitor



Pixelated detector volume

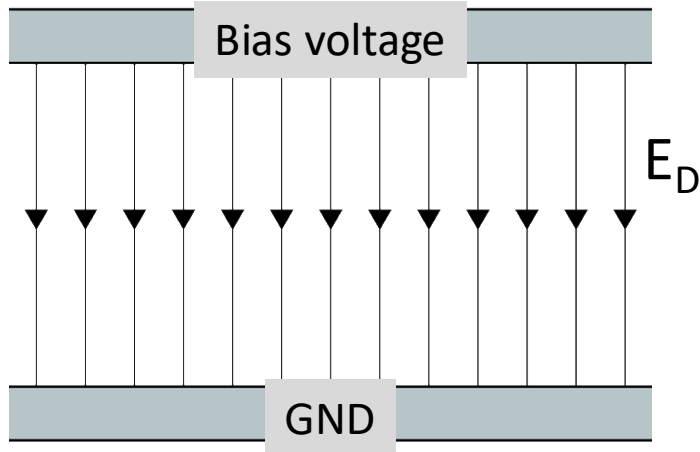


For a full derivation and discussion, see e.g. [H. Spieler's lectures](https://www-physics.lbl.gov/~spieler/)
<https://www-physics.lbl.gov/~spieler/>

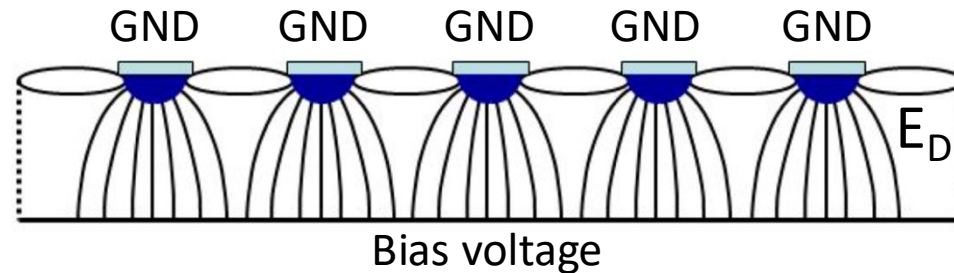
Shockley-Ramo Theorem

- Calculate the actual field that the charges will drift in: E_D
 - The electrode configuration as it is in the detector

Parallel plate capacitor



Pixelated detector volume



- The drift field and weighting field will look very different
 - The drift field determines the trajectory of the particles
 - Special case: they are identical for a parallel plate capacitor

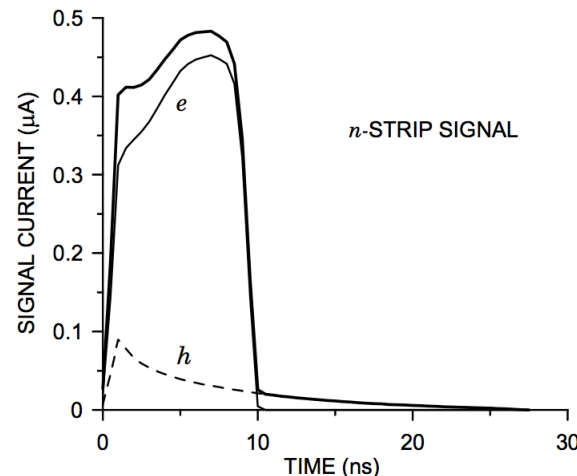
Shockley-Ramo Theorem

- Use these two fields to calculate the induced current
 - Velocity vector of the charge drifting along the field lines

$$\vec{v}_q(t) = \mu \cdot \vec{E}_D(\vec{x}(t)), \quad \text{where } \begin{cases} \mu \text{ is the mobility} \\ E_D \text{ is the drift field} \end{cases}$$

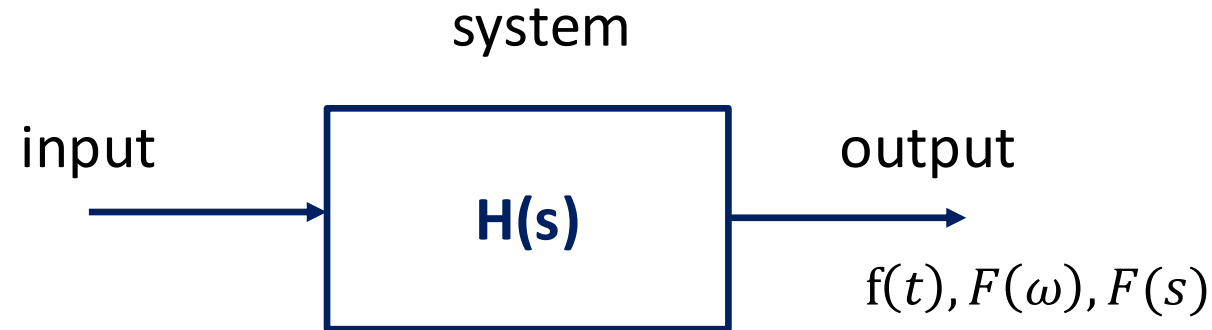
- Shockley-Ramo Theorem
 - The time-dependent induced current is given by

$$i_{sig}(t) = q \cdot \vec{v}(t) \cdot \vec{E}_W(\vec{x}(t)), \quad \text{where } E_W \text{ is the weighting field}$$



Example of a calculated current from electrons and holes in a silicon detector (note the difference in mobility)

Transfer functions



Transfer function: $h(t), H(s)$

Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Laplace transform

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

where $s = \sigma + i\omega$

Properties of the Laplace transformation

- Laplace transforms of derivatives:

$$f'(t) \quad sF(s) - f(0)$$

$$f^{(n)}(t) \quad s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

- Transforms (linear) differential equations to polynomial equations
- Solve the equations and find poles and zeroes
 - Defines the dynamical behaviour of the system
- Solution becomes combinations of exponential and sinus functions

$$e^{at} \quad \frac{1}{s-a}$$

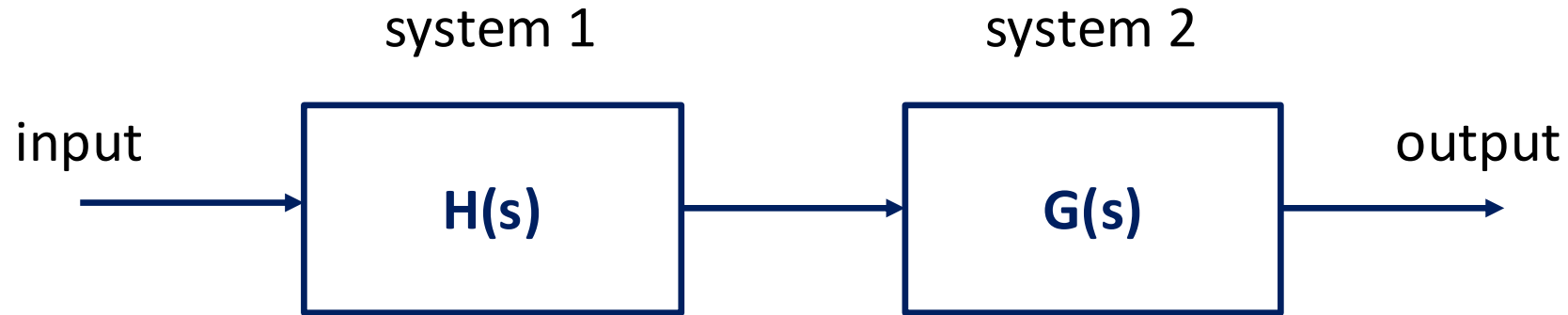
$$\sin(at)$$

$$t \sin(at)$$

$$\frac{a}{s^2 + a^2}$$

$$\frac{2as}{(s^2 + a^2)^2}$$

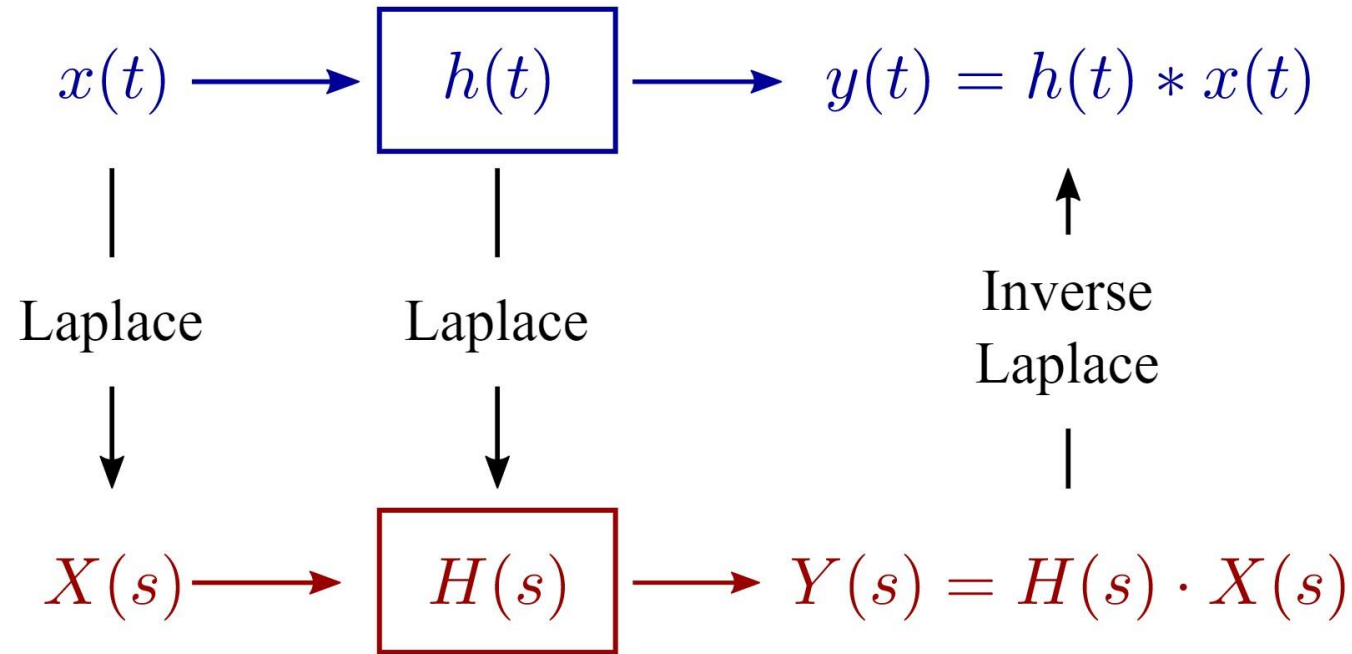
Combining the transfer functions of two systems



- Time domain: convolution integral
 - $f(t) = \int_{-\infty}^{\infty} h(\tau) \cdot g(t - \tau) d\tau$
- Laplace (also frequency) domain: multiplication
 - $F(s) = H(s) \cdot G(s)$
 - Deconvolute: $H(s) = F(s)/G(s)$

Transfer functions – time and frequency domain

Time domain



Frequency domain

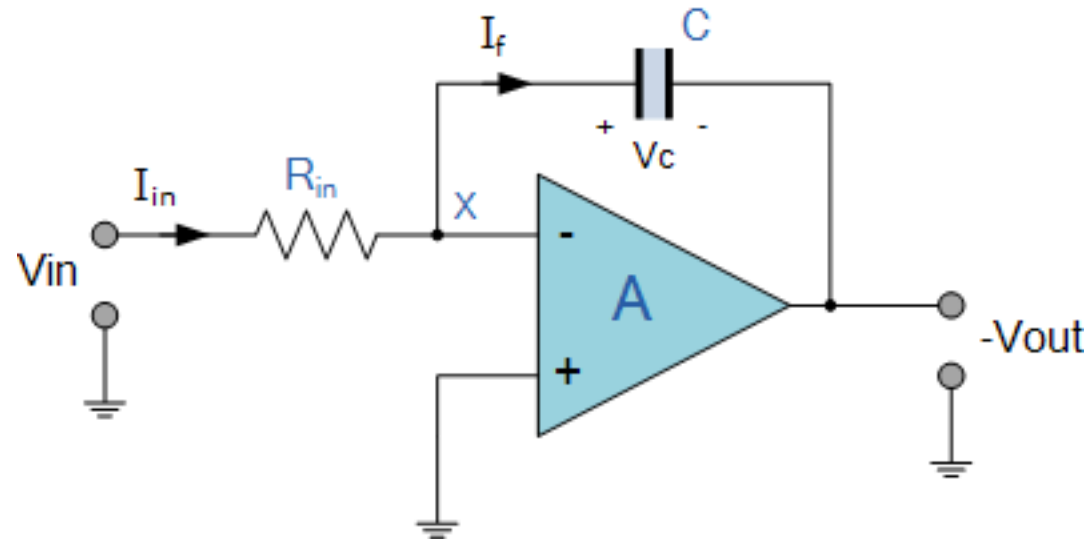
Fourier transformation

Table of Common Functions and their Fourier Transforms

Function name	Function in the time domain	Fourier Transform (in the frequency domain)
	$w(t)$	$\hat{W}(f)$
Dirac delta	$\delta(t)$	1
Constant	1	$\delta(f)$
Cosine	$\cos(2\pi f_0 t)$	$\frac{\delta(f - f_0) + \delta(f + f_0)}{2}$
Sine	$\sin(2\pi f_0 t)$	$\frac{\delta(f - f_0) - \delta(f + f_0)}{2j}$
Unit step function	$u(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases}$	$\frac{1}{j\omega}$ (for $\omega = 2\pi f$)
Decaying exponential (for $t > 0$)	$e^{-\alpha t} u(t)$	$\frac{1}{\alpha + j2\pi f}$, $\alpha > 0$
Box or rectangle function	$\text{rect}(at) = \begin{cases} 0, & \text{if } at > \frac{1}{2} \\ 1, & \text{if } at \leq \frac{1}{2} \end{cases}$	$\frac{1}{ a } \text{sinc}\left(\frac{f}{a}\right) = \frac{\sin(\pi f/a)}{\pi f/a}$
Sinc function	$\text{sinc}(at) = \frac{\sin(\pi at)}{\pi at}$	$\frac{1}{ a } \text{rect}\left(\frac{f}{a}\right)$
Comb function	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$
Gaussian	e^{-at^2}	$\sqrt{\frac{\pi}{a}} e^{-\frac{(\pi f)^2}{a}}$

Charge integrating amplifier

- Integrating amplifier is commonly used to integrate the induced current
 - Gives a signal proportional to the ionised charge
 - Shaping required to bring the signal back to the baseline



Simple common emitter stage

