

From QCD to Quantum gravity and Back

A review on recent developments in S-matrix bootstrap

Puzzles in the Quantum Gravity Landscape
Graz, September 2026

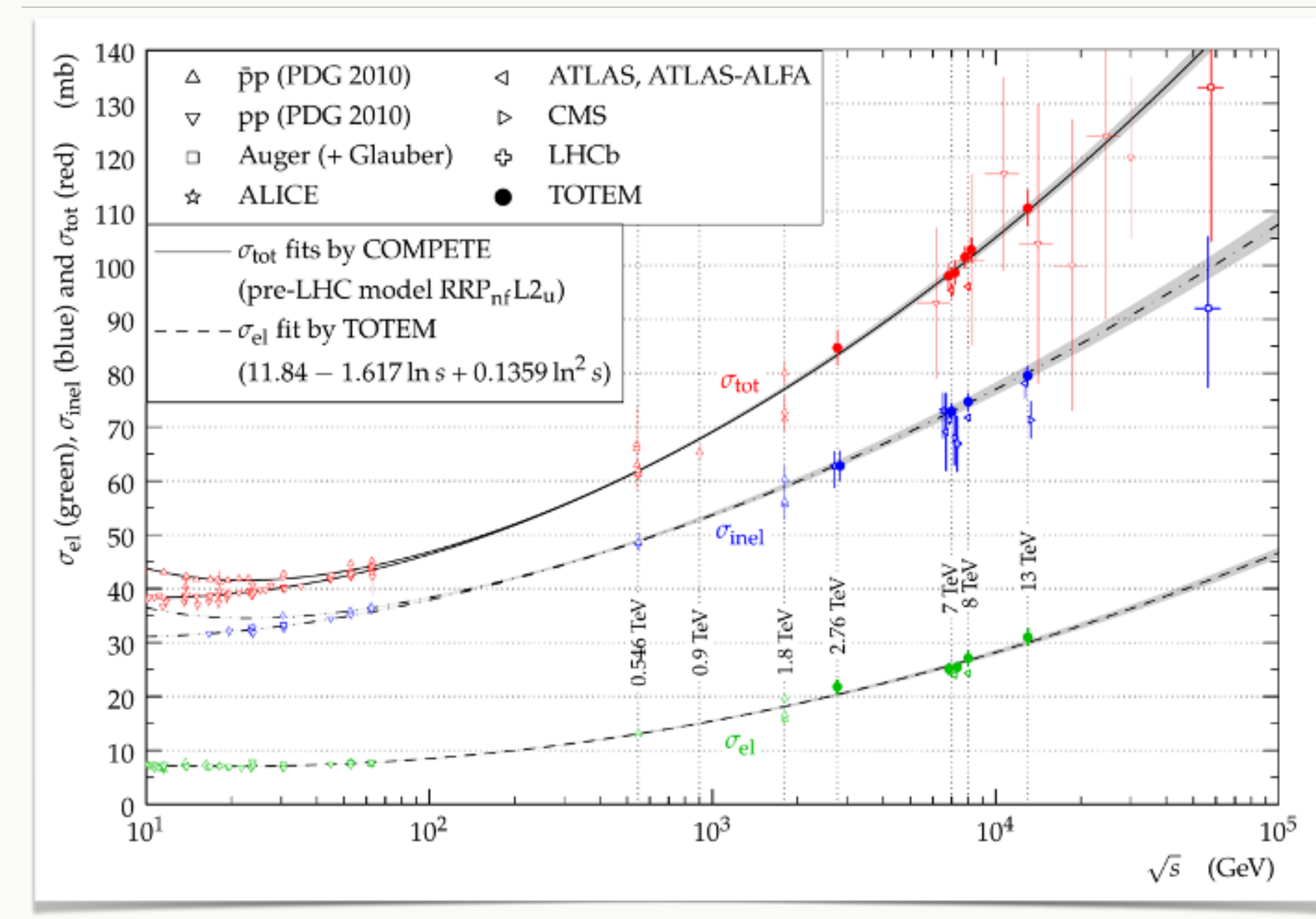
Piotr Tourkine, LAPTh, Annecy, France



Relativistic interactions

Quantum mechanics and Relativity are barely compatible

- 20th century — weak coupling. Interacting quantum field theories (gauge theories), the Standard Model.
- 21st century — strong coupling. Two problems with no perturbative handle at all:
 - soft hadronic scattering
 - **gravity at the Planck scale** — *what does $2 \rightarrow 2$ graviton scattering look like?*
- The bootstrap (ideas from 60s): take the consistency conditions as the definition of the theory, and map the space of what is possible.



σ_{tot} , σ_{inel} , σ_{el} for pp up to 13 TeV — Regge-inspired fits



part I

**dynamics
from
first principles**

Relativistic interactions

- Anti-particles, CPT, spin-statistics
forced by Lorentz invariance, locality and unitarity
- Massless spin 1 must couple to conserved charges
- Massless spin 2 must couple universally to everything
- *(hierarchy problems for spin 0)*
- No local unitarity interacting massless spins > 2 in flat space
- No mixing internal / spacetime symmetries
- Is this too restrictive? Ana's talk and ∞ -dim symmetries

Consistency fixes some of the structure

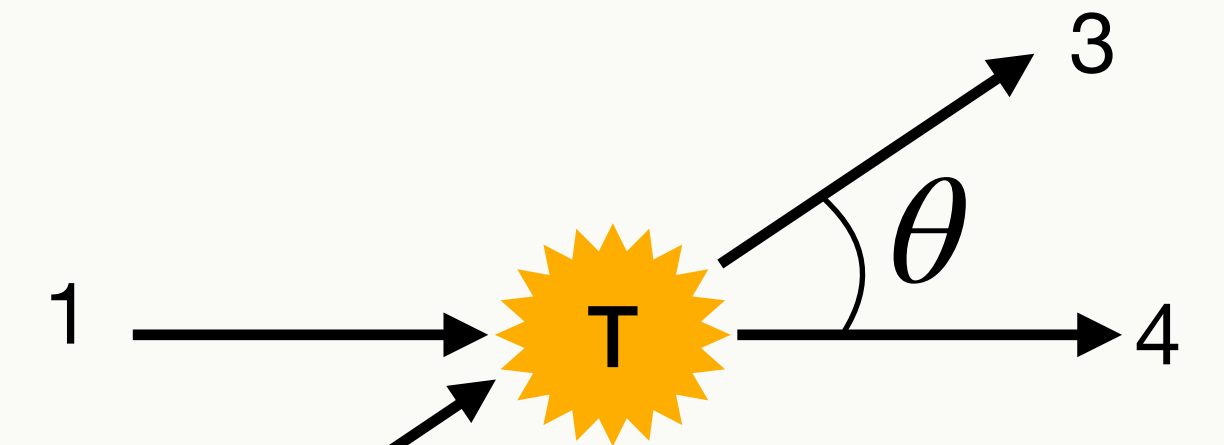
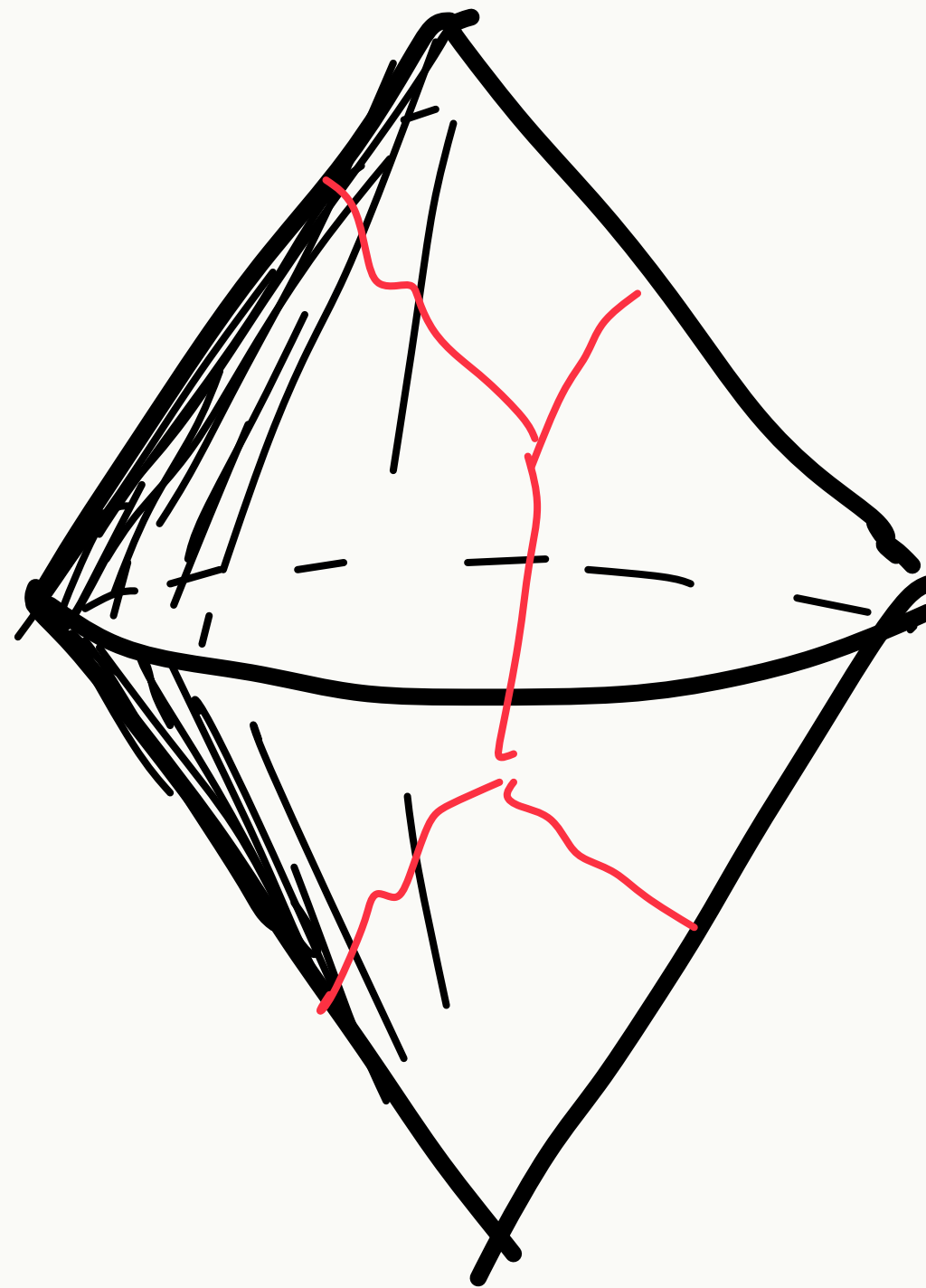
Dynamics : the S-matrix

interactions

- $\hat{S} : |\text{in}\rangle \mapsto |\text{out}\rangle$
- $\hat{S} = 1 + i\hat{T}$
- $2 \rightarrow 2$: a function $T(s, t, u)$ (up to polarisations)

No local gauge-invariant observables in quantum gravity $\rightarrow$ S-matrix.

This object can perhaps be compared across approaches



$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

$$(E, \theta) \mapsto (s, t, u)$$

$$s + t + u = \sum_{i=1}^4 m_i^2$$

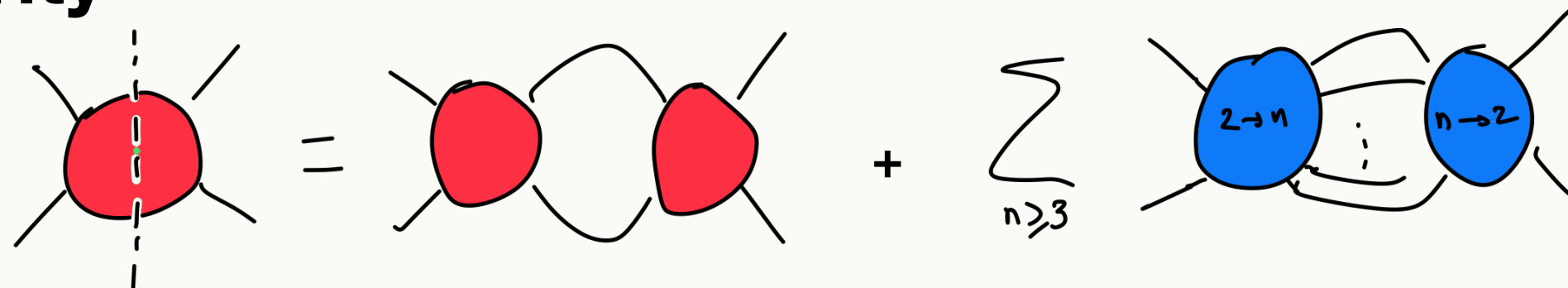
Axioms

For gapped theories, axioms on rigorous footing:

A **Analyticity** $\Leftrightarrow$ causality. T analytic in s at fixed t up to cuts (and bound-state poles).

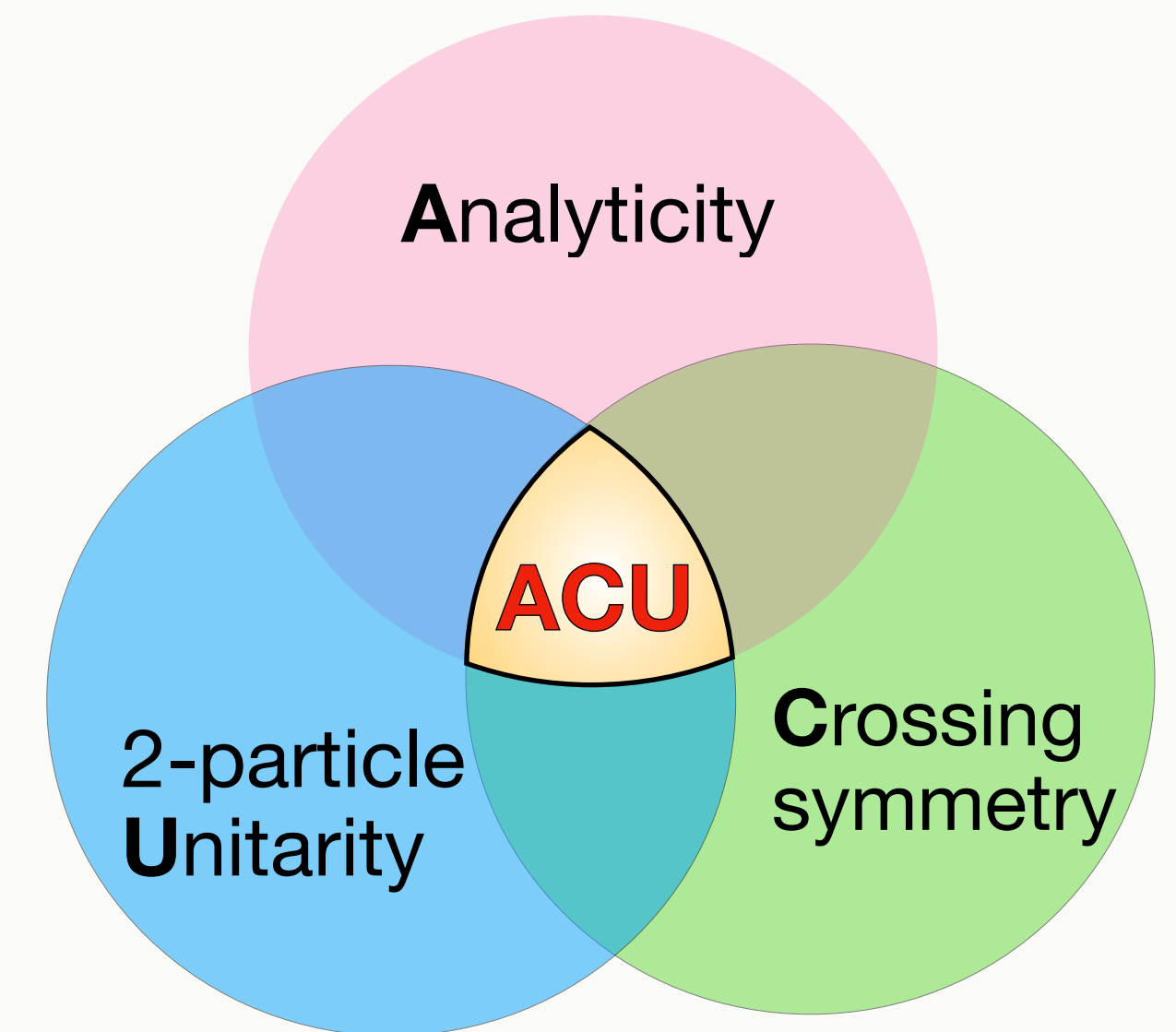
C **Crossing** $T(s, t) = T(t, s) = T(u, t)$ one function for all three channels.
Bros-Epstein-Glaser '60s

U **Unitarity**

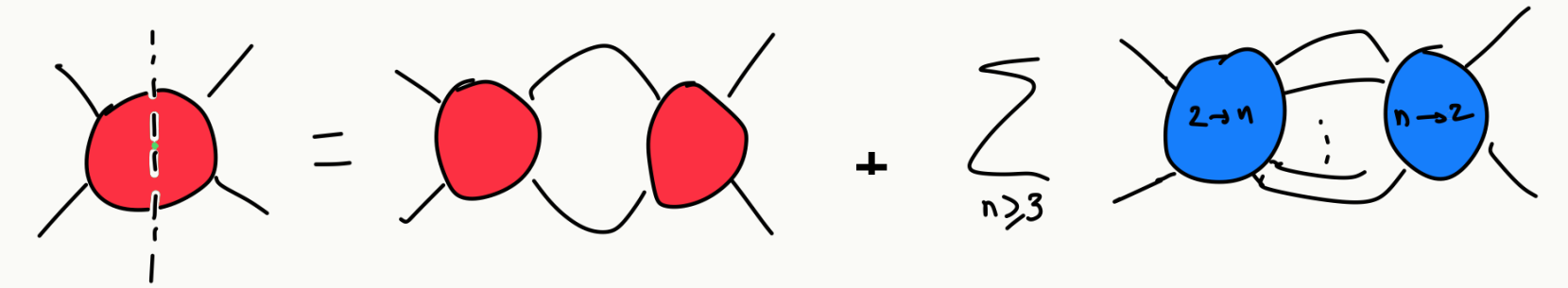


Theorem: in $d > 2$, scattering requires production

Aks '64



Unitarity in partial waves



A fancy Fourier expansion that diagonalizes unitarity

massive & massless*

$$T(s, t) = 16\pi \sum_{J=0}^{\infty} (2J+1) f_J(s) P_J(\cos(\theta))$$

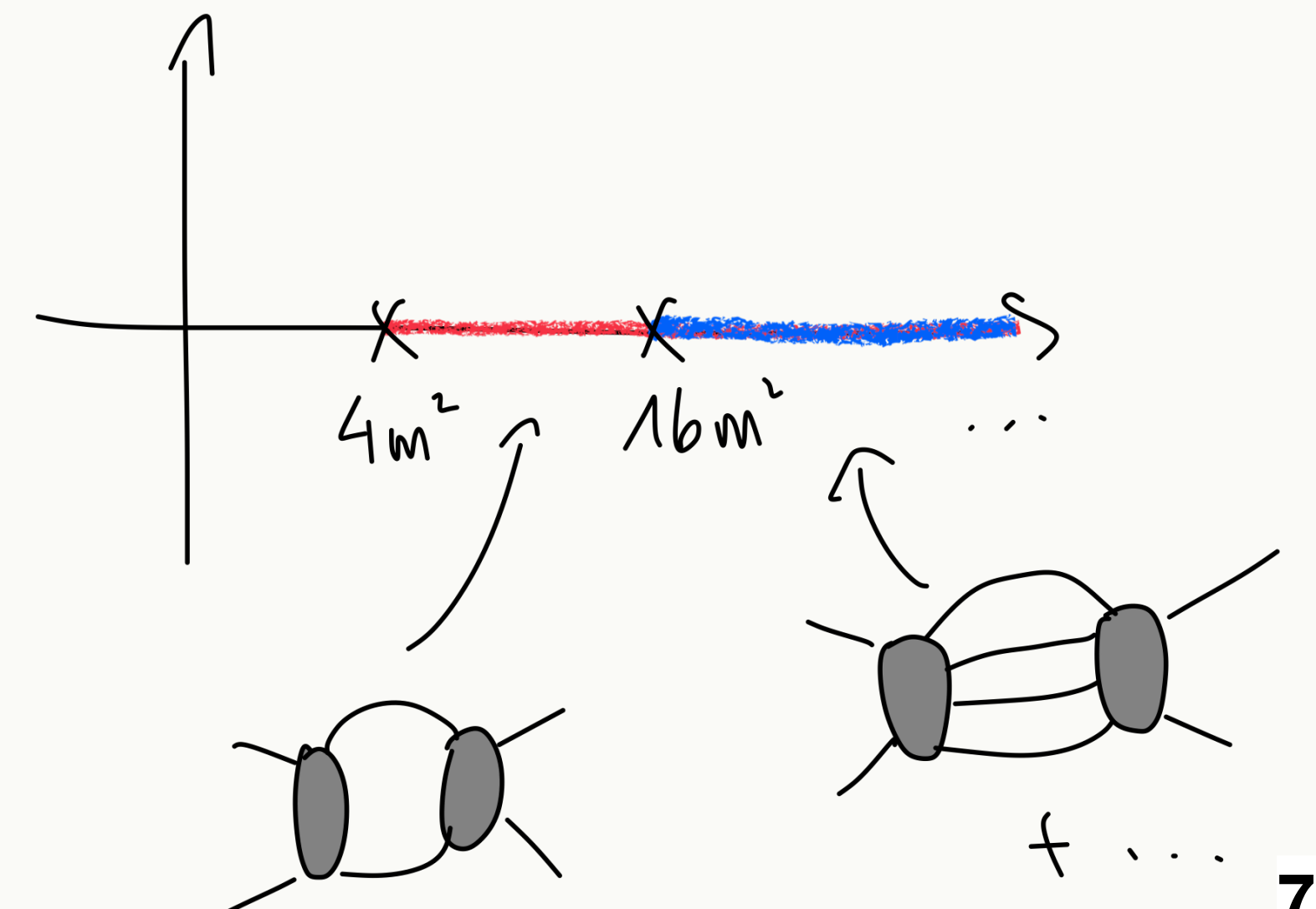
$$S_J(s) = 1 + ic(s) f_J(s)$$

- $|S_J(s)|^2 = 1, s \in [4m^2; s_{MP}]$

Elastic unitarity
massive

- $|S_J(s)|^2 \leq 1, s \in [s_{MP}; +\infty[$

- Straightforward to check



Boundedness

(locality)

- Proven: polynomial boundedness for gapped theories (60's, Epstein Glaser Martin)
- Froissart bound: assume massive exchange and polynomial force. Comes from long distance physics
 - Radius of interaction : $E^n \sim \exp(-mr)$ $\rightarrow$ $r \sim \log(E)$
 - $\hookrightarrow \sigma \sim \log(E)^2$
- Do they same in gravity: long-distance is semi-classical, under good control (Amati Ciafaloni Veneziano, 90's)
- Using this, Häring-Zhiboedov '22: proved $|T(s, t)| < s^2$ (for $d > 4$)

Space of S-matrices

How does ACU restrict the space of possible interactions? → Bounds.

Remarks:

Go away from the lamppost

Test proposals

Test conjectures

Locate where tensions are



Space of S-matrices

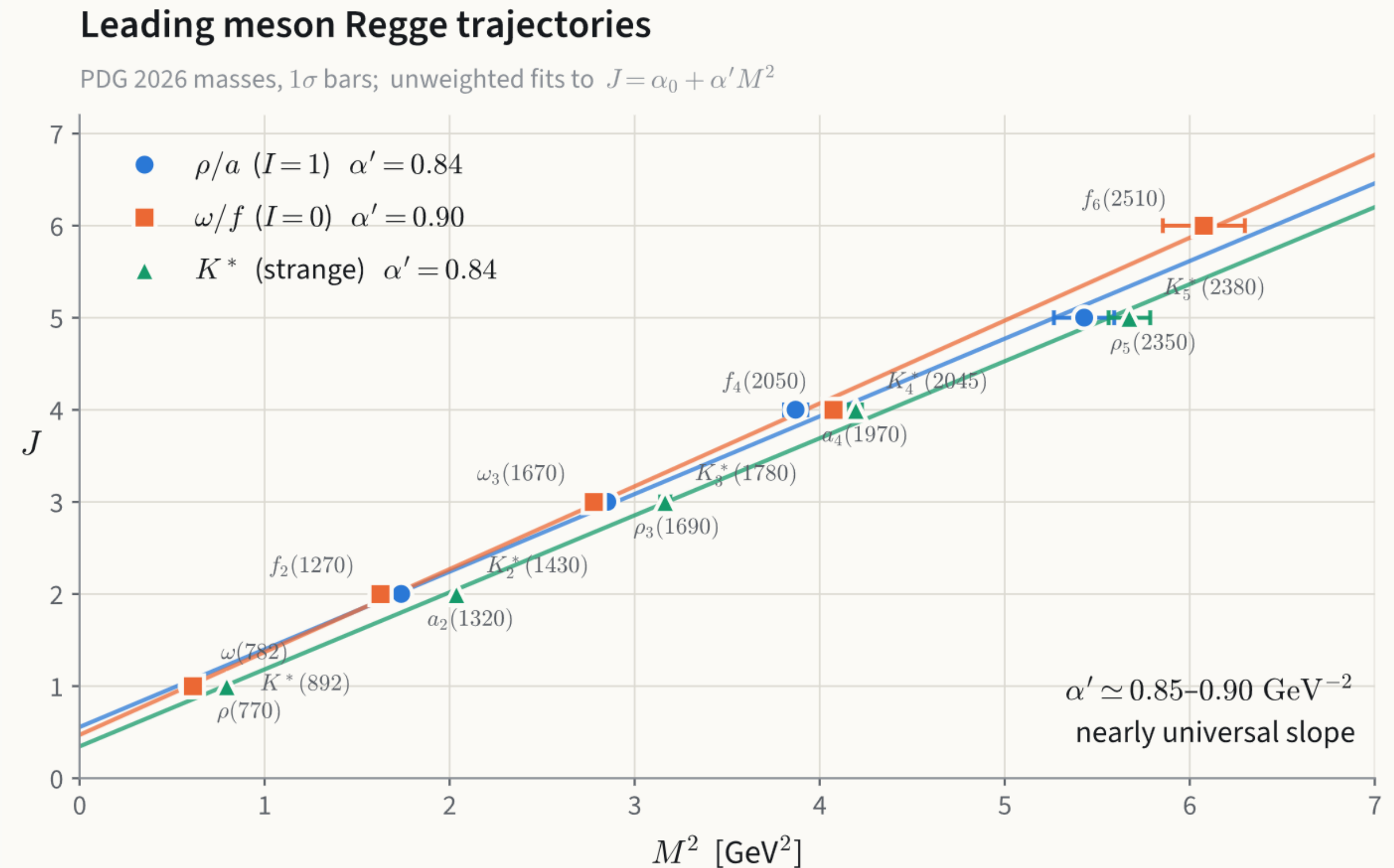
How does ACU restrict the space of possible interactions? → Bounds.

Space of S-matrices

How does ACU restrict the space of possible interactions? → Bounds.

A remarkable fact: *dynamics* emerges from that question.

“Chew Frautschi” plot



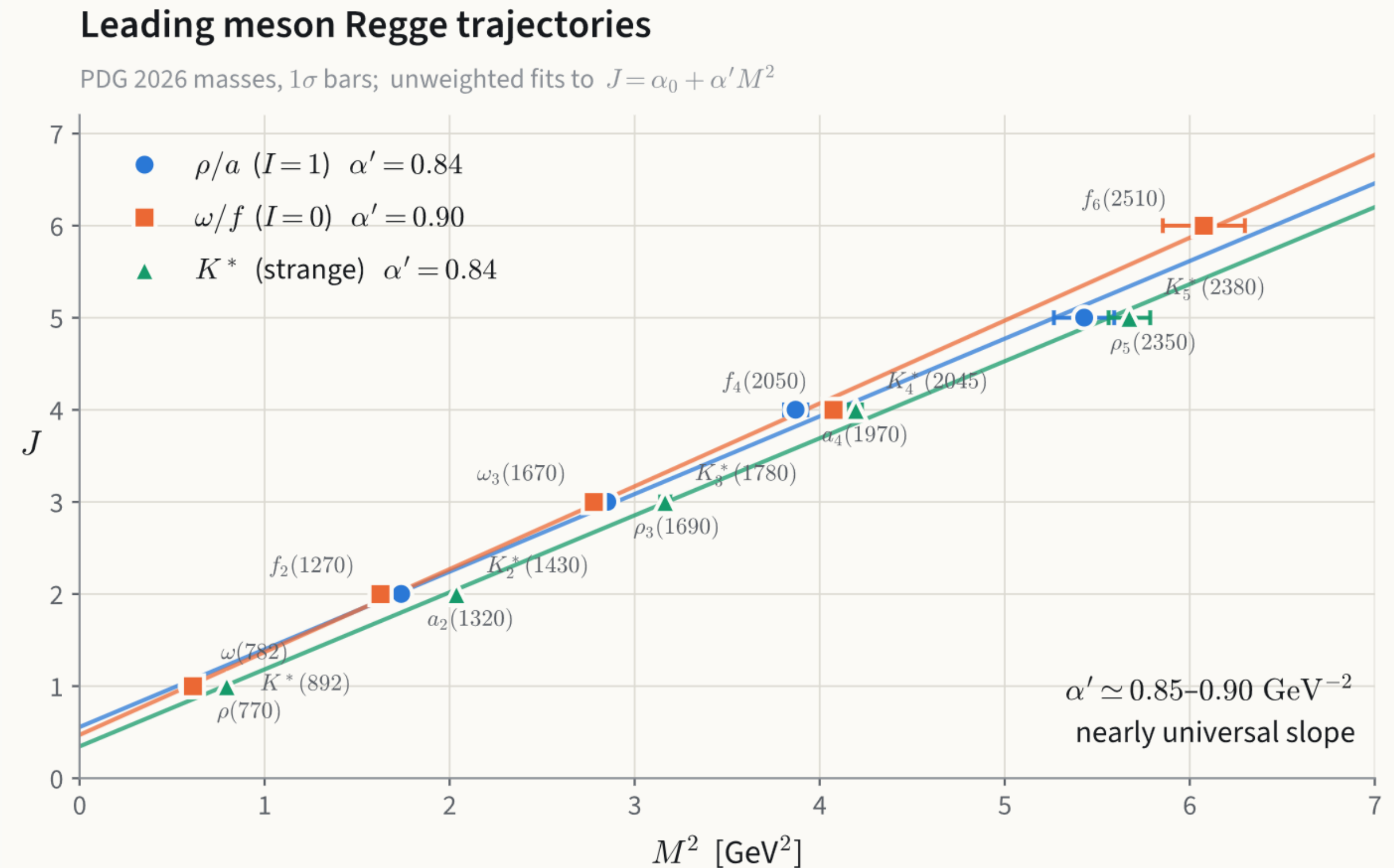
Space of S-matrices

How does ACU restrict the space of possible interactions? -> Bounds.

A remarkable fact: *dynamics* emerges from that question.

- **Gravity:** $T \simeq 8\pi G \frac{s^2}{-t}$ grows like s^2 . Eventually breaks unitarity.
- Tree-level unitarisation in string theory by infinite exchange of higher-spin modes

$$s^2 \rightarrow s^{2+\alpha't} \ll s^2 \text{ (at } t < 0)$$
- Causality constraints on graviton 3-pt coupling: make this a theorem at weak coupling ([Anna's talk](#); CEMZ)
- Recent new element: polynomial boundedness forces infinitely many trajectories ([Eckner Figueroa Metayer PT'25](#))
- *Non-perturbatively?*
- Similar questions in hadronic physics.
- Now is a perfect time to revisit these problems



part II

Bootstraps

Bound from inside and outside

- **Primal.** Build explicit amplitudes satisfying ACU
- **Dual.** Linear functionals that certify "no such amplitude exists": *aims to rigorous exclusion* (Guerrieri, Sever 2021; Anna's talk)
- **Positivity.** Dispersive/positivity bounds are the dual problem with unitarity linearised, $\text{Im } T \geq 0$. Full non-linear unitarity, $|S_J| \leq 1$, is strictly stronger — and non-perturbative. (Anna's talk)

Bound from inside and outside

- **Primal.** Build explicit amplitudes satisfying ACU
- **Dual.** Linear functionals that certify "no such amplitude exists": *aims to rigorous exclusion* (Guerrieri, Sever 2021; Anna's talk)
- **Positivity.** Dispersive/positivity bounds are the dual problem with unitarity linearised, $\text{Im } T \geq 0$. Full non-linear unitarity, $|S_J| \leq 1$, is strictly stronger — and non-perturbative. (Anna's talk)

The convex way

primal

- Simplify unitarity to $|S_J(s)|^2 \leq 1$ (★)
- Equivalent to a semi-definite problem $\alpha^2 + \beta^2 \leq 1 \iff \begin{pmatrix} 1 + \alpha & \beta \\ \beta & 1 - \alpha \end{pmatrix} \succeq 0$
- Use SDPB — good for arbitrary precision
- Impose (★) on an ansatz with A+C:
- $T_N(s, t) \sim \sum_{i,j,k}^N \alpha_{i,j,k} s^i t^j u^k \rightarrow$ gives explicit amplitudes with ACU and partial waves $S_J(s) = \sum \tilde{\alpha}_{i,j,k} s^i$
- Constraints discretized on grids, spin cut-off: $|S_J(s_i)|^2 \leq 1$ for $J = 0, \dots, J_{max}$ and $s_i \in (s_0, \dots, s_{grid})$

Numerical bootstrap: the convex way

Example

What is the space of analytic functions $f(z)$ with $|f(z)| \leq 1$ on $|z|=1$.

Choose ansatz: $f_N(z) = \sum_{i=0}^N \alpha_i z^i$

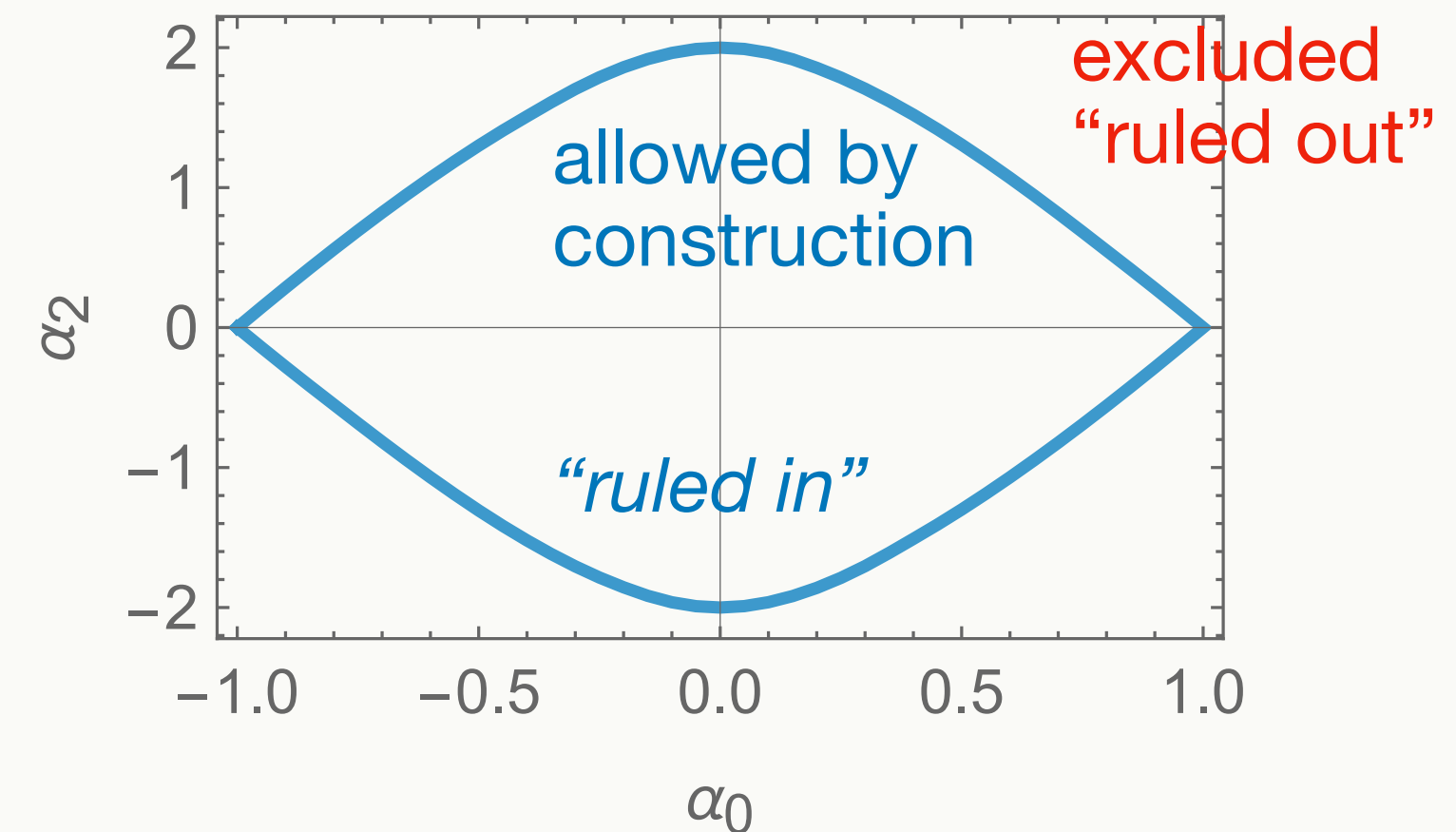
Impose constraints $|f(e^{i\theta_k})| \leq 1$, $\theta_k = \frac{2\pi}{k}$

Choose observables:

Max/Minimize e.g. α_0, α_2

Send N to infinity.

Result:

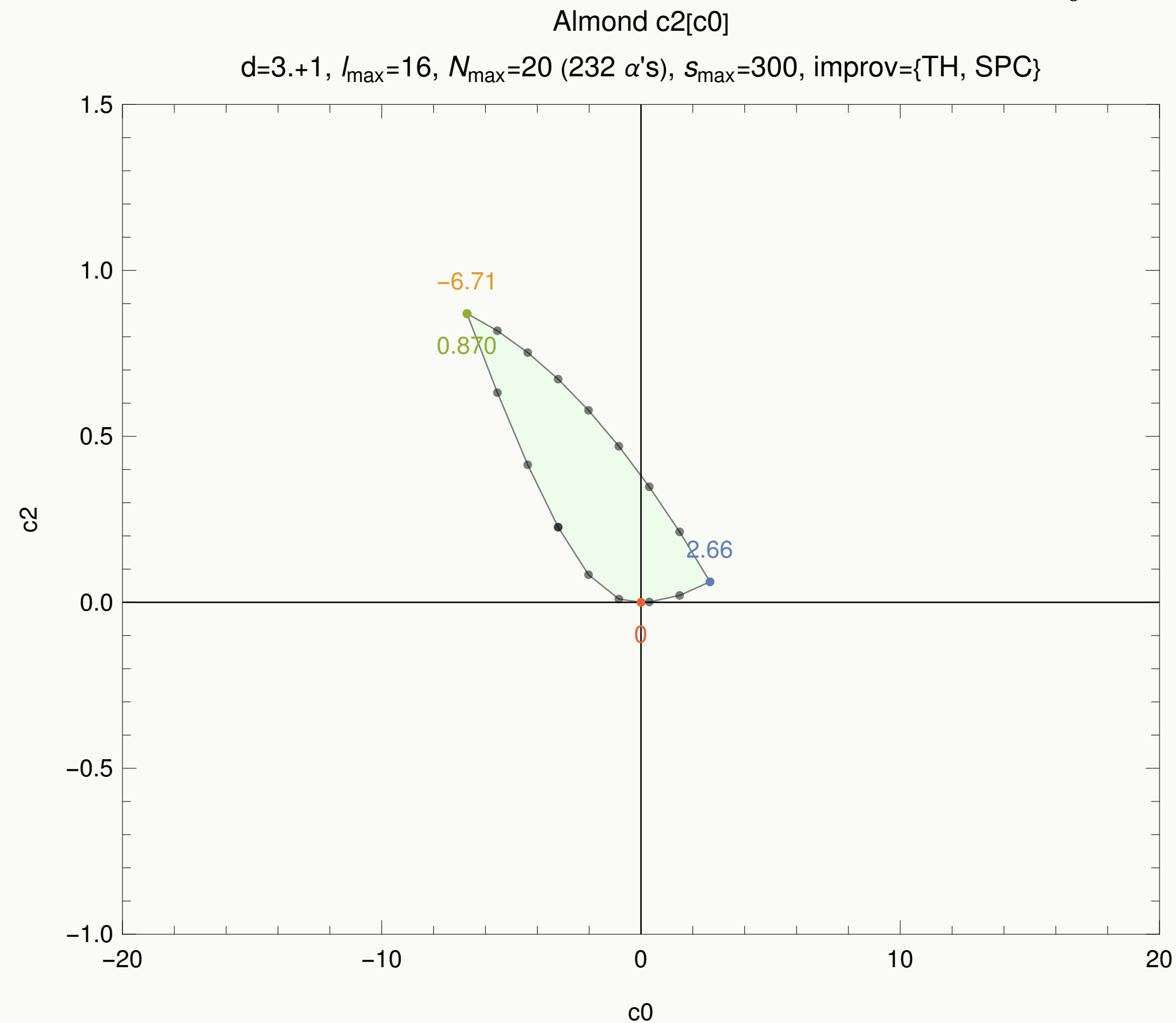


Simple example in d=4

- Massive, identical scalars. Simple observables : $T(s_0, t_0)$, $\partial_s^2 T(s, t_0) \big|_{s=s_0}$. Min / max values ?

Simple example in d=4

- Massive, identical scalars. Simple observables : $T(s_0, t_0), \partial_s^2 T(s, t_0) \big|_{s=s_0}$. Min / max values ?



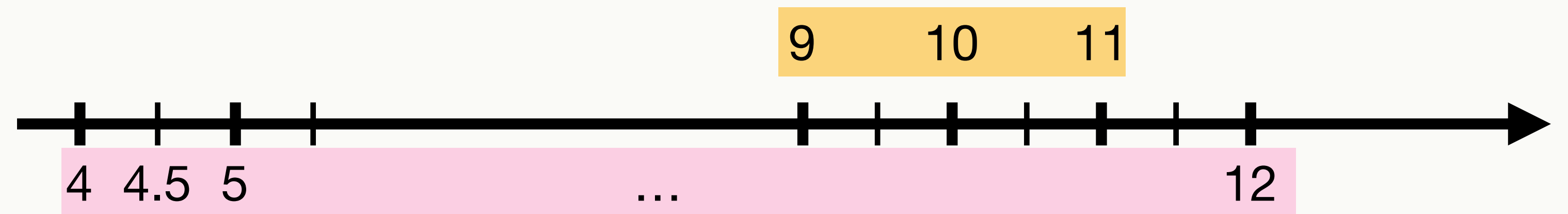
part III

Two applications:

quantum gravity

gapped QFT

Guerrieri Penedones Vieira, +Murali, 2021, 2022



Gumus Metayer **PT** 2026

Gravity, non perturbatively

- **Setting.** *Maximal supersymmetry, $d=9,10,11$:* every $2 \rightarrow 2$ amplitude of the graviton multiplet is a fixed polarisation factor times one function $T(s,t,u)$.

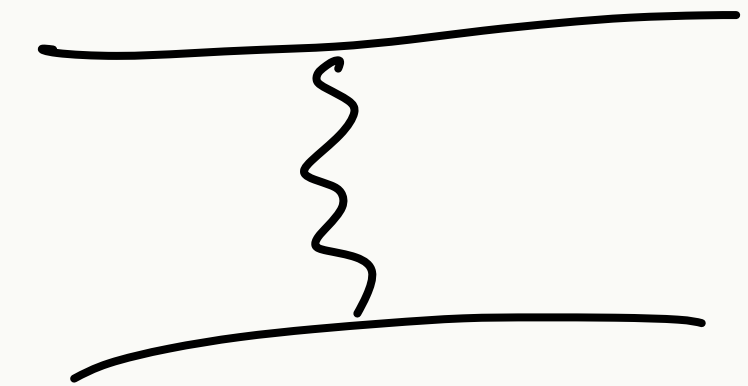
- **Low energy is known:**

$$T = 8\pi G_N s^4 \left(\frac{1}{stu} + \alpha \ell_P^6 + \dots \right)$$

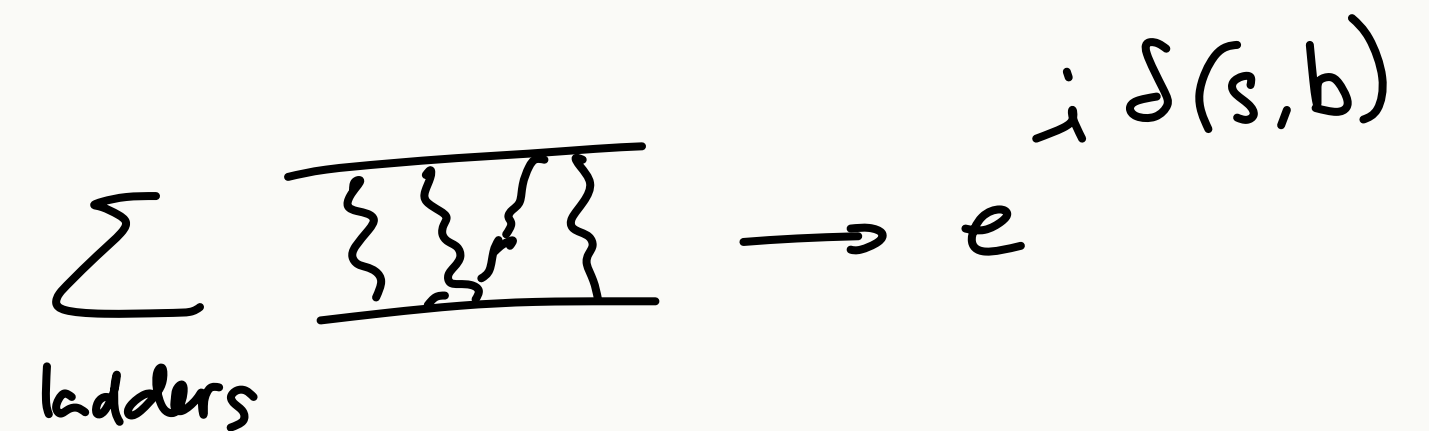
- the graviton pole, then α — the R^4 coefficient. Known in string/M-theory.
- *Which α admit any UV completion?*

QG amplitudes, 3 regimes:

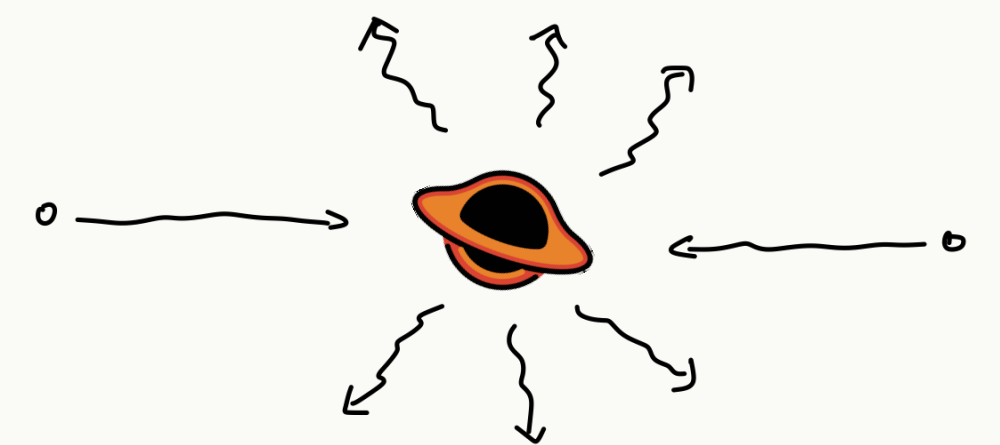
Born



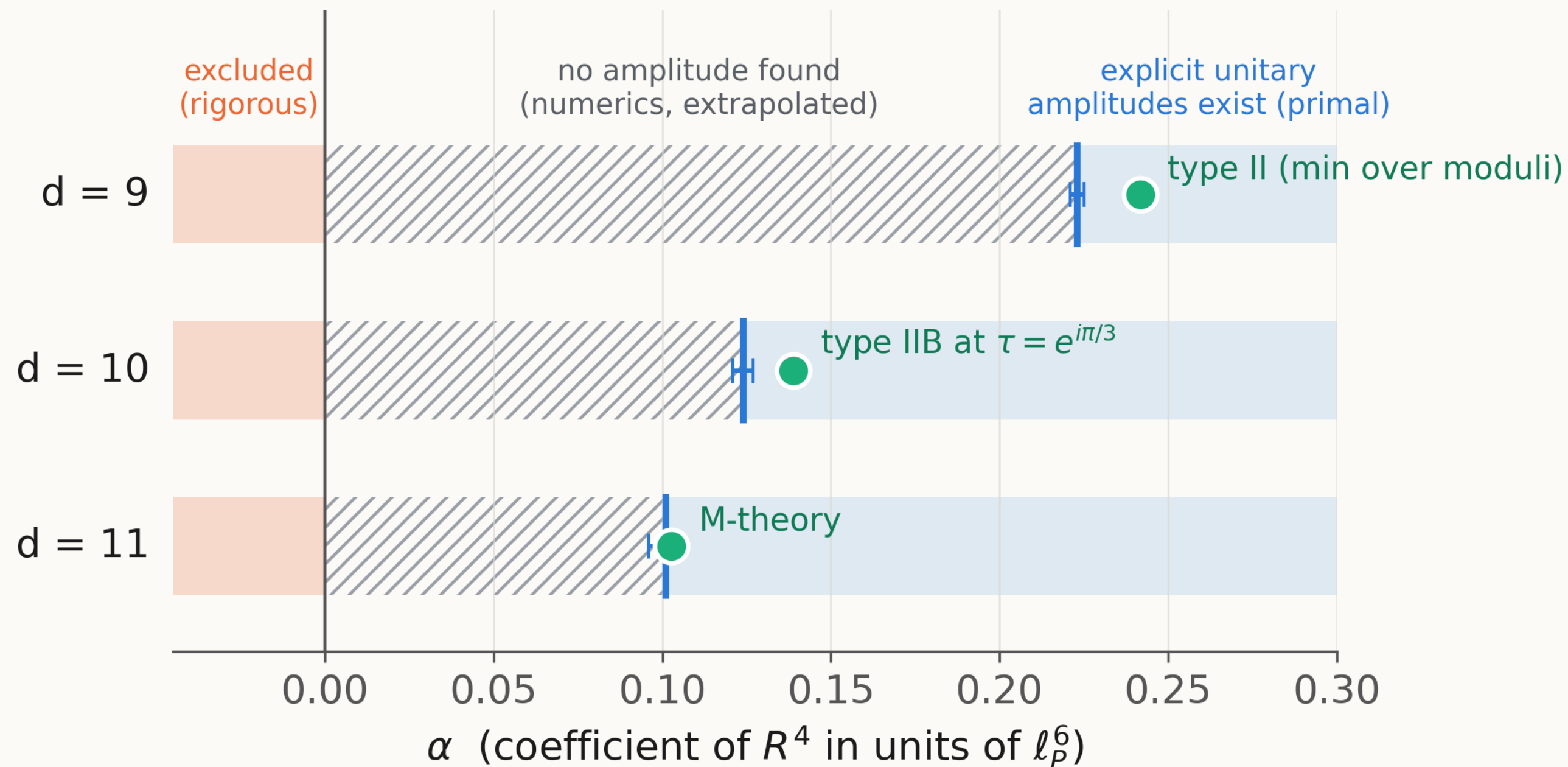
Eikonal



Strong gravity



Where is string/M theory?



Bootstrap lower bound on the R^4 coefficient vs. the actual string/M-theory value minimised over moduli. Guerrieri–Murali–Penedones–Vieira '22

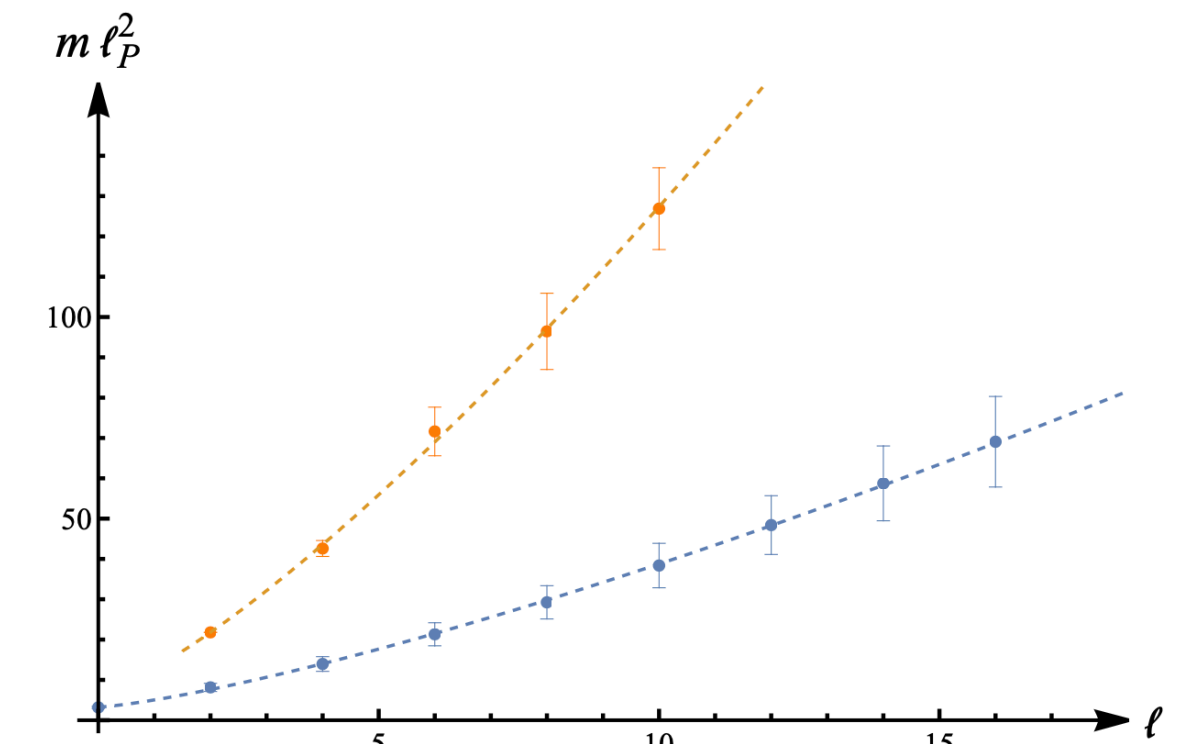
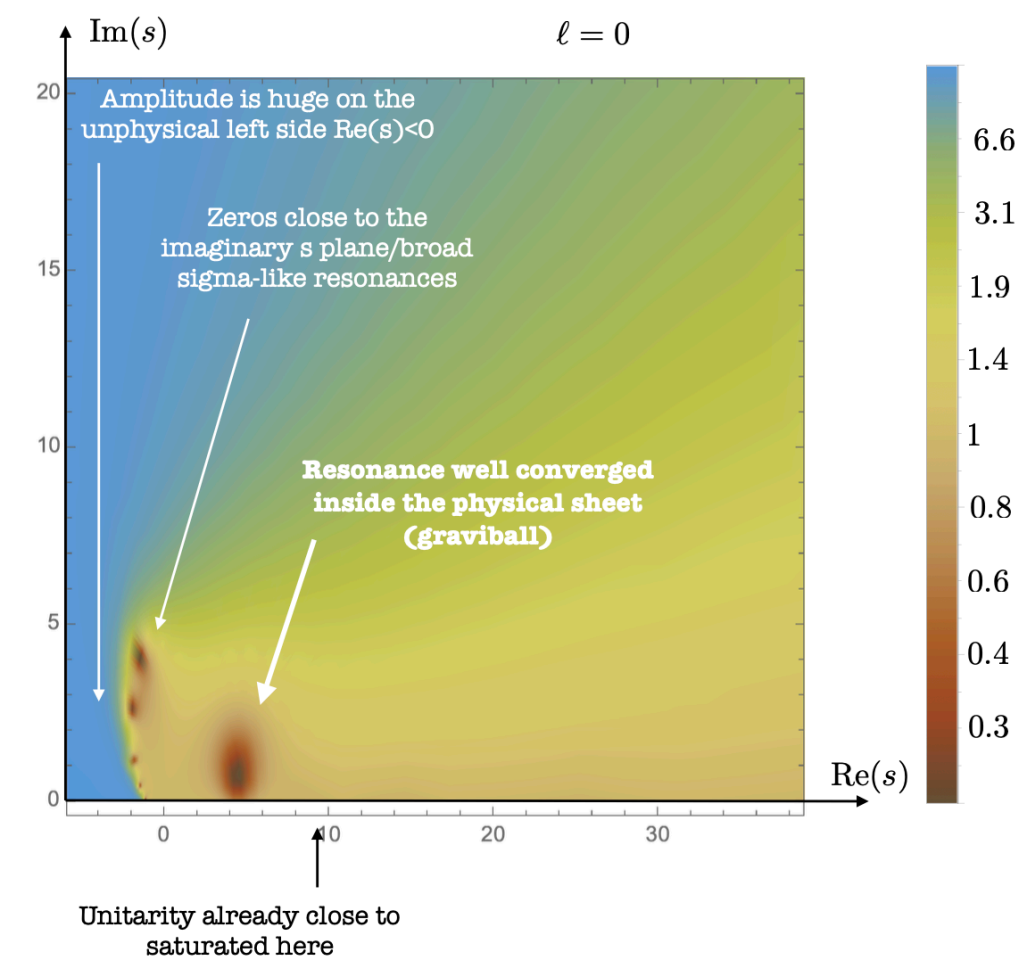
- Linearised unitarity : $\alpha \geq 0$. Full non-linear unitarity: **string and M-theory sit at the boundary**
- In 11d the gap between the bound and M-theory is $\sim 2\%$; the residual gap $\leftrightarrow$ inelasticity ? *invisible to an elastic ansatz.*
- **The extremal amplitudes:** strongly coupled completion with infinite tower of resonances modes.

... A strongly coupled string theory amplitude?

Emergent dynamics

Strongly coupled resonances

- From extremal amplitude, extract physical data:
- Resonances = zero of partial waves in complex plane $|S_J(s^*)| = 0$
- Scattering lengths, phase shifts, etc.
- UV: not completely controlled.



Next steps :

other d

include/obtain black-hole entropic suppression terms $S_J(s) \sim e^{-\delta_{BH}(s,J)}$? Chaotic phase shifts ?

control gravitational radiation

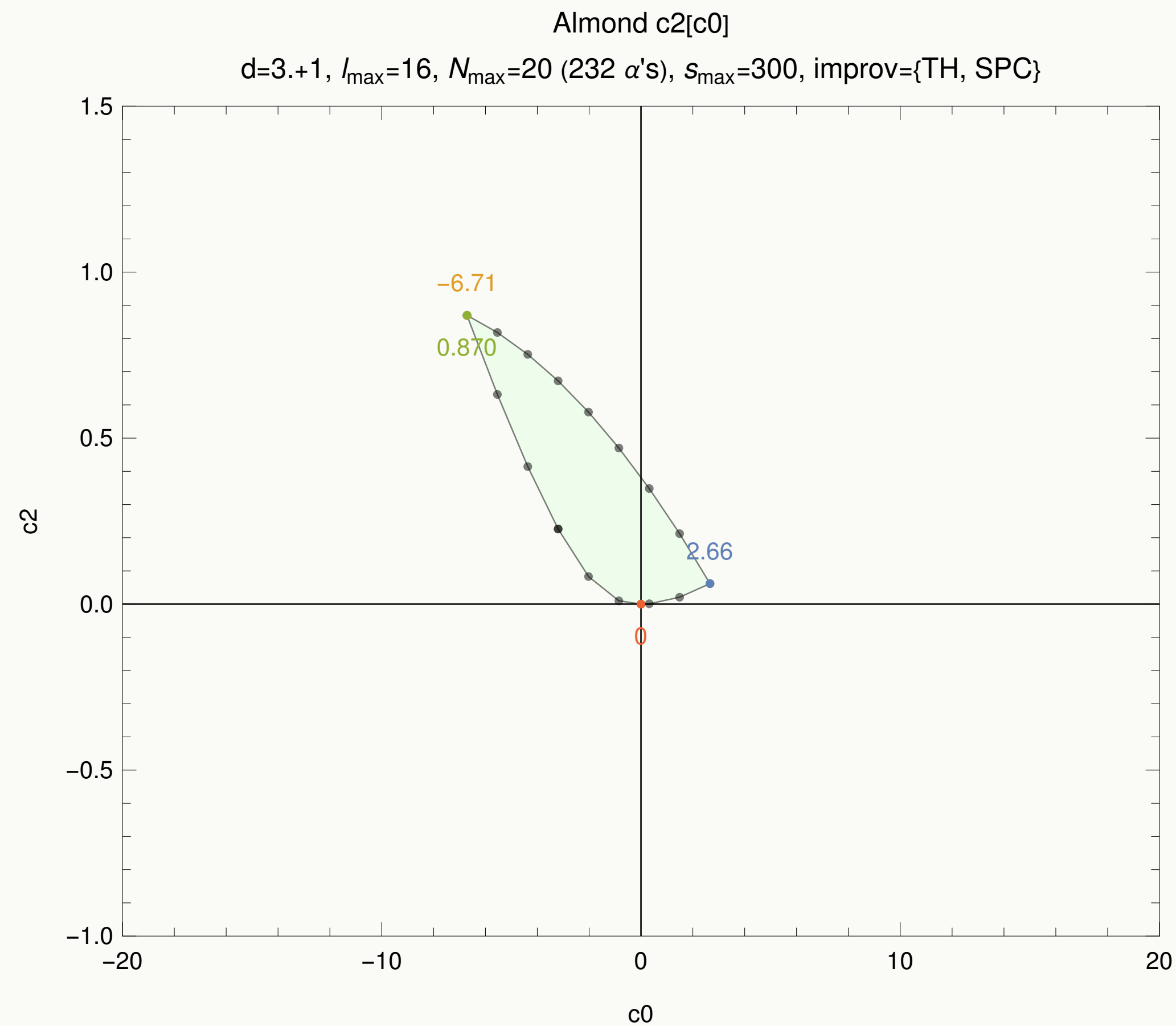
Beyond susy: scattering across dimensions OUR WORK

identical scalars

Beyond susy: scattering across dimensions OUR WORK

identical scalars

Remember:

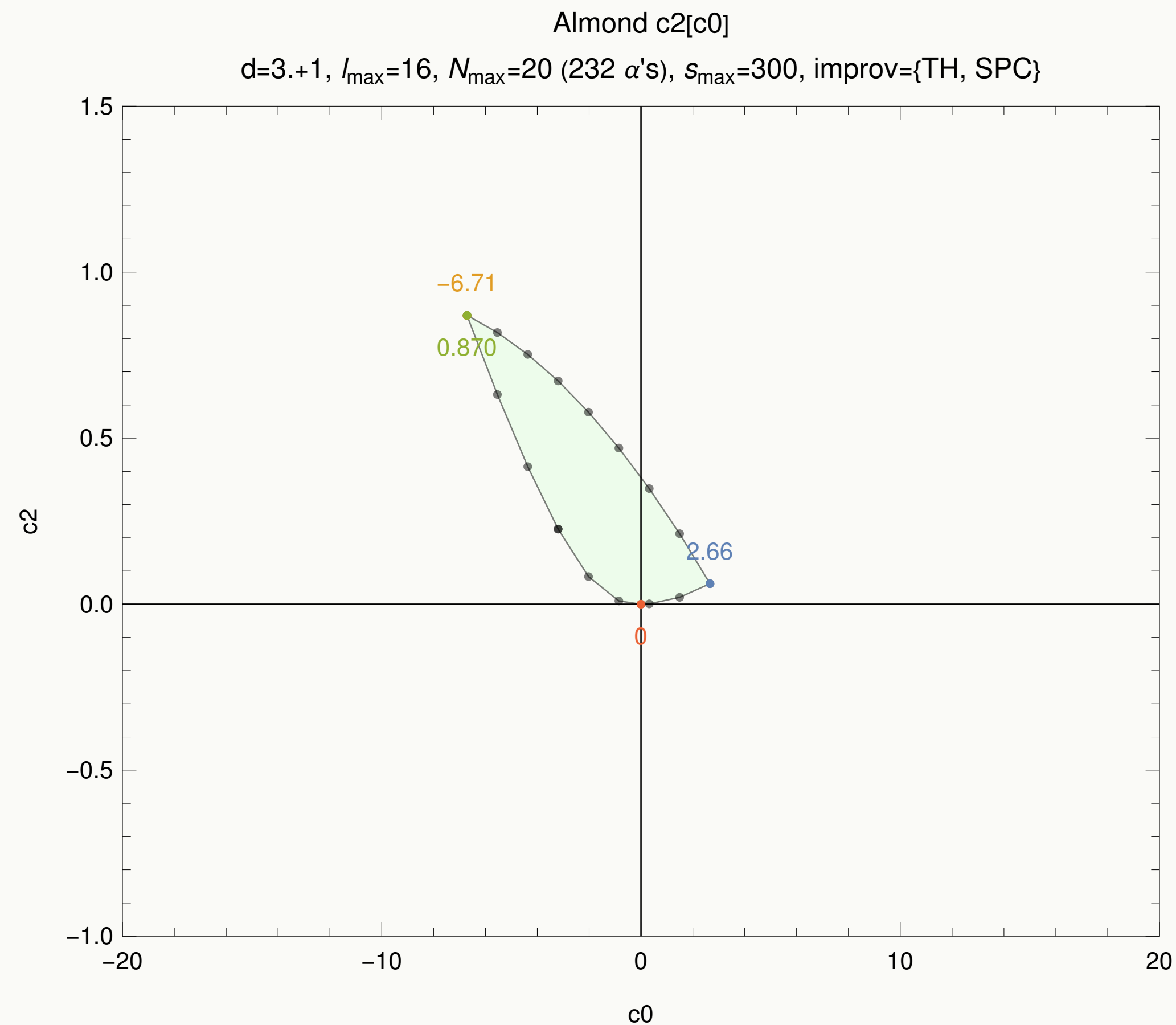


Beyond susy: scattering across dimensions OUR WORK

identical scalars

Remember:

d appears everywhere
analytically



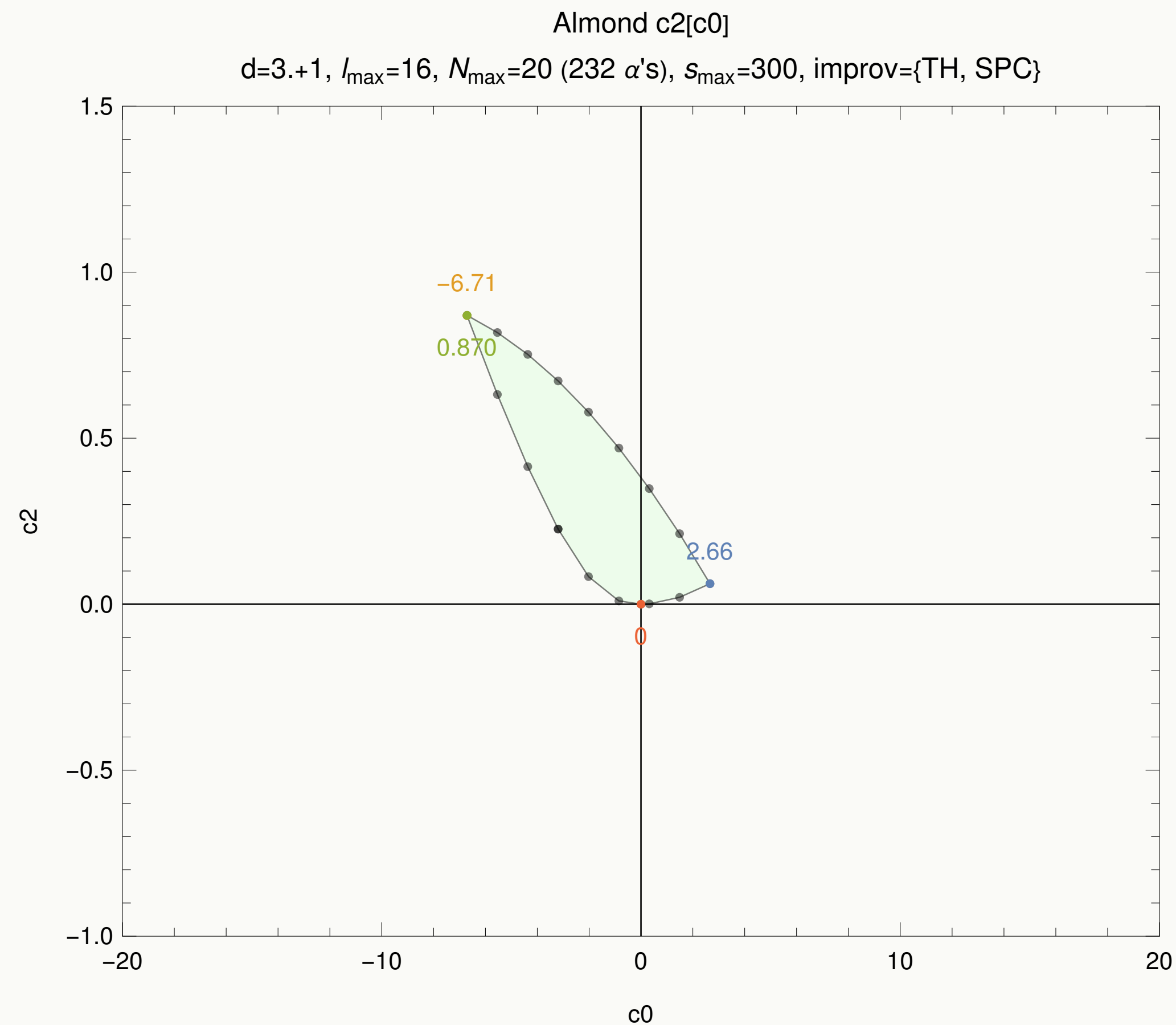
Beyond susy: scattering across dimensions OUR WORK

identical scalars

Remember:

d appears everywhere
analytically

explore space of d 's !



Beyond susy: scattering across dimensions OUR WORK

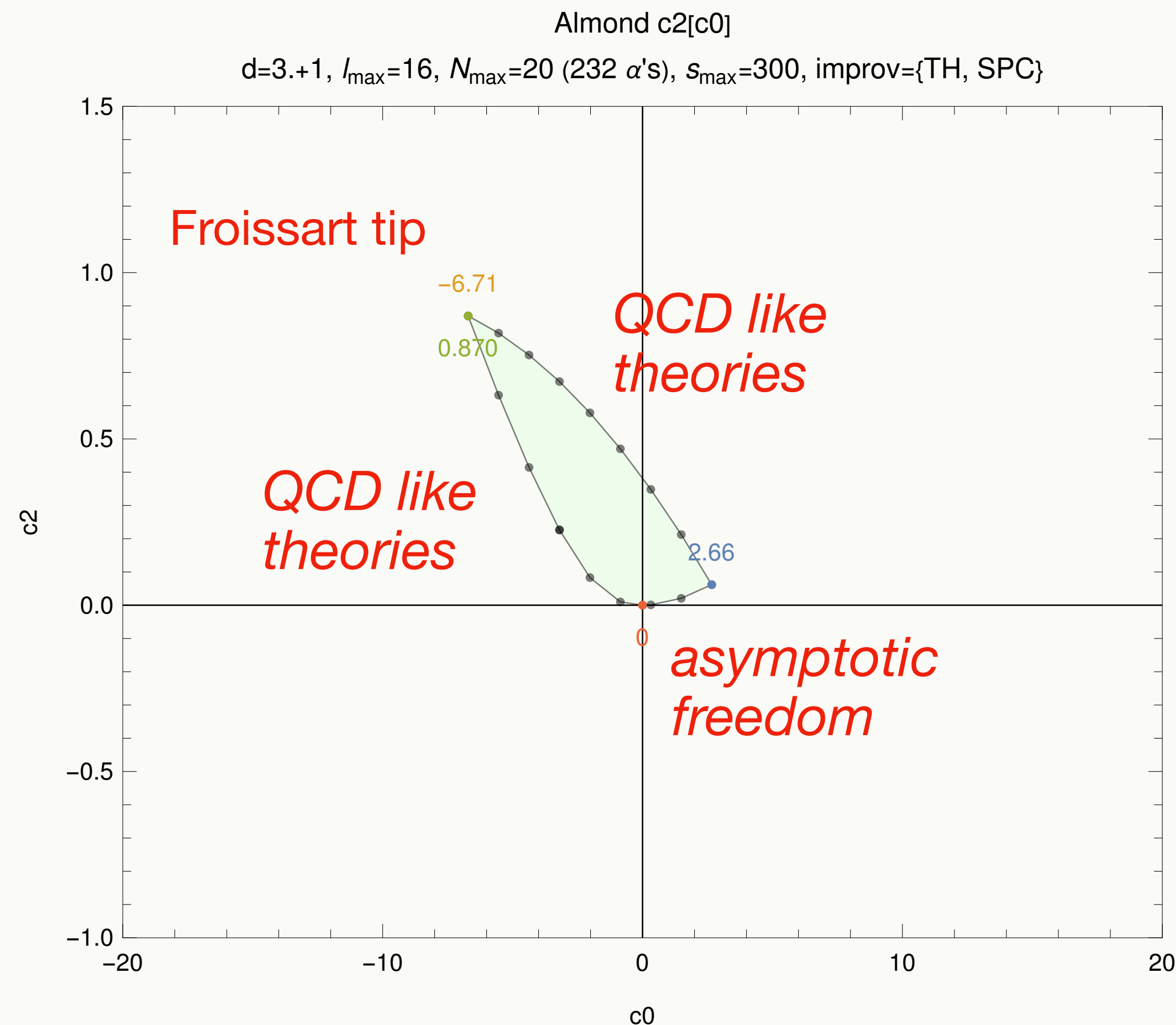
identical scalars

Remember:

d appears everywhere
analytically

explore space of d 's !

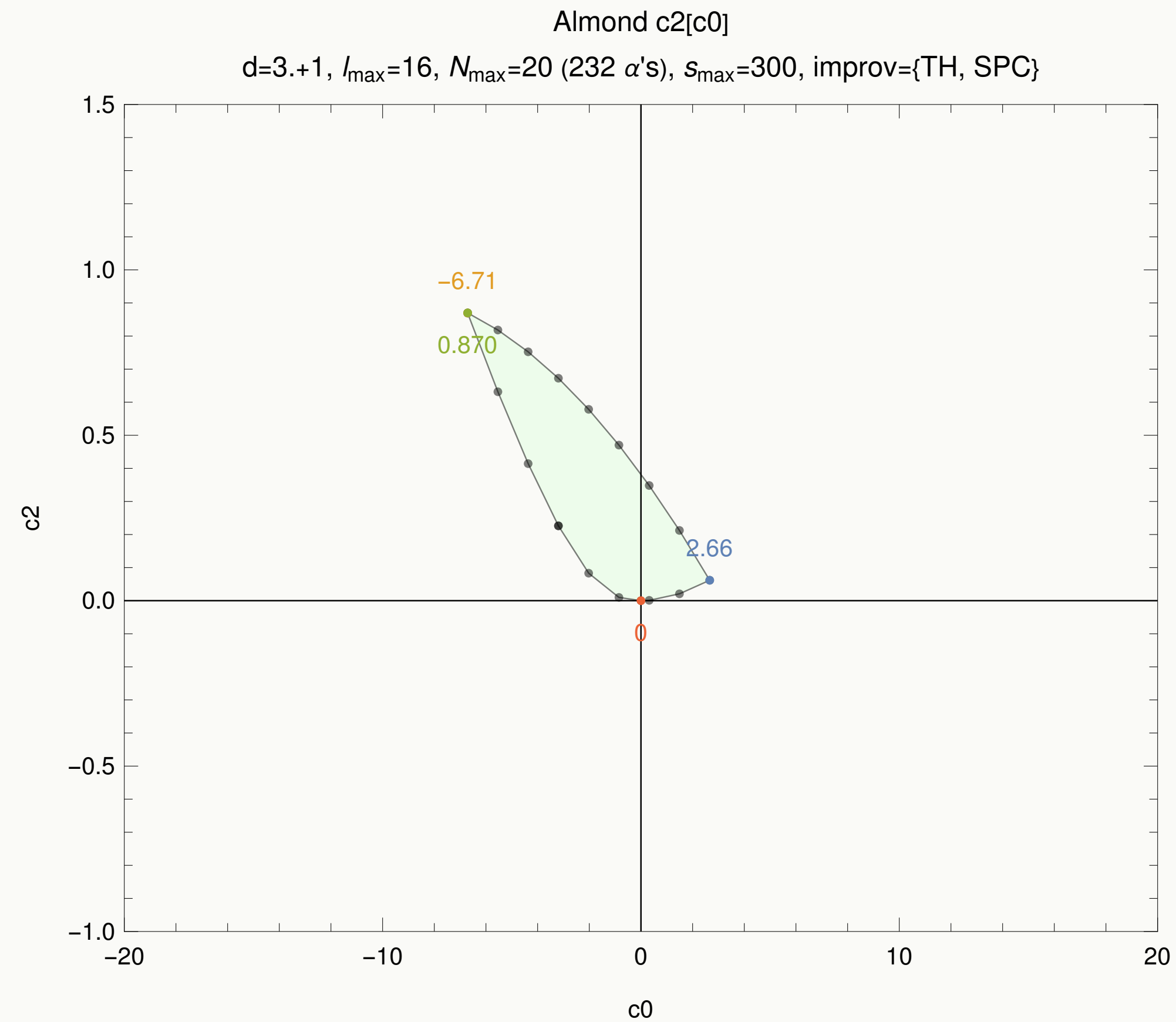
what physics to expect ?



*Do local, non-gravitational
QFTs exist in higher d ?*

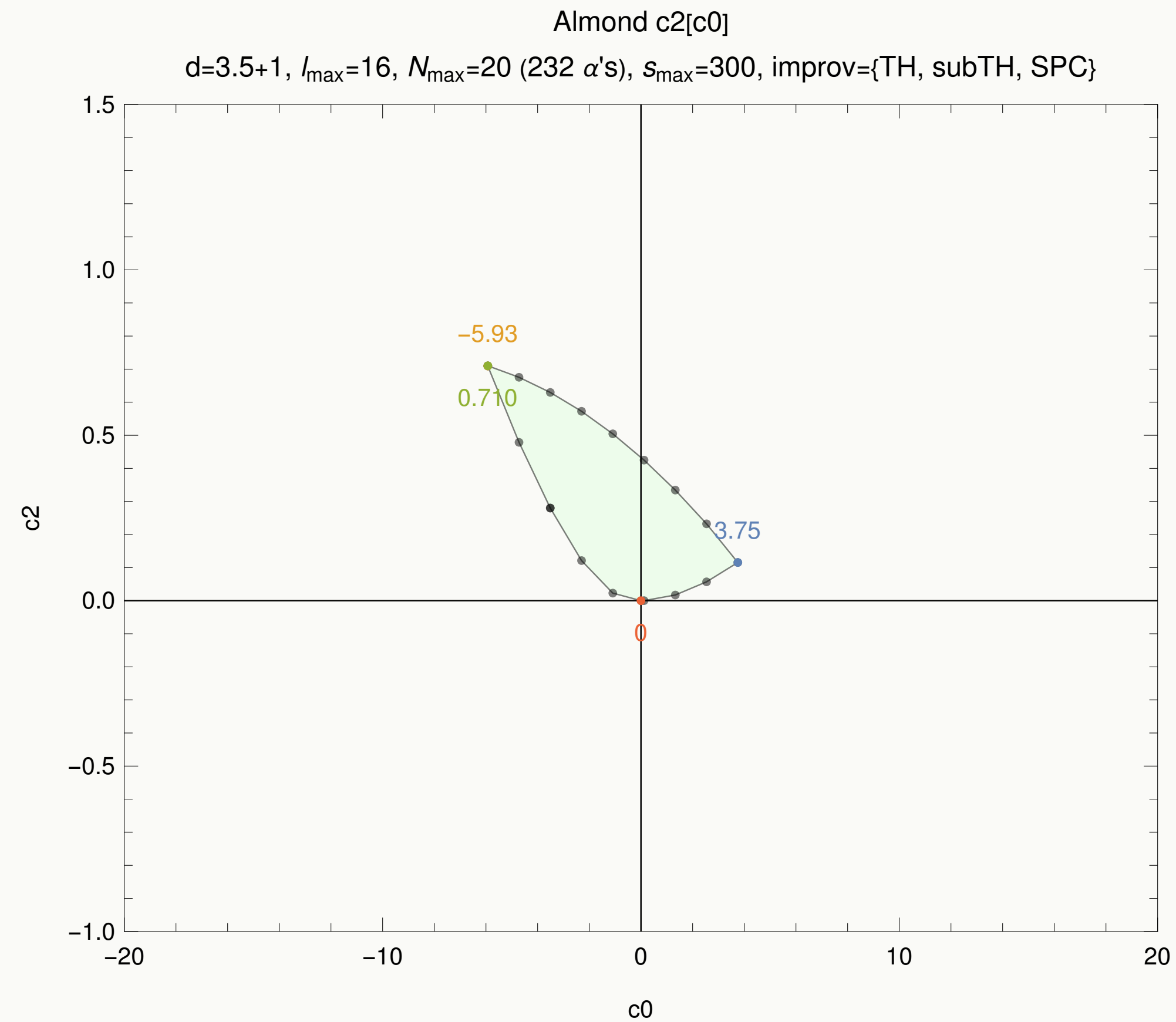
Beyond susy: scattering across dimensions

d=4

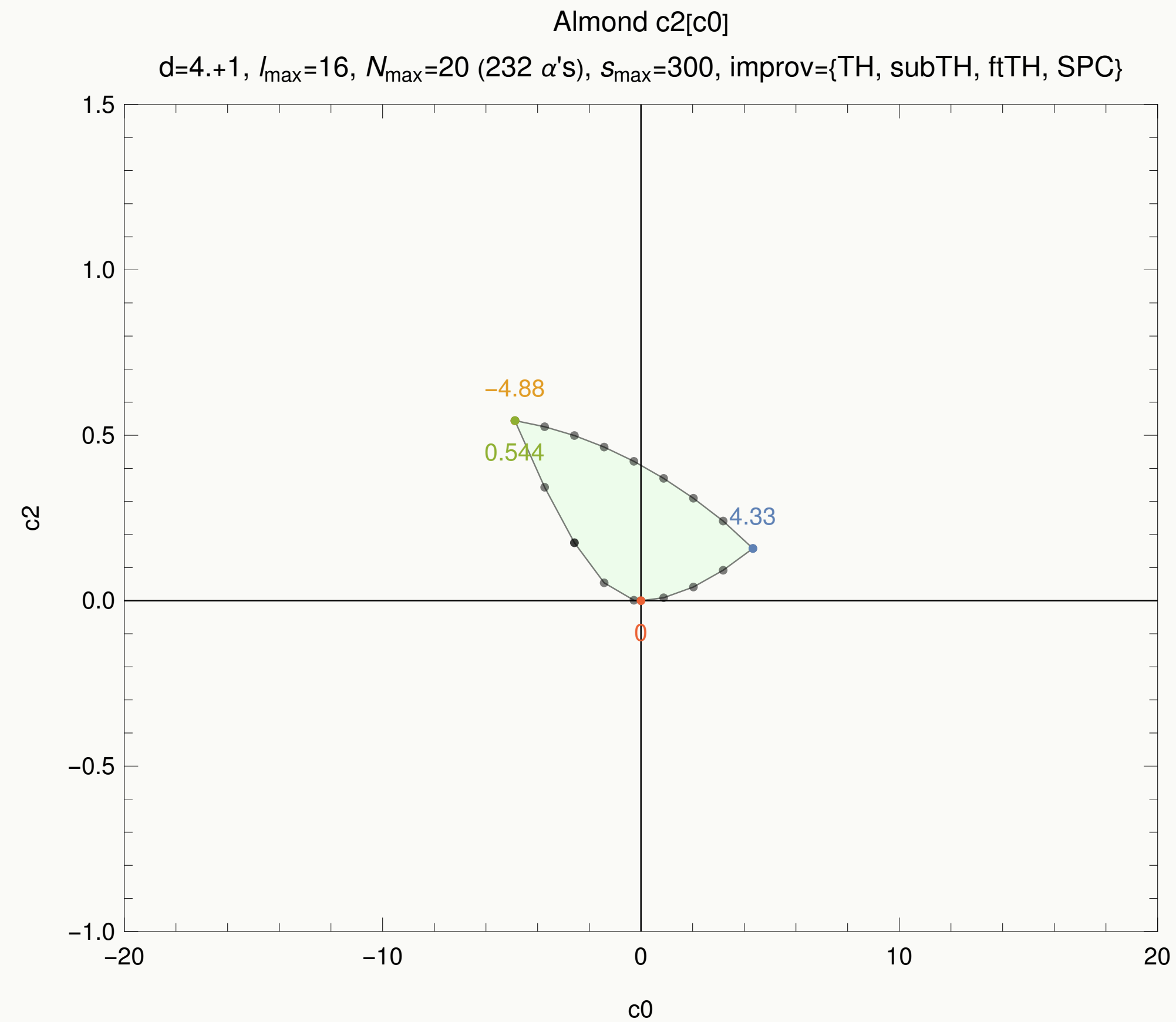


Beyond susy: scattering across dimensions

d=4.5

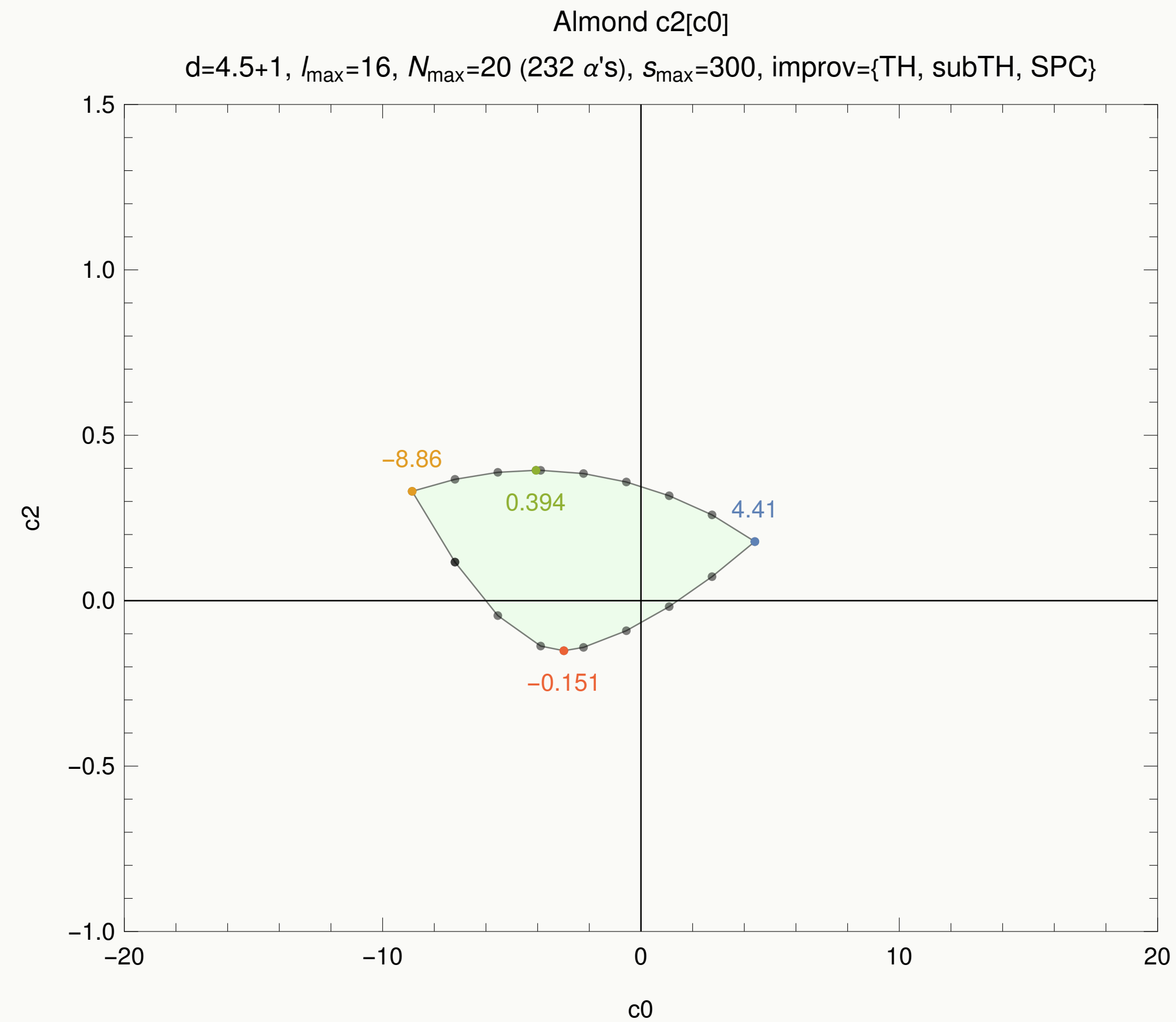


Beyond susy: scattering across dimensions



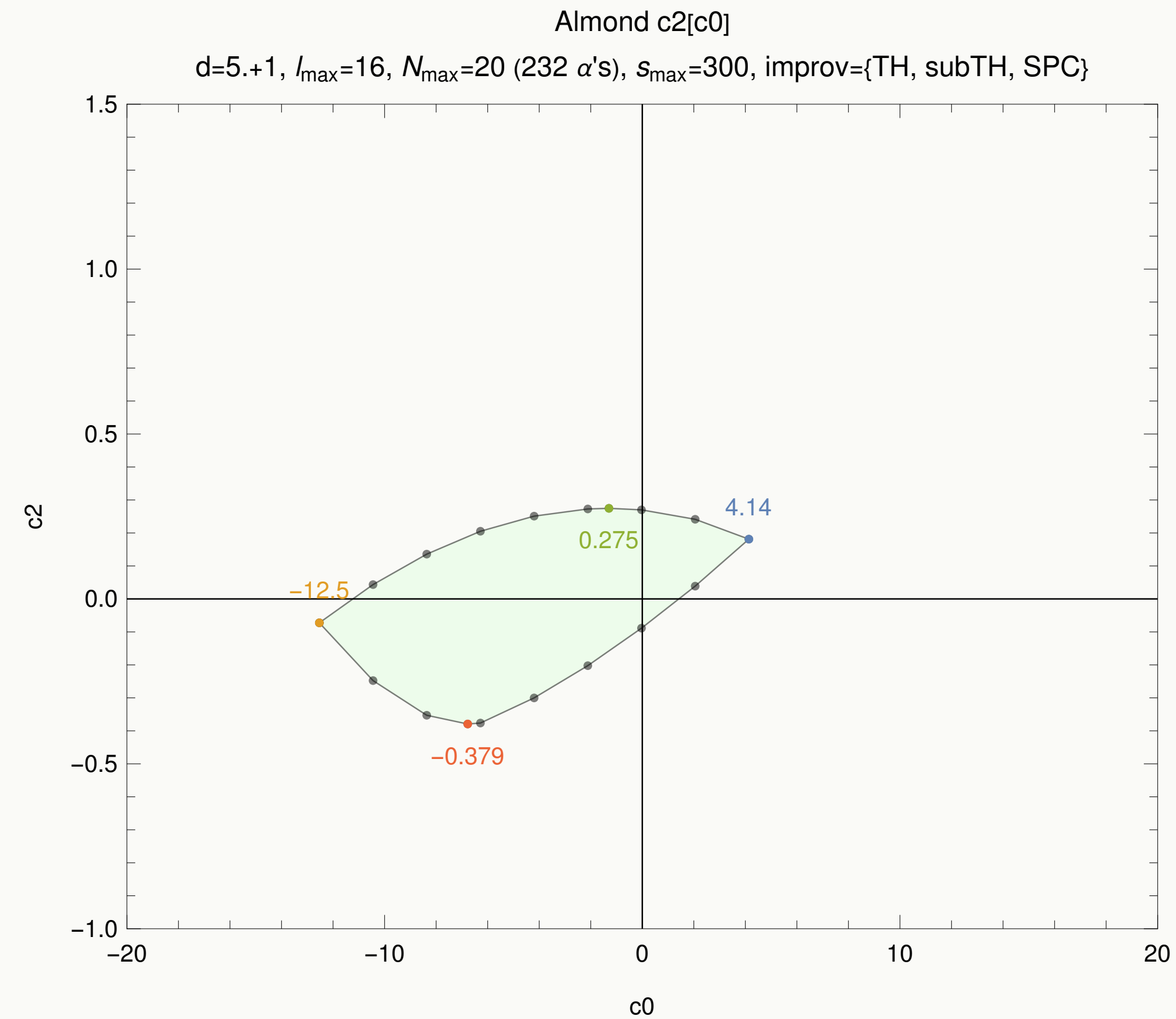
Beyond susy: scattering across dimensions

$d=5.5$



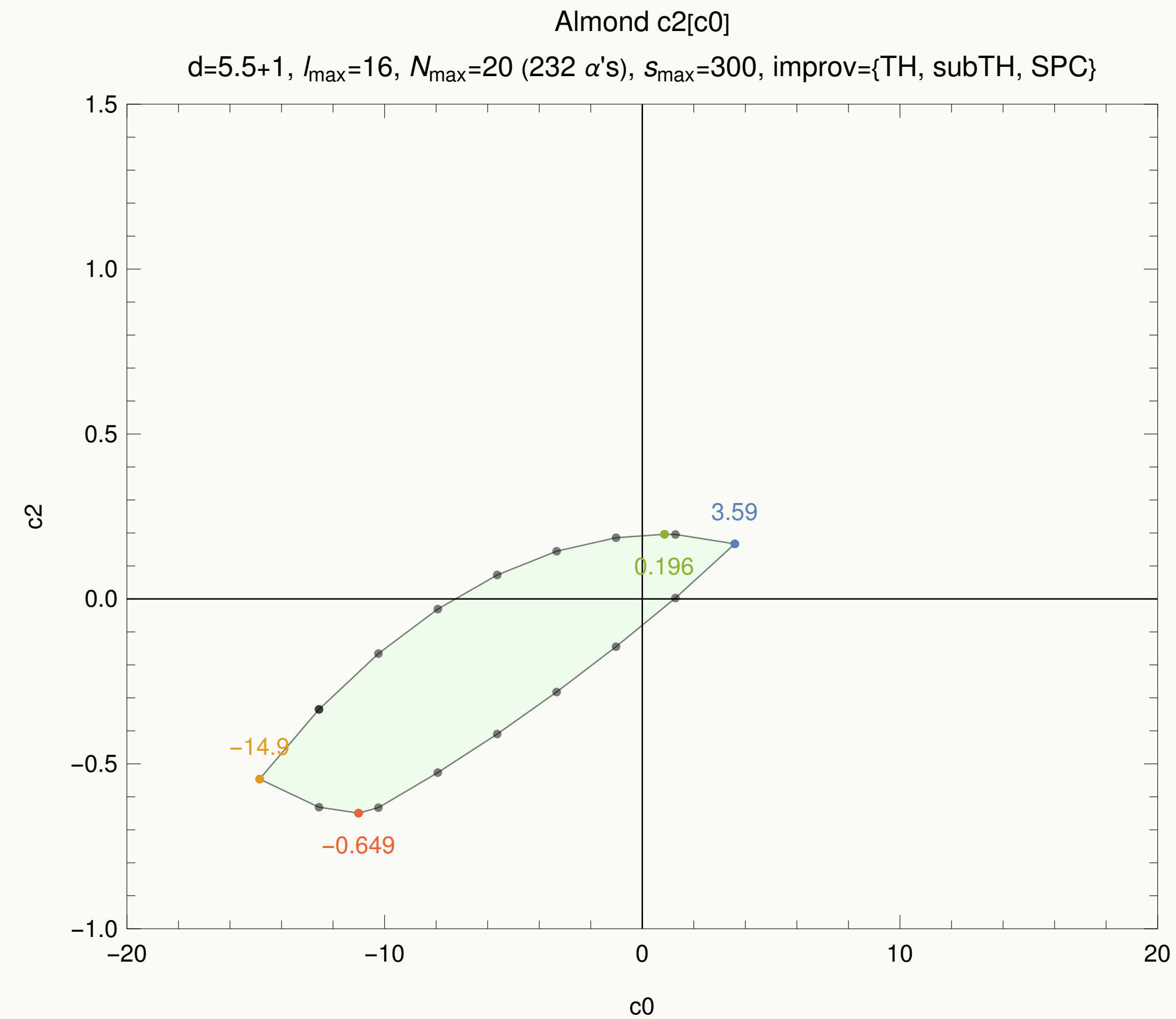
Beyond susy: scattering across dimensions

d=6



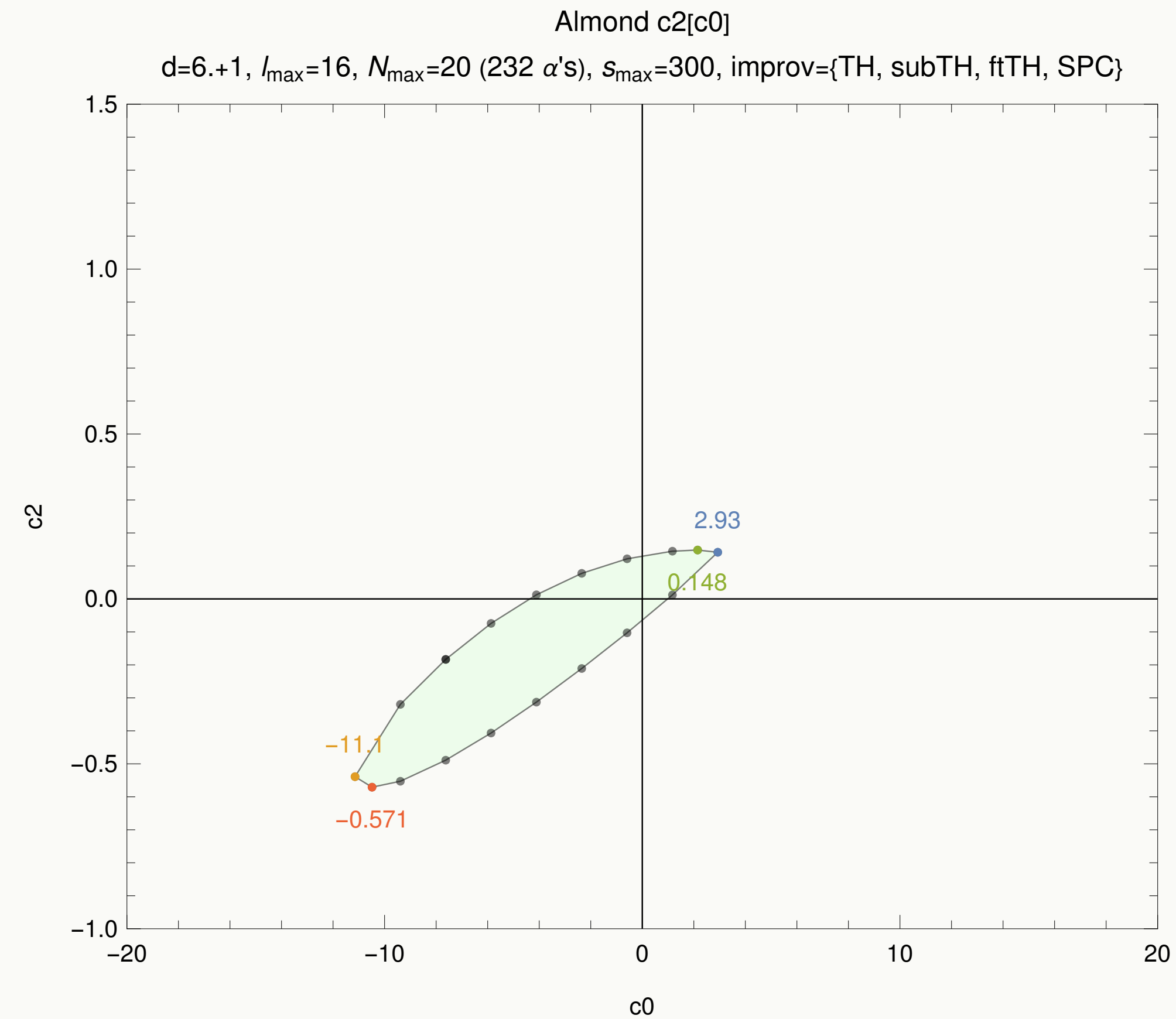
Beyond susy: scattering across dimensions

d=6.5



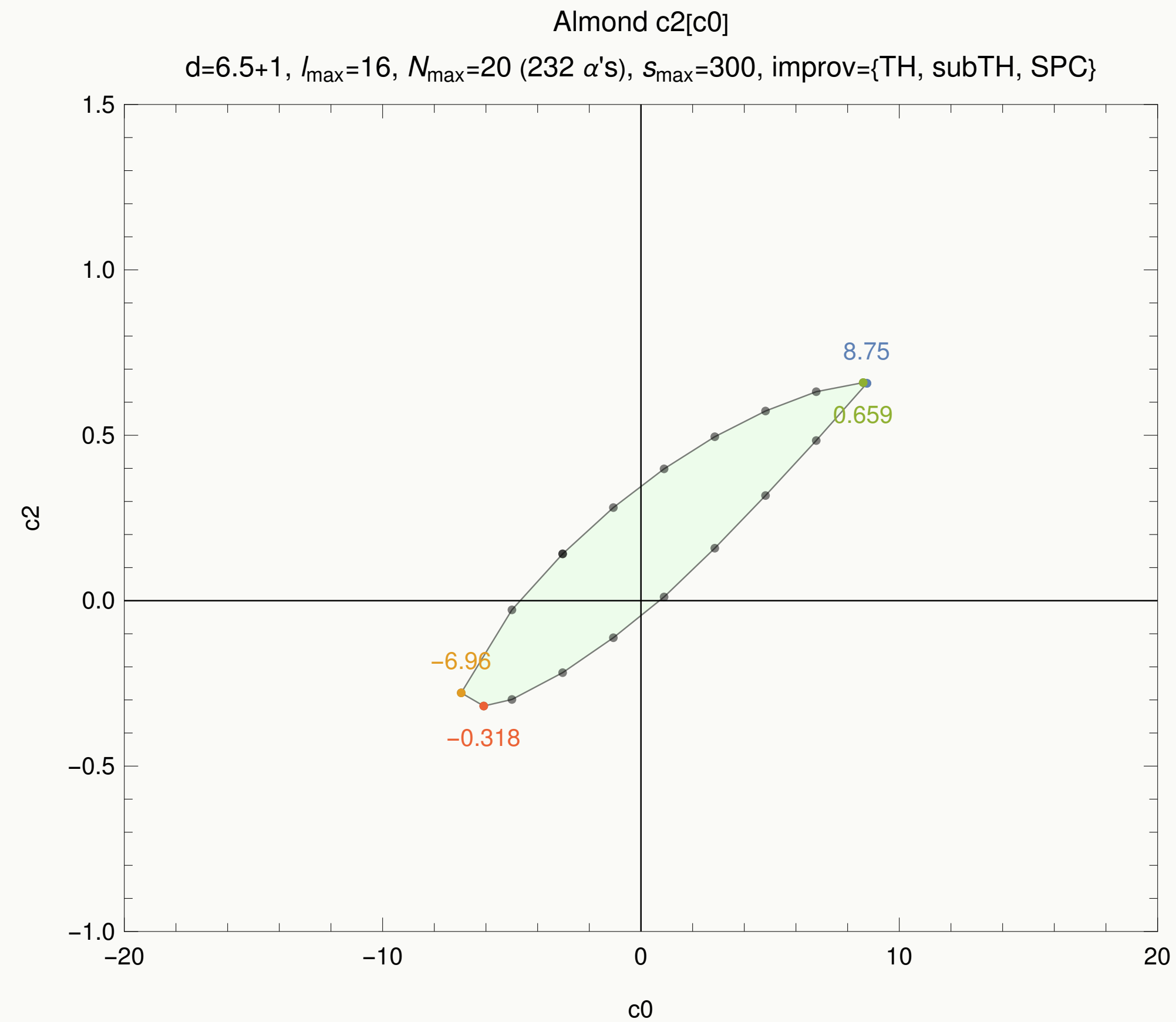
Beyond susy: scattering across dimensions

$d=7$



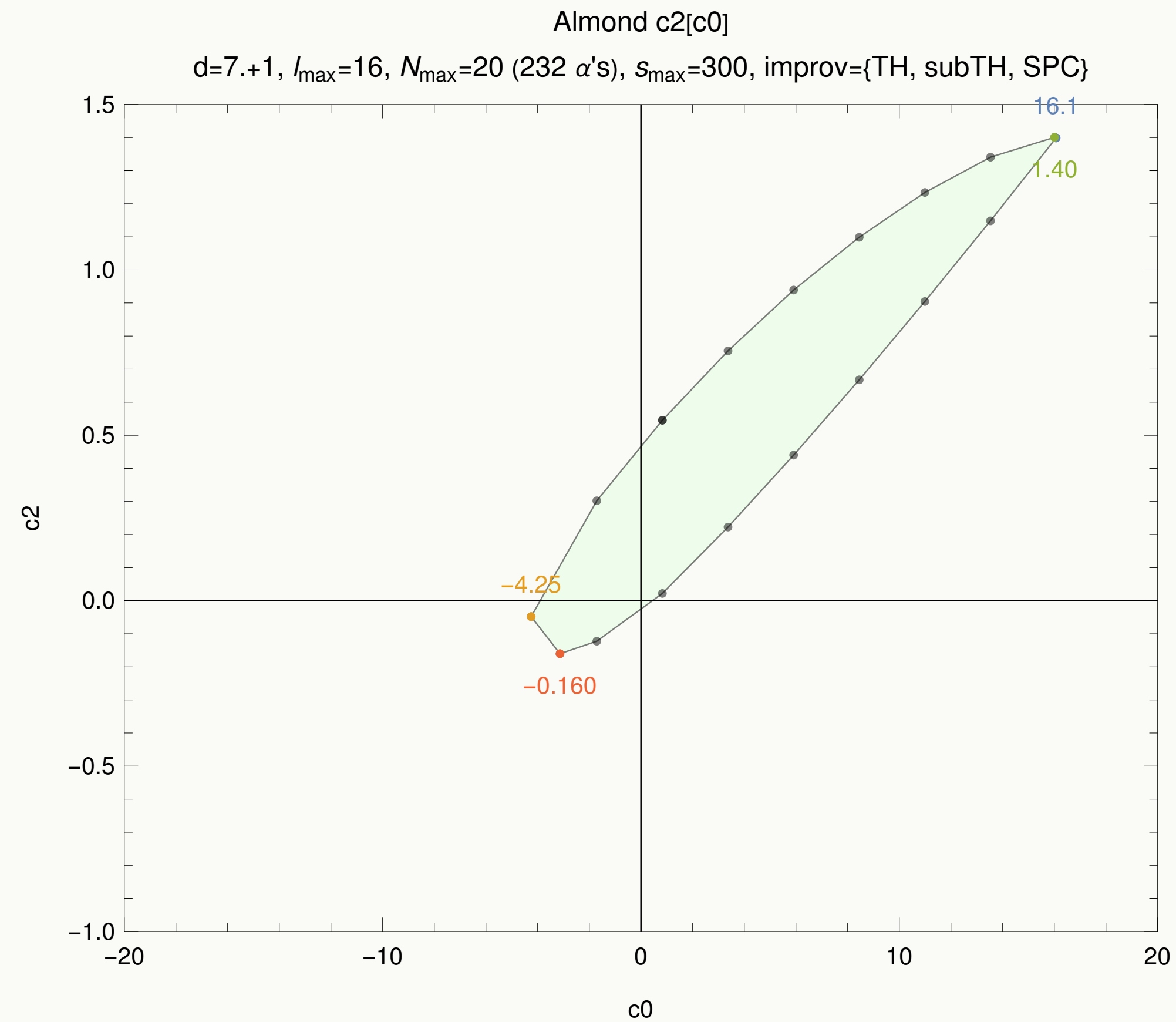
Beyond susy: scattering across dimensions

$d=7.5$



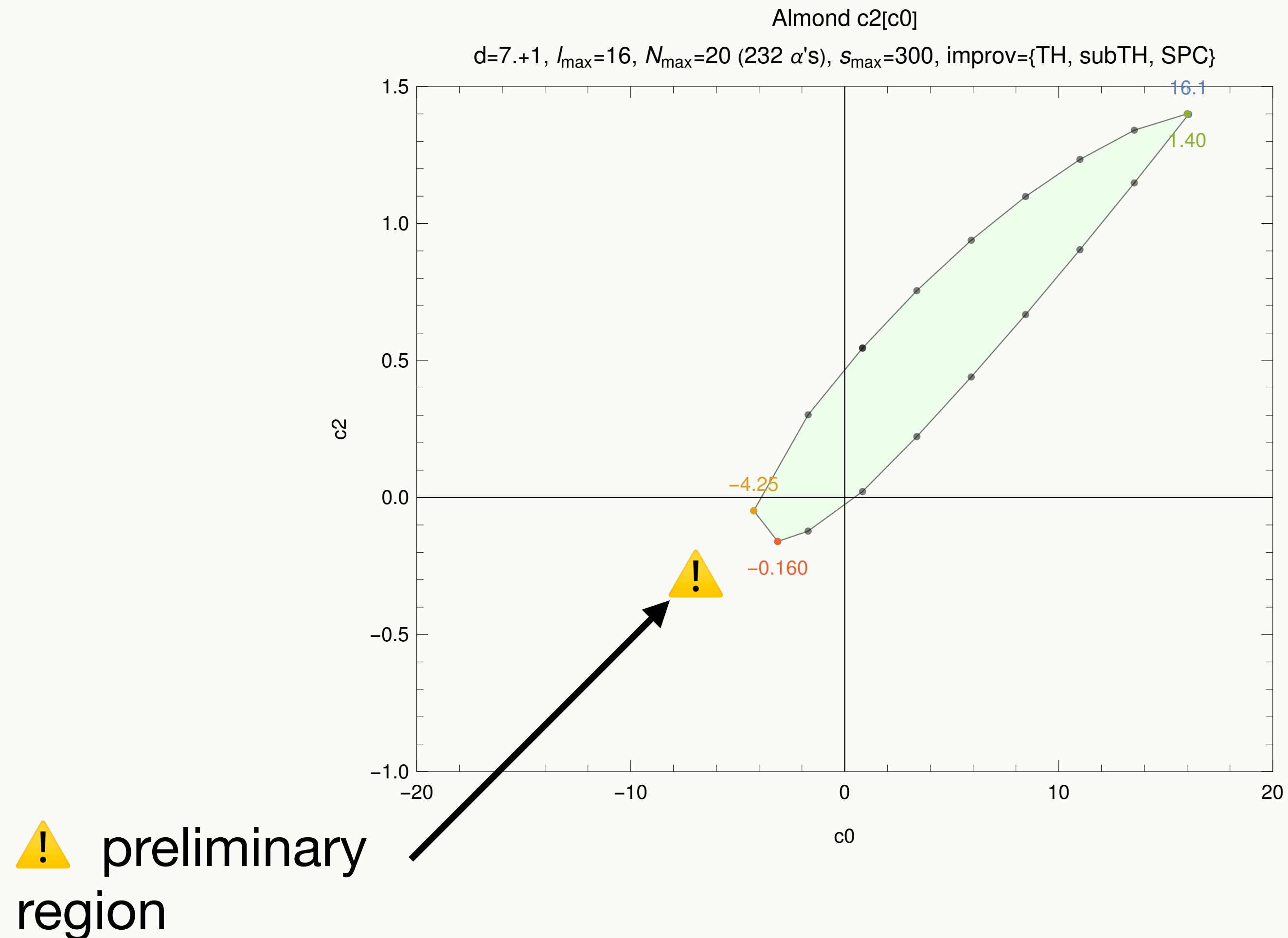
Beyond susy: scattering across dimensions

d=8

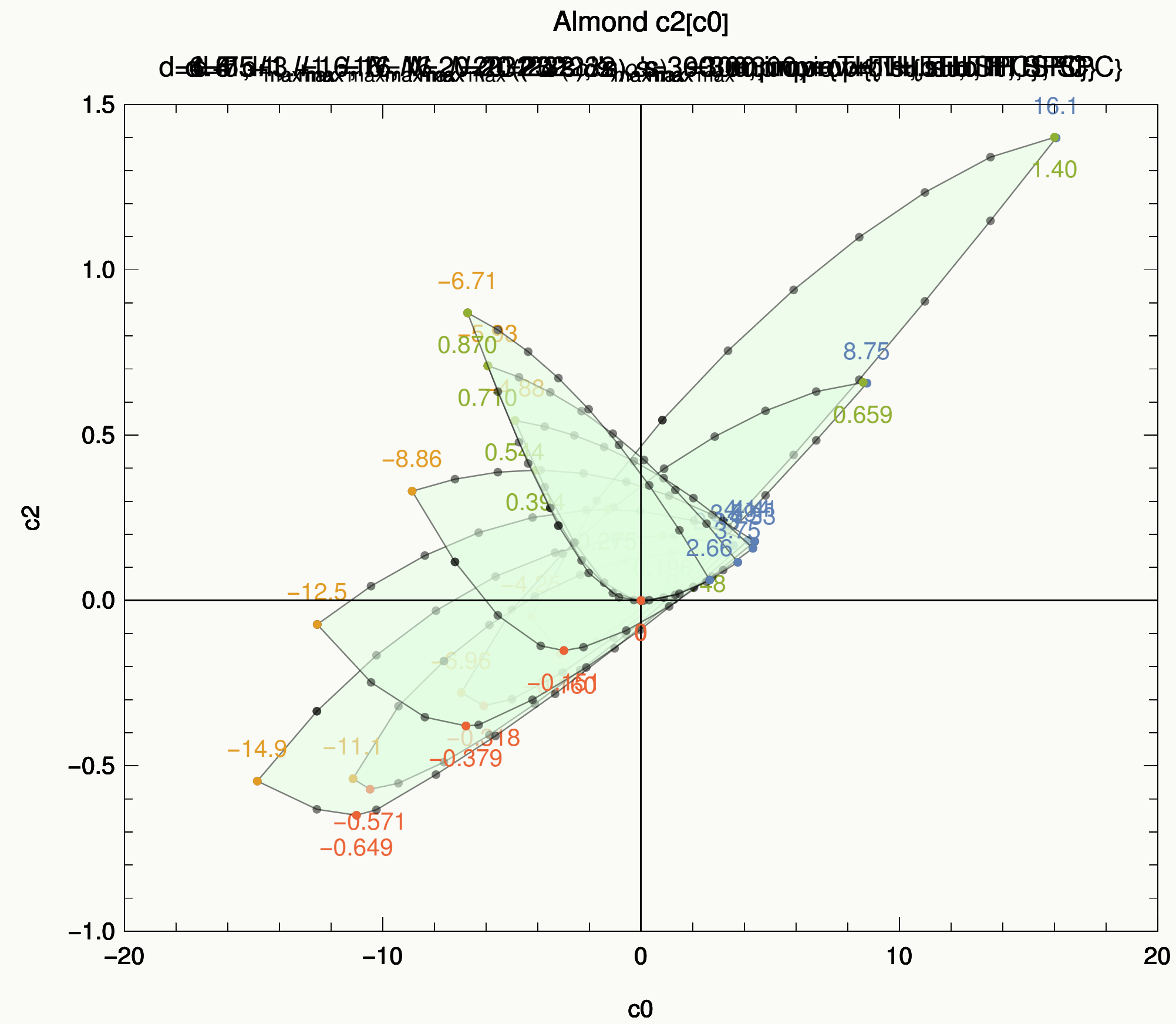


Beyond susy: scattering across dimensions

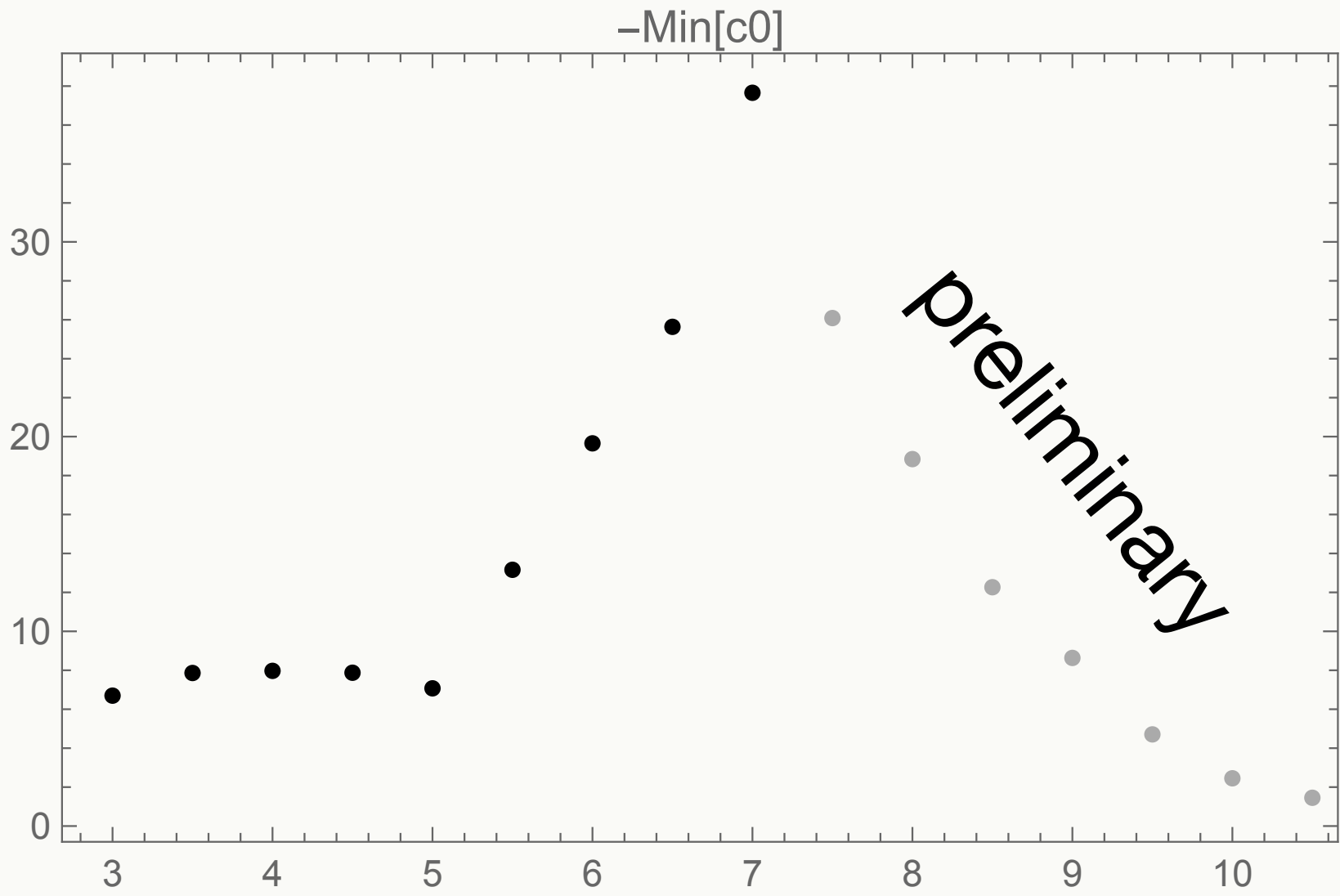
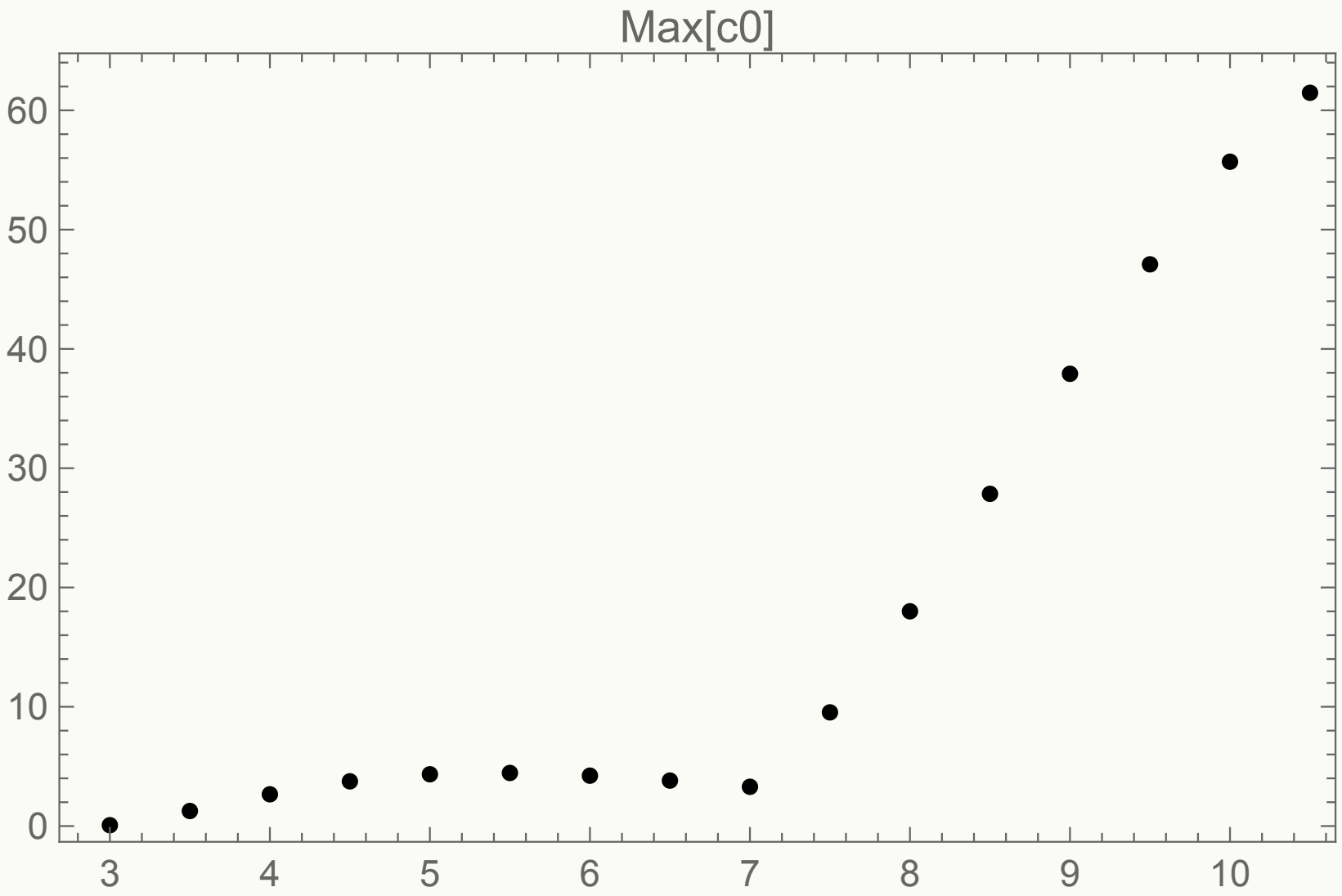
d=8



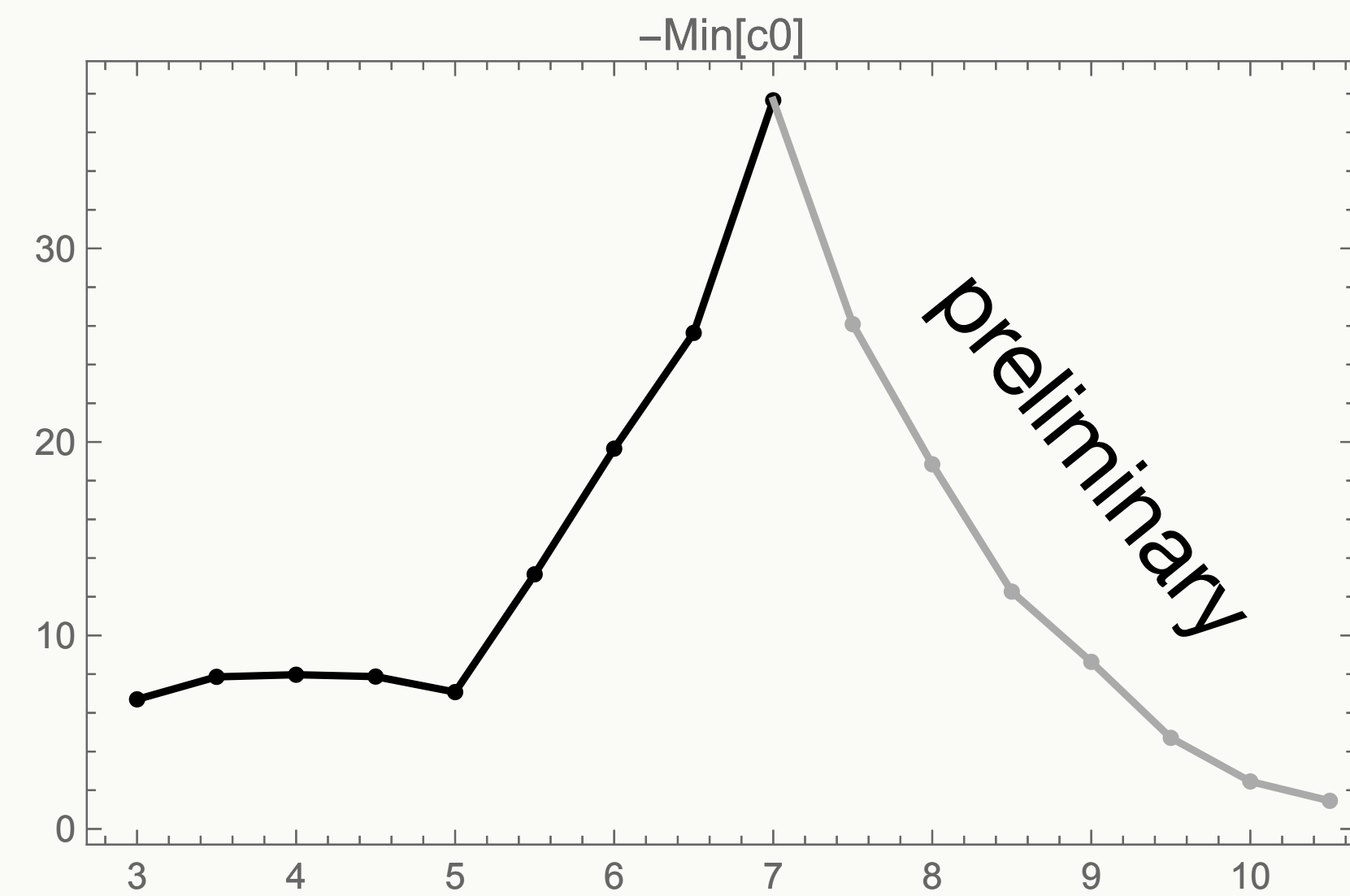
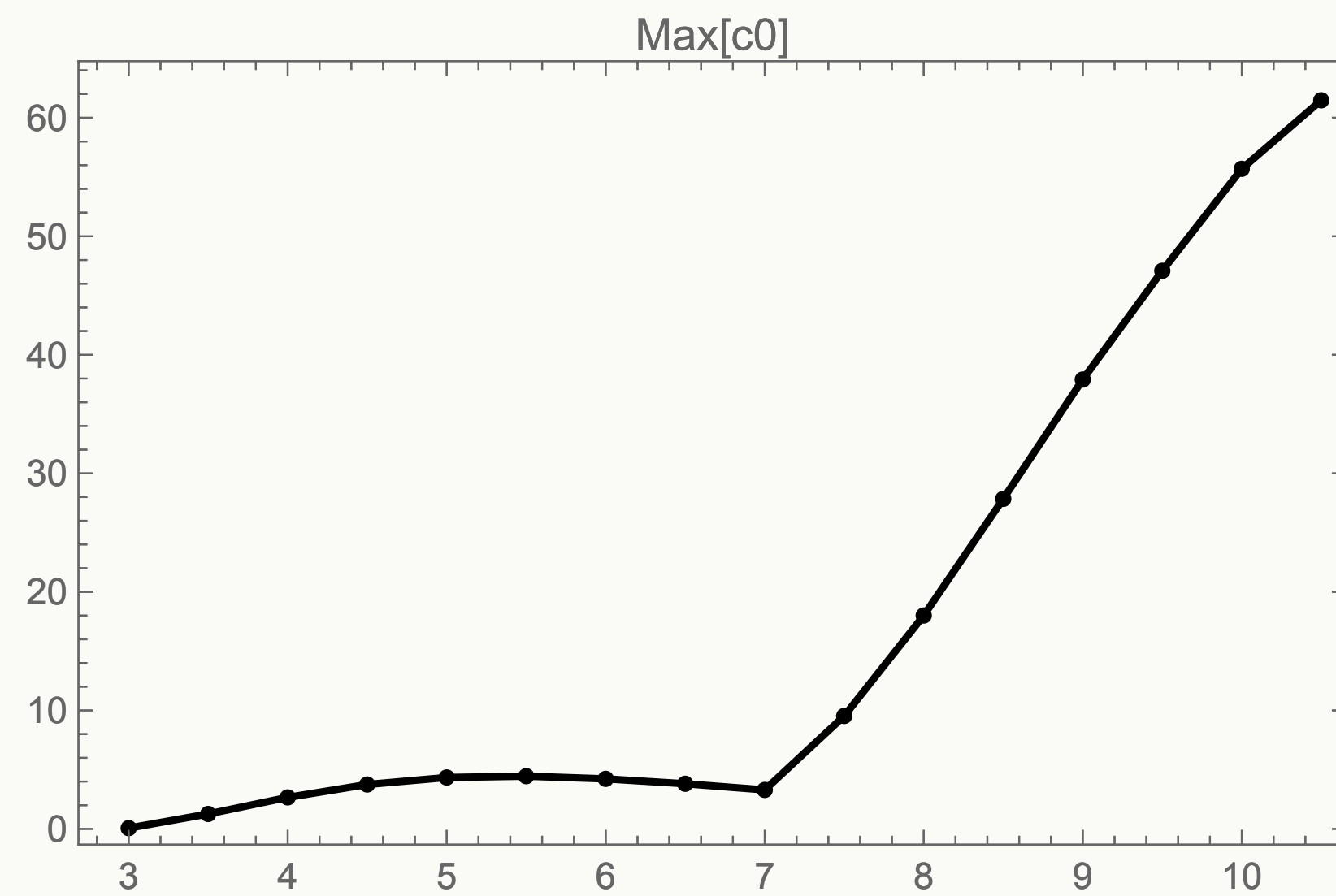
Extremal spaces



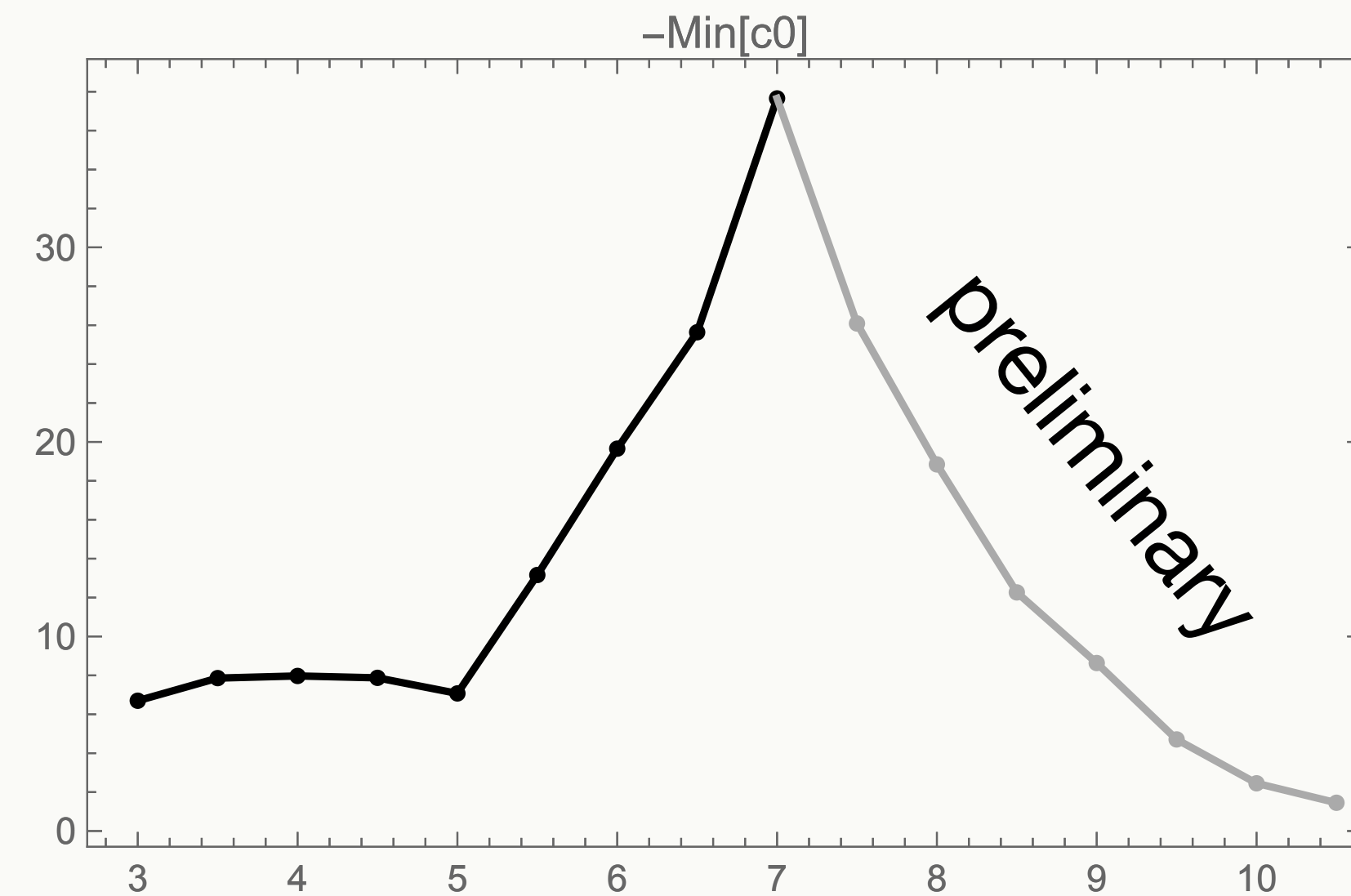
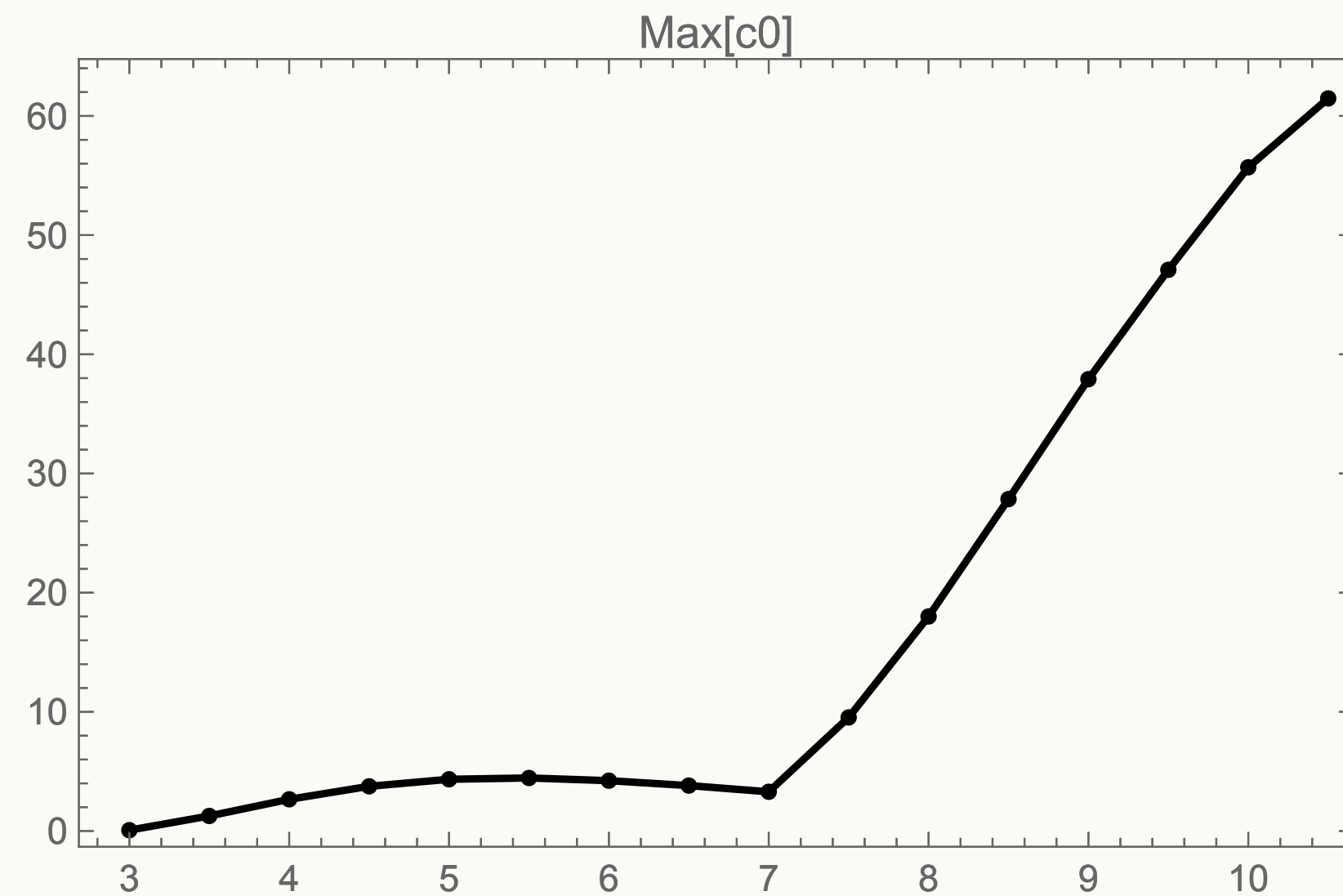
Extremal bounds



Extremal bounds



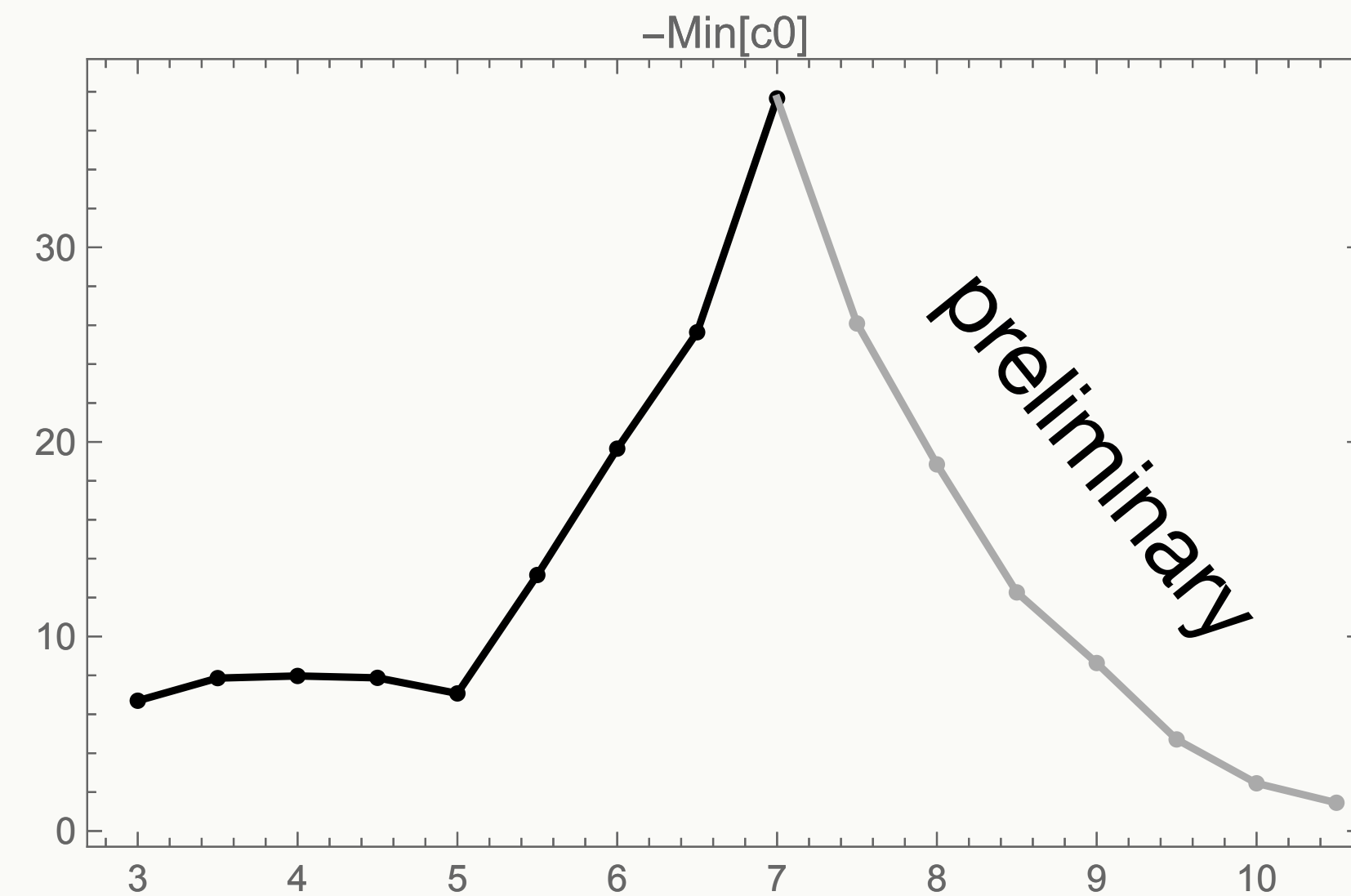
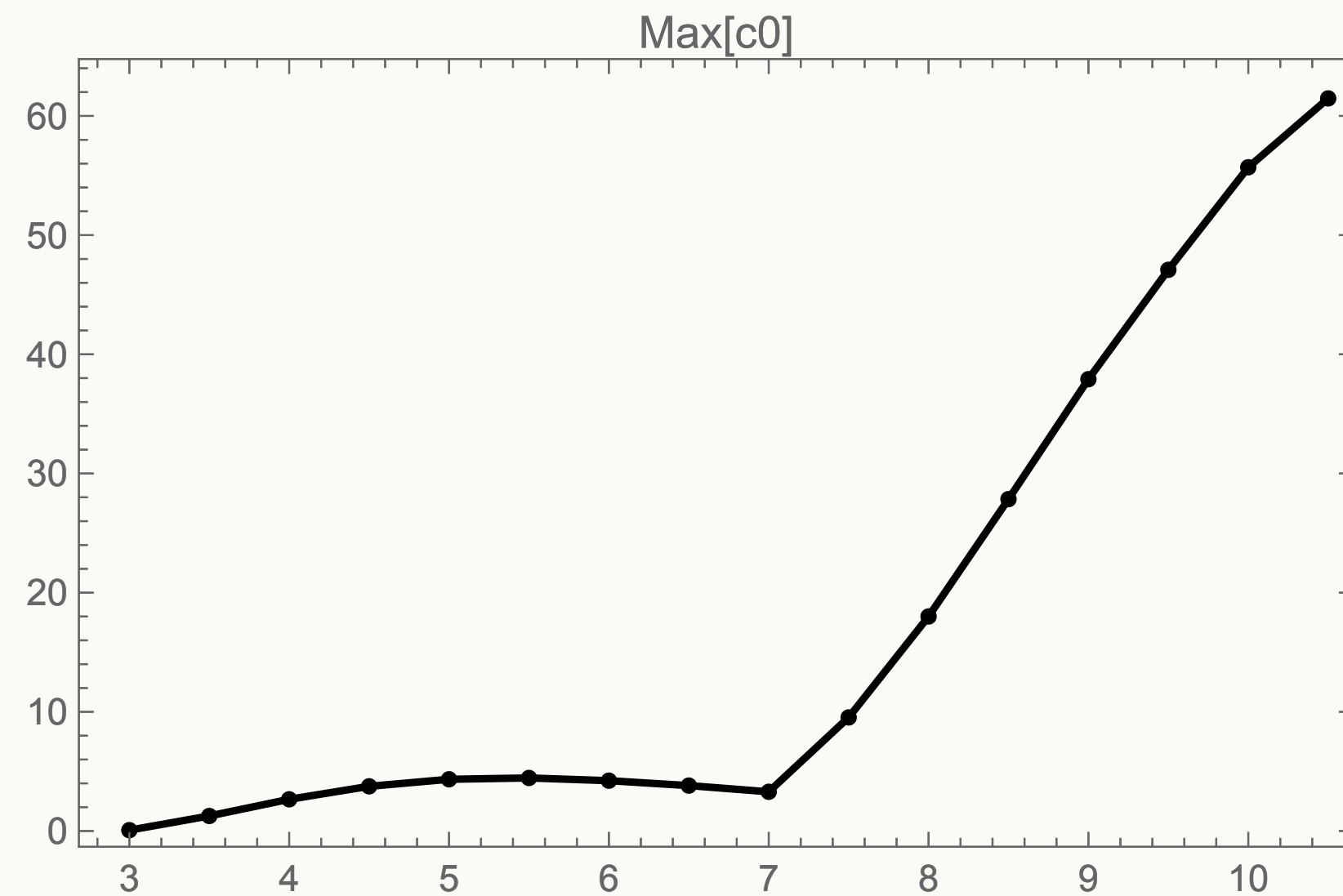
Extremal bounds



Sharp kinks at $d = 5$ and $d = 7$

How to explain?

Extremal bounds



Sharp kinks at $d = 5$ and $d = 7$

How to explain?

No microscopic origin, but clear amplitude structure (threshold effects)

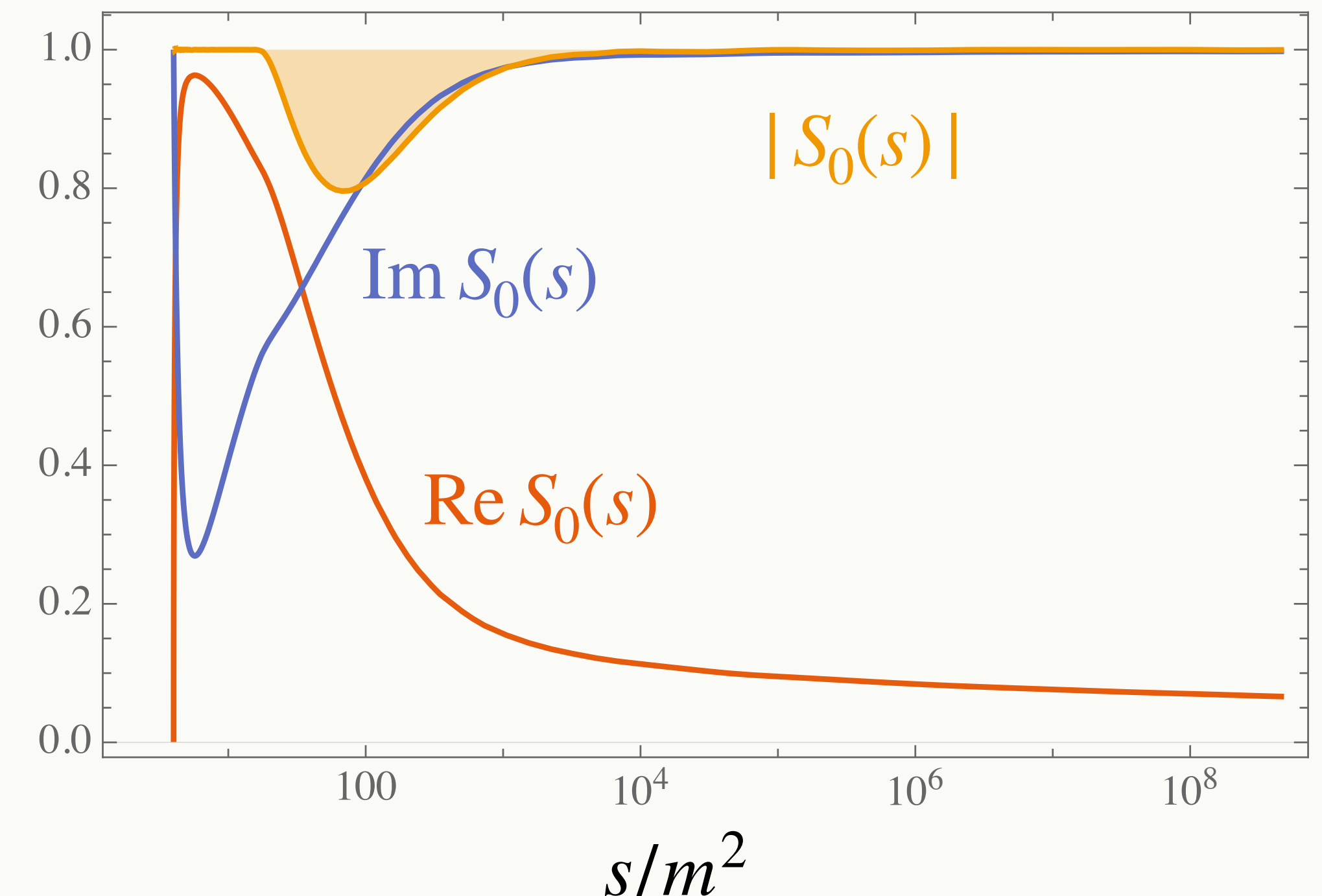
The Achilles heel: particle production

The Achilles heel: particle production

- **Primal bootstrap amplitudes saturate unitarity**, $|S_J|=1$: the space we bound is a superset, and **no theory physical theory lives on its boundary**
- *Aks tension*
- A non-perturbative handle on inelasticity : **PT**, [Zhiboedov, +Leflot, Gumus 2021, 23, 24, 26](#)
- Rigorous construction, **solves elastic unitarity exactly**
- Modern approach with machine learning
- Roughly, solve full non-linear equation with **dynamical particle production** (fixed by IR constraints)

$$\text{scattering} = \text{scattering}^2 + \text{production}$$

Example from our 2026 paper



Decaying amplitude (asympt. free)
Growing amplitude? Gravity?

Conclusion

- Map out space of possible non-perturbatively consistent interactions
- Open problem: treat inelasticity
- Now is a perfect time to revisit these questions (numerics and concepts)
- Large d phases ?
- Go to $d=4$ — problem of soft radiation & charges ([Ana's and Anna's talks](#))
- Energy-Energy correlators are IR safe observables (but loose parts of crossing)

[Moult-Zhu 2506.09119](#)

[Chicherin-Korchensky-Sokatchev-Zhiboedov](#)
Energy correlators in four-dimensional gravity 2512.23791