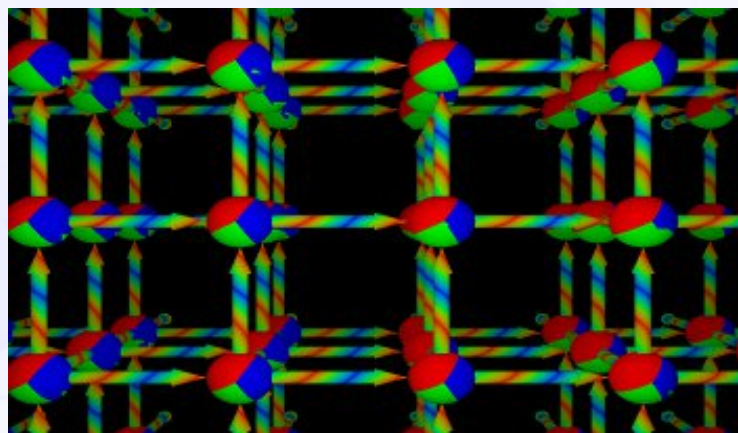
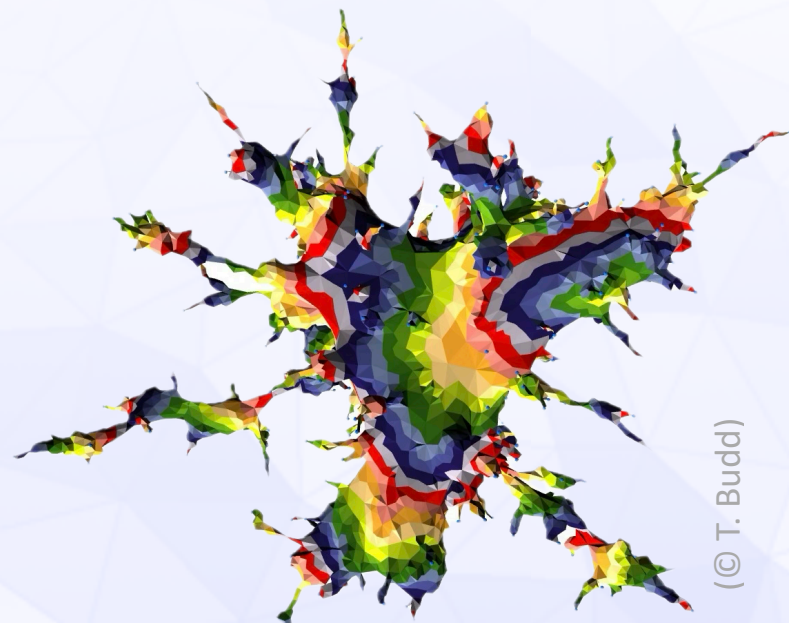


CDT: Lattice Quantum Gravity Reloaded

Why quantum gravity still isn't easy, but different



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Renate Loll, Radboud University

Puzzles in the QG Landscape

Graz, 15 Sep 2026

What's the problem with quantum gravity?

We are still searching for a fundamental quantum theory of gravity and spacetime (and matter?), which unifies the principles of quantum field theory (QFT) and general relativity (GR), and produces *numbers*.

Root problem: perturbative Quantum Gravity (QG) based on the split $g_{\mu\nu}(x) = \eta_{\mu\nu}^{Mink} + h_{\mu\nu}(x)$ of the spacetime metric is not renormalizable

flat metric

perturbation

G. 't Hooft & M. Veltman (1974), M. Goroff & A. Sagnotti (1985), A. v.d. Ven (1992)

⇒ **Option 1:** QG as a QFT, but beyond perturbation theory (how?)

Option 2: come up with a “radically new idea” (which one?)

Most of the community chose **2**, leading to the birth of QG approaches (aka Pandora's box), often invoking “exotic” ingredients (d.o.f., symmetries, fundamental principles) *beyond* those of GR and QFT.

⇒ 40 yrs of ever richer approaches, largely detached from physical phenomenology, without a clear bottom line, *and* a growing wishlist of questions that QG should answer

J. Armas, “Conversations on Quantum Gravity”, CUP 2021

A tale of two worlds

Collectively, these expectations seem *seriously overblown*, given (i) how little we know about QG beyond perturbation theory, (ii) the absence of observational data, (iii) the limitations of our theoretical/mathematical/numerical tools, and (iv) the smallness of $\hbar$ -quantum corrections/effects.

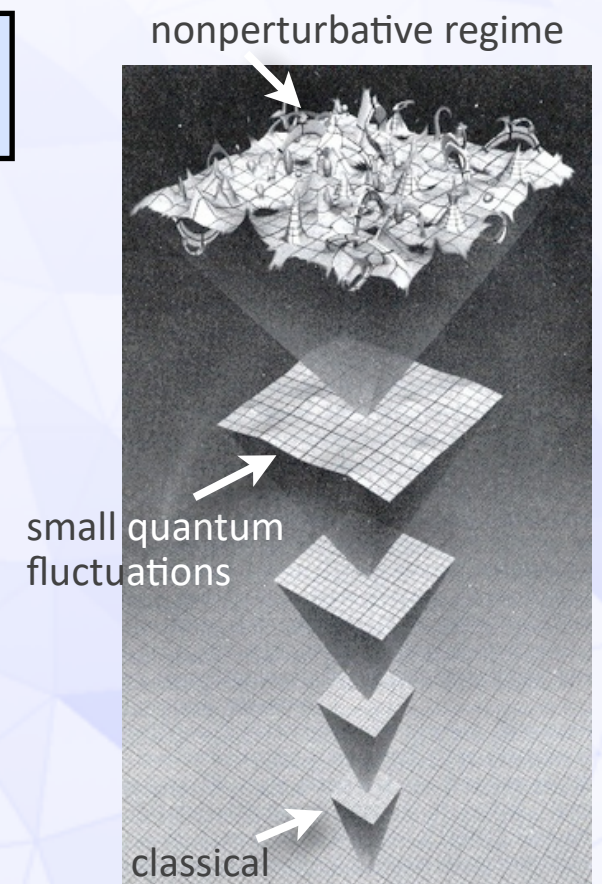
⇒ Reset expectations about *solving quantum gravity*.

- by general consensus, QG has a “*quantum foam regime*” near ℓ_{Pl} , beyond perturbation theory
- quantum fluctuations *of spacetime itself* are large, tensor calculus & standard GR tools are *not applicable*
- nevertheless, most work is based on *semiclassical, effective QG* (“ $g_{\mu\nu}$ -physics”)

J. F. Donoghue, arXiv:2211.09902

Given the lack of approach-independent progress, should we worry?

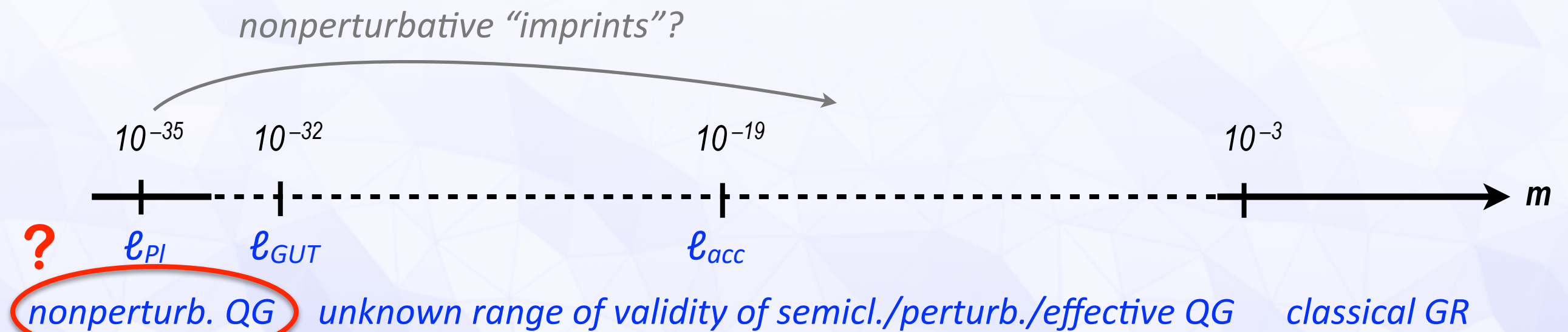
⇒ *Take quantum foam more seriously!*



Searching for signatures of quantum gravity

“Characteristic” Planck scales? $\Rightarrow \ell_{Pl}$ is very small, M_{Pl} is very large

Scales of (quantum) gravity:



- hope for “low”-energy imprints of fundamental quantum gravity @ ℓ_{Pl}
- are there observable observables? ($\hbar$ -quantum corrections are tiny)
- unclear how far can we trust UV-extrapolations of $g_{\mu\nu}$ -physics

Even if radically new ideas, perturbation theory and effective descriptions have not (yet) succeeded, there is still ...

... Option 1: no-frills, nonperturbative QG

What if ...

... the (trans-)Planckian quantum foam regime could be reached and described purely by QFT methods, without any extraneous ingredients,

... a fully nonperturbative QFT formulation of gravity was the ***primary or only gateway*** to this strongly coupled quantum regime @ ℓ_{Pl} ,

... despite the fact that many fine-grained $g_{\mu\nu}$ -questions, which require a background metric or reference system are not operationally defined there (prominent example: “*What is the entropy of a black hole?*”),

... and, rather than dealing with “ $\hat{R}^{\kappa}_{\lambda\mu\nu}$ ”, physics was described by ***critical exponents, universal scaling coefficients*** and other ***global observables***,

... nevertheless allowing us —in the best of all worlds— to connect to ***early-universe cosmology***, e.g. via the scaling of curvature correlators,

... well, then ***quantum gravity would still be exciting, but different.***

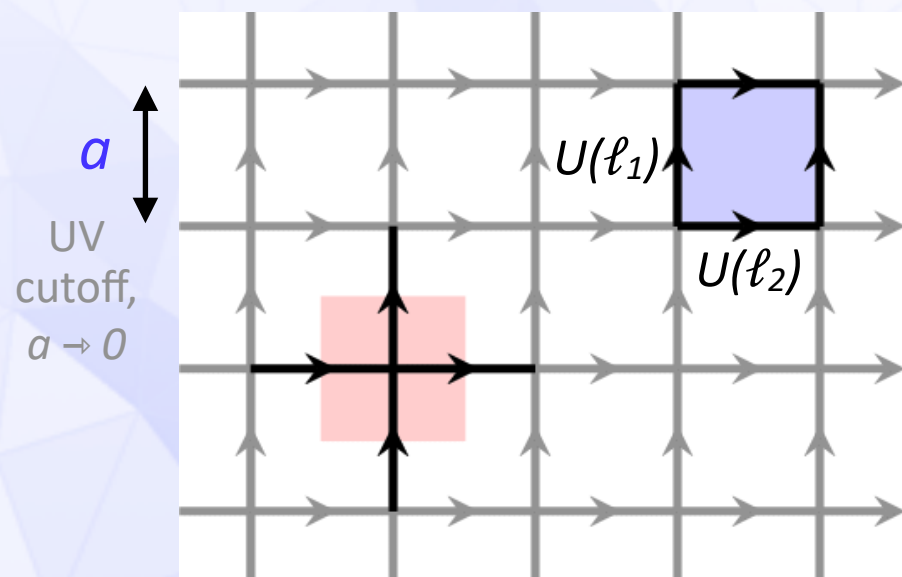
This is “Lattice Quantum Gravity, Reloaded”

Put the *gravitational path integral* (Feynman’s *sum over histories*),

$$Z = \int \mathcal{D}g \, e^{iS_{\text{grav}}[g]}$$

“geometry” $\rightarrow g \in \frac{\text{Lor}(M)}{\text{Diff}(M)}$

on the lattice, following the immensely successful lattice QCD à la Wilson.
— Unfortunately, this leads to a clash of fundamental principles.



cubic lattice representing flat spacetime,
with gauge fields U living on 1D edges ℓ

When one begins to think about formulating general relativity on a lattice one runs into an immediate conceptual difficulty. In general relativity the geometry of spacetime itself is dynamical and subject to quantum fluctuations and it is difficult to imagine how to make a lattice theory in which the lattice structure itself is dynamical. This

L. Smolin, “Quantum Gravity on the Lattice”, NPB 148 (1979) 333

Lattice QG on a *fixed* lattice has never worked.
(Recall ***quantum gravity is not easy***).

Game changer: *dynamical, causal* lattices

- **Breakthrough I:** use *piecewise flat simplicial lattices* T to regularize curved spacetimes; each triangulation T is defined by its connectivity and two possible (space-/time-like) edge lengths $\ell_s^2 = a^2$, $\ell_t^2 = -\alpha a^2$, $\alpha > 0$.
- originally, *Dynamical Triangulations* (“DT”) used equilateral triangles in the 2D Euclidean path integral (QG toy model w/ Riemannian surfaces)

F. David, NPB 257 (1985) 45; J. Ambjørn, B. Durhuus & J. Fröhlich, *ibid*, 433; V. Kazakov et al, PLB 157 (1985) 295

- however, quantum gravity needs *4D Lorentzian spacetime lattices*

- **Breakthrough II:** **Causal Dynamical Triangulations (“CDT”)**

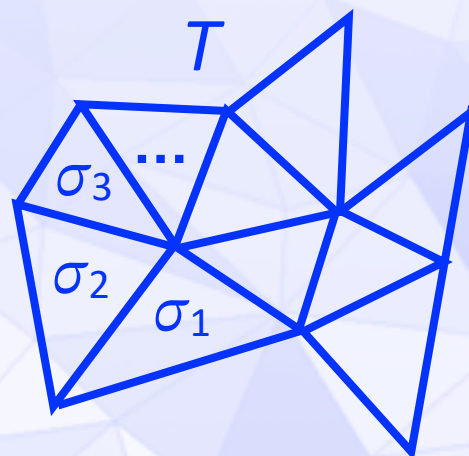
together with a *well-defined Wick rotation* $\alpha \mapsto -\alpha$

J. Ambjørn & R. Loll, NPB 536 (1998) 407, J. Ambjørn, J. Jurkiewicz & R. Loll, NPB 610 (2001) 347

2D:

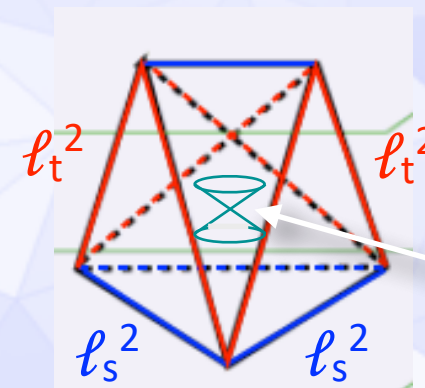


simplicial equilateral Euclidean
building block in 2D DT



configuration T in the nonperturbative
path integral of 2D Euclidean QG

4D:



simplicial Minkowskian (3,2)-building
block in 4D CDT
[(4,1)-building block not shown]

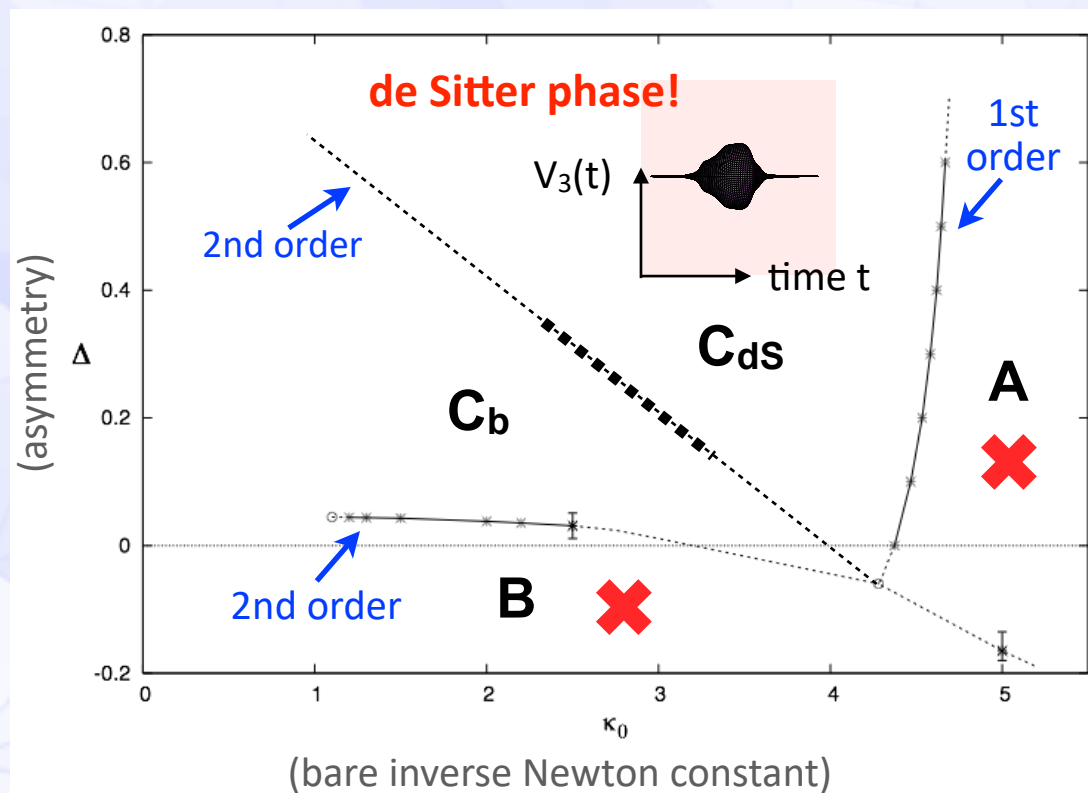
$\ell^2 \propto a^2$
NB: UV cutoff

Lattice quantum gravity à la CDT in a nutshell

$$Z = \lim_{a \rightarrow 0} \sum_{\text{causal triang. } T} \frac{1}{C(T)} e^{iS^R[T]}, \quad S^R[T] = -(\kappa_0 + 6\Delta)N_0(T) + \kappa_4 N_4(T) + \Delta N_{41}(T)$$

“bare” lattice version of Einstein-Hilbert action — **very simple!**
 κ_0 = bare coupling constants N_0 = # vertices κ_4 = bare coupling constants N_4 = # four-simplices
 Δ = bare coupling constants N_{41} = # four-simplices

- **continuum limit** $a \rightarrow 0$, $N_4 \rightarrow \infty$, for finite physical volume $V_4 = N_4 a^4$
- **causal structure**: path integral histories obey global hyperbolicity, implemented in terms of a geometrically defined “proper time”
- transfer matrix is reflection positive $\Rightarrow$ **unitary** continuum theory
- Z is computed by **Monte Carlo methods** after analytic continuation

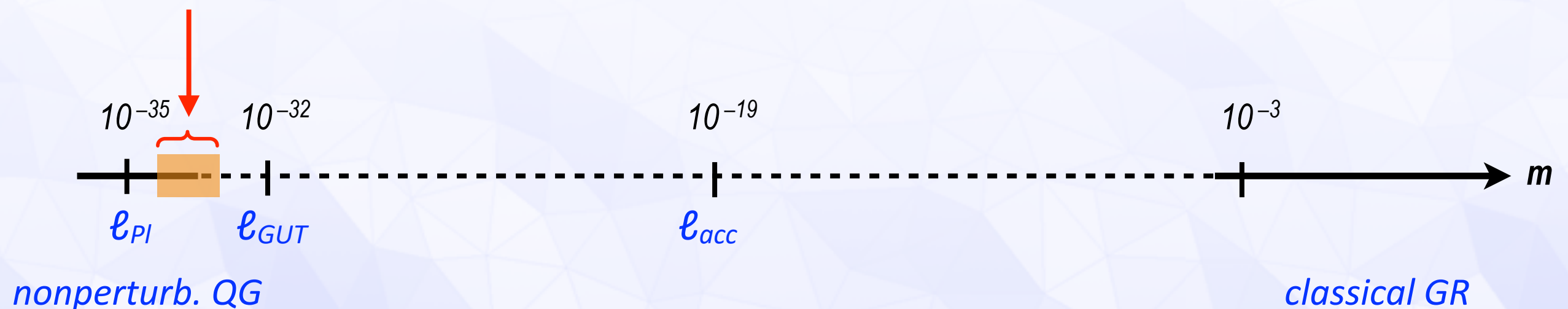


- the infamous **conformal divergence** is tamed nonperturbatively
- phase diagram: **2nd-order transitions**
- good evidence of universality; search for evidence of **UV fixed points** (asymptotic safety) under way

S. Weinberg (1979), M. Reuter & F. Saueressig, PRD 65 (2002) 065016; CDT: J. Ambjørn et al, PRD 110 (2024) 126006

A window on Planckian physics!

“observational” window
of lattice QG (*CDT*)



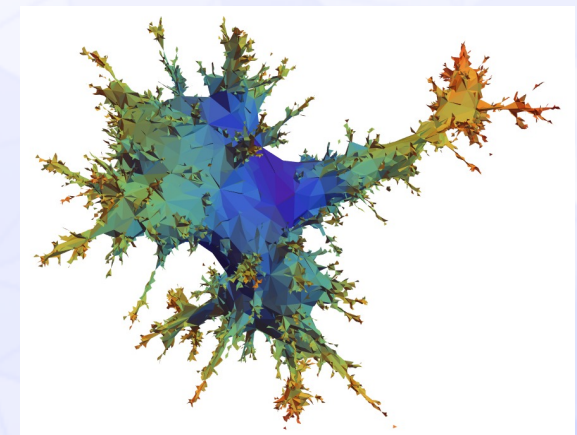
N.B.: relation between lattice and physical units is not ad hoc, but
from comparing observables (three-volume correlator)

Lattice quantum gravity à la CDT: Results

- CDT's ingredients are simple, but their nonperturbative dynamics *is not*. Chunks of quantum spacetime of linear size $\lesssim 20 \ell_{Pl}$ can be studied *quantitatively* with state-of-the-art simulations for volumes $N_4 \lesssim 10^6$.
- physical properties of quantum spacetime are captured by the expectation values of (nonlocal) *quantum observables* $\hat{\mathcal{O}}$:

$$\langle \hat{\mathcal{O}} \rangle = \lim_{a \rightarrow 0} \frac{1}{Z} \sum_{\text{causal triang. } T} \frac{1}{C(T)} \mathcal{O}[T] e^{-S_{\text{grav}}[T]}, \quad Z = \lim_{a \rightarrow 0} \sum_{\text{causal triang. } T} \frac{1}{C(T)} e^{-S_{\text{grav}}[T]}$$

- configurations T are nonsmooth, but have notions of *geodesic (link) distance* and *volume*
- large quantum fluctuations and the absence of meaningful reference frames $@\ell_{Pl}$ dictate the nature of (diffeomorphism-invariant) observables



(© T. Budd)

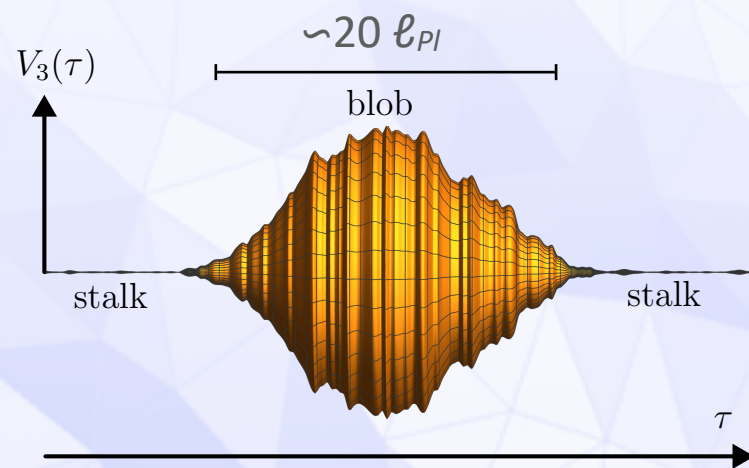
2D quantum geometry:
nowhere differentiable as $a \rightarrow 0$

REVIEWS: J. Ambjørn, A. Görlich, J. Jurkiewicz & R. Loll, Phys. Rep. 519 (2012) 127, arXiv:1203.3591
R. Loll, Class. Quant. Grav. 37 (2020) 013002, arXiv:1905.08669,
J. Ambjørn & R. Loll, Encycl. Math. Phys., arXiv:2401.09399; Scholarpedia, arXiv:2604.05641

Breakthrough result: the nonperturbative 'de Sitter' vacuum

The measured *large*-scale spectral and Hausdorff dimensions, the shape $\langle V_3(\tau) \rangle$ (3-volume as function of proper time) and the average curvature of the quantum universe match those of a classical 4D de Sitter space, although *no background or symmetry assumptions were ever put in*.

J. Ambjørn, A. Görlich, J. Jurkiewicz, R. Loll, PRL 100 (2008) 091304; N. Klitgaard & R. Loll, Eur. Phys. J. C80 (2020) 990



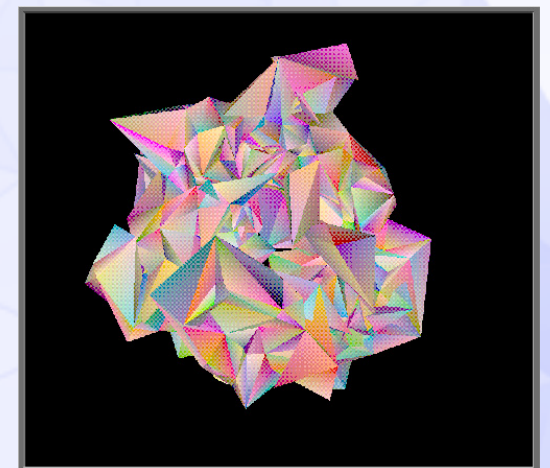
MC snapshot of $\langle V_3 \rangle$ as function
of Euclidean proper time τ

One finds $\langle V_3(\tau) \rangle \propto \cos^3(c\tau)$, like for a (Euclidean) de Sitter universe, but **locally** this quantum space is **not** described by the symmetry-reduced metric

$$ds^2 = d\tau^2 + c^2 \cos^2(\tau/c) d\Omega^2$$

of a homogeneous, isotropic de Sitter cosmology, but

resembles a wildly fluctuating quantum foam.



Very suggestive! — can we “do” cosmology?

- Can we provide *first-principles QG derivations* and justifications of standard assumptions made in early-universe cosmology?
 - In what way is the quantum universe obtained from the nonperturbative path integral only “*quasi de Sitter*”? (exact dS disfavoured physically)
 - Can we find *quantum-gravitational seeds of structure*? Can we do without inflaton fields?
- note very different settings: QFT on curved background spacetime vs nonperturbative lattice quantum gravity
 - nature of observables is different, e.g. diffeomorphism-invariant two-point functions of local scalar operators $\mathcal{O}$:

$$\langle \mathcal{O}\mathcal{O} \rangle(r) = \frac{1}{Z} \int \mathcal{D}g \, e^{-S[g]} \underbrace{\int d^4x \sqrt{g(x)}}_{\text{x-integral}} \underbrace{\int d^4y \sqrt{g(y)}}_{\text{y-integral}} \underbrace{\mathcal{O}(x)\mathcal{O}(y)}_{\text{operator insertions at x \& y}} \underbrace{\delta(d_g(x,y)-r)}_{\text{additional operator dependence!}}$$

Research program: observables in progress

recovery of classical symmetries: is quantum spacetime approximately homogeneous and isotropic on suitably coarse-grained scales?

- *nonperturbative homogeneity measures* A. Silva & R. Loll, PRD 107 (2023) 086013
- *discrete ‘quantum’ analogues of translations/rotations* w.i.p. w/ T. O’Shea

effective topology: is Planckian spacetime really a quantum foam? use tools from TDA to measure ➤ *quantum Betti numbers*

J. van der Duin, R. Loll, M. Schiffer & A. Silva, Eur. Phys. J. C 86 (2026) 2, PLB 881 (2026) 140857 and w.i.p.

quantifying fluctuations/inhomogeneities: via NP two-point functions



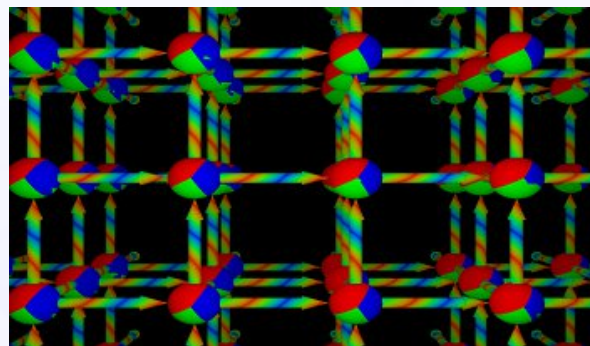
this is technically and conceptually challenging



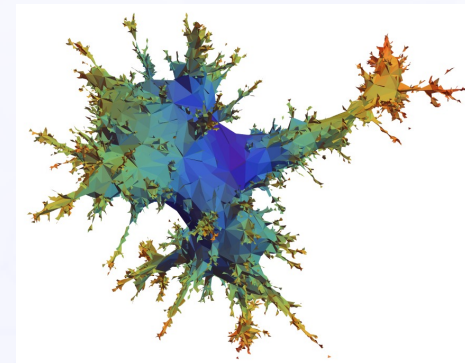
- *curvature correlators*, starting in 2D J. van der Duin & R. Loll, Eur. Phys. J. C84 (2024) 7
- *quenched scalar field correlators* in 4D J. van der Duin & R. Loll & M. Schiffer, w.i.p.
 - > see also Jesse’s poster
- related work by the Graz group! A. Maas, S. Plätzer & F. Pressler, PLB 879 (2026) 140600

Summary and outlook

- The alleged incompatibility between the fundamental principles of GR and QFT is overcome by using ***causal, dynamical lattices*** (it took a while).



rigid cubic lattice (for QCD)



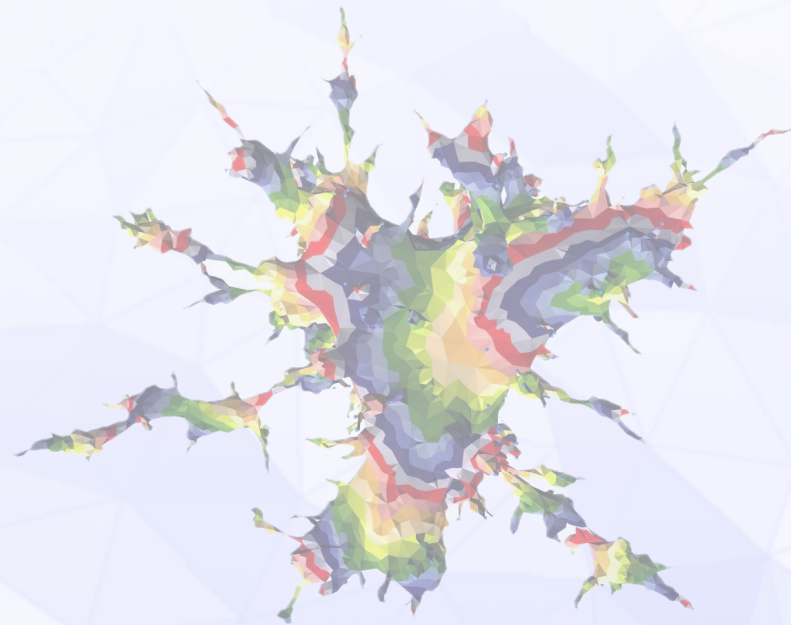
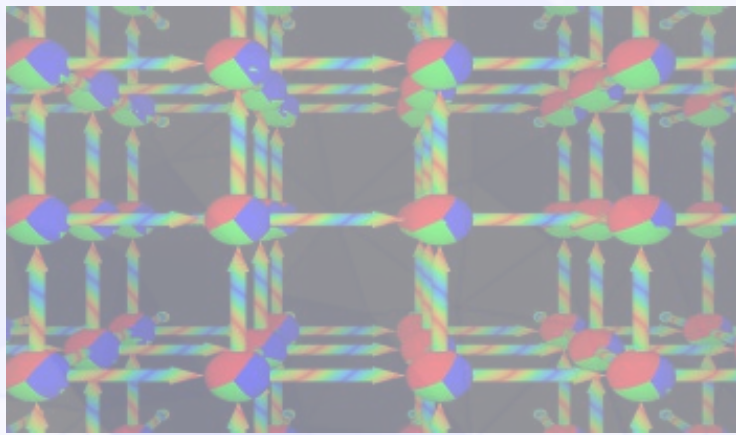
dynamical lattice (for QG)

- ***Quantum gravity is (still) not easy***, but reloaded lattice QG has opened a unique observational window near ℓ_{PI} , where the path integral can be computed, and conceptual & technical questions be addressed explicitly.
- CDT lattice QG is just nonperturbative QFT, not a “QG approach”; it may be the ***primary gateway*** to the strongly coupled quantum regime $@\ell_{PI}$.
- Being able to compute from first principles has given us unprecedented *blueprints of quantum spacetime and the emergence of classicality*, and

a new perspective on what quantum gravity can realistically deliver.

CDT: Lattice Quantum Gravity Reloaded

Why quantum gravity still isn't easy, but different



Thank you!