

From groups and graphs to black holes and big bounce

Principles and current perspectives of loop quantum gravity

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Some basics of loop quantum gravity



Loop Quantum gravity

Quantum theory of gravity.

Some of the **assumptions**:

- * space-time 3+1 dimensional
- * Formulation of GR based on **frames**
- * general covariance important (—> use background structures as little as possible)



Some of the **consequences**

- * QFT of geometry
- * discrete, combinatorial picture: “Atoms” of space(-time)
- * not easy to interpret operationally, no easy classical limit



Classical starting point: Formulation using a frame e

$$g_{\mu\nu}(x) := e_{\mu}^I(x) e_{\nu}^J(x) \eta_{IJ}$$

Local frame rotations are gauge trafos:

$$e_{\mu}^I(x) \longmapsto \Lambda^I{}_J(x) e_{\mu}^J(x) \quad \Lambda(x) \in SO(3, 1)$$

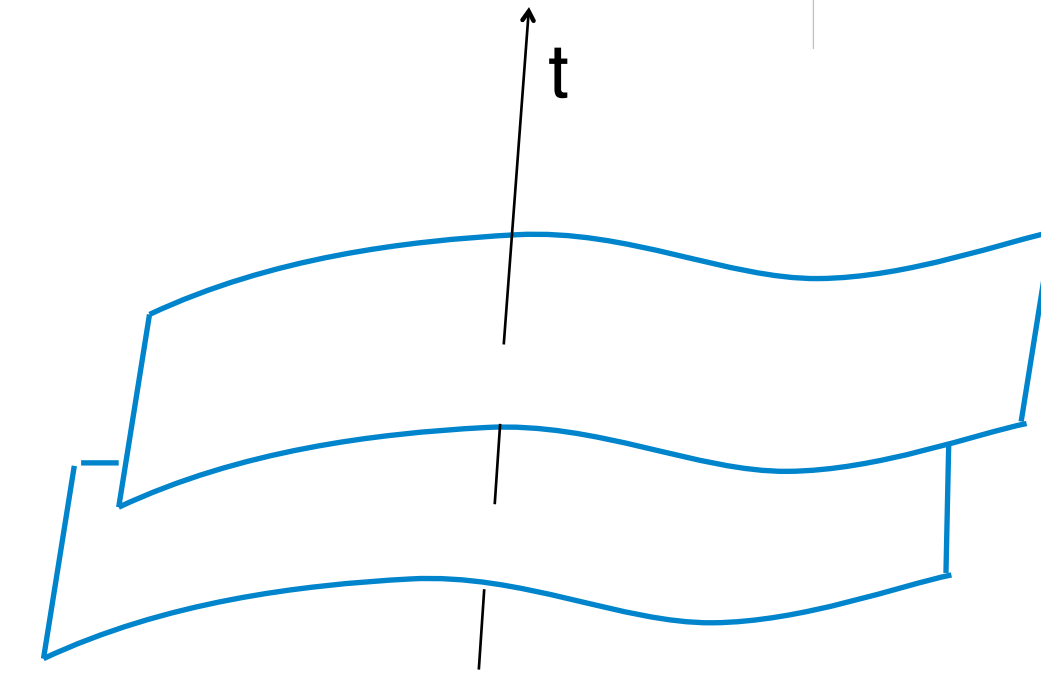
Holst action: for 4d GR

$$S[\omega, e] = \frac{1}{32\pi G} \int_M \left[\star(e \wedge e)^{IJ} - \frac{1}{\beta} e^I \wedge e^J \right] \wedge F(\omega)_{IJ}$$

- ✱ $\delta S[\omega, e] = 0 \implies g_{\mu\nu}$ fulfills Einstein equations
- ✱ Invariant under [gauge transformations](#) and [diffeomorphisms](#)
- ✱ Similar description available for 3d GR.

Legendre trafo

- * space/time split
- * covering group $SO(3,1) \rightarrow SL(2,C)$
- * partial gauge fixing (time gauge) $SL(2,C) \rightarrow SU(2)$



$$S = \frac{1}{8\pi\beta G} \int dt \int_{\Sigma_t} E_i^a \dot{A}_a^i - \omega_0^i G_i + N^a C_a + NC$$

$$i, j, \dots : SO(3) \text{ (or } SU(2))$$

Canonical variables

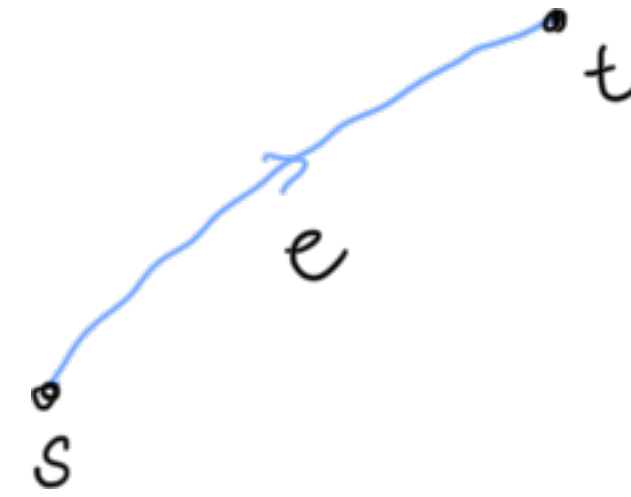
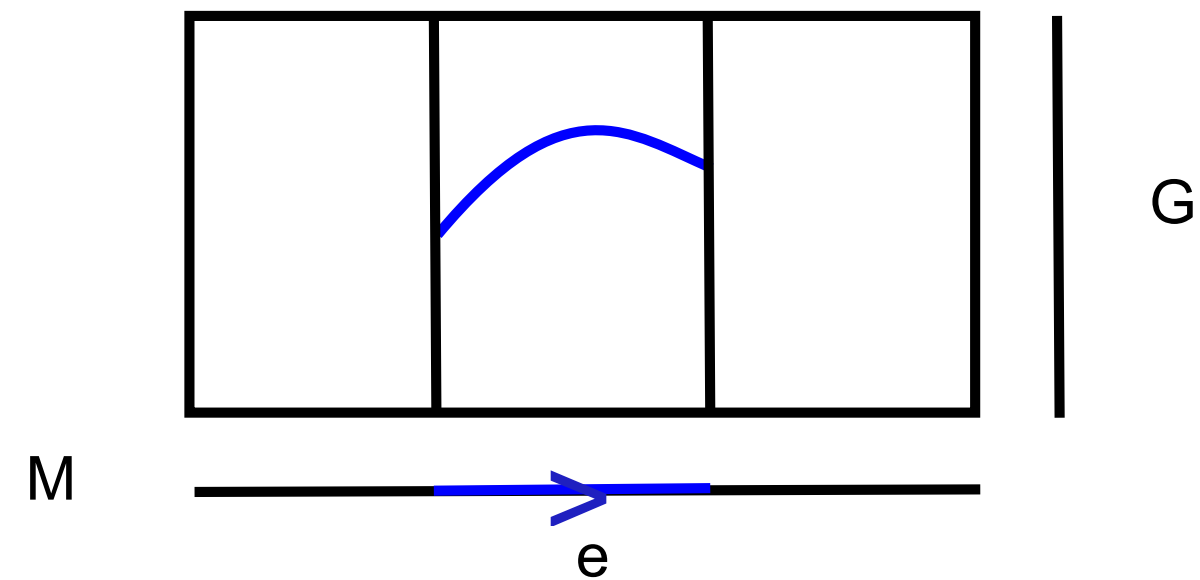
$$E_i^a = \frac{1}{2} e_b^j e_c^k \epsilon^{abc} \epsilon_{ijk} = \sqrt{|\det q|} e_i^a,$$

$$A_a^i = (*\omega)_a^{0i} + \beta \omega_a^{0i}$$

Phase space of GR as subspace of $SU(2)$ Yang-Mills. [Ashtekar, Barbero, Sen]

Similar description available for 3d GR.

Holonomies: Think of A in terms of parallel transport



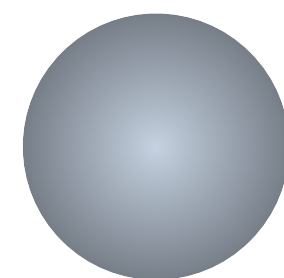
$$\left. \begin{array}{l} \text{Connection } A \\ \text{Path } e \end{array} \right\} \longrightarrow h_e[A] := \mathcal{P} \exp - \int_e A \quad \text{“holonomy along } e\text{”}$$

SU(2) valued (under local trivialization). Under change of trivialization (gauge trafo)

$$h_e \longrightarrow g(t) h_e g^{-1}(s)$$

Spatial area and volume:

$$V_{\mathcal{R}} = \int_{\mathcal{R}} \sqrt{\det E}$$



$$A_{\mathcal{S}} = \int_{\mathcal{S}} |E|$$





Canonical loop quantum gravity



Quantum theory [Rovelli, Smolin, Ashtekar, Lewandowski]: (unique under some assumptions)

Holonomies create excitations of geometry from vacuum

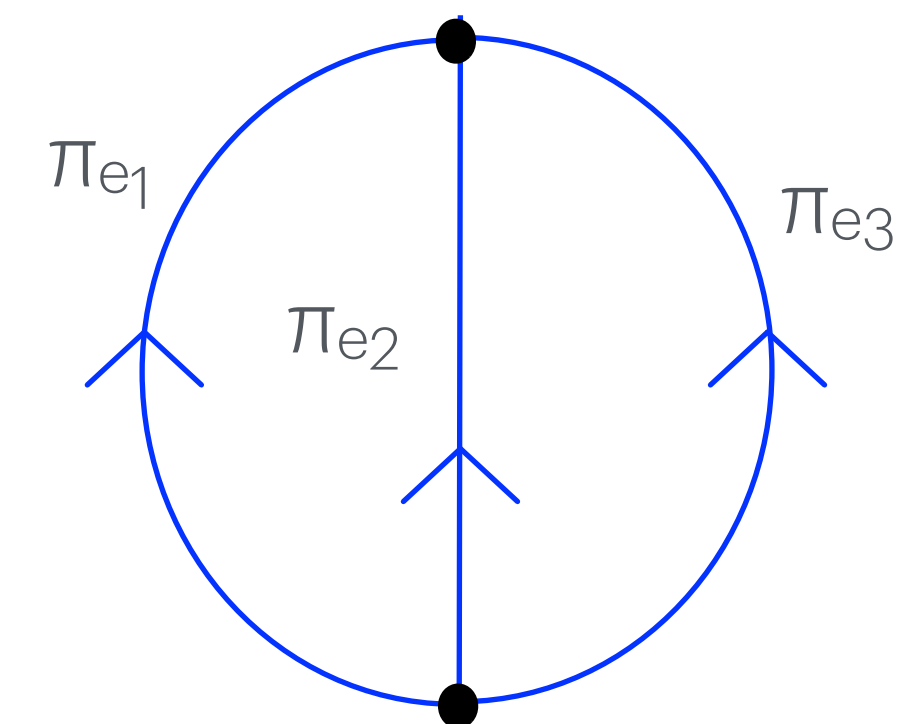
$$\text{tr } \pi_j(h_{\text{loop}}) \left| \right\rangle = \left| \text{loop}_j \right\rangle \quad A \left| \text{loop}_j \right\rangle = 8\pi\beta\ell_P^2 \sqrt{j(j+1)} \left| \text{loop}_j \right\rangle$$

Basis for states: Labeled by

- * graph γ
- * for each edge e an irreducible rep π_{je} of G
- * for each vertex v in γ an intertwiner (equivariant map/invariant tensor) I_v

$$\pi_{j_1} \otimes \dots \otimes \pi_{j_n}(g) I_v = I_v$$

→ **Spin network basis**



Spatial area: [Rovelli+Smolin, Ashtekar+Lewandowski]

$$\hat{A}_{\triangle} | \bigcirc_j \rangle = 8\pi\beta l_p^2 \sqrt{j(j+1)} | \bigcirc_j \rangle$$

More complicated in general.

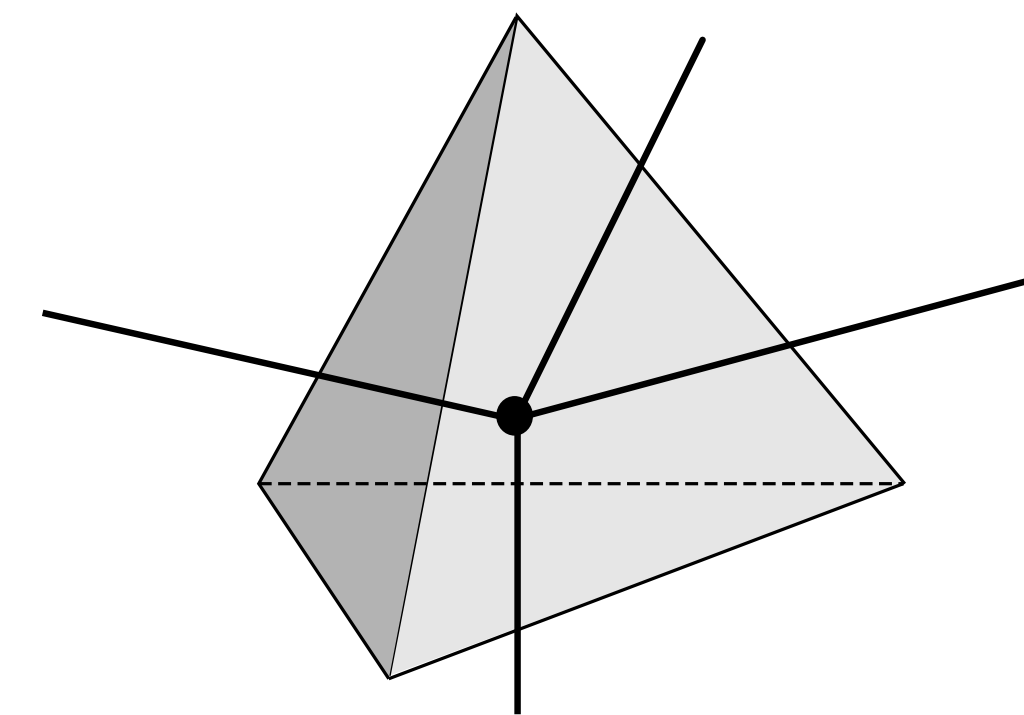
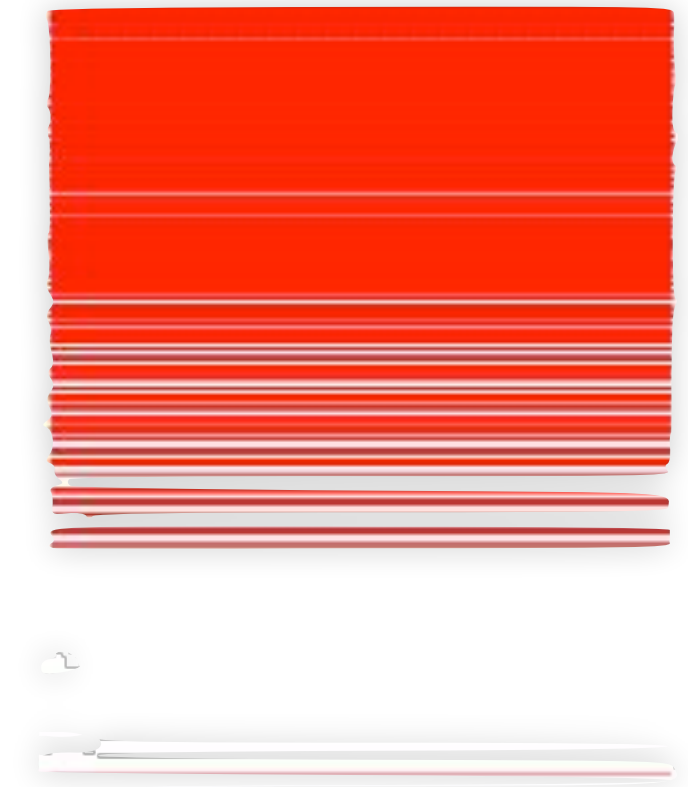
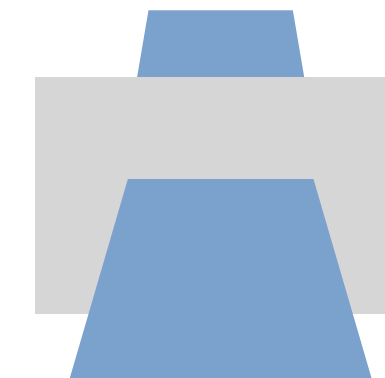
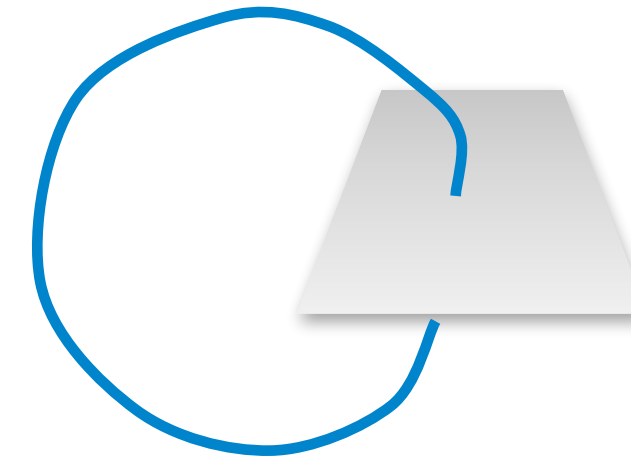
$$[\hat{A}_{\triangle}, \hat{A}_{\square}] \neq 0$$

Volume: [Rovelli+Smolin, Ashtekar+Lewandowski]

Action on intertwiner spaces

Picture: [Baez, ... , Bianchi, Dona,Speziale]

n valent vertex $\longleftrightarrow$ quantized flat polyhedron



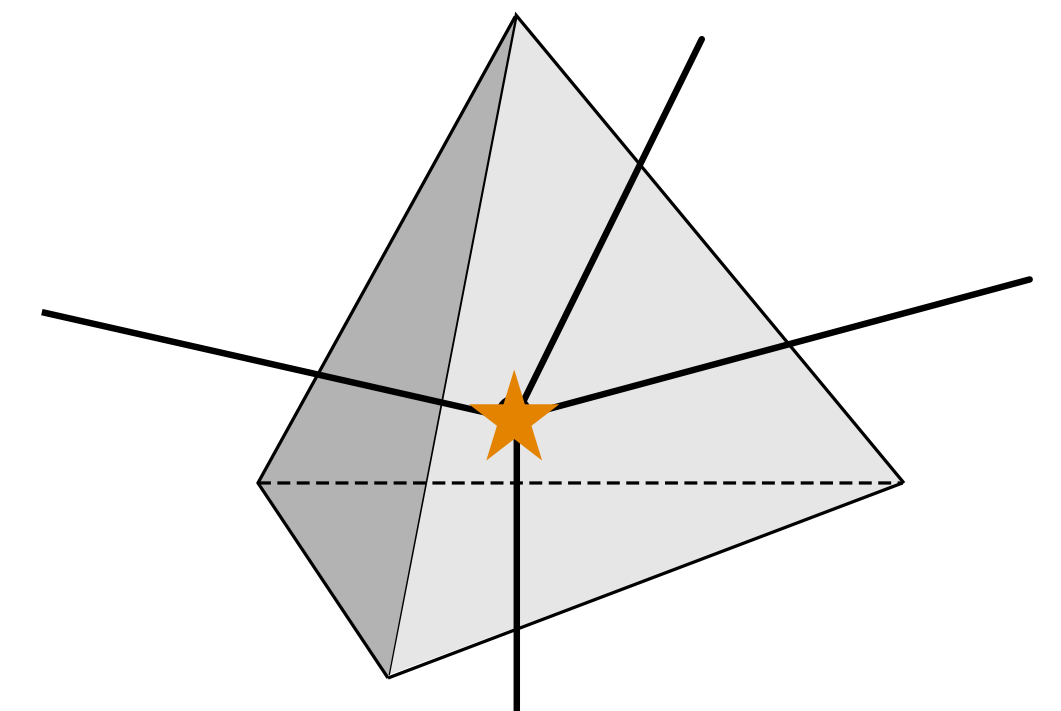
Fermions

Must couple to gravity [Corichi, Rovelli, Thiemann,...]

$$\dots h[A]_C{}^B c_B^\dagger(p) |0\rangle$$

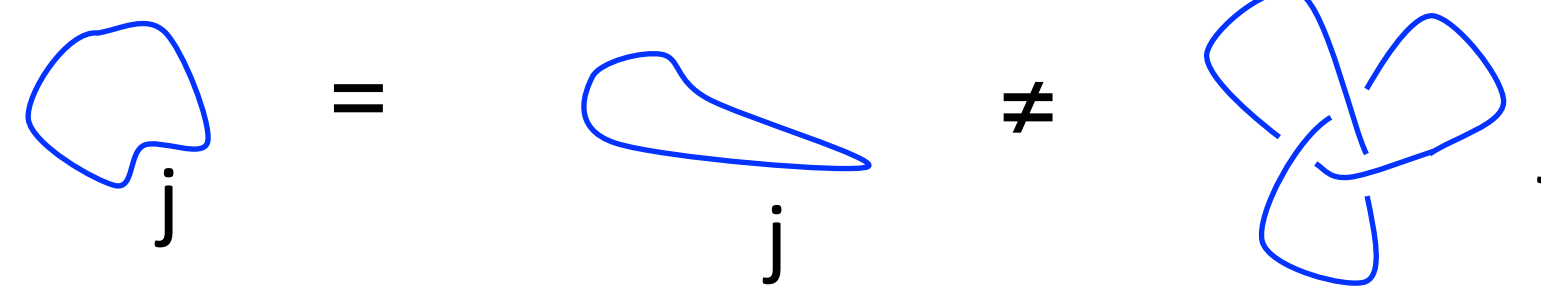


Affect geometry [Mansuroglu, HS]

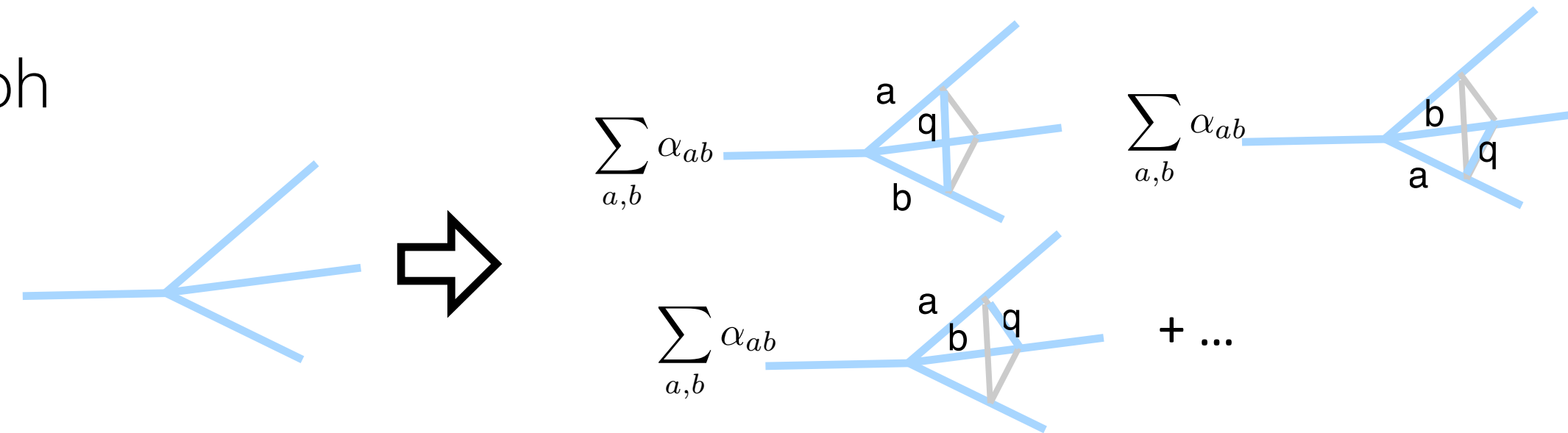


Constraints (quantum Einstein equations)

- * Diff constraint: Roughly speaking

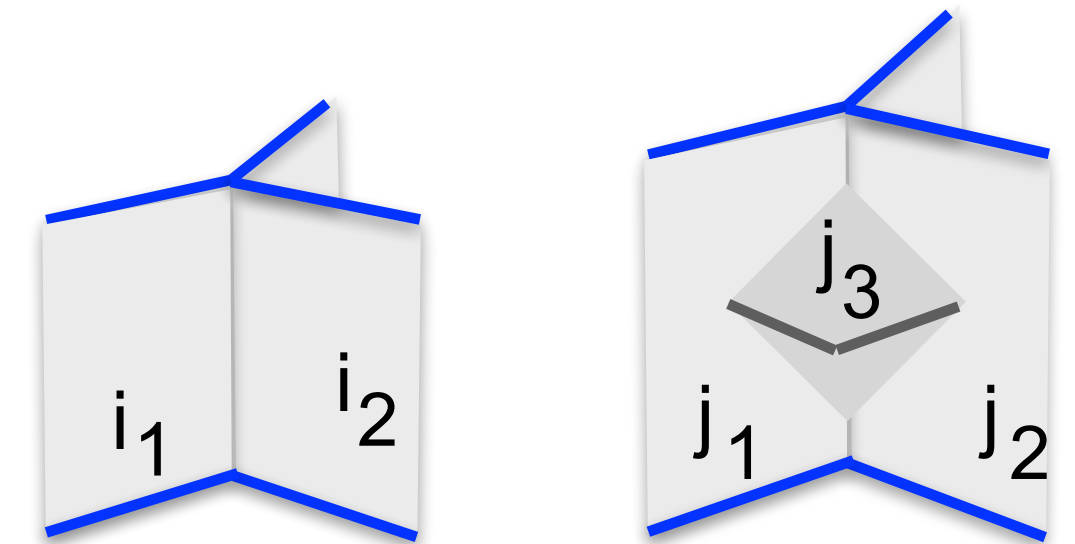


- * Hamilton constraint: Changing the graph



- * Matrix elements of projector on solutions: have expansion labeled by coloured 2-complexes

$$\begin{aligned} \langle S | P_{\text{phys}} | S' \rangle &= \langle S | S' \rangle + i \int DN \int dx N(x) \langle S | \hat{C}(x) | S' \rangle \\ &\quad - \frac{1}{2} \int DN DN' \int dx dx' N(x) N'(x') \langle S | \hat{C}(x) \hat{C}(x') | S' \rangle + \dots \end{aligned}$$



- * Similar picture from discretized path integral —> Spinfoam (covariant) loop quantum gravity



Covariant loop quantum gravity



Spin foam [Ponzano-Regge, Baez, Rovelli+Reisenberger,.....]

For group G :

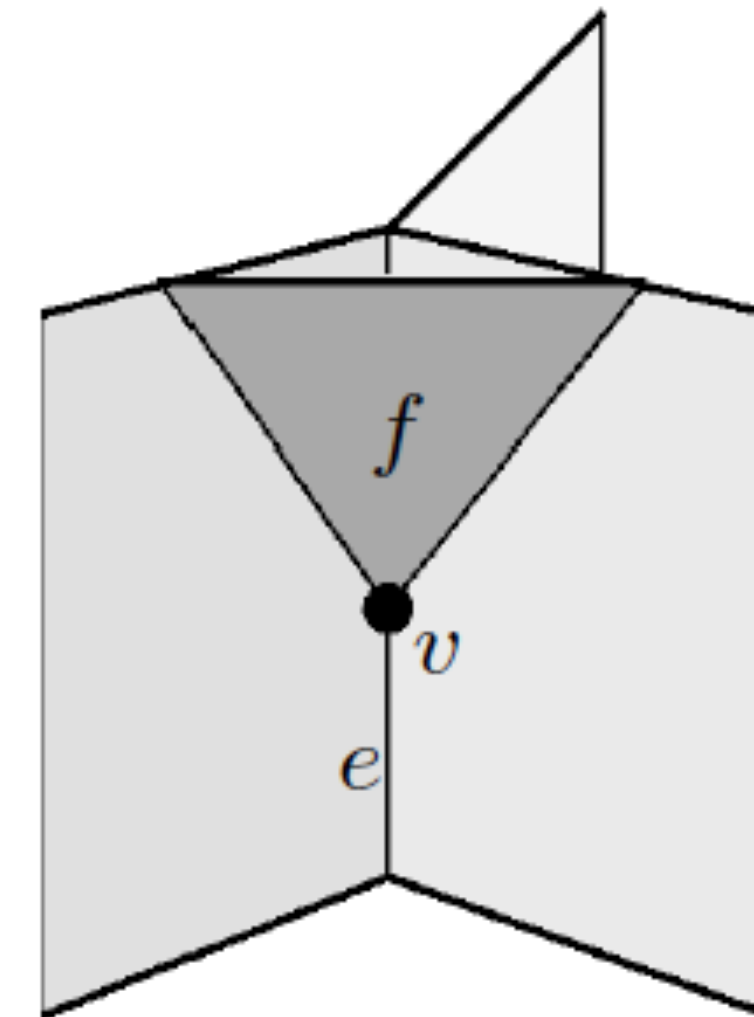
- *oriented 2-complex (with boundary) C
- *labeling of faces with irreps of G
- *labeling of edges with interwiners between face-reps

Amplitude:

$$Z : \{ C, \text{ boundary data} \} \longrightarrow \mathbb{C}$$

Typically:

$$Z = \sum_{\{f\} \rightarrow \text{Irrep } G} \sum_{\iota} \prod_f A_f(\pi) \prod_v A_v(\pi_e, \iota_n)$$



Can rewrite **Holst action** as

$$S[B, \omega, \phi] = \int B \wedge F(\omega) + \phi B \wedge B$$

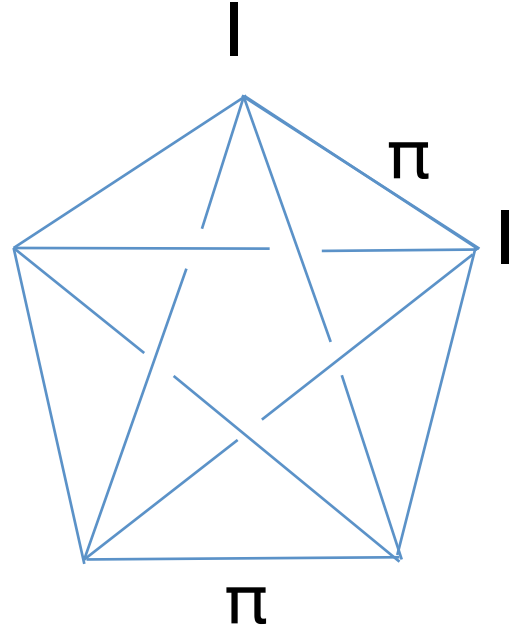
where ϕ enforces *twisted simplicity*

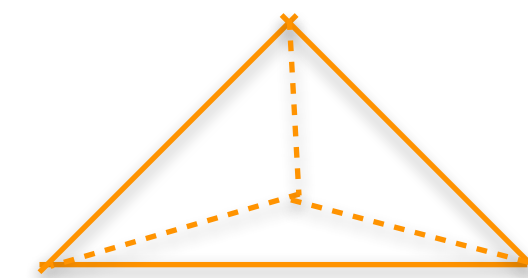
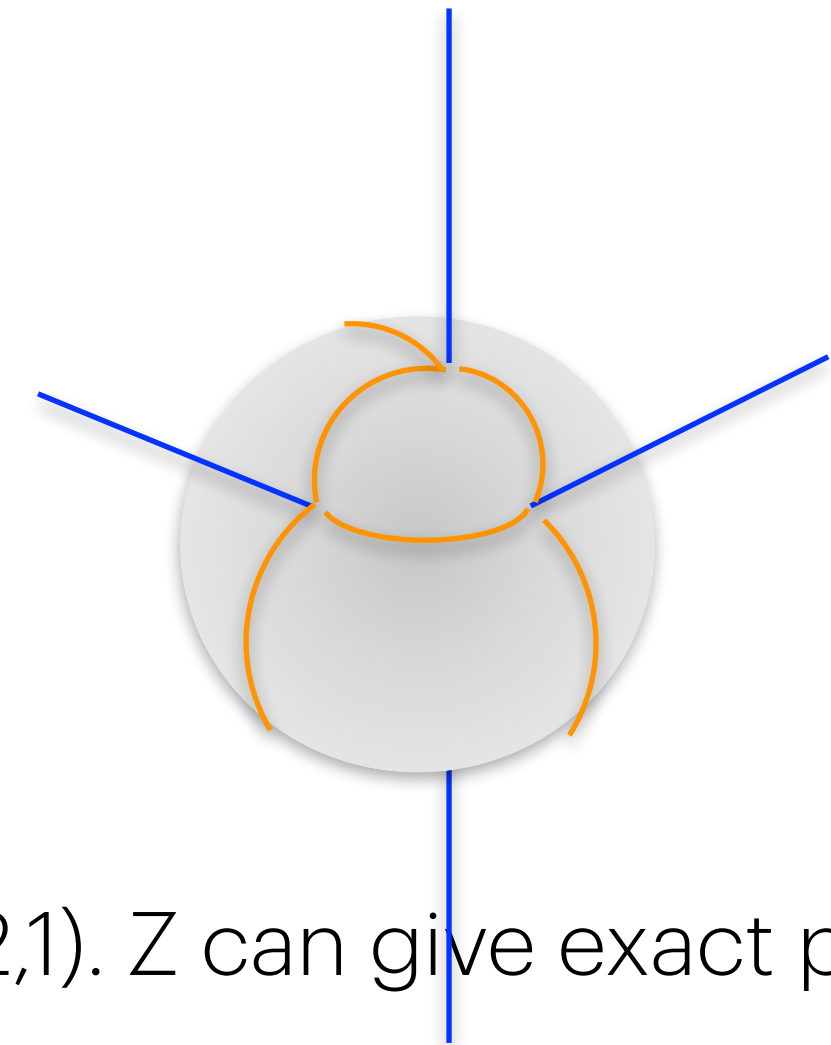
$$B = *(e \wedge e) + \frac{1}{\beta} e \wedge e$$

BF theory

$$S[B, \omega] = \int \text{tr}[B \wedge F(\omega)]$$

topological theory with well known quantum theory. In 2+1 equal to gravity for $SO(2,1)$. Z can give exact path integral:

$$Z = \sum_{\{\pi_f\}} \sum_{\{I_e\}} \prod_f A_f(\pi_f) \prod_v \quad \text{(II)}$$


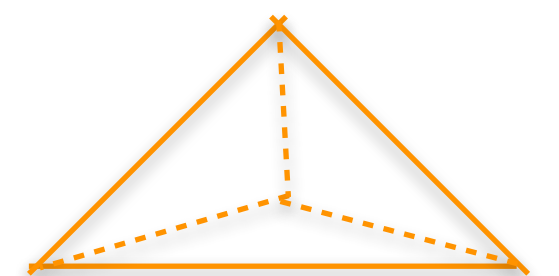
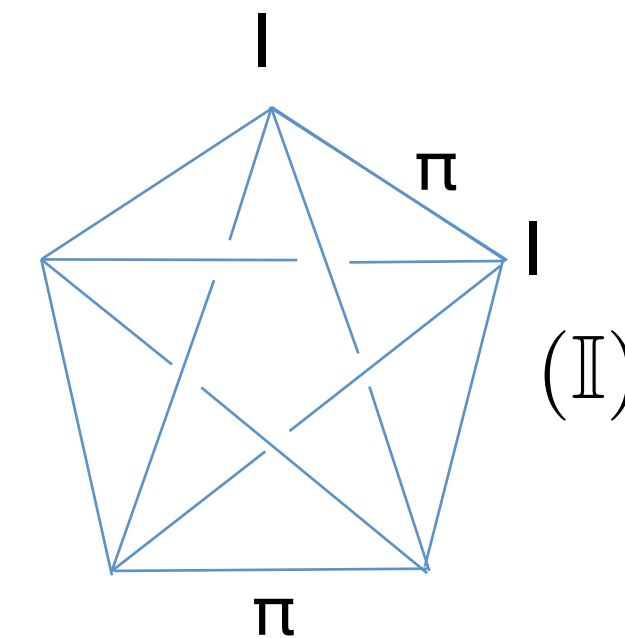
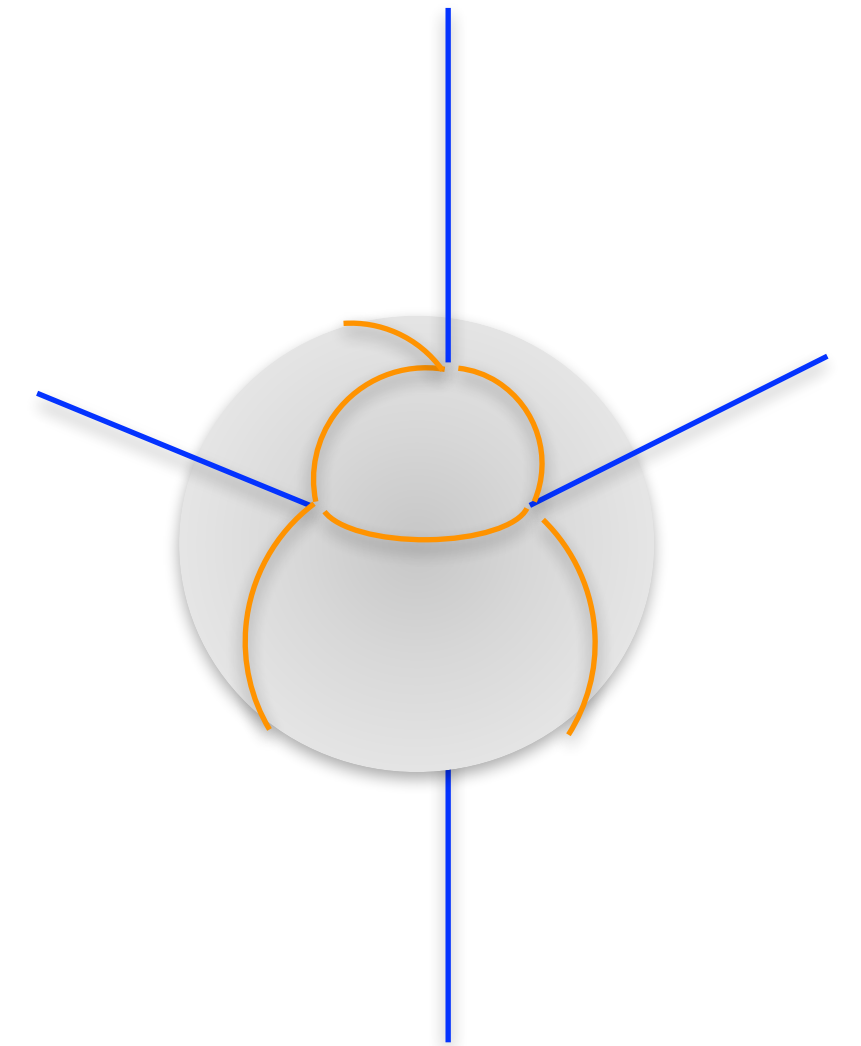


Can more or less match covariant and canonical approach

EPRL-FK vertex amplitude: [Engle, Pereira, Rovelli, Livine, Freidel, Krasnov, ...]

- ✱ T: vertex spin network
- ✱ Y: map $SU(2)$ spin nets $\rightarrow$ $SL(2, \mathbb{C})$ spin nets
- ✱ For choice of $SU(2)$ subgroup

$$A_v(j_e, \iota_n) = (YT_{\{j_e, \iota_n\}})(\mathbb{I}_{SL(2, \mathbb{C})})$$



Different perspective on canonical and covariant LQG:

Group field theory [Ambjorn, Rovelli, Oriti, Gielen, ...]

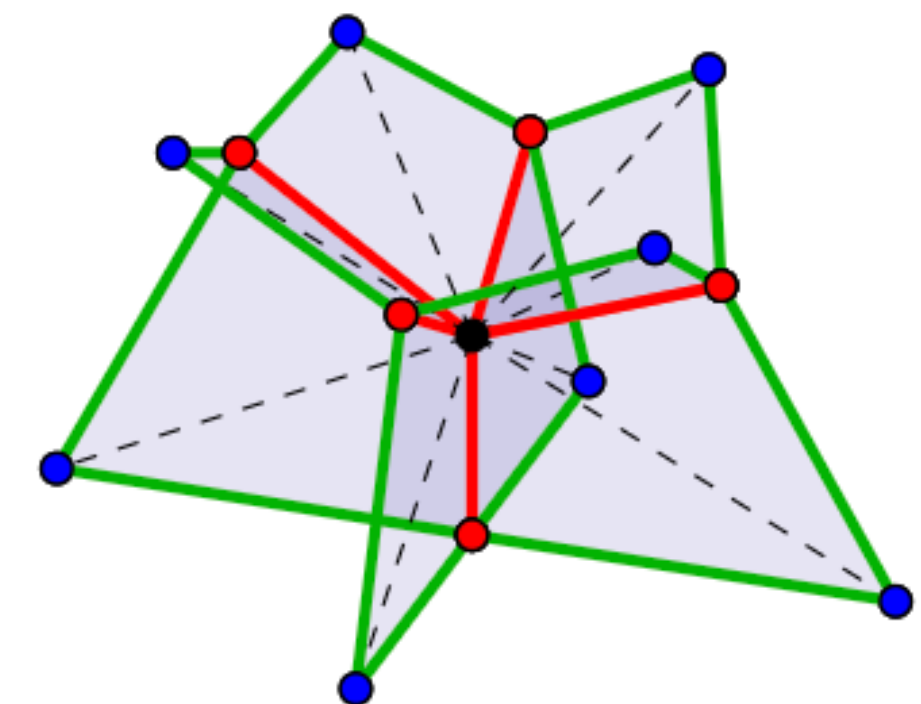
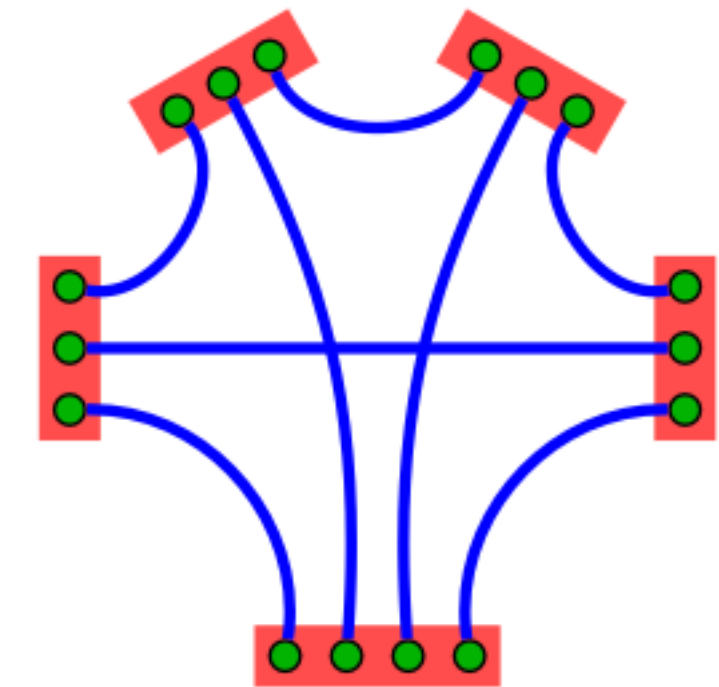
QFT of a fields on product of groups

$$\phi(g_1, g_2, \dots, g_D) : G^{\otimes D} \rightarrow \mathbb{C}$$

For suitable kinematics, action, in principle

- * Creation operators create spin network vertices (atoms of space)
- * Feynman expansion = spin foam expansion

Technical matching with LQG Hilbert spaces not always clear, but beautiful complementary perspective on LQG kinematics and dynamics.



Oriti, Ryan, Thürigen 2015



Why could it be right? The classical answers



Some aspects of classical limit under good control

Canonical formalism: [Thiemann+Winkler, HS, Ashtekar+Lewandowski, Lewine+Speziale,...]

Heat kernel coherent states

$$\psi[A] = \left[e^{-t\Delta_{\mathcal{A}}} \delta_{A'}(A) \right]_{A' \rightarrow C}$$

- * approximate classical metric in expectation values
- * small fluctuations
- * Approximate solutions of constraints

Covariant formalism: [Barrett et al '11,]

Vertex amplitude asymptotes to Regge-action when evaluated on boundary coherent state

$$\sum_{j_e} \sum_{\iota_n} A_v^{\text{EPRL}}(j_e, \iota_n) \Psi_{\partial\Delta}(j_e, \iota_n) \propto e^{iS_{\text{Regge}}(\partial\Delta)} + e^{-iS_{\text{Regge}}(\partial\Delta)}$$

$$S_{\text{Regge}}(\partial\Delta) = \sum_{i < j} A_{ij} \theta_{ij}$$

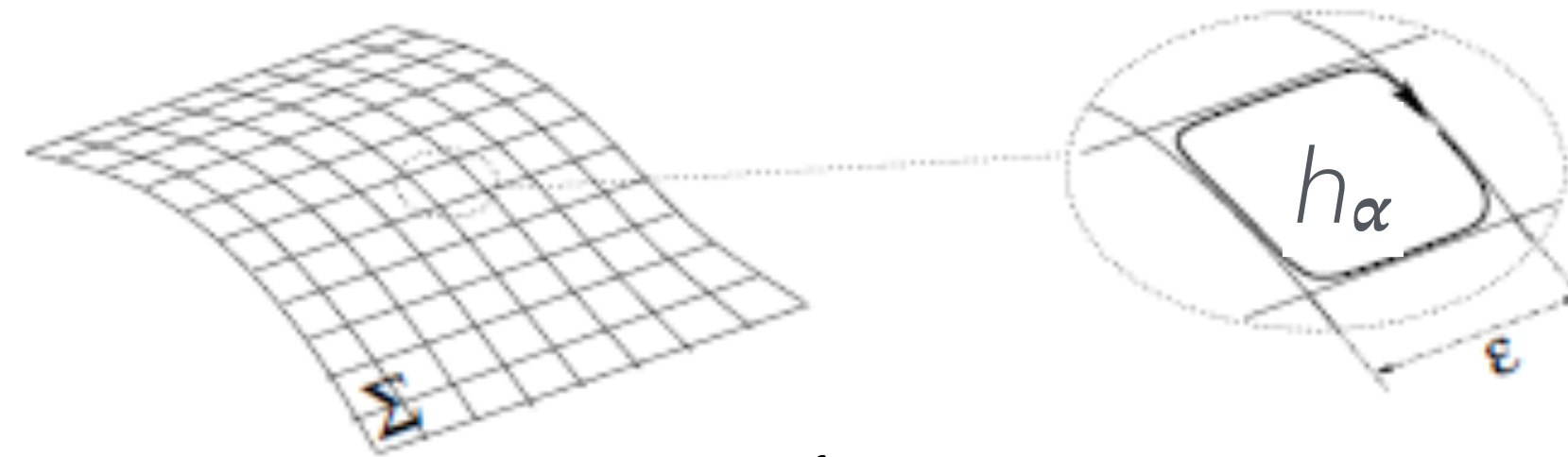
where Ψ is peaked on boundary geometry $\partial\Delta$ of Lorentzian large 4-simplex Δ .

(Euclidean) 3d Quantum Gravity can be described this way

[Ponzano, Regge, Barrett, Crane, ...] [Freidel, Livine, Noui, Perez,]

Canonical

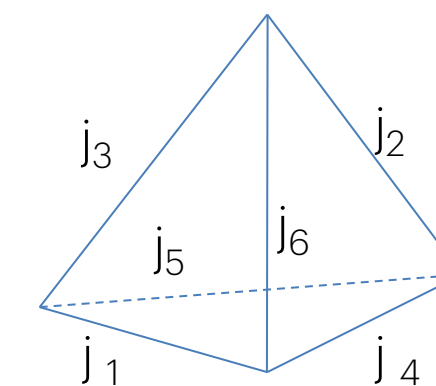
$$C[N] = \prod_{\alpha} \text{tr}(N_{\alpha} h_{\alpha})$$



from Perez 06

Reisenberger-Rovelli expansion of projector gives Ponzano-Regge [Perez+Noui]

$$\left\langle \begin{array}{c} 1 \\ \diagup \quad \diagdown \\ 2 \quad 3 \end{array}, \begin{array}{c} 5 \quad 2 \\ \diagup \quad \diagdown \\ 1 \quad 6 \\ \diagdown \quad \diagup \\ 4 \quad 3 \end{array} \right\rangle_{ph} = \sqrt{\Delta_4 \Delta_5 \Delta_6} \left\{ \begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array} \right\} \times \text{rest}$$



Black holes

Isolated horizons: [Ashtekar et al, Engle+Noui+Perez]

Quasilocal notion of BH horizon, strong enough for BH thermodynamics.

Boundary condition at horizon

$$F^I(A^\beta) \Big|_{\leftarrow} = \frac{(1 - \beta^2)\pi}{a_H} \sum_{\leftarrow}^I$$

Symplectic structure acquires CS boundary term.

Quantum theory: [Ashtekar+Baez+Krasnov, Engle+Noui+Perez, many others]

CS on boundary, LQG in bulk $\mathcal{H}_{(j_1, j_2 \dots)} = \mathcal{H}_{CS}(j_1, j_2 \dots) \otimes \mathcal{H}_{\text{bulk}}$

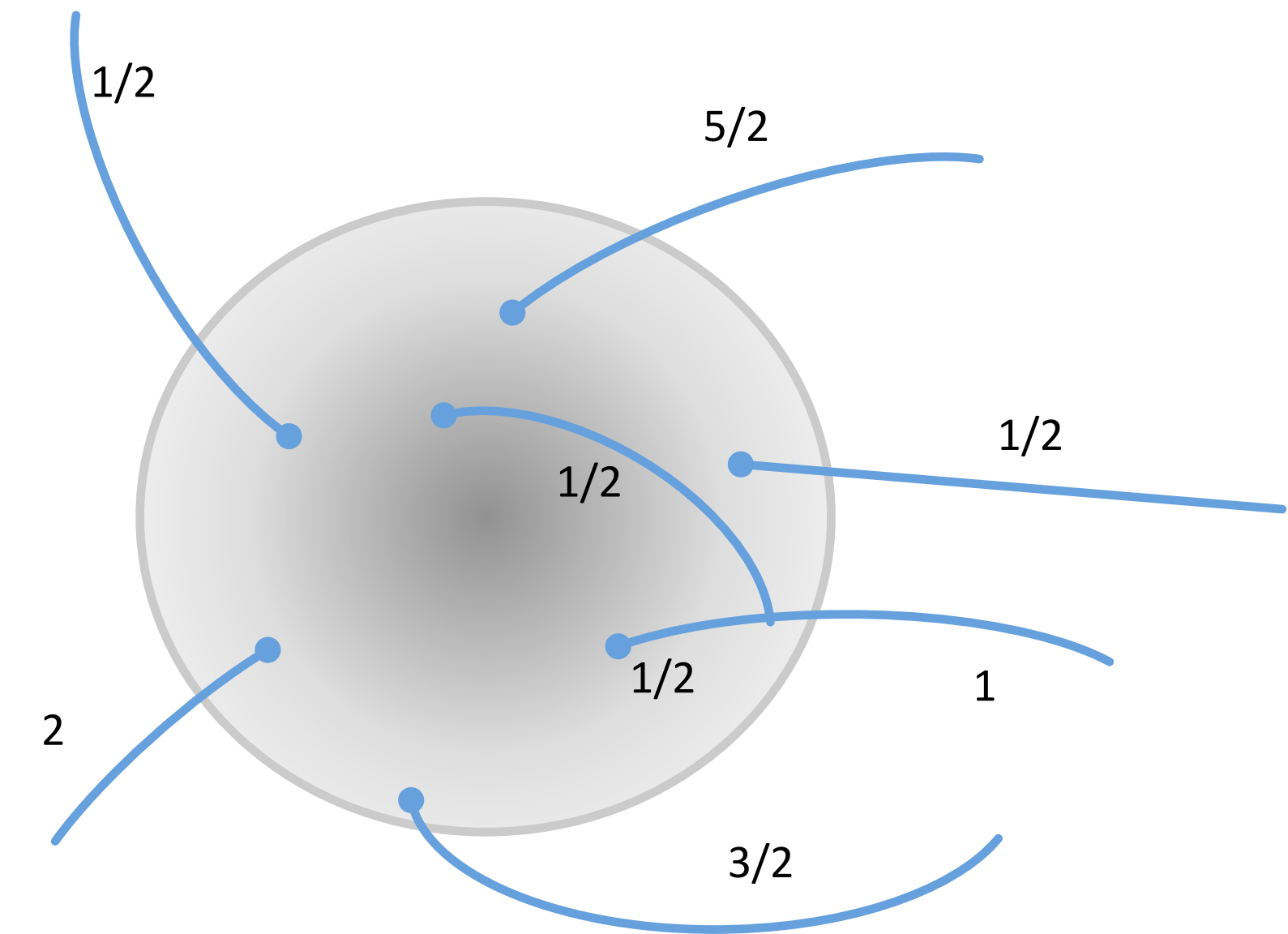
$$\mathcal{H}_{a_H} = \bigoplus_{4\pi\beta \sum_i \sqrt{j_i(j_i+1)} \approx a_H} \mathcal{H}_{(j_1, j_2 \dots)}$$

Entropy:

$$\ln N_{\leq}(a) = \frac{\tilde{\beta}}{2\pi\beta} \frac{a_H}{4l_P^2} - \frac{3}{2} \ln(a_H/l_P^2) + O(a_H^0)$$

reproduces Bekenstein-Hawking for correct choice of β .

Similar results for more general types of BH with same choice of β .



Symmetric sectors of gravity as testbed for LQG

Example: **Loop quantum cosmology**

[Bojowald, Ashtekar, Pawłowski, Singh, many others]

FRW with massless scalar: Canonical variables

$$A_a^I = c \omega_a^I, \quad E_I^a = p \sqrt{q_0}^0 e_I^a$$

Quantization in terms of holonomies $\hat{=}\exp(ilc)$, Fluxes $\hat{=}\ p$

$$p|l\rangle = \frac{8\pi\beta l_P^2}{3}, \quad \exp(il'c)|l\rangle = |l + l'\rangle$$

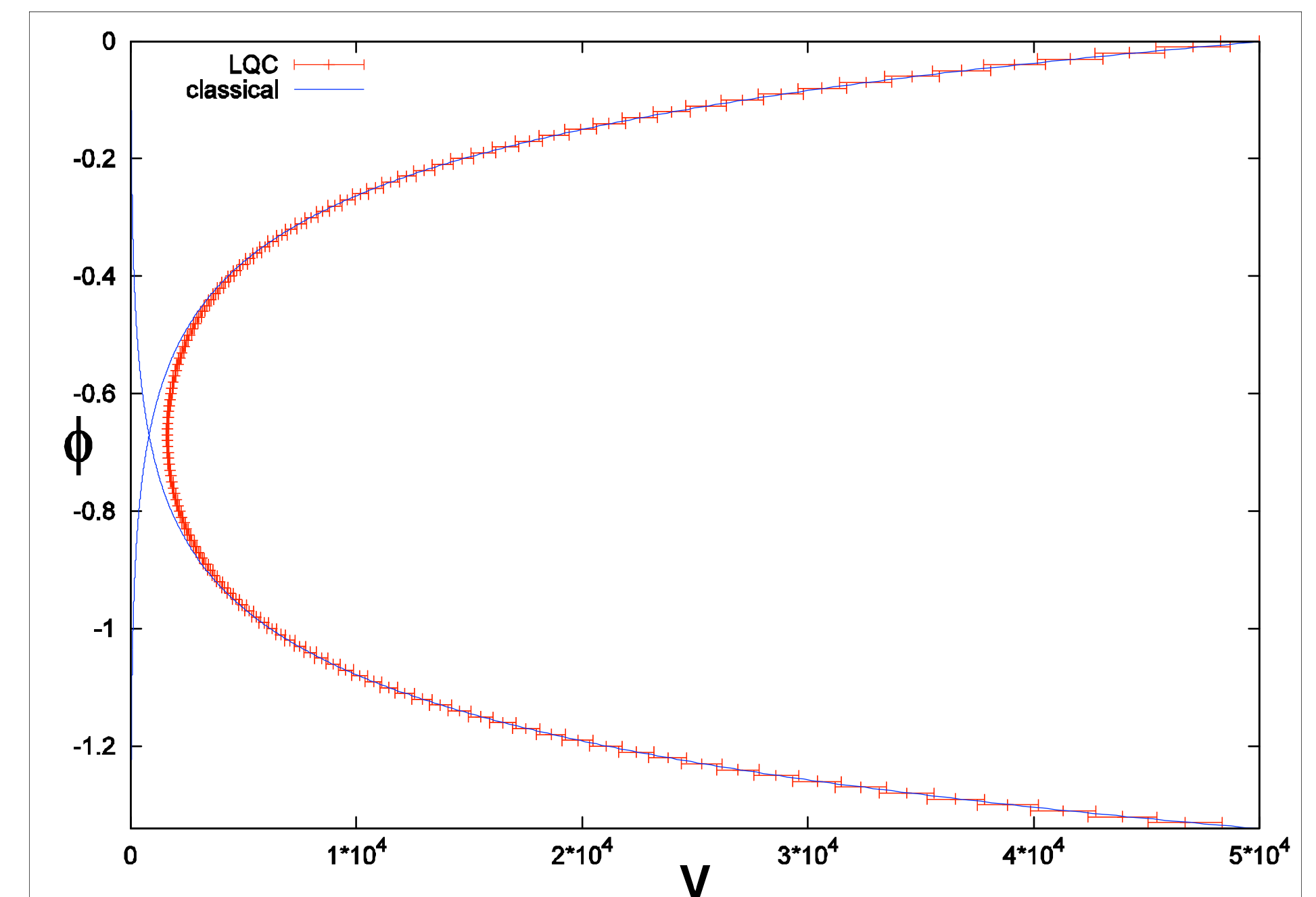
Quantum constraint

$$\partial_\phi^2 \Psi(\phi, l) = \hat{C} \Psi(\phi, l)$$

$$\hat{C} |l\rangle = f_0(l) |l\rangle + f_+(l) |l + \delta_+(l)\rangle + f_-(l) |l - \delta_-(l)\rangle$$

scalar = time, physical Hilbert space = positive frequency space

- ✳ Can GR be reproduced far away from the singularity?
- ✳ What happens at the big bang singularity?
- ✳ Some aspects derivable from full theory.



From Ashtekar, Pawłowski, Singh



Recent progress



Better understanding of Spinfoams

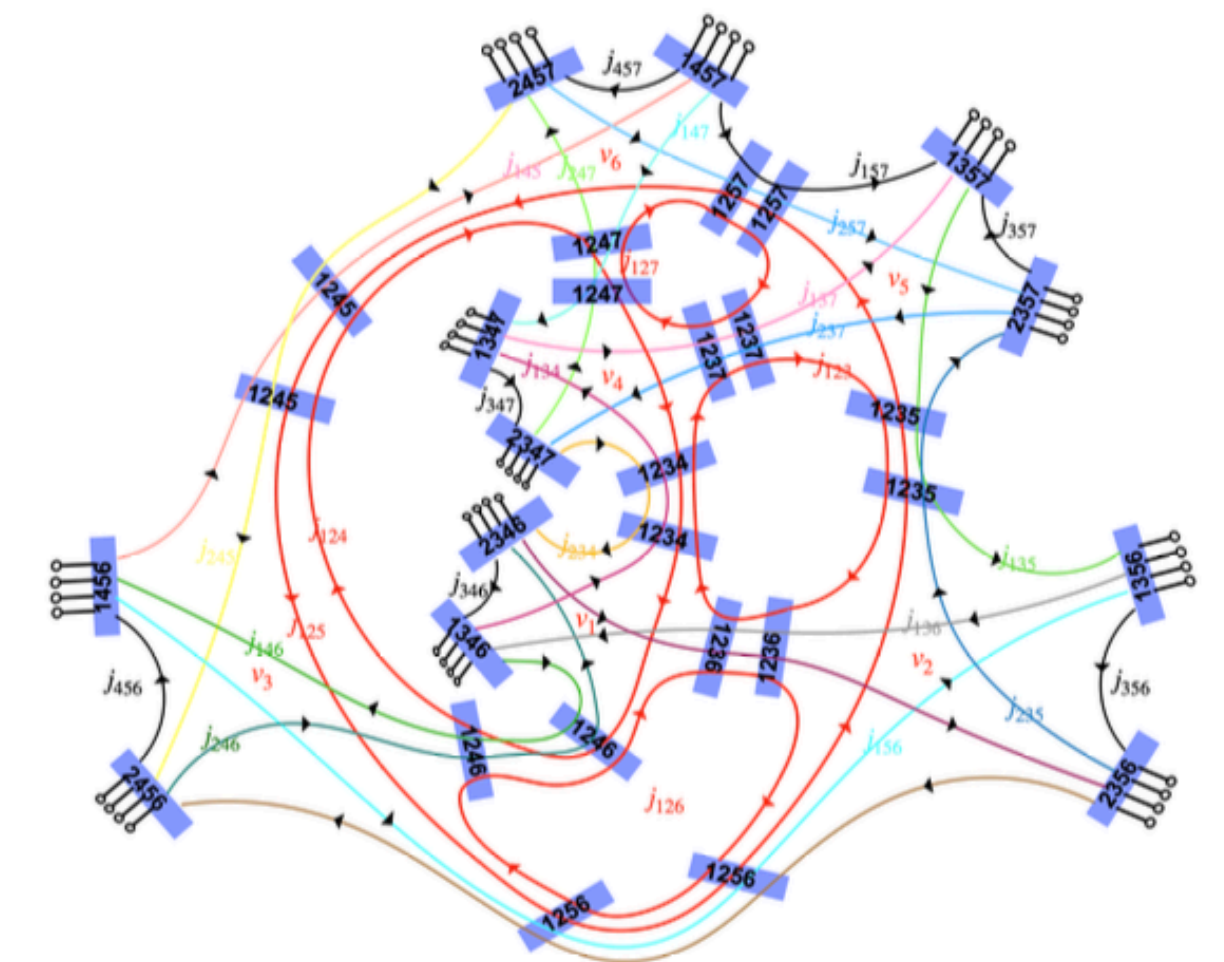
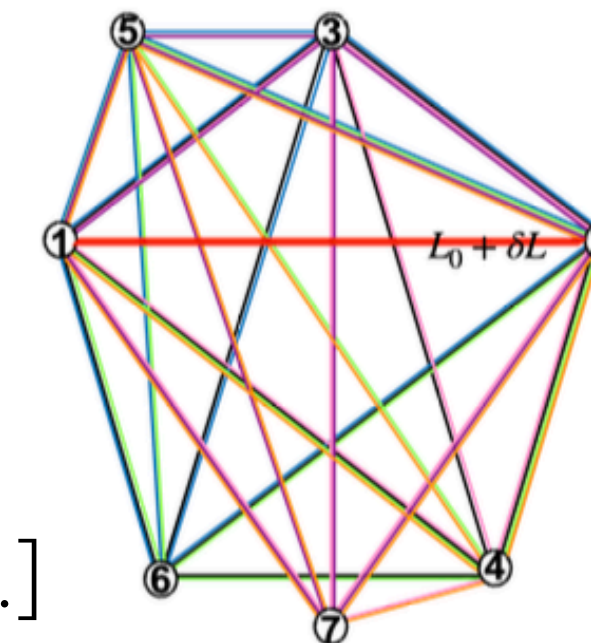
Amplitudes, critical points for foams with multiple 4-simplices? Flatness problem?

Various approaches and tools

- ✱ Effective spinfoams [Asante, Dittrich, Kogios, Padua-Argüelles, Steinhaus,...]: Computationally tractable models incorporating key features of LQG
- ✱ numerical methods [Asante, Han, Dittrich, Dona, Liu, Speziale, Steinhaus, ..]: sl2cfoam,
- ✱ Simultaneous large j and refinement limit [Engle, Han, Liu, Rovelli,...]
- ✱ Area-metric gravity [Borissova, Dittrich, Padua-Argüelles,...]

show: understanding semiclassical consistency, propagating DOF is subtle, but no obvious showstopper. Also:

- ✱ spinfoam cosmology [Bianchi, Han, Kisielowski, Rovelli, Vidotto,...]
- ✱ Alternative vertex amplitudes: Causality/orientation, simplicity constraints [Bianchi, Engle, Oriti,...]

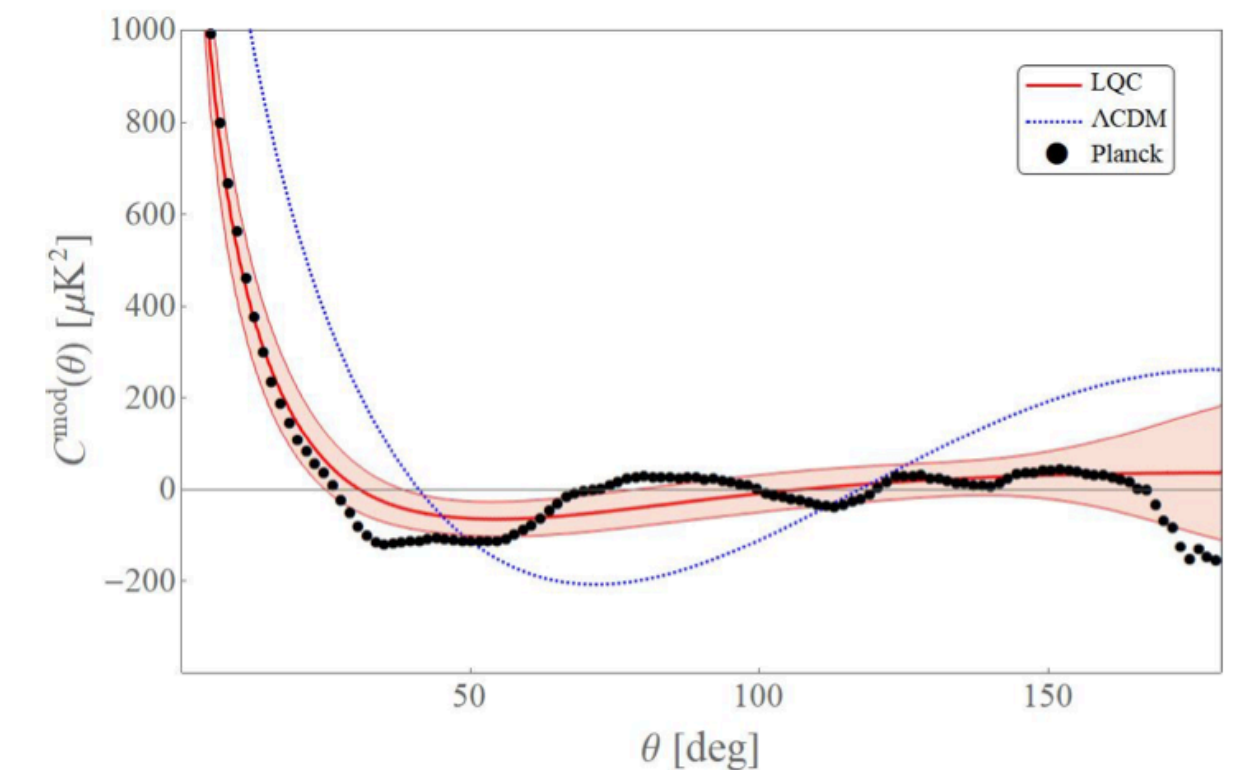
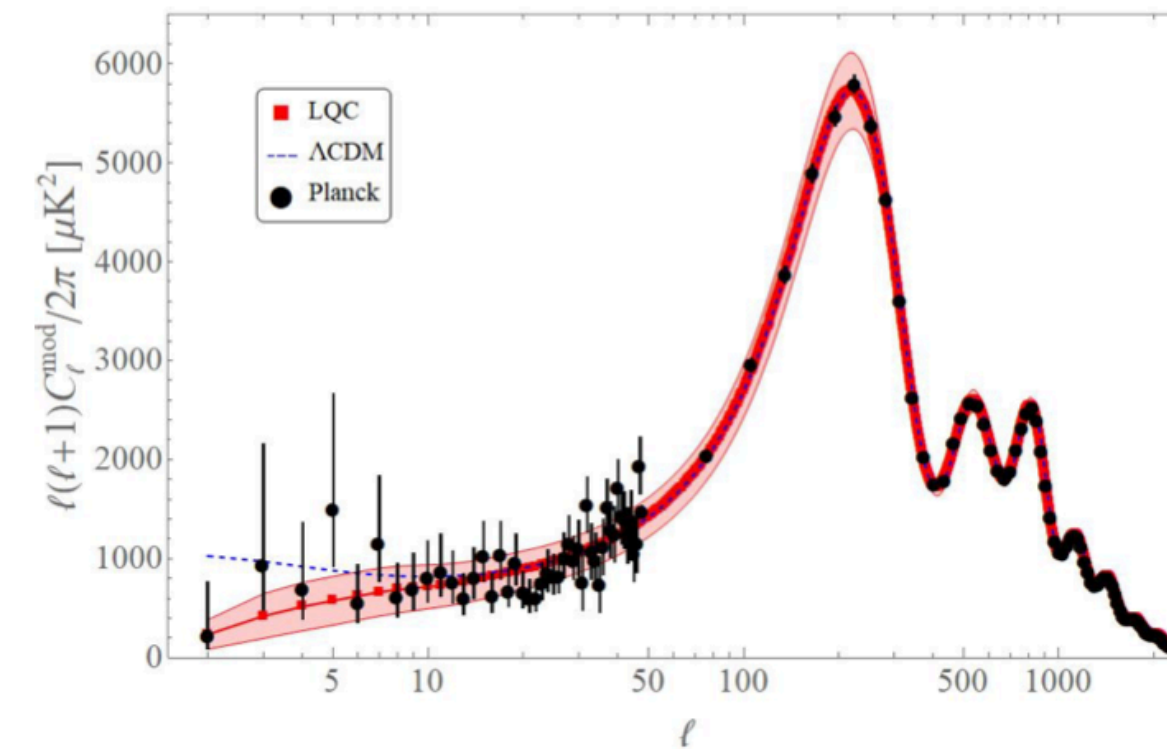


from Han, Liu, Qu 2023

Extended work on symmetry reduced models

Sophisticated models based on LQC

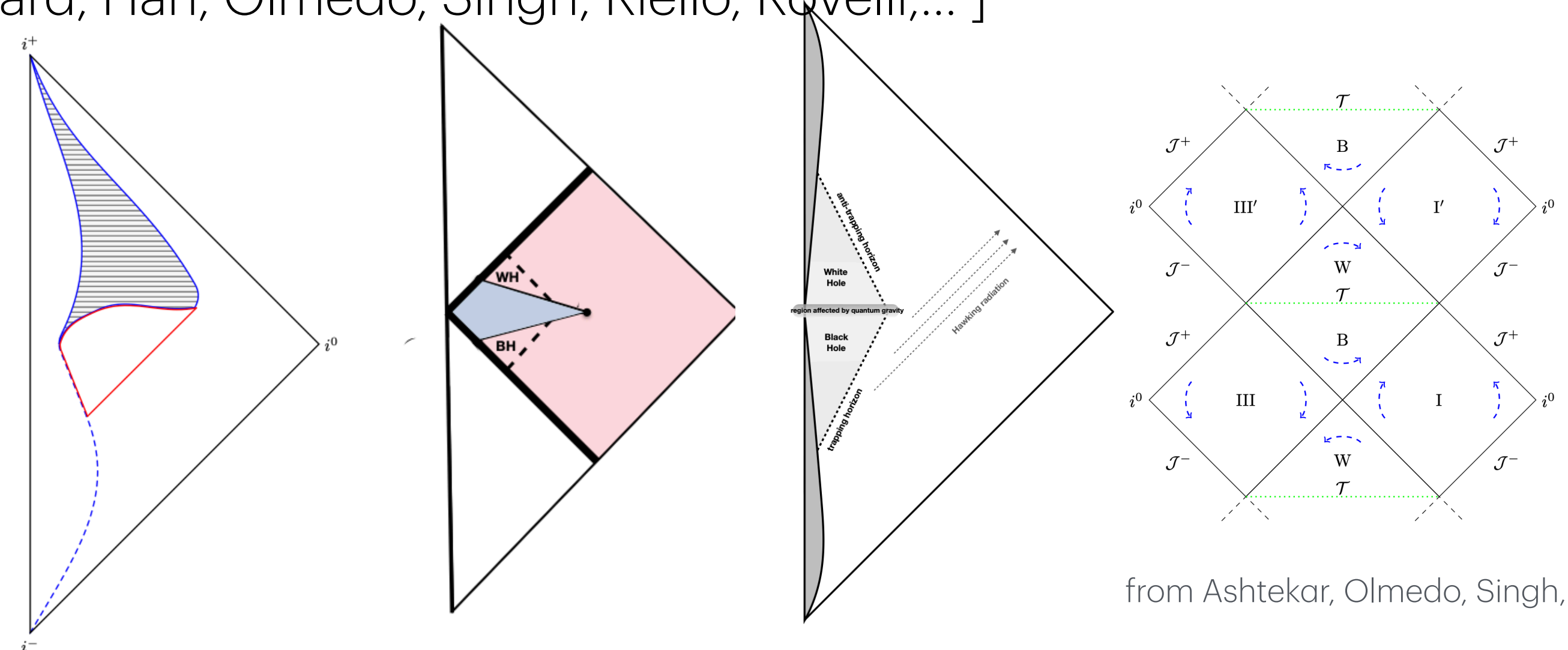
- ✳ Bounce effects can alleviate anomalies
[Ashtekar et al, Agullo et al, Mena et al,...]



Black hole to white hole transition, matter bounce

- ✳ Effective models with LQC techniques, SF amplitudes [Ashtekar, Haggard, Han, Olmedo, Singh, Riello, Rovelli,...]
- ✳ Notion of general covariance of effective theory [Giesel, Lewandowski, Liu, Ma, ...]
- ✳ Bounce with matter [Giesel, Fazzini, Husain, Wilson-Ewing,]
- ✳ Remnants? [Rovelli, Vidotto,...]

from Agullo, Krasas, Sreenath, 2021



from Ashtekar, Olmedo, Singh, 2023

Other notable areas

Quantum information

- ✱ Entanglement in states of LQG [Bianchi, Han, Livine, Speziale, HS,...]
- ✱ Aspects of holography, AdS/CFT [Chirco, Colafranceschi, Oriti, Han, HS,...]
- ✱ Quantum reference frames vs. observables in gauge theories, gravity [Brukner, Dittrich, Giacomini, Hoehn, Thiemann, ...]

Group field theory

- ✱ GFT condensates for quantum cosmology [Hoehn, Gielen, Oriti,...]

SUGRA

- ✱ LQG in higher dimensions [Thiemann, ...]
- ✱ LQG with partially manifest SUGRA [Eder, Livine, HS,...]



Own recent work

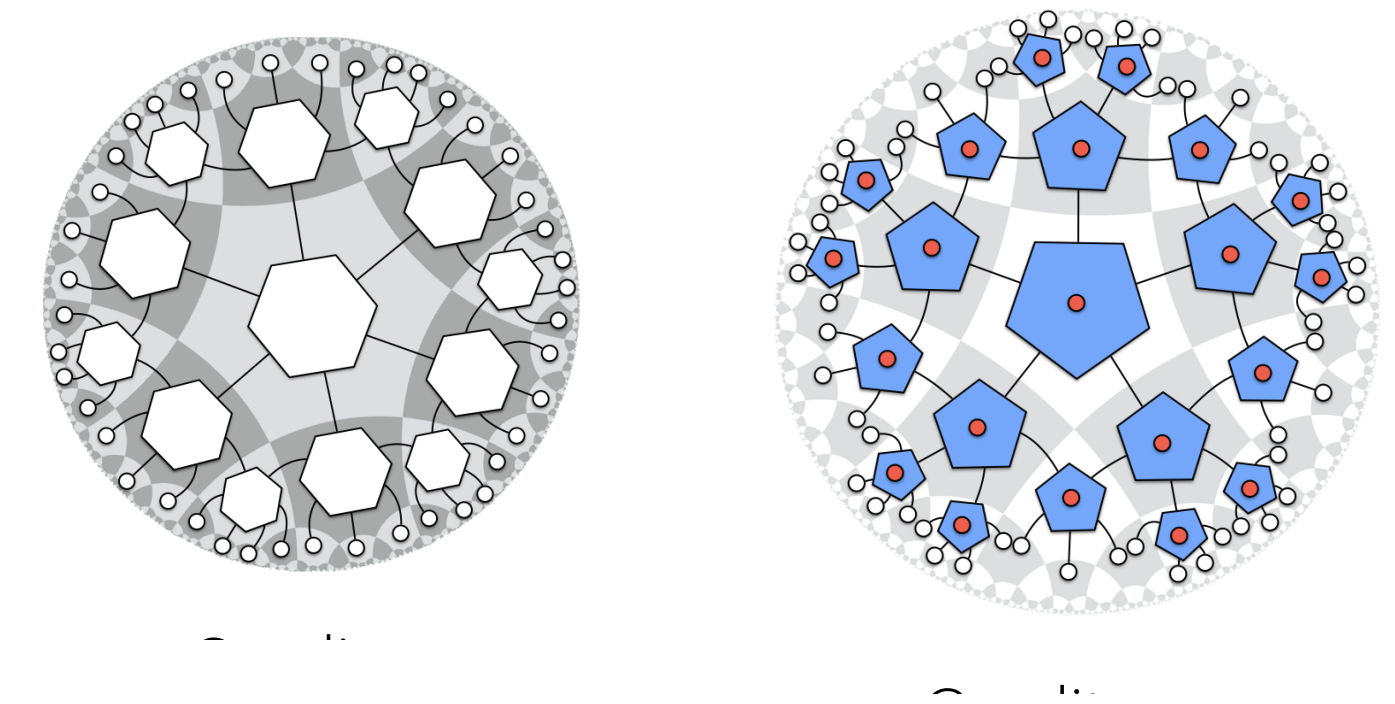


An AdS/CFT Model from LQG principles [Otto, Mansuroglu, Schuch, Gühne, HS]

Holographic tensor networks: [Pastawski et al. 2015]

Models for AdS/CFT in 2+1 dimensions:

Tensor:
[[5,1,3]]₂ perfect code



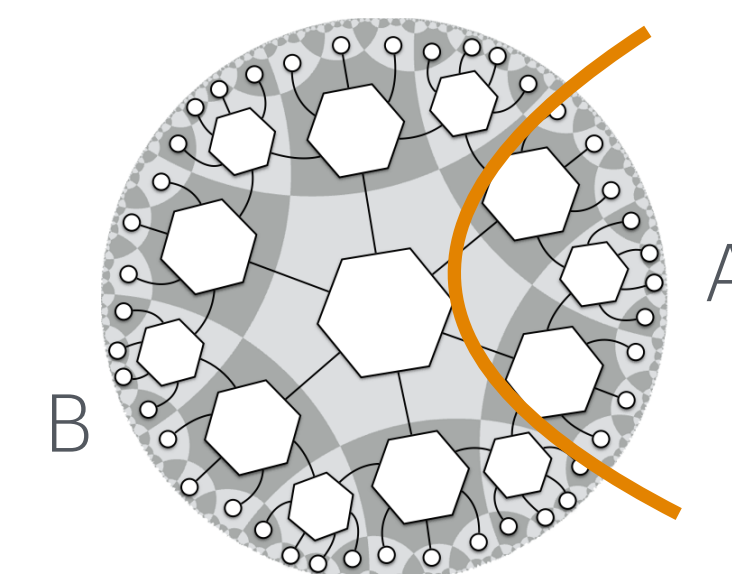
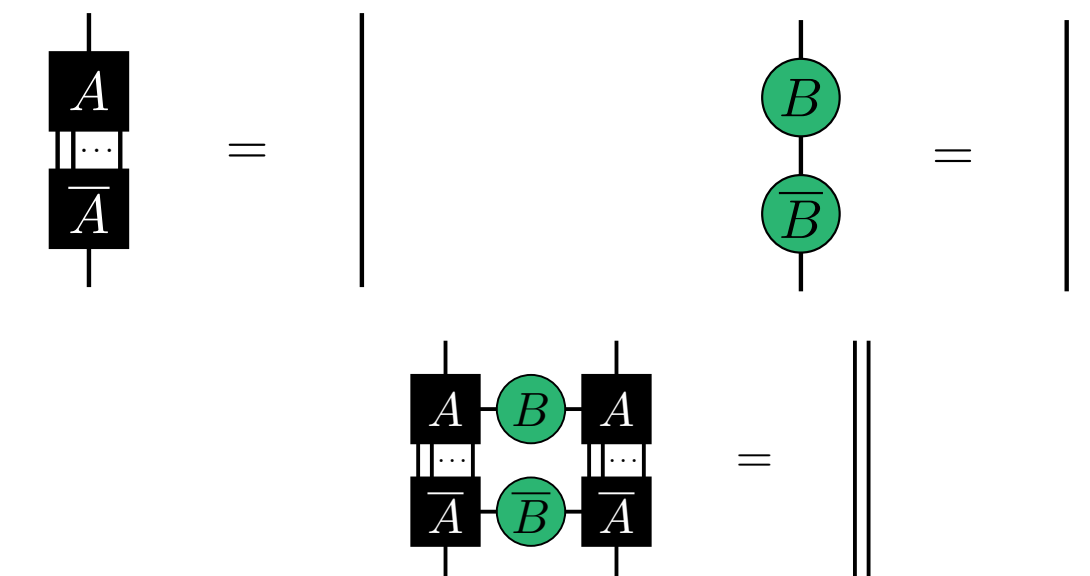
Holographic tensor networks from LQG

- ✱ Use linear combination of spin nets that give hyperinvariant tensor networks
- ✱ Ryu-Takayanagi property
- ✱ Genuine quantum state, with fluctuations of geometry

$$\text{Var}(L_\gamma) = \langle L_\gamma^2 \rangle - \langle L_\gamma \rangle^2 = L_\Gamma(\gamma) \left(\sum_j \ell_j \left(\sqrt{j(j+1)} - c_A \right) \right)$$

$$\frac{\langle L_\gamma^2 \rangle - \langle L_\gamma \rangle^2}{\langle L_\gamma \rangle^2} \simeq \frac{\ell_{\text{Planck}}}{\langle L_\gamma \rangle}$$

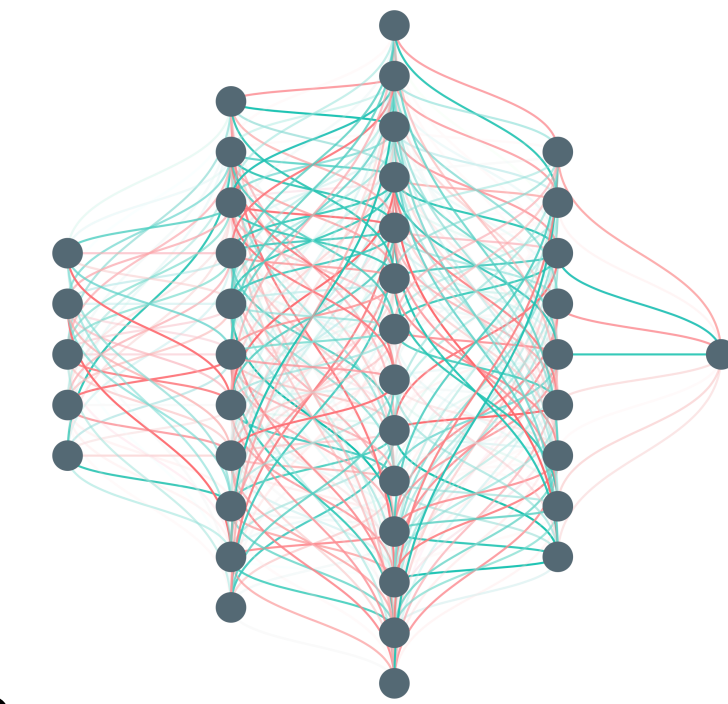
- ✱ Constraints, Bulk DOF: WIP



Machine learning in LQG [Sherif, HS]

Example: Solving constraints via neural network quantum states

$$\langle \Psi_{w,b} | \mathcal{C} | \Psi_{w,b} \rangle \stackrel{!}{=} \min_{(\underline{m}_1, \underline{m}_2, \dots, \underline{m}_N)} \Psi_{w,b}(\underline{m}_1, \underline{m}_2, \dots, \underline{m}_N)$$



Prosaic interpretation: Variational Ansatz for physical state

Romantic interpretation: Special kind of brain learns about gauge invariance etc.

Challenge:

- ✱ Complexity of the constraints

$$\begin{aligned} \hat{H}^\epsilon(N) f_\gamma &= \frac{16}{3i\kappa\ell_p^2} \sum_{v \in V(\gamma)} \frac{N(v)}{E(v)} \sum_{v(\Delta)=v} \epsilon^{IJK} \text{tr}(h_{\alpha_{IJ}(\Delta)} h_{e_K(\Delta)} [h_{e_K(\Delta)}^{-1}, \hat{V}(U_\epsilon(v))]) f_\gamma \\ &+ \frac{64}{3\kappa(i\ell_p^2)^3} \sum_{v \in V(\gamma)} \frac{N(v)}{E(v)} \sum_{v(\Delta)=v} \epsilon^{IJK} \text{tr}(h_{e_I(\Delta)} [h_{e_I(\Delta)}^{-1}, \hat{K}^\epsilon] h_{e_J(\Delta)} [h_{e_J(\Delta)}^{-1}, \hat{K}^\epsilon] h_{e_K(\Delta)} [h_{e_K(\Delta)}^{-1}, \hat{V}(U_\epsilon(v))]) f_\gamma \end{aligned}$$

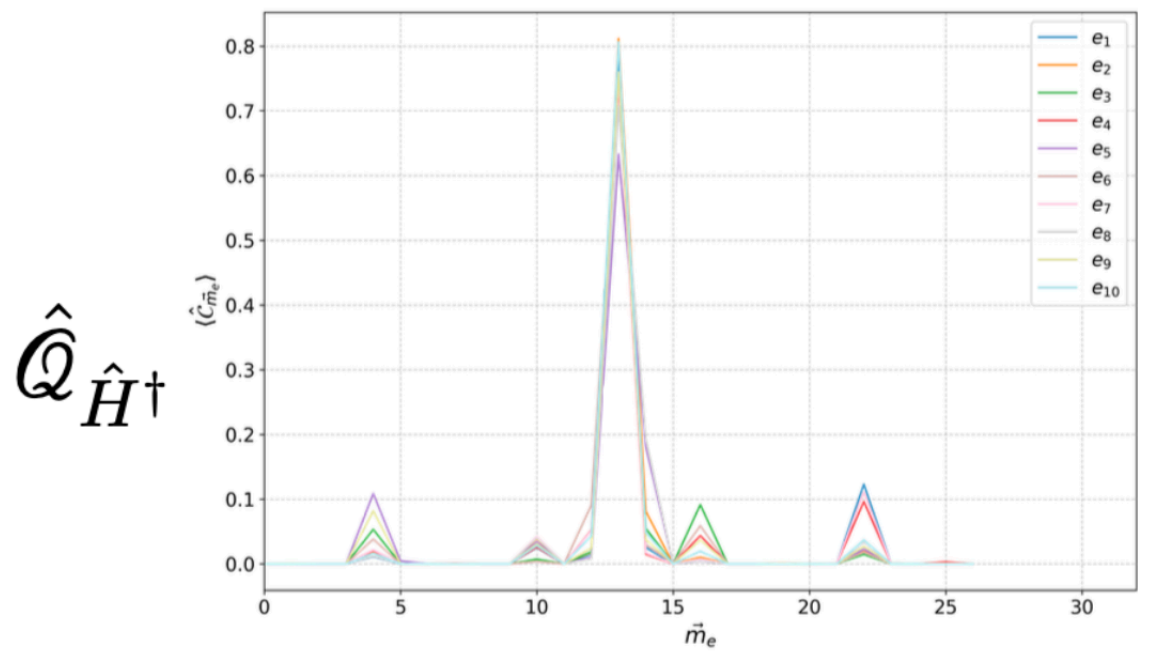
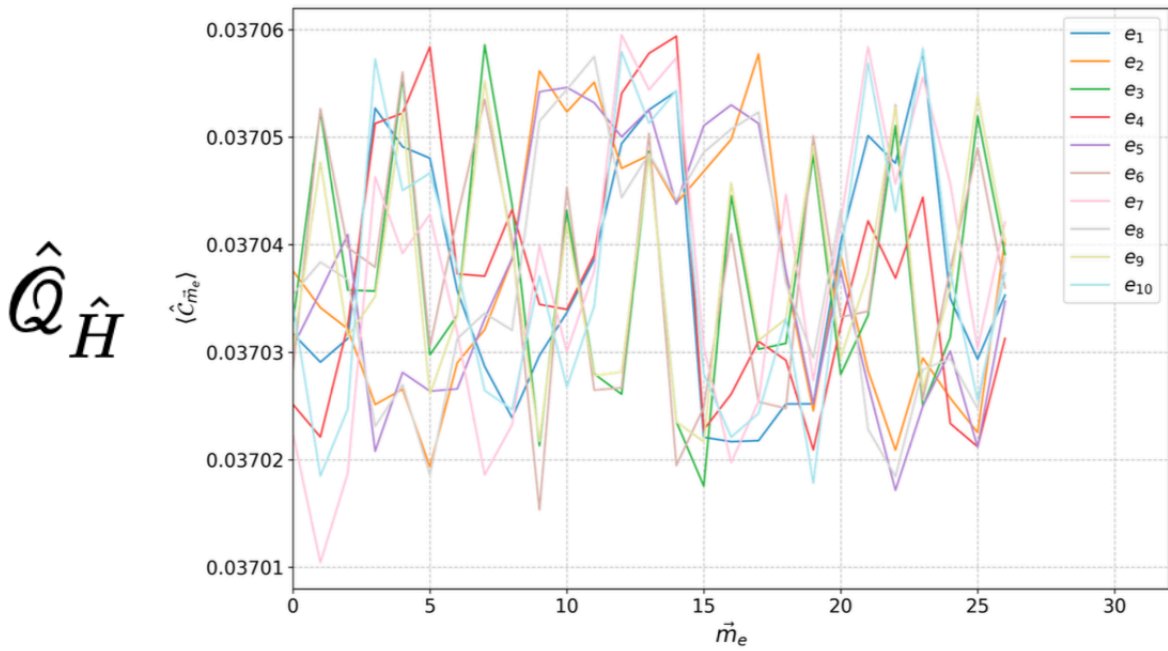
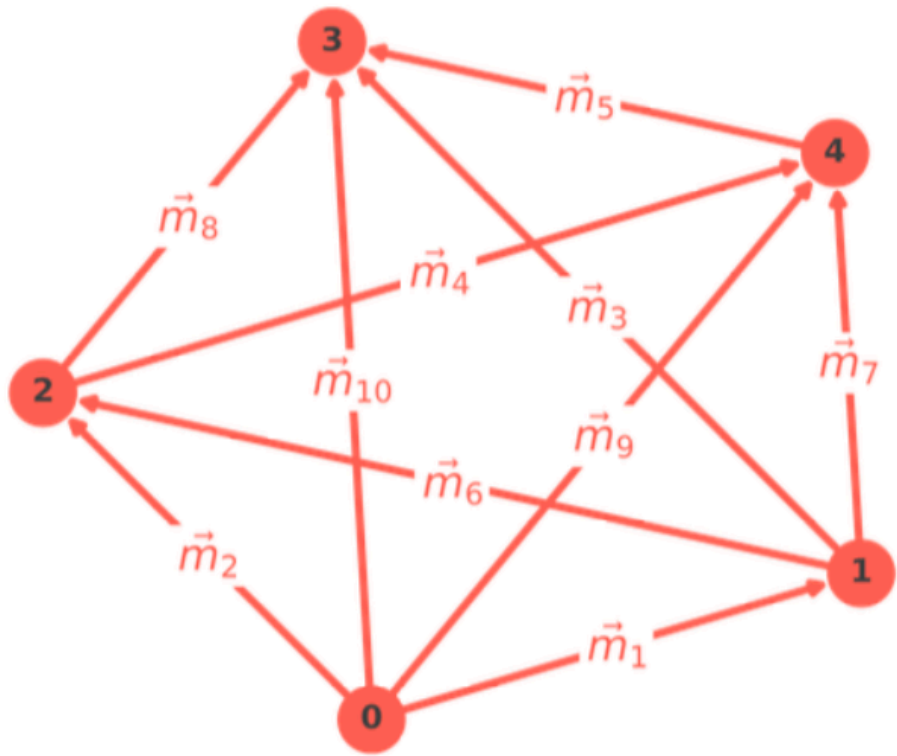
Thiemann 1998

- ✱ Exponential growth of Hilbert space with cutoffs

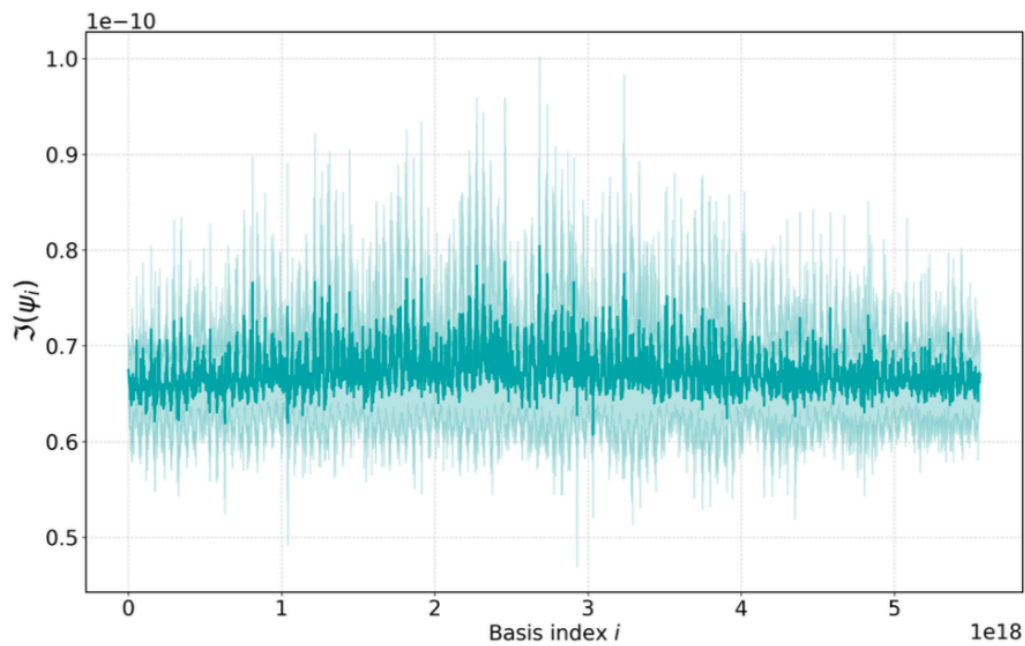
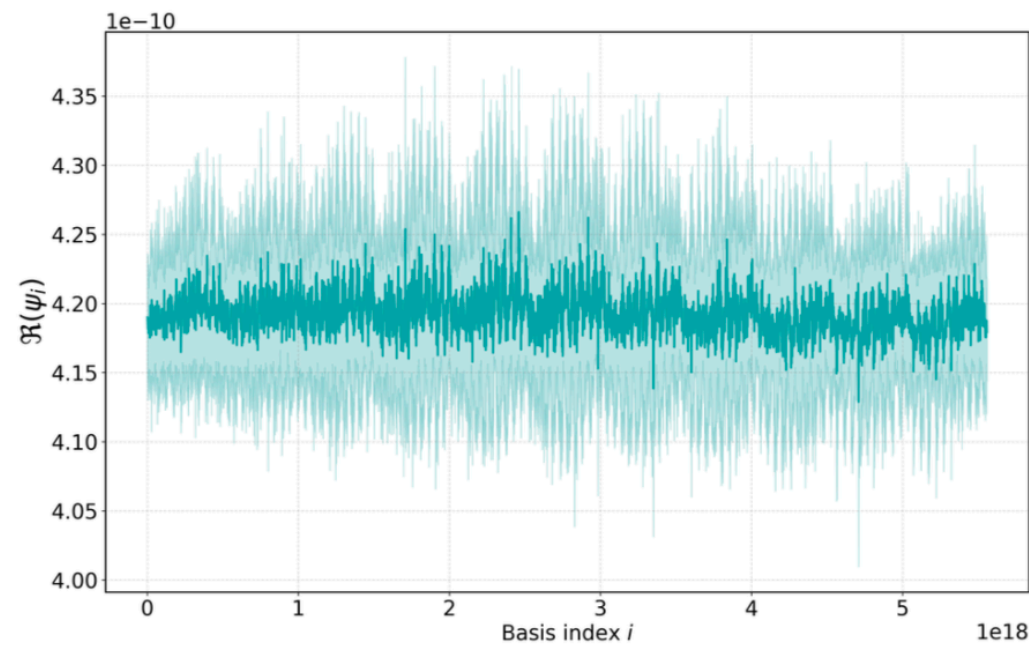


Can analyze simplified $U(1)^3$ Euclidean model on K5

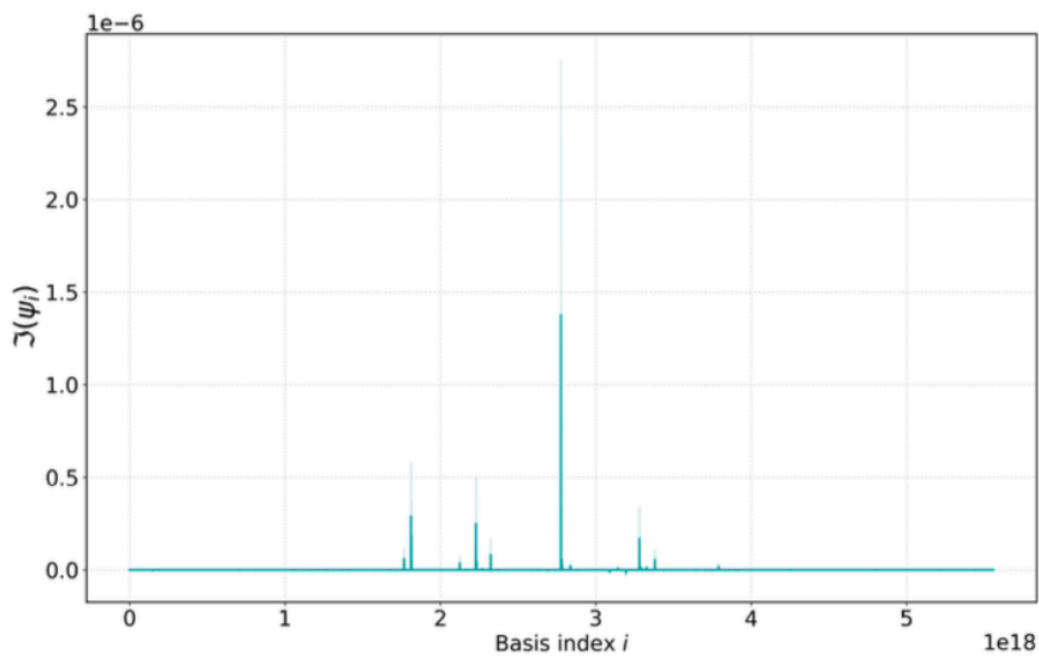
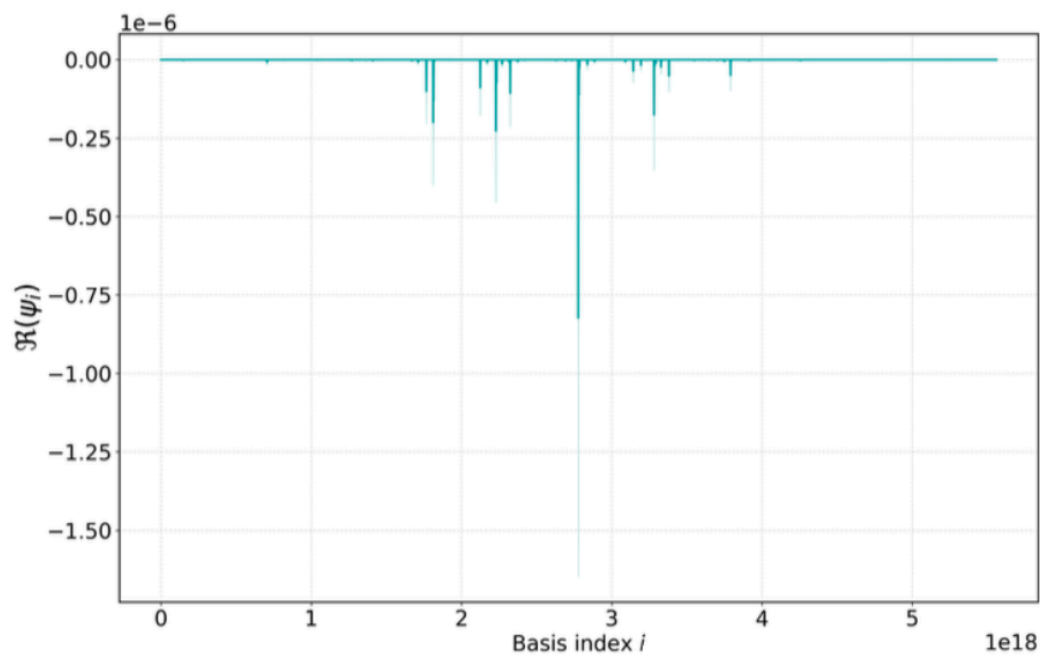
- *dim H up to 10^{18}
- *Expectation values, correlations, geometry
- *ordering ambiguities



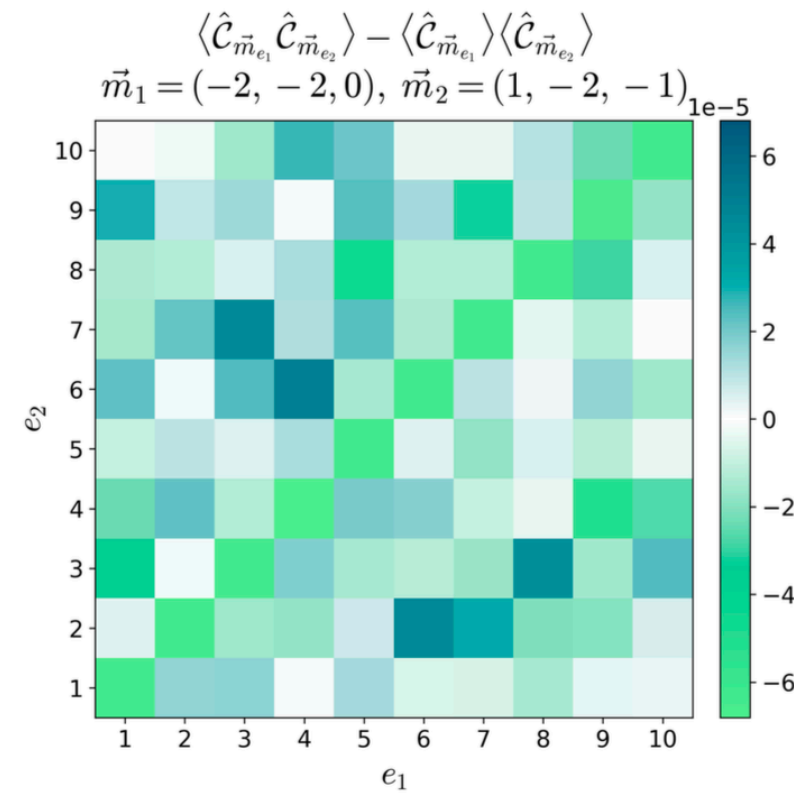
$\hat{Q}_H$



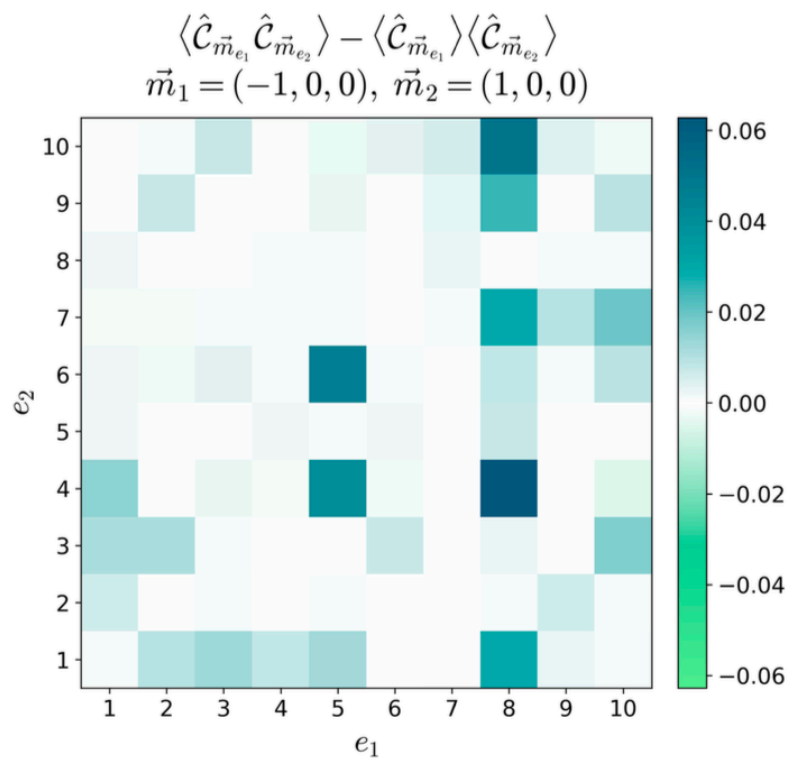
$\hat{Q}_{H^\dagger}$



$\hat{Q}_H$



$\hat{Q}_{H^\dagger}$

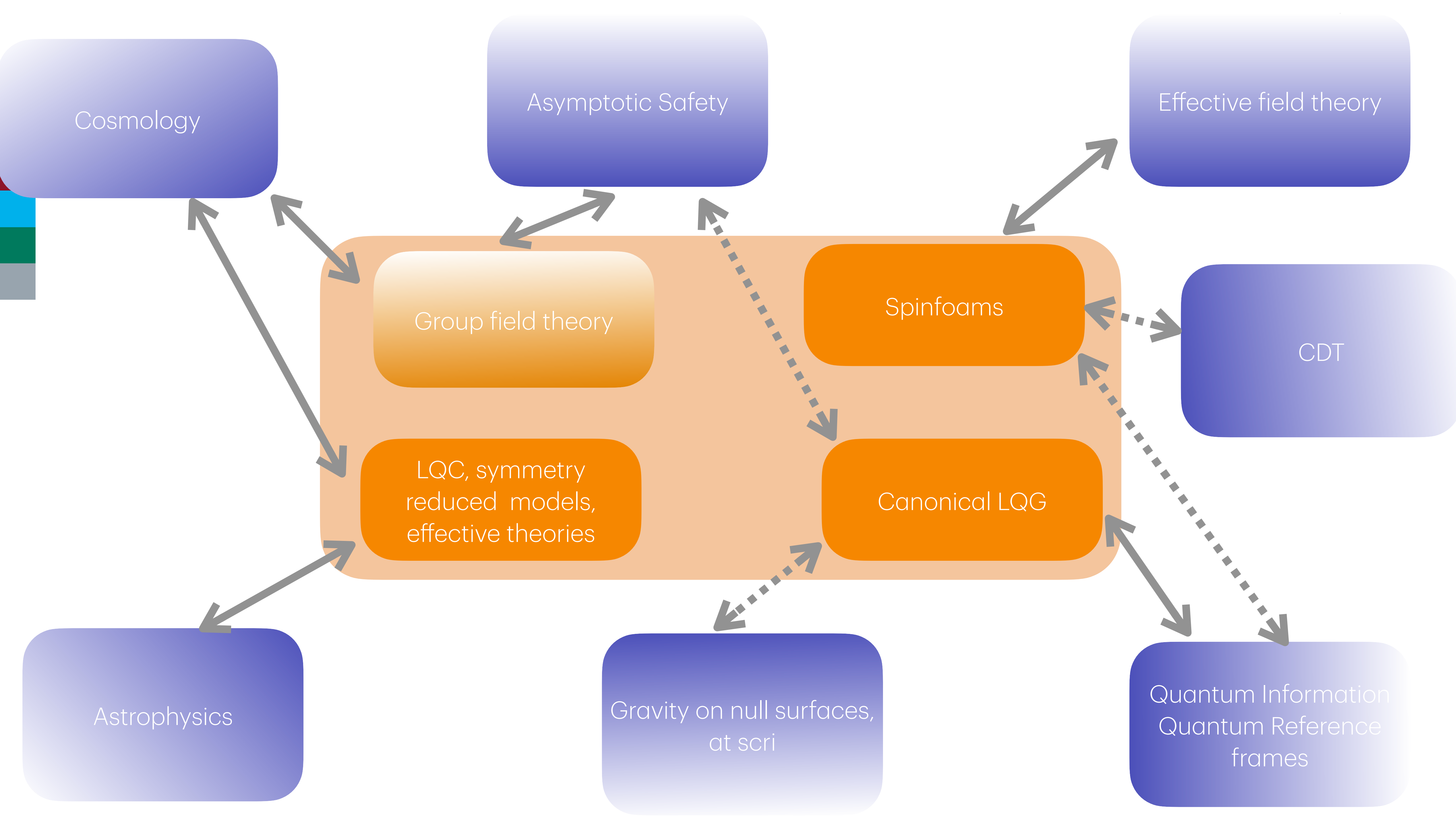


$$G_{\vec{m}_1, \vec{m}_2}(e_1, e_2) := \langle \hat{\mathcal{C}}_{\vec{m}_1}^{(e_1)} \hat{\mathcal{C}}_{\vec{m}_2}^{(e_2)} \rangle - \langle \hat{\mathcal{C}}_{\vec{m}_1}^{(e_1)} \rangle \langle \hat{\mathcal{C}}_{\vec{m}_2}^{(e_2)} \rangle$$



Connection to other approaches







Important challenges



Connections: LQC+symmetry reduced models — spinfoams — canonical LQG

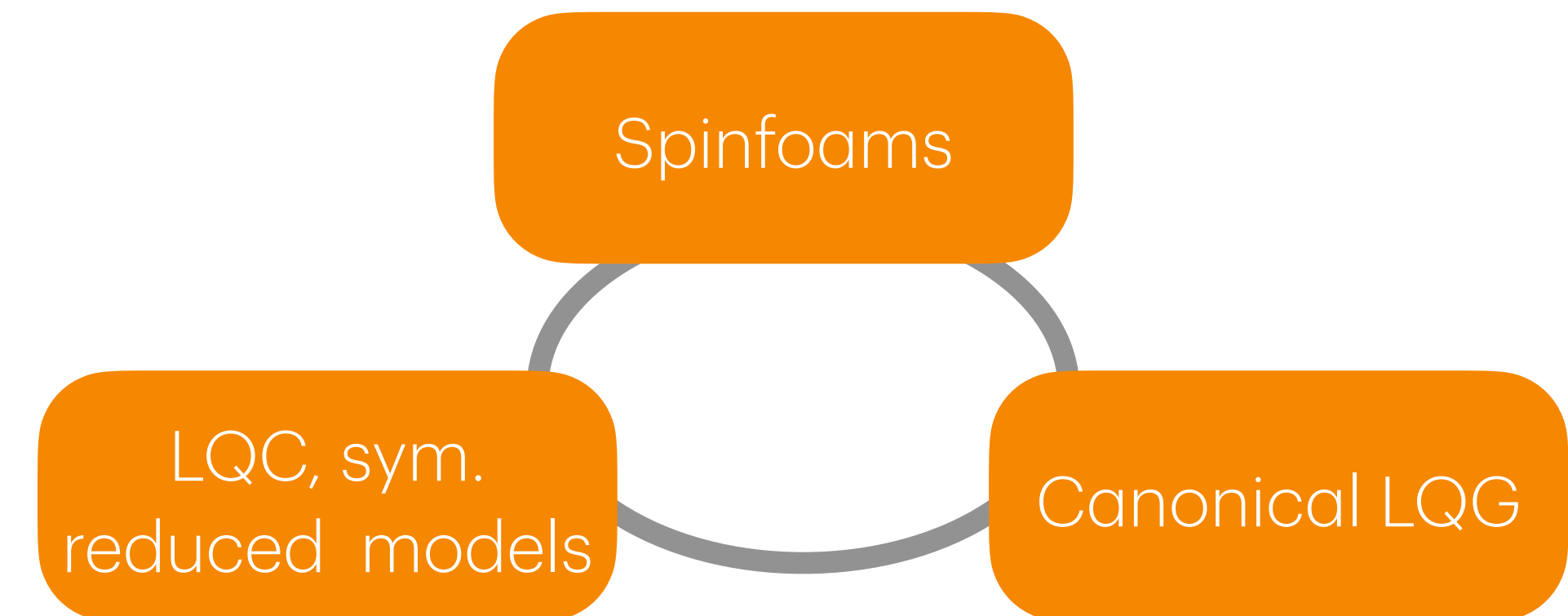
- * H-constraint vs. SF-amplitude?
- * Unequivocal derivation of LQC, other symmetric models?

Coupling matter:

- * Ambiguities?
- * Any restrictions?

Predictions:

- * Unambiguous “zero order prediction”? ((Quantized area? Remnants? Parity violation? ...))
- * Unambiguous “first order predictions”? ((Corrections to dispersion relation? CMB B-modes?...))





Bonus slides

