

Effective Quantum Gravity for the Beginning

Alberto Salvio



14 – 18 September 2026

Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches
Institute of Physics, Graz

Mainly based on

Andrea Ambrosi de Magistris, AS, 2605.27505

Effective theories

→ tool to exploit large separation of scales

e.g.

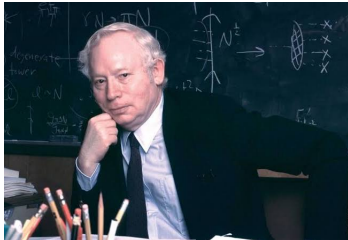
- ▶ motion of planets without caring about animals
- ▶ motion of animals without caring about atoms
- ▶ ...

Effective theories

→ tool to exploit large separation of scales

e.g.

- ▶ motion of planets without caring about animals
- ▶ motion of animals without caring about atoms
- ▶ ...
- ▶ Example of Effective Field Theory (EFT):
dynamics of strong interactions below the confinement scale in terms of composite particles rather quarks and gluons



Physica A: Statistical Mechanics and its
Applications

Volume 96, Issues 1–2, April 1979, Pages 327–340



Phenomenological Lagrangians ☆

Steven Weinberg

→ exploits the separation between observable microscopic scales and the Planck scale

PHYSICAL REVIEW D

VOLUME 50, NUMBER 6

15 SEPTEMBER 1994

General relativity as an effective field theory: The leading quantum corrections

John F. Donoghue

Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01002

(Received 26 May 1994)

I describe the treatment of gravity as a quantum effective field theory. This allows a natural separation of the (known) low energy quantum effects from the (unknown) high energy contributions. Within this framework, gravity is a well-behaved quantum field theory at ordinary energies. In studying the class of quantum corrections at low energy, the dominant effects at large distance can be isolated, as these are due to the propagation of the massless particles (including gravitons) of the theory and are manifested in the nonlocal and/or nonanalytic contributions to vertex functions and propagators. These leading quantum corrections are parameter-free and represent necessary consequences of quantum gravity. The methodology is illustrated by a calculation of the leading quantum corrections to the gravitational interaction of two heavy masses.

EFT of quantum gravity

→ exploits the separation between observable microscopic scales and the Planck scale

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Wait! Isn't quantum gravity a problem?

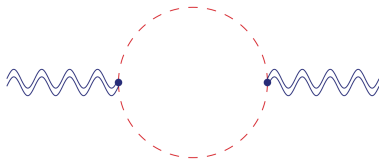
“quantum gravity problem”

find a mathematically consistent quantum theory of gravity which is valid at all energies and in agreement with all experimental and observational data

Wait! Isn't general relativity non renormalizable?

Infinite (!) corrections involving higher-curvature invariants (**not present in the Einstein-Hilbert action**) are generated at 2-loops [*Goroff, Sagnotti (1985,1986)*]

And matter does it already at 1-loop [*Utiyama, DeWitt (1962)*]:



$$\rightarrow S_{\text{quad}} = \int d^4x \sqrt{-g} \mathcal{L}_{\text{quad}}$$
$$\mathcal{L}_{\text{quad}} = c_2 R^2 + c'_2 R_{\mu\nu} R^{\mu\nu}$$

↙ ↘
infinite!

Phenomenological Lagrangian for quantum gravity

Donoghue noted that “non-renormalizable theories are actually just as renormalizable as renormalizable theories as long as we include all interactions allowed by symmetries” (quote from [Weinberg](#)’s book on quantum field theory)

$$\rightarrow \mathcal{L}_{\text{eff}} = -\underbrace{CC}_{\swarrow} + \bar{M}_{Pl}^2 \left(-\frac{1}{2}R + \frac{c_2}{\bar{M}_{Pl}^2}R^2 + \frac{c'_2}{\bar{M}_{Pl}^2}R_{\mu\nu}R^{\mu\nu} + \frac{c_3}{\bar{M}_{Pl}^4}R^3 + \frac{c'_3}{\bar{M}_{Pl}^4}R_{\mu\nu}R^{\mu\nu}R + \dots \right)$$

$\approx 2.4 \times 10^{18} \text{ GeV}$

- Includes *all (infinite) terms* compatible with the invariance under general coordinate transformations in the gravity and matter sector too

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- ▶ Includes *all (infinite) terms* compatible with the invariance under general coordinate transformations in the gravity and matter sector too
- ▶ *one treats perturbatively*
 - ▶ the extra terms in addition to the Einstein-Hilbert, the CC, the standard matter terms
 - ▶ the infinite interactions of $\tilde{h}_{\mu\nu} = \bar{M}_{Pl}(g_{\mu\nu} - \eta_{\mu\nu})$

So all these *infinite* terms do not lead to extra d.o.f. besides the massless spin-2 graviton and the matter d.o.f.

The EFT of gravity captures the low-energy effects of all UV completions of general relativity below a maximal energy (curvature) scale above which it breaks down (**cutoff**):

$$\Lambda \equiv \min(\bar{M}_{Pl}, \bar{M}_{Pl}/^{2(i-1)}\sqrt{c_i^{\text{max}}})$$

Expected cutoff values

For typical UV completions one expects $\Lambda \approx \bar{M}_{Pl}$, but much lower values may be well motivated, e.g. by Higgs naturalness

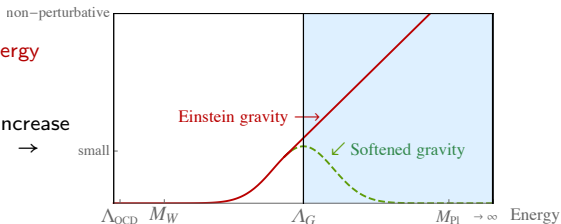
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(Einstein) gravitational interactions increase with energy

Idea (*softened gravity*):

consider theories where the increase of the gravitational coupling $\rightarrow$ stops at some $\Lambda_G \ll M_{Pl}$.



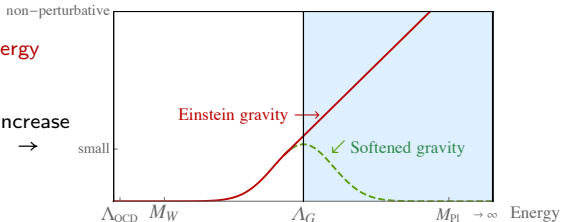
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The gravitational contribution to the Higgs mass is

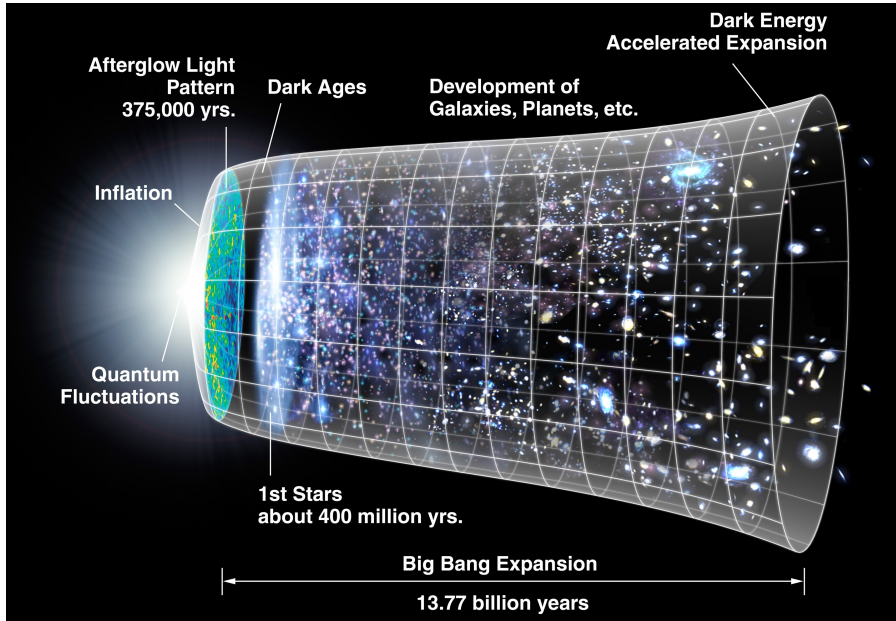
$$\delta M_h^2 \approx \frac{G_N \Lambda_G^4}{(4\pi)^2}$$

Requiring $\delta M_h \sim M_h \rightarrow \Lambda_G \lesssim 10^{11} \text{ GeV}$ [Giudice, Isidori, Salvio, Strumia (2014)]

In a softened-gravity UV completion $\Lambda \sim \Lambda_G$

Possible examples: quadratic gravity [Stelle (1979); Salvio (2019)] or string theories with low string scale [Antoniadis (1990)]

Can we apply the EFT during the whole history of the universe?



Inflation

[Starobinsky (1980); Guth (1981); Linde (1982); Albrecht and Steinhardt (1982)]



What it can solve: horizon, flatness, monopole problems

To solve these problems inflation, quantified by the scale factor $a(t)$, should last enough $\rightarrow$ lower bounds on

$$\ln \left(\frac{a(t_{\text{end}})}{a(t_{\text{in}})} \right) \equiv \text{number of } e\text{-foldings}$$

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How it is implemented (slow-roll inflation):

- ▶ we assume a scalar field ϕ (the inflaton)
- ▶ at some early time the potential energy density is large, but quite flat ...
- ▶ $\rightarrow$ the Hubble constant changes slowly $\rightarrow$ nearly exponential expansion

Leading low-energy effective action for inflation

$$S_I = \int d^4x \sqrt{-g} \left[\frac{K_{ij}(\phi)}{2} \partial_\mu \phi^i \partial^\mu \phi^j - U(\phi) - \frac{\bar{M}_{Pl}^2}{2} R \right]$$

where K_{ij} is a field metric (K is a positive-definite matrix)

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A “non-minimal coupling” $\sqrt{-g}f(\phi)R$ can be reabsorbed in a redefinition of K_{ij} and U through a Weyl transformation of the metric.

- ▶ “Jordan frame”: original field variables, where $\sqrt{-g}f(\phi)R$ is present
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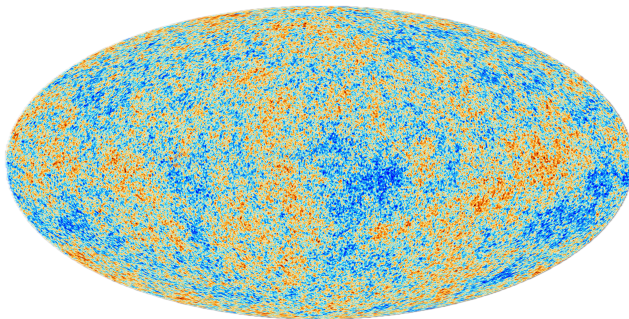
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Goal: understand if/when higher-derivative corrections (suppressed by Λ) give sizable contributions

Near homogeneity and isotropy

The universe appears very homogeneous and isotropic on large scales, up to small fluctuations

Temperature map of the cosmic microwave background (CMB):



source: Nasa website (Planck data)

→ adopt a homogeneous and isotropic (FLRW) metric in first approximation and add small corrections

Inflationary fields

$$ds^2 = a(\eta)^2 \left\{ (1 + 2\Phi(\vec{x}, \eta)) d\eta^2 - 2V_i(\vec{x}, \eta) d\eta dx^i - [(1 - 2\Psi(\vec{x}, \eta))\delta_{ij} + h_{ij}(\vec{x}, \eta)] dx^i dx^j \right\}$$

$$\eta = \int_{t_0}^t \frac{dt'}{a(t')} \quad (\text{conformal time})$$

small corrections parameterized by scalar (Φ, Ψ) , vector V_i and tensor h_{ij} , in conformal Newtonian gauge

$$\partial_i V_i = 0, \quad h_{ij} = h_{ji}, \quad h_{ii} = 0, \quad \partial_i h_{ij} = 0$$

$$\phi^i(\vec{x}, \eta) \rightarrow \underbrace{\phi^i(\eta)}_{\text{homogeneous isotropic}} + \underbrace{\varphi^i(\vec{x}, \eta)}_{\text{small correction}}$$

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small corrections $\rightarrow$ linearized equations $\rightarrow$ use of spatial Fourier transform ($\vec{x} \leftrightarrow \vec{q}$)

- ▶ $\vec{q}$: “co-moving momentum”
- ▶ $\vec{q}/a$: “physical momentum” (inflation stretches wavelengths)

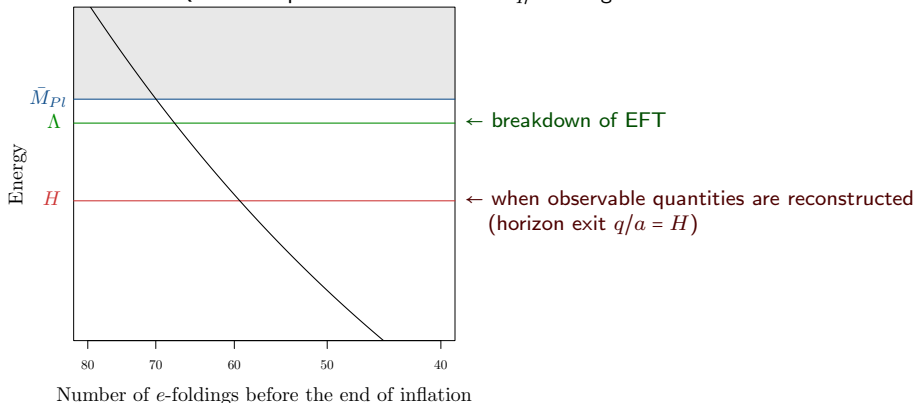
A possible issue: the sub-horizon limit $q/a \gg H$

Follow the inflationary history in reverse, $a \rightarrow 0$

During inflation the Hubble rate H is nearly constant

$$H \equiv \frac{\dot{a}}{a} \equiv \frac{1}{a} \frac{da}{dt} = \frac{1}{a} \mathcal{H} \equiv \frac{1}{a} \frac{a'}{a} \equiv \frac{1}{a^2} \frac{da}{d\eta}$$

Qualitative plot of the evolution of q/a during inflation:



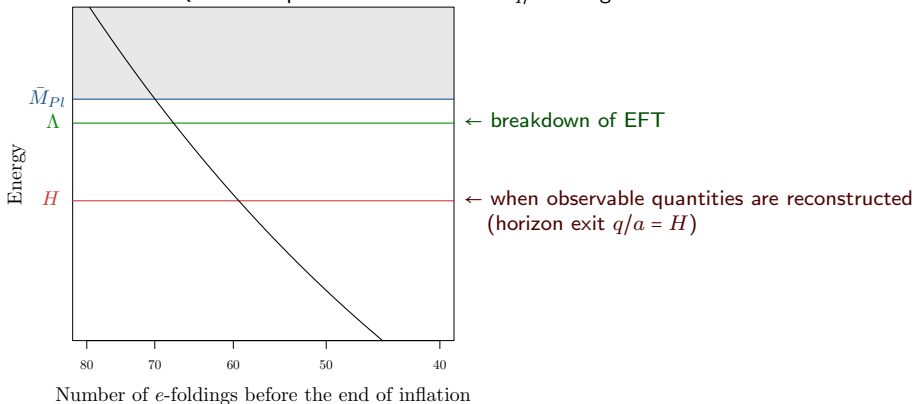
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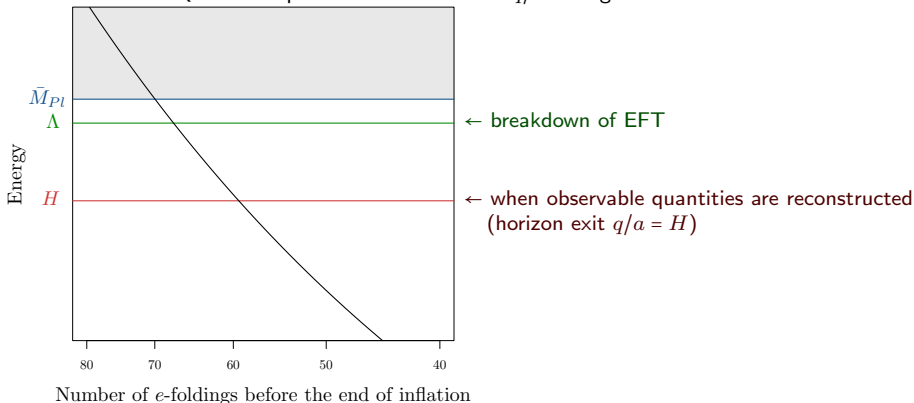
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Qualitative plot of the evolution of q/a during inflation:



The sub-horizon limit $q/a \gg H$ is used to determine the Hilbert space and the amplitude of quantum fluctuations, if conceptually needed the validity of the EFT is limited!! $\rightarrow$ Relative uncertainty increases from $(H/\Lambda)^2$ up to $(q/(a\Lambda))^2$

Recipe to avoid the sub-horizon limit $q/a \gg H$

- ▶ Express the annihilation operators a_i as functionals of the fields
at an arbitrary time η_*
- ▶ The fields are quantized via canonical equal-arbitrary-time commutators at η_*
- ▶ The canonical commutators are non-homogeneous operator relations, one then fixes the amplitude of quantum fluctuations and find the usual commutation relations $[a_i, a_j] = 0$ and $[a_i, a_j^\dagger] = \delta_{ij}$, which determine the Hilbert space

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Tensor perturbations are the easiest non-trivial case

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$$C_1 \equiv \Pi_\Psi + 3a^2 K_{ij} \phi'^i \varphi^j = 0$$

- ▶ Requiring $C_1 = 0$ at any time leads to the secondary constraint

$$C_2 \equiv \mathcal{H} \Pi_\Psi - \phi'^i \Pi_{\varphi_i} + 2a^2 \bar{M}_{Pl}^2 \vec{\nabla}^2 \Psi - 6a^2 \bar{M}_{Pl}^2 (\mathcal{H}^2 - \mathcal{H}') \Psi - a^4 U_{,i} \varphi^i + \frac{a^2}{2} K_{mn,j} \phi'^m \phi'^n \varphi^j = 0$$

Tensor perturbations

$$h_{ij}(\vec{x}, \eta) = \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q} \cdot \vec{x}} \sum_{\lambda=\pm 2} \overbrace{e_{ij}^{\lambda}(\hat{q})}^{\text{polarization tensor}} h_{\lambda}(\vec{q}, \eta)$$

for $\hat{q}$ along the third axis the $e_{ij}^{\lambda}(\hat{q})$ are given by

$$e_{11}^{+2} = -e_{22}^{+2} = 1/2, \quad e_{12}^{+2} = e_{21}^{+2} = i/2, \quad e_{3i}^{+2} = e_{i3}^{+2} = 0, \quad e_{ij}^{-2} = (e_{ij}^{+2})^*$$

$$h_{ij}(\vec{x}, \eta) = \sum_{\lambda=\pm 2} \int \frac{d^3 q}{(2\pi)^3} \left[e_{ij}^{\lambda}(\hat{q}) \underbrace{h_q(\eta)}_{\text{classical solution}} e^{i\vec{q} \cdot \vec{x}} \underbrace{b_{\lambda}(\vec{q})}_{\text{annihilation operators}} + e_{ij}^{*\lambda}(\hat{q}) h_q^*(\eta) e^{-i\vec{q} \cdot \vec{x}} b_{\lambda}^{\dagger}(\vec{q}) \right]$$

annihilation operators

expressed as functionals
of the fields

$$\rightarrow b_{\lambda}(\vec{q}) = \frac{e_{ij}^{*\lambda}(\hat{q})}{h_q \overleftrightarrow{\partial}_{\eta} h_q^*(\eta)} \int d^3 x e^{-i\vec{q} \cdot \vec{x}} h_{ij}(\vec{x}, \eta) \overleftrightarrow{\partial}_{\eta} h_q^*$$

Canonical (non-vanishing) commutators (no constraints in this sector)

$$\left[h_{\lambda}(\vec{q}, \eta), \Pi_{\lambda'}(\vec{k}, \eta) \right] = i \delta_{\lambda\lambda'} \delta^{(3)}(\vec{q} - \vec{k})$$

$$\Pi_{\lambda}(\vec{q}, \eta) = \frac{a^2(\eta) \bar{M}_{Pl}^2}{4(2\pi)^3} h_{\lambda}^{\dagger}(\vec{q}, \eta)$$

$$\left[h_{\lambda}(\vec{q}, \eta), h_{\lambda'}^{\dagger}(\vec{k}, \eta) \right] = \frac{4i(2\pi)^3}{a^2(\eta) \bar{M}_{Pl}^2} \delta_{\lambda\lambda'} \delta^{(3)}(\vec{q} - \vec{k})$$

Scalar perturbations

$$\varphi^i(\vec{x}, \eta) = \sum_{r=1}^N \int d^3q \left[\varphi_{qr}^i(\eta) e^{i\vec{q} \cdot \vec{x}} \alpha_r(\vec{q}) + \varphi_{qr}^{*i}(\eta) e^{-i\vec{q} \cdot \vec{x}} \alpha_r^\dagger(\vec{q}) \right]$$

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Defining $\xi_{qr} \equiv \{\varphi_{qr}^1, \dots, \varphi_{qr}^N, \Pi_{qr}^1, \dots, \Pi_{qr}^N\}$, in momentum space

$$\xi^i(\vec{q}) \equiv \xi_{qr}^i \alpha_r(\vec{q}) + \xi_{qr}^{*i} \alpha_r^\dagger(-\vec{q})$$

annihilation operators expressed
as functionals of the fields $\rightarrow$

$$\alpha_r(\vec{q}) = i \xi_{qr}^{*i}(\eta_*) \Omega_{ij} \xi^j(\vec{q})$$

Ω is an arbitrary real, antisymmetric, non-degenerate matrix

Choosing Ω appropriately and using the canonical commutators one finds

$$[\alpha_r(\vec{q}), \alpha_s^\dagger(\vec{k})] = \delta_{rs} \text{ and } [\alpha_r(\vec{q}), \alpha_s(\vec{k})] = 0$$

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$\rightarrow$ Neglecting higher-derivative terms in the EFT produce an uncertainty $\sim (H/\Lambda)^2$.

Scalar perturbations

$$\varphi^i(\vec{x}, \eta) = \sum_{r=1}^N \int d^3q \left[\varphi_{qr}^i(\eta) e^{i\vec{q} \cdot \vec{x}} \alpha_r(\vec{q}) + \varphi_{qr}^{*i}(\eta) e^{-i\vec{q} \cdot \vec{x}} \alpha_r^\dagger(\vec{q}) \right]$$

$$\Pi_\varphi^i(\vec{x}, \eta) = \sum_{r=1}^N \int d^3q \left[\Pi_{qr}^i(\eta) e^{i\vec{q} \cdot \vec{x}} \alpha_r(\vec{q}) + \Pi_{qr}^{*i}(\eta) e^{-i\vec{q} \cdot \vec{x}} \alpha_r^\dagger(\vec{q}) \right]$$

Defining $\xi_{qr} \equiv \{\varphi_{qr}^1, \dots, \varphi_{qr}^N, \Pi_{qr}^1, \dots, \Pi_{qr}^N\}$, in momentum space

$$\xi^i(\vec{q}) \equiv \xi_{qr}^i \alpha_r(\vec{q}) + \xi_{qr}^{*i} \alpha_r^\dagger(-\vec{q})$$

annihilation operators expressed
as functionals of the fields $\rightarrow$

$$\alpha_r(\vec{q}) = i \xi_{qr}^{*i}(\eta_*) \Omega_{ij} \xi^j(\vec{q})$$

Ω is an arbitrary real, antisymmetric, non-degenerate matrix

Choosing Ω appropriately and using the canonical commutators one finds

$$[\alpha_r(\vec{q}), \alpha_s^\dagger(\vec{k})] = \delta_{rs} \text{ and } [\alpha_r(\vec{q}), \alpha_s(\vec{k})] = 0$$



no need to take the sub-horizon limit to determine the Hilbert space and the amplitude of quantum fluctuations

$\rightarrow$ Neglecting higher-derivative terms in the EFT produce an uncertainty $\sim (H/\Lambda)^2$.

[Planck (2018)] gives $H < 2.5 \cdot 10^{-5} \bar{M}_{Pl} \approx 6.1 \cdot 10^{13} \text{ GeV}$

$\rightarrow$ for $\Lambda \gg 6.1 \cdot 10^{13} \text{ GeV}$ the uncertainty is small

Scalar canonical commutators

Determined by Dirac brackets: $[A_1, A_2] = i\{A_1, A_2\}_D$

The non-vanishing ones are

$$\begin{aligned}
 \{\varphi^i(\vec{x}), \Pi_{\varphi j}(\vec{y})\}_D &= \delta_j^i \delta^{(3)}(\vec{x} - \vec{y}) - \frac{3}{2\bar{M}_{Pl}^2} K_{jl} \phi'^i \phi'^l G(\vec{x} - \vec{y}) \\
 \{\Psi(\vec{x}), \Pi_{\Psi}(\vec{y})\}_D &= \frac{3K_{ij}}{2\bar{M}_{Pl}^2} \phi'^i \phi'^j G(\vec{x} - \vec{y}) \\
 \{\varphi^i(\vec{x}), \Psi(\vec{y})\}_D &= \frac{\phi'^i}{2a^2 \bar{M}_{Pl}^2} G(\vec{x} - \vec{y}), \\
 \{\Pi_{\varphi i}(\vec{x}), \Pi_{\Psi}(\vec{y})\}_D &= 3a^2 K_{ij} \phi'^j \delta^{(3)}(\vec{x} - \vec{y}) - 9a^2 K_{ij} \phi'^j (\mathcal{H}^2 - \mathcal{H}') G(\vec{x} - \vec{y}), \\
 \{\Psi(\vec{x}), \Pi_{\varphi i}(\vec{y})\}_D &= \frac{G(\vec{x} - \vec{y})}{2\bar{M}_{Pl}^2} \left(3\mathcal{H} K_{ij} \phi'^j + a^2 U_{,i} - \frac{1}{2} K_{mn,i} \phi'^m \phi'^n \right), \\
 \{\Pi_{\varphi i}(\vec{x}), \Pi_{\varphi j}(\vec{y})\}_D &= \frac{3a^2}{2\bar{M}_{Pl}^2} G(\vec{x} - \vec{y}) \left[(a^2 U_{,i} - \frac{1}{2} K_{mn,i} \phi'^m \phi'^n) K_{jl} \phi'^l \right. \\
 &\quad \left. - (a^2 U_{,j} - \frac{1}{2} K_{mn,j} \phi'^m \phi'^n) K_{il} \phi'^l \right],
 \end{aligned}$$

where

$$G(\vec{x} - \vec{y}) = - \int \frac{d^3 q}{(2\pi)^3} \frac{e^{i\vec{q} \cdot (\vec{y} - \vec{x})}}{q^2}$$

Slow-roll inflation

Slow-roll inflation: estimating the EFT uncertainty

Single inflaton χ (canonically normalized)

The amplitude of quantum fluctuations along $\chi(t)$ is encoded in *[Planck (2018)]*

$$A_s \approx \frac{U/\epsilon}{24\pi^2 \bar{M}_{Pl}^4} \approx 2.1 \cdot 10^{-9}, \quad \left(\text{where } \epsilon \equiv \frac{\bar{M}_{Pl}^2}{2} \left(\frac{1}{U} \frac{dU}{d\chi} \right)^2 \right)$$

Since $H^2 \approx U/(3\bar{M}_{Pl}^2)$

$$\left(\frac{H}{\Lambda} \right)^2 \approx 8\pi^2 A_s \epsilon \frac{\bar{M}_{Pl}^2}{\Lambda^2} \approx 1.7 \cdot 10^{-7} \epsilon \frac{\bar{M}_{Pl}^2}{\Lambda^2}$$

Neglecting higher-derivative terms is better than the slow-roll approximation if

$$\Lambda \gtrsim 4.1 \cdot 10^{-4} \min \left(1, \sqrt{\frac{\epsilon}{|\eta|}} \right) \bar{M}_{Pl}, \quad \left(\text{where } \eta \equiv \frac{\bar{M}_{Pl}^2}{U} \frac{d^2 U}{d\chi^2} \right)$$

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Multi inflatons ϕ^i

The amplitude of quantum fluctuations along $\phi^i(t)$ is still encoded in

$$A_s \approx \frac{2H^2}{\pi^2 r \bar{M}_{Pl}^2}, \quad \left(\text{where } r \text{ is the tensor-to-scalar ratio} \right)$$

$$\left(\frac{H}{\Lambda} \right)^2 \approx \frac{\pi^2}{2} A_s r \frac{\bar{M}_{Pl}^2}{\Lambda^2} \approx 1.0 \cdot 10^{-8} r \frac{\bar{M}_{Pl}^2}{\Lambda^2}$$

Models

Some relevant slow-roll models

Starobinsky scalar ϕ_S (canonically normalized)

$$U(\phi_S) = \frac{3f_0^2 \bar{M}_{Pl}^4}{8} \left(1 - e^{-2\phi_S/\sqrt{6}\bar{M}_{Pl}}\right)^2$$

The Higgs field ϕ_H [Bezrukov, Shaposhnikov (2007)]

$$U(\phi_H) = \frac{\lambda_H \phi_H^4}{4\Omega^4(\phi_H)}, \quad K_{HH}(\phi_H) = \frac{\Omega^2(\phi_H) + 6\xi^2 \phi_H^2 / \bar{M}_{Pl}^2}{\Omega^4(\phi_H)}, \quad \Omega^2(\phi_H) \equiv 1 + \xi \phi_H^2 / \bar{M}_{Pl}^2$$

Natural inflaton ϕ_N (a pseudo-Nambu Godstone boson)

[Freese, Frieman, Olinto (1990); Salvio, Sciusco (2024)]

$$U(\phi_N) = \frac{V(\phi_N)}{F^2(\phi_N)}, \quad K_{NN}(\phi_N) = \frac{2F(\phi_N) + 3\bar{M}_{Pl}^2 \left(\frac{dF}{d\phi_N}(\phi_N)\right)^2}{2F^2(\phi_N)}$$

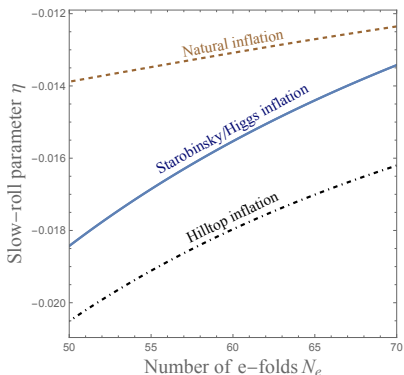
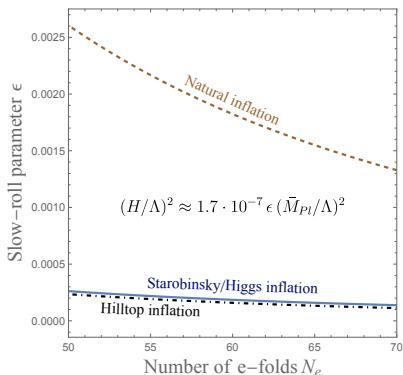
where $V(\phi_N) \equiv \Lambda_N^4 [1 + \cos(\phi_N/f)]$, $F(\phi_N) \equiv 1 + \alpha \left[1 + \cos\left(\frac{\phi_N}{f}\right)\right]$,

e.g. $f = 6\bar{M}_{Pl}$ and $\alpha = 0.2$

Hilltop, ϕ_h (canonically normalized) [Boubekeur, Lyth (2005); Salvio (2019)]

$$U(\phi_h) = \frac{\lambda_h}{4} \cosh^4 \left(\frac{\phi_h}{\sqrt{6}\bar{M}_{Pl}} \right) \left(v^2 - 6\bar{M}_{Pl}^2 \tanh^2 \left(\frac{\phi_h}{\sqrt{6}\bar{M}_{Pl}} \right) \right)^2, \quad \text{e.g. } v = 2.43\bar{M}_{Pl}$$

EFT and slow-roll uncertainties for these models



Left: The relative size $(H/\Lambda)^2$ of the corrections due to higher-derivative terms in the EFT is given in terms of ϵ

Right: the second slow-roll parameter η , which is significantly larger than ϵ for these models, so neglecting higher-derivative terms is better than the slow-roll approximation if

$$\Lambda \gtrsim 4.1 \cdot 10^{-4} \sqrt{\frac{\epsilon}{|\eta|}} \bar{M}_{Pl}$$

Open question: what is the inflationary quantum state?

Such state is usually chosen to be the vacuum, but other choices remain possible

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Another possibility: the inflationary state is a **thermal** distribution of particles

[Bhattacharya, Mohanty, Rangarajan (2005)]

- ▶ the predictions can significantly change in this case,
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Perhaps a consistent UV-complete theory of quantum gravity will determine the quantum state at the beginning

Conclusions

- ▶ EFTs are useful tools to exploit large separation of scales
- ▶ Quantum gravity can be formulated as an EFT at energies $\ll \Lambda \leq \bar{M}_{Pl}$
- ▶ Inflation, “the beginning”, is the most critical era for the validity of this EFT
- ▶ We can determine the Hilbert space and amplitude of quantum inflationary fluctuations considering only times from horizon crossing $q/a = H$ on **(avoiding the dangerous sub-horizon limit, $q/a \gg H$)**
- ▶ The uncertainty of the inflationary EFT is of order $(H/\Lambda)^2$ which is smaller than the slow-roll uncertainty for

$$\Lambda \gtrsim 4.1 \cdot 10^{-4} \min \left(1, \sqrt{\frac{\epsilon}{|\eta|}} \right) \bar{M}_{Pl},$$

- ▶ Can we determine the quantum inflationary state?



Thank you very much for your attention!