

Overview of group field theory

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Tétraèdres – Paul Sérusier (ca 1910)

Introduction

Group field theory (GFT) is a background-independent approach to quantum gravity which combines discrete and continuum aspects in a unique way.

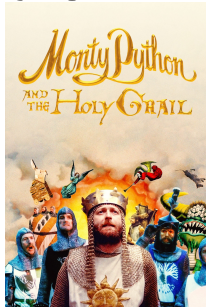
A group field theory is, in a sense, a quantum field theory, but one that lives on an abstract space (**not** spacetime!)

GFT connects to many discrete approaches, in particular spin foams, loop quantum gravity and matrix models, while incorporating methods of (relativistic and non-relativistic) quantum field theory.

I have the daunting task of giving a brief overview over some of many ideas suggested in GFT – I have made a personal selection.

In GFT, as in other research programmes, the continuum geometry of Einstein is **emergent** from different (“non-spatiotemporal”) fundamental degrees of freedom.

The holy grail of quantum gravity: a good definition of the path integral



$$Z = \sum_{\text{topology?}} \int \mathcal{D}g \, e^{iS[g]},$$

Perturbative definitions of Z lead to non-renormalisable divergences. **Non-perturbative** definition needed, perhaps a discrete one (cf. lattice gauge theory).

spin connection $A_a^i(x)$, triad $E_j^b(x)$.

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A model for (2+1)d gravity [Ponzano–Regge 1968]

In three spacetime dimensions and Riemannian signature, can define Ashtekar-like $SU(2)$ variables and use an action

$$S[E, A] = \int E_i \wedge F[A]^i.$$

The equation of motion for A is $F[A] = 0$, i.e., flat connections.

Define path integral on a (dual) triangulation of a manifold: replace A by finite group elements g_e along edges and impose flatness around closed loops.

Then

$$Z = \int dg_e \prod_f \delta(g_{e_1^f} \cdots g_{e_n^f}) = \sum_{j_f} \prod_f (2j_f + 1) \prod_v \{6j\}_v$$

using an expansion of $\delta(g)$ in terms of irreducible representations (spins) j_f . Geometrically, j_f represent edge lengths; for large j , $\{6j\} \sim \cos(S_{\text{Regge}}[j])$.

Group field theory: Boulatov model

Basic idea: define an **abstract** field theory whose perturbative Feynman expansion gives a quantum gravity sum over geometries.

In the **Boulatov model** [Boulatov 1992] one defines a **group field** φ and action

$$\varphi : \mathrm{SU}(2)^3 \rightarrow \mathbb{R}, \quad \varphi(g_1, g_2, g_3) = \varphi(g_2, g_3, g_1) = \varphi(g_3, g_1, g_2),$$

$$S[\varphi] = \frac{1}{2} \int d^3g \, \varphi(g_i)^2 - \frac{\lambda}{4!} \int d^6g \, \varphi(g_1, g_2, g_3) \varphi(g_1, g_4, g_5) \varphi(g_2, g_5, g_6) \varphi(g_3, g_6, g_4).$$

Notice the “nonlocal” structure of the interaction term. Then

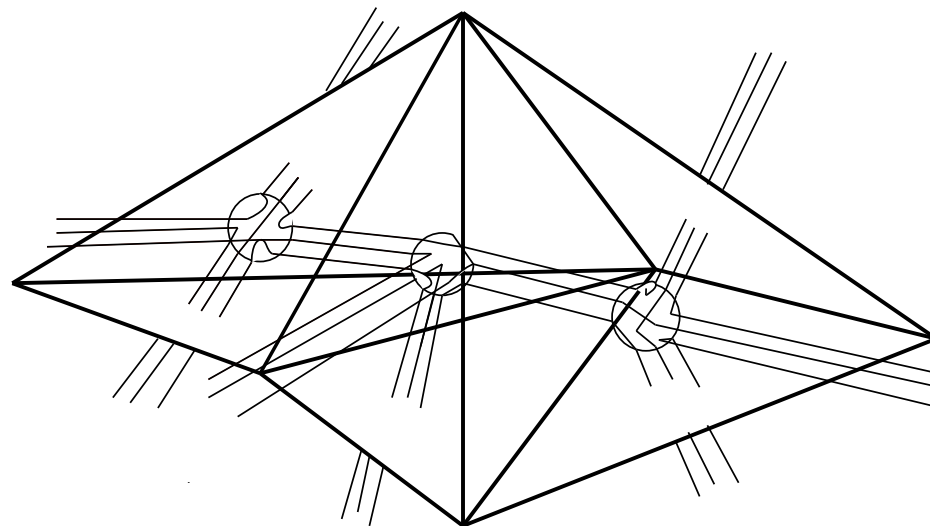
$$\int \mathcal{D}\varphi e^{-S[\varphi]} = \sum_C \lambda^{N_v(C)} \sum_{j_f} \prod_f (2j_f + 1) \prod_v \{6j\}_v$$

is a sum over 3-dimensional simplicial complexes C containing $N_v(C)$ vertices.
 $\Rightarrow$ Ponzano–Regge model plus “sum over topologies” !

An attempt at visualisation

Just as in standard QFT, we can think of the group field $\varphi(g_i)$ as creating “particles” with some labels g_i . In (2+1) dimensions, the particle is a triangle with g_i defining a discrete connection through its edges.

Interaction term is defined to glue four triangles to form a tetrahedron:



Idea: **spacetime emerges from (the degrees of freedom of) Feynman graphs**

GFT as a quantum field theory

- Sum over (discrete) geometries and topologies has been rewritten as Feynman expansion of an auxiliary field theory (GFT).
- This is an expansion around the trivial “no-space” vacuum; simple Feynman graphs contain few building blocks.
- Continuum limit $\rightarrow$ matrix models would require analogue of double-scaling limit, to have control over the infinite sum of Feynman graphs.
- QFT is not just defined by its Feynman expansion!

Most research connecting GFT to quantum gravity uses the last point as an observation to incorporate different QFT methods.

GFT renormalisation

What we have written down is a bare (tree-level) GFT action. Radiative corrections can be studied with standard methods, taking into account the unusual GFT domain space.

For example, radiative corrections require addition of a nontrivial kinetic term [Ben Geloun, Bonzom 2011]. Including them turns

$$S_{\text{bare,kin}}[\varphi] = \frac{1}{2} \int d^3g \, \varphi(g_i)^2$$

into

$$S_{\text{renorm,kin}}[\varphi] = \frac{1}{2} \int d^3g \, \varphi(g_i) \left(\sum_{s=1}^3 \Delta_{(s)} + m^2 \right) \varphi(g_i) .$$

Much more like a standard QFT (conventional propagator structure, etc).

GFT renormalisation (II)

Renormalisation of GFT has been studied for a variety of models. Analysis is complicated by nonlocal interactions and nontrivial (compact or non-compact) domain space, so truncations and approximations are needed.

Goal: identify phase transitions to an effective continuum (“condensate”) phase, away from trivial vacuum $\varphi = 0$, described by interacting fixed point.

Many papers. Shown that models with compact group domain do not allow for phase transition [Pithis, Thürigen 2021].

Indications for existence of condensate phase using mean-field theory and Landau–Ginzburg analysis [Marchetti, Oriti, Pithis, Thürigen 2021–23] in models based on $SL(2, \mathbb{C})$ and with additional matter (scalar field) degrees of freedom.

Basis for phenomenology based on “condensate” mean-field description of φ .

Ingredients in model building

The GFT domain space is determined by the degrees of freedom needed to decorate Feynman graphs ($\Leftrightarrow$ discrete spacetimes).

In the Boulatov model, we had $\varphi : \text{SU}(2)^3 \rightarrow \mathbb{R}$ for three-dimensional discrete geometry with $\text{SU}(2)$ Yang–Mills-like spin connection.

Other popular choices include, e.g., $\text{SO}(4)$ (Riemannian) or $\text{SL}(2, \mathbb{C})$ (Lorentzian) in four dimensions. $\text{SU}(2)$ still appears in Ashtekar–Barbero variables.

Matter fields appear as additional data on the graph, require extra arguments of the group field. Adding N scalar fields extends the domain to $G^d \times \mathbb{R}^N$.

Interactions determined by combinatorial principles of Feynman graphs (inspired by “gluing” of simplices), usually nonlocal. Extra terms usually generated by renormalisation. Symmetries can be imposed to restrict the theory space.

Hilbert space structure; canonical quantisation?

Standard definition of GFT via functional integral. Definition of Hilbert space not obvious (attempt at a review: [SG 2025]).

In general, functional integral definition not expected to be equivalent to a “canonical” quantisation (cf. discussions in higher-derivative QFT).

In standard case (compact gauge group plus scalar field), construction simplifies:

$$\begin{aligned} S_{\text{kin}}[\varphi] &= \frac{1}{2} \int d^4g \, d\chi \, \varphi(g_i, \chi) (\mathcal{K}^0 + \mathcal{K}^2 \partial_\chi^2) \varphi(g_i, \chi) \\ &= \frac{1}{2} \sum_J d\chi \, \varphi_J(\chi) (\mathcal{K}_J^0 + \mathcal{K}_J^2 \partial_\chi^2) \varphi_J(\chi) \end{aligned}$$

after decomposition into Peter–Weyl (discrete Fourier) modes J . J modes decouple and each one is a harmonic oscillator or upside-down oscillator.

“Spacetime emergence”: free theory [Oriti, Sindoni, Wilson-Ewing 2016, . . .]

Promote $\varphi_J(\chi)$ and its momentum $\pi_J(\chi)$ to operators and define ladder operators

$$[\hat{a}_J(\chi), \hat{a}_{J'}^\dagger(\chi)] = \delta_{JJ'}$$

seen as creating and annihilating tetrahedra with given labels J . Hamiltonian is

$$\hat{\mathcal{H}}_J = \pm\omega_J \left(\hat{a}_J^\dagger \hat{a}_J + \frac{1}{2} \mathbf{1} \right) \quad \text{or} \quad \hat{\mathcal{H}}_J = \frac{1}{2}\omega_J \left(\hat{a}_J^\dagger \hat{a}_J^\dagger + \hat{a}_J \hat{a}_J \right) .$$

Each J Hamiltonian corresponds to either a harmonic oscillator or an upside-down harmonic oscillator, or **squeezing operator**.

Particle number $\langle \hat{N}_J \rangle = \langle \hat{a}_J^\dagger \hat{a}_J \rangle$ conserved for the harmonic oscillator; for the squeezing operator we have an exponential instability:

$$\frac{d}{d\chi} \langle \hat{N}_J \rangle = -i \langle [\hat{N}_J, \hat{\mathcal{H}}_J] \rangle = -i\omega_J \langle \hat{a}^\dagger \hat{a}^\dagger - \hat{a} \hat{a} \rangle, \quad \frac{d^2}{d\chi^2} \langle \hat{N}_J \rangle = 4\omega_J^2 \langle \hat{N}_J \rangle + 2\omega_J^2 .$$

An example of spacetime emergence: free theory (II)

Loop quantum gravity suggests that each “particle” (tetrahedron) comes with a given volume v_J , so we can define a volume operator

$$\hat{V}_J(\chi) = v_J \hat{N}_J(\chi).$$

This quantity evolves in the matter clock time given by χ . One can show that

$$\left(\frac{\langle \hat{V}'_J(\chi) \rangle}{\langle \hat{V}_J(\chi) \rangle} \right)^2 = 4\omega_J^2 \left(1 + \frac{v_J}{\langle \hat{V}_J(\chi) \rangle} - \frac{c}{\langle \hat{V}_J(\chi) \rangle^2} \right)$$

where $c \geq 0$ depends on initial conditions.

For a generic state, $\langle \hat{V}_J(\chi) \rangle$ satisfies an equation similar to the Friedmann equation of standard cosmology, with the correct low-energy limit (if $\omega_J^2 = 3\pi G$) and a “bounce” resolving the singularity! 🍾

Cosmological phenomenology from GFT?

Many attempts to extend this simple result beyond the truncation to free GFT. Interactions can be included perturbatively, e.g., (out last week!)

Emergent dark sector in group field theory cosmology

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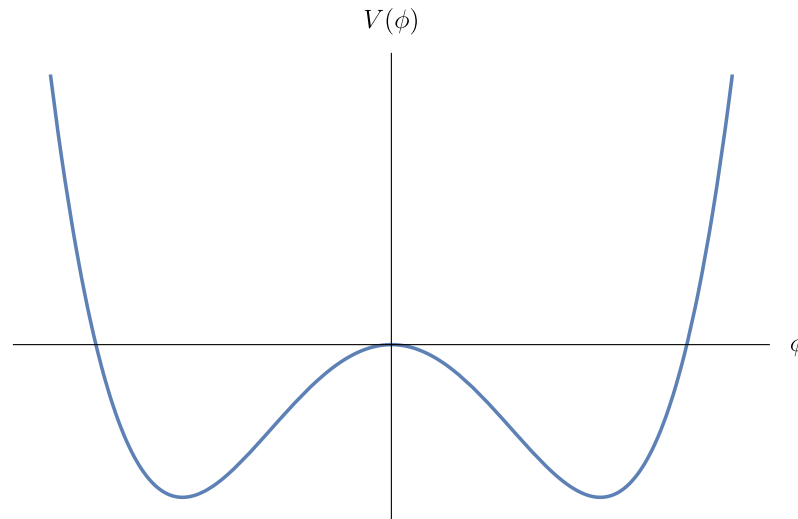
We develop an analytical treatment of the emergent cosmological dynamics induced by local polynomial interactions in a deparametrised group field theory model, moving beyond the usual non-interacting approximation. For a single field mode with quartic and sextic couplings we obtain a closed-form generalised Friedmann equation within a controlled Gaussian regime. The dynamics preserves the quantum bounce while generating effective dark matter and dark energy contributions as collective quantum-geometric phenomena, providing the first derivation of both components from a single model. Matching the resulting dark-energy-to-dark-matter density ratio to observations

Since free theory is unstable, perturbation theory not viable beyond relatively short time; new (non-perturbative? numerical??) methods needed.

To go beyond homogeneity, more observables than total volume $\hat{V}(\chi)$ needed. Main idea: use four scalar matter fields as reference frame (coordinate system) for space and time, construct “local” observables to probe effective geometry.

A simple, workable example with interactions [SG, Santacruz 2023]

In a model where interactions stabilise the field, one can find ground states corresponding to **static** cosmology.



At small coupling, ground state is sharply peaked in particle number $\hat{N}_J$! Can be well approximated by coherent state with field peaked on the potential minimum. Quantum fluctuations small and become arbitrarily small at weak coupling.

GFT condensate: mean field an excellent approximation near the minimum.

A bold proposal: spacetime metric in group field theory [SG, Mickel 2024]
 GFT model with four massless scalar fields, and translational (shift) symmetry $\chi^A \rightarrow \chi^A + \epsilon^A$. Symmetries lead to a conserved GFT energy-momentum tensor

$$\partial_A T^{AB} = 0.$$

This conservation law is the analogue of the **classical** conservation law

$$\partial_\mu (j^\mu)^A = 0, \quad (j^\mu)^A = -\sqrt{-g} g^{\mu\nu} \partial_\nu \chi^A$$

arising from shift symmetry. (This is the Klein–Gordon equation!)
 Matching these conserved quantities suggests an identification

$$T^{AB} \leftrightarrow (j^A)^B$$

in a coordinate system formed by the massless scalars χ^A . But in **this** coordinate system

$$(j^A)^B = -\sqrt{-g} g^{AB} \quad !$$

Connections to other approaches

GFT has many (structural, methodological) connections to other approaches:

- Kinematical variables of spin foams and loop quantum gravity, (Feynman) amplitudes as a representation of spin foam amplitudes, GFT action as a generating function similar to matrix models.
- Renormalisation group flow techniques can be applied to a continuum QFT.
- Construction of relational observables using scalar field coordinates connects to loop quantum cosmology and gravity, many other quantum gravity constructions, conceptual questions of “problem of time”.
- Friedmann equations similar to those of loop quantum cosmology can be derived without canonical LQG (a major achievement!).

Strength or weakness? GFT as Frankenstein’s monster of quantum gravity?

Challenges for GFT (my personal opinion)

- Lack of background spacetime makes constructions of observables (e.g., based on local curvature or emergent causal structure) difficult.
- No clear agreement on which principles lead to promising models for four-dimensional quantum gravity.
- Renormalisation studies promising in leading to models with “condensate” phase, but many approximations needed. Lack of observables makes it hard to say whether such models include “quantum gravity” in a meaningful sense.
- Numerical methods essentially not used at all so far; GFT overly reliant on analytical approximations, often pushed outside their regime of validity.
- Coupling of spinors? Proposal for coupling (realistic) matter? Status of Λ ? Significance of spacetime dimension/choice of gauge group/...?

Thank you!