

Quantum Null Geometry: A bottom-up perspective on QG

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GRAZ-Puzzles in the QG landscape
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*Based on 2309.03932, 2404.11132
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2510.26589 & 2604.02228
Josh Kirklin



Bottom-Up vs Top-Down

- In the past a lot of effort have been put into developing top down-up perspective in Quantum gravity: A fundamentally Planckian description of QG physics from first principles (String-Theory, LQG, Causal sets, Asymptotic safety,...)

- Advocacy for a **Bottom-up perspective**:

Focus on general QG features common to several approaches, but not exclusive to one. Focus on physical predictions

- **Local Holography**: (AdS/CFT, edge mode Quantization, Black-Hole Physics,...)
- **Area as a Quantum operator** (LQG, Quantum Horizons, QIQG,..)
- **Dimensional reduction** at high energy : (Causal set, CDT, AS, High energy scattering)
- Symmetry deformations (ST, LQG, ...)
- Quantum Causality (Causal Set, QI, Canonical Quantization,...)

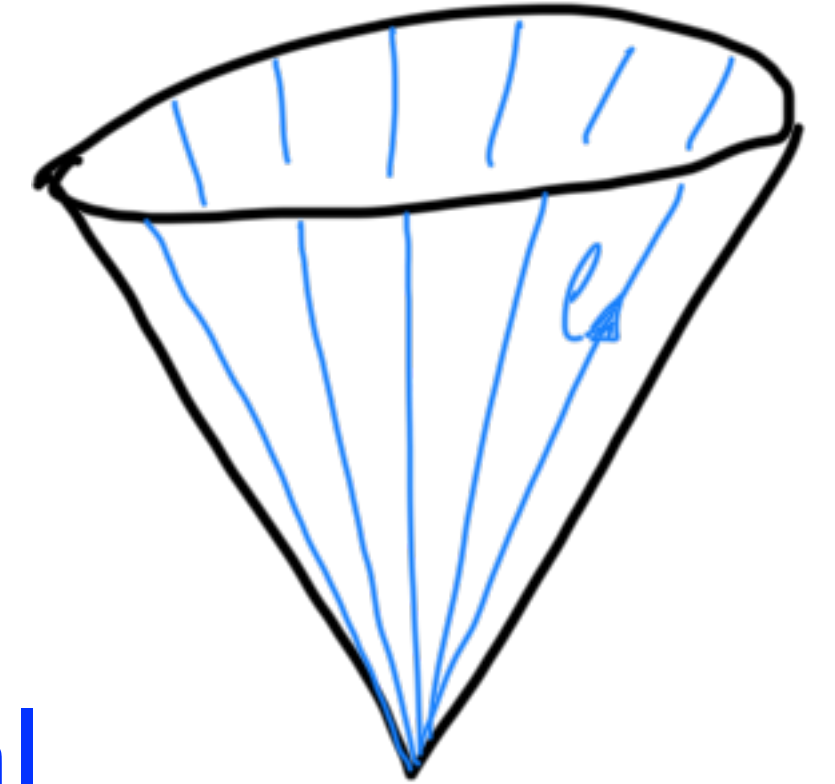
Mesoscopic Quantum Gravity

- What are the physical consequences of the presence of a physical UV-cutoff ?
- Fundamental departure from the EFT paradigm (UV-IR mixing)
- Potential experimental test of QG UV completion
- Theory agnostic: Find universal structures, unravel physical effects due to existence of fundamental constituents and build new observables.
- **Key Guideline** : Symmetry $\longrightarrow$ equivalence principle

Background independence, Gauge invariant observables, Quantum reference frames, Corner symmetries, edge modes, cardinal renormalization

Null geometry

- **Null rays** are universal and foundational elements of classical and quantum geometry.
- Null surfaces can be understood as initial value data [Bondi, Sachs, Penrose]
- The constraint equations along null surfaces can be **solved exactly** as charge conservation laws
- Raychaudhuri has bring essential insights into the nature of **gravitational entanglement** and positive energy conditions
- At the quantum level the property of null ultra-locality of entanglement leads to many insights
- Test case for Canonical quantization of quantum gravity and the study of **Corner symmetries, quantum references frames (QRF), Dressing fields and Cardinal renormalization**



[Wall '11. Bousso]

Plan

- Null ray geometry and Raychaudhuri constraint
- Dressing time and classical gauge-invariant observables
- Quantum obstruction : normal ordering and the Virasoro anomaly
- Covariant normal ordering and the quantum dressing map, anomaly cancellation
- Noise interferometry

Geometry of Null Hypersurfaces

[Levy-Leblond '64, Ashtekar '78
-'24, Henneaux '81, Dautcourt '97,
Duval-Gibbons-Horvarthy '14, ...]

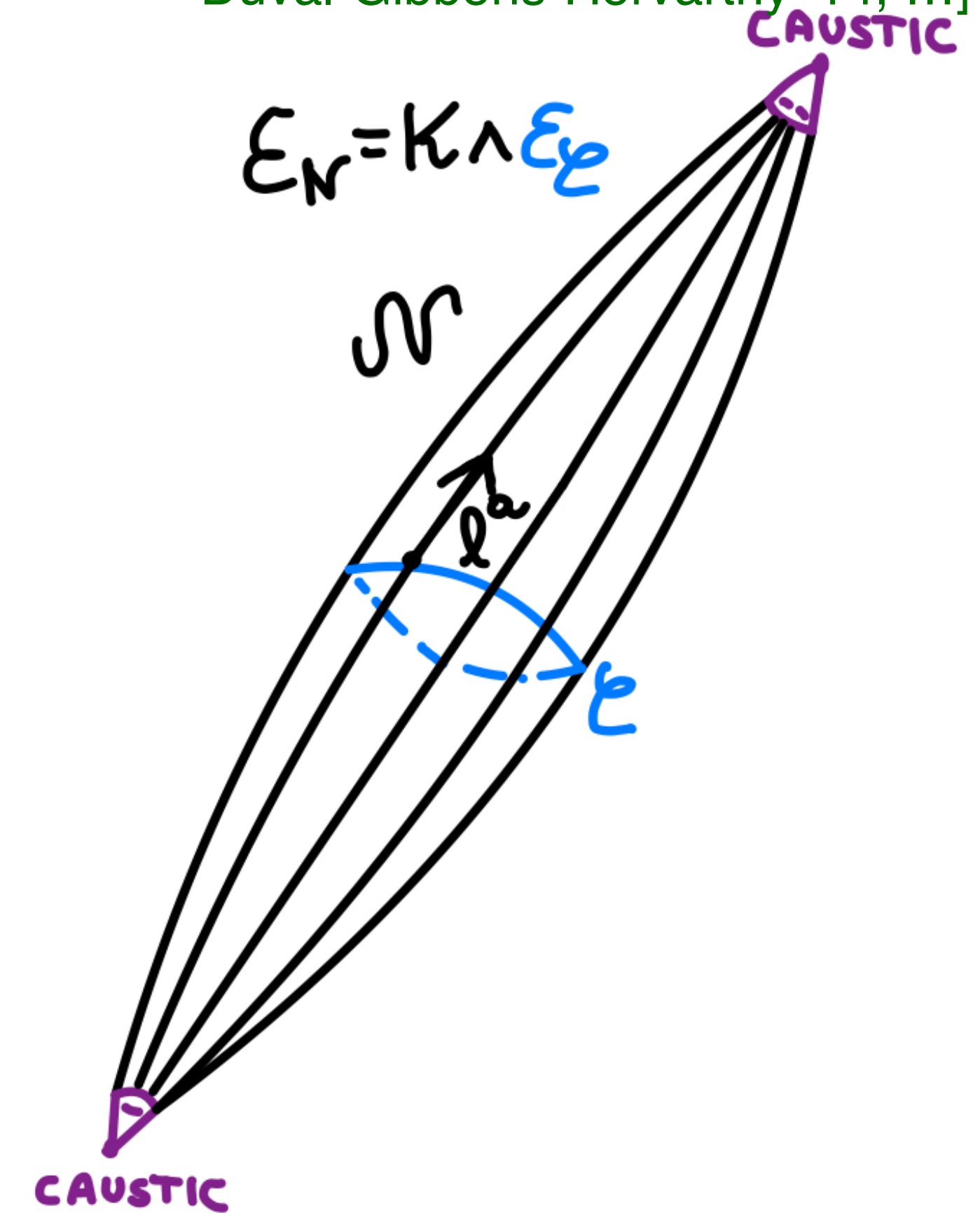
$\mathcal{N}$ 3d Null Manifold

Carrollian Structure (ℓ^a, q_{ab}) with $\ell^a q_{ab} = 0$ $\ell^a \partial_a = \partial_v$

Expansion Tensor $\theta_{ab} = \frac{1}{2} \mathcal{L}_\ell q_{ab}$ such that $\theta_a{}^b = \overset{\text{Expansion}}{\rightarrow} \frac{\theta}{2} q_a{}^b + \overset{\text{Shear}}{\downarrow} \sigma_a{}^b$

Area form: $\Omega = \sqrt{\det^*(q)}$

Surface tension: $\mu = \overset{\text{Inaffinity}}{\rightarrow} \kappa + \frac{\theta}{2}$ $\kappa = D_\ell \ell$



Dynamics on Null Hypersurfaces

[Ciambelli-Petropoulos '19
Donnay-Marteau '19,
Chandrasekaran-Flanagan-
Shehzad-Speranza '21]
[Freidel-Jai-akson '22]

Null Brown York Tensor $T_a{}^b := D_a \ell^b - \delta_a^b D_c \ell^c$

Einstein Gravity projected to $\mathcal{N}$: $D_b T_a{}^b = 8\pi G T_{a\ell}^{\text{mat}}$

Projected along ℓ^a : $(\mathcal{L}_\ell + \theta)\theta = \mu\theta - \sigma_a{}^b \sigma_b{}^a - 8\pi G T_{\ell\ell}^{\text{mat}}$ Null Raychaudhuri Equation

Projected along $q_c{}^a$: $(\mathcal{L}_\ell + \theta)\pi_a = D_a \mu - D_b \sigma_a{}^b + 8\pi G T_{a\ell}^{\text{mat}}$ Damour Equation

$$\mu = \kappa + \frac{\theta}{2} \quad \text{Surface Tension}$$

Fluid-Gravity perspective, Membrane paradigm

[Price-Thorne '88]

Symplectic structure

$$\Theta^{\text{can}} = \frac{1}{8\pi G} \int_{\mathcal{N}} \left(\frac{1}{2} \Omega \sigma^{ab} \delta q_{ab} - \mu \delta \Omega + \Omega \partial_v \phi \delta \phi \right)$$

Spin2 : q_{ab} and $\Omega \sigma^{ab}$ are conjugate variables : Radiation modes

Spin0 : Ω and μ are conjugate variables : Newtonian modes

Scalar matter : ϕ and $\partial_v \phi$ are conjugate variables

Symplectic structure and dynamics

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Spin2 : q_{ab} and $\Omega \sigma^{ab}$ are conjugate variables : Radiation modes

Spin0 : Ω and μ are conjugate variables : Newtonian modes

Scalar matter : ϕ and $\partial_v \phi$ are conjugate variables

Raychauduri constraint $C = \partial_v^2 \Omega - \mu \partial_v \Omega + \Omega (\sigma^2 + 8\pi G T_{vv}^{\text{mat}}) = 0$

expresses the gauge invariance under time reparametrization

$$\delta_f \Omega = f \partial_v \Omega, \quad \delta_f \phi = f \partial_v \phi, \quad \delta_f \mu = \partial_v (f \mu) + \partial_v^2 f$$

Quantum Null geometry

- A Key quantum property of Null ray data on Null surfaces is **Ultralocality**
- Different null rays are causally disconnected therefore their correlation functions decouple

$$\langle \partial_v \phi(v_1, \theta_1) \partial_v \phi(v_2, \theta_2) \rangle = \frac{\delta^{(2)}(\theta_1, \theta_2)}{(v_1 - v_2 + i\epsilon)^2}$$

Kay-Wald 91
Janssen-Verch 25

- Entanglement is concentrated along null rays: A generic property of Hadamard states, i.e. an expression of the equivalence principle at the quantum level.
- The densitized stress tensor $T := \Omega T_{vv}(v, \theta)$ of matter, gravitons and geometric degrees of freedom (spin 0) **generates** quantum **reparametrization symmetry**
- It develops a central charge c

$$\langle T(v_1, \theta_1) T(v_2, \theta_2) \rangle \simeq \frac{c}{2} \frac{\delta^{(2)}(\theta_1, \theta_2)}{(v_1 - v_2 + i\epsilon)^4}$$

Ciambelli, Leigh, LF 25

Quantum Null geometry

- The projected stress tensor $T_{vv}(v, \theta)$ of matter, gravitons and geometric degrees of freedom develop a central charge

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- c is a diffeomorphism **anomaly**.
- It is **dimensionless** and counts the number of degrees of freedom on $S^\perp$

Classically $c = 0$: Problem of time

In EFT $c_{QFT} = \infty$: Fundamental UV divergence

In mesoscopic QG: c_{QG} is finite Area form = molecular density

Ciambelli, Leigh, LF 25

Cardinal renormalization

Reuter-Becker 21.

Leigh-Minic-Kowalski-G, LF 22

Percacci-Fererro 24

Branchina et al 25

- Ultralocality + finiteness of c means that **we can study each null ray independently** as a gauged quantum **2d Chiral CFT** : The embadon
- Effective high energy dimensional reduction

Carlip 98

Solodukhin 98

Carlip 17

Decoupling

If we assume that radiation is perturbative $\sigma \rightarrow \partial_v \phi_i$

We can **decouple** the spin 0 sector from the radiative sector (spin2+ matter)

$$\varphi_i = \sqrt{\Omega} \phi_i \qquad \beta = \mu + \frac{4\pi G}{\sqrt{\Omega}} \partial_v (\sqrt{\Omega} \sum_i \phi_i^2)$$

Such that

$$C = \underbrace{\frac{1}{8\pi G} (\partial_v^2 \Omega - \beta \partial_v \Omega)}_{:=\tau} + \underbrace{\sum_i (\partial_v \varphi_i)^2}_{:=T^{\text{rad}}} \hat{=} 0$$

This decoupling allows one to use the technology of **Quantum Reference Frames (QRF)** in a field theoretical context $\rightarrow$ quantize C and construct gauge invariant (GI) observables

spin 0 = Physical clock for the radiative degrees of freedom $C = \tau + T^{\text{rad}} \hat{=} 0$

Dressing time

[Ciambelli-Leigh-LF, Wieland]

- To construct Gauge invariant observables we need to find a **Quantum Reference Frame** (QRF) for time reparametrization:

This is a field $V : v \rightarrow V(v)$ which **commutes** with T^{rad} and is **conjugate** to the spin 0 Hamiltonian $\mathcal{T}$. And which is such that its boost charge $S(V) := \Omega - V \partial_V \Omega$ defines an **entropy** $\partial_V S(V) \geq 0$

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It is given by the **dressing time** defined by

V is determined by β up to affine transformations
by two **edge modes**

$$\frac{\partial_v^2 V}{\partial_v V} = \beta$$

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Given V we can trade canonical variables for the spin 0 sector $(\Omega, \beta) \rightarrow (\tau, V)$

$$\Theta_0 = \int_{\mathbb{R}} \Omega \delta \beta = \int_{\mathbb{R}} \tau \frac{\delta V}{\partial_v V}$$

The dressing time is a quantum reference frame **(QRF)** for time reparametrization

Classical Dressing

The integrated constraint $C_f = \int_{\mathbb{R}} dv f C$ generates diffeomorphism

The symplectic action $\{C_f, \cdot\}$ exponentiate to an action of diffeomorphism

$$F \triangleright (\Omega, V, \varphi) = (\Omega \circ F, V \circ F, \varphi \circ F), \quad F = \exp(f \partial_v) \in \text{Diff}^+(\mathbb{R})$$

In the classical theory we can **construct Gauge Invariant** observables by pushing forward the fields through the dressing time:

$$\tilde{\Omega} := \Omega \circ V^{-1}, \quad \tilde{\varphi}_i := \varphi_i \circ V^{-1}. \quad \text{are gauge-invariant.}$$

GI observables are obtained by replacing $\Omega \rightarrow \tilde{\Omega}, \varphi \rightarrow \tilde{\varphi}$

$$O[\Omega, \varphi] \longrightarrow D_{\text{cl}}[O] = O[\tilde{\Omega}, \tilde{\varphi}]$$

All gauge-invariant observables are obtained this way

Classical Summary

- We have achieved **decoupling** in the gravitational perturbative regime

$$C = \tau + T^{\text{rad}} \hat{=} 0$$

- We can construct a **dressing time observable** conjugated to the spin 0 stress

$$(\Omega, \beta) \rightarrow (\tau, V) \qquad \tau = \frac{1}{8\pi G} (\partial_v^2 \Omega - \beta \partial_v \Omega) \qquad \frac{\partial_v^2 V}{\partial_v V} = \beta$$

- We can use it to construct the set of classical **gauge invariant observables**

$$D_{\text{cl}}[O] = O[\tilde{\Omega}, \tilde{\varphi}]$$

- Dressed area $D(\tau) = D(\partial_v^2 \Omega)$

Canonical Quantization

The dressing time commutes with itself $[V(v), V(v')] = 0$

The canonical conjugate to V is $\Pi(v) = \frac{\tau}{\partial_v V}$

We can straightforwardly quantize the fields to obey

$$[\Pi(v), V(v')] = -i\hbar\delta(v, v') \quad [\partial_v\varphi_i(v), \varphi_j(v')] = -\delta_{ij}\frac{i\hbar}{2}\partial_v\delta(v, v')$$

With a vacuum state **annihilated** by **positive frequency** modes

$$P_+\Pi|0\rangle = P_+V|0\rangle = P_+\varphi_i|0\rangle = 0$$

The projectors are $(P_\pm\varphi)(v) = \pm \int \frac{du}{2\pi i} \frac{\varphi(u)}{u - v \mp i\epsilon}$

The kinematical quantization possesses **negative norm states** !

These are due to the spin zero sector

Quantum Dressing

[Kirklin-LF]

Anomalies arise at the quantum level due to renormalization of composite observables

Composite operators are obtained as product of linear field at the same point.

Such products are defined by subtracting the vacuum expectation value.

usually by normal ordering

$$O[\Omega, \varphi_i] \rightarrow :O[\Omega, \varphi_i]:$$

Normal ordering places $P_+\varphi$ to the right and $P_-\varphi$ to the left

The projectors are not invariant under diffeomorphisms

$$U[F]^\dagger (P_\pm \varphi) U[F] = (P_\pm^F \varphi)$$

$$(P_\pm^F \varphi)(v) = \pm \int \frac{dF(u)}{2\pi i} \frac{\varphi(u)}{F(v) - F(u) \mp i\epsilon}$$

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This means that normal ordering is also not invariant

Hollands-Wald
Rejzner

$$U[F]^\dagger :O[\tilde{\Omega}, \tilde{\varphi}]: U[F] =:O[\tilde{\Omega}, \tilde{\varphi}]:_F \neq O[\tilde{\Omega}, \tilde{\varphi}]:$$

For instance $:C:_F =:C: + \frac{c\hbar}{24\pi} \{F, v\}$ ← **Schwarzian derivative**

$$:= \partial_v \left(\frac{\partial_v^2 F}{\partial_v F} \right) c - \frac{1}{2} \left(\frac{\partial_v^2 F}{\partial_v F} \right)^2$$

$c = 2 + c_{\text{rad}}$ represents a diffeomorphism anomaly

This is an **obstruction** to simple quantization of gauge-invariant observables.

Three Quantum Challenges

Quantizing the gauge invariant sector requires solving three **connected challenges**

These are **specific to QFT**: Unlike QM all states share infinite vacuum entanglement

- **Covariance**: ordinary normal ordering is background-dependent.
- **Positivity**: the spin-0 sector creates negative-norm states
- **Anomaly**: Prevent the imposition of the constraints

$$\frac{1}{i\hbar}[C_f, C_g] = C_{[f,g]_W} + \frac{\hbar c}{48\pi} \int_{\mathbb{R}} (\partial_v f \partial_v^2 g - \partial_v g \partial_v^2 f) \quad \hat{C}_f |\Psi\rangle \neq 0$$

In QRF language these challenges stem from extending the perspective neutral approach beyond locally compact groups

In the work [2604.02228](#) Josh Kirklin and I resolve these three puzzles

Resolution

The dressing time QRF plays a triple role

- It dresses fields and produces **localized gauge-invariant observables**
- It dresses the vacuum subtraction, giving **covariant normal ordering**
- It is necessary to **deform** the classical phase space and **cancel anomalies**

Covariant Normal ordering

- The solution of the first puzzle is to use the dressing time instead of the background time to define the normal ordering.
- This provides a notion of **covariant normal ordering** ie a **background-independent** one

For radiative fields this is direct since V commutes with the radiative fields $[V, \varphi_i] = 0$
we define

$$:O[\varphi_i]:_V = U_{\text{rad}}[V] :O[\varphi_i]: U_{\text{rad}}^{-1}[V]$$

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It is such that

$$:O[\varphi_i]:_V =: \exp \left(\int du dv G_V(u, v) \frac{\delta}{\delta \varphi_i(u)} \frac{\delta}{\delta \varphi_i(v)} \right) O[\varphi_i]:$$

Where $G_V(u, v) := \ln \left(\frac{V(u) - V(v)}{u - v} \right)$ is a smooth kernel.

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Define the **quantum dressing map** by combining classical dressing with covariant normal ordering:

$$D(:O[\Omega, \varphi]:) = :O[\tilde{\Omega}, \tilde{\varphi}]:_V$$

All gauge-invariant operators can be obtained using this quantum dressing procedure.

Gauge -invariant algebra

The gauge invariant algebra is a **Virasoro cross-product**

$$\mathcal{A}^G = \text{Vir}_M \ltimes \mathcal{A}_{\text{rad}}$$

$\uparrow$ $\uparrow$
 $D(\tau)$ $D(O[\varphi_i])$

Connes; Takesaki '70

Vest, Eccles, Hohn, Kirklin '24

Fewster Janssen, Loveridge, Rejzner, Waldron '24

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- If $\mathcal{O}$ is purely radiative then the dressing map is an **algebra morphism**

$$D(O_{\text{rad}})D(O'_{\text{rad}}) = D(O_{\text{rad}}O'_{\text{rad}})$$

- The spin 0 stress action on radiative fields is

$$[\tilde{\tau}(u), D(O_{\text{rad}})] = -D([T_{\text{rad}}(u), O_{\text{rad}}])$$

- The spin 0 stress $\tilde{\tau} = D(\tau)$ generates a Virasoro algebra at central charge $c = 24$

Physical Hilbert space

- The physical Hilbert space $\mathcal{H}_{\text{phys}}$ can be constructed through the GNS representation of $\mathcal{A}^G$

$$\mathcal{H}_{\text{phys}} = \text{span}(\mathcal{A}^G|0\rangle) / \mathcal{N}_{\text{null states}}$$

- We show $\mathcal{H}_{\text{phys}} \simeq \mathcal{H}_{\text{rad}} \otimes \mathcal{V}_{24-c^{\text{rad}}}$  Virasoro Verma module at central charge $24 - c_{\text{rad}}$

Spurious dof that wouldn't be there if we had gauge fixed before quantizing,
negative norm states if $c_{\text{rad}} > 24$

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Spurious dof that wouldn't be there if we had gauge fixed before quantizing, negative norm states if $c_{\text{rad}} > 24$

- Remarkably, one can remove these by deforming the classical phase space the Raychaudhuri constraint and the dressing : (similar to Green-Schwarz mechanism)

$$C = \tau + T^{\text{rad}} - \frac{c_{\text{cl}}\hbar}{48\pi} \partial_v \left(\frac{\partial_v^2 V}{\partial_v V} \right)$$

- One chooses $c_{\text{cl}} = 24 - c_{\text{rad}}$ → $c_{\tilde{\tau}} = c_{\text{rad}}$ → No quantum anomaly

Dressed quantization

The covariant normal ordering constructs the algebra,
the counterterm cancels the anomaly,
and the physical Hilbert space reduces to the radiative Hilbert space

Noise interferometry

A new type of interferometry experiment such as QUEST at Cardiff and GQuest at Caltech are trying to measure the noise spectra of proper time fluctuation through single photon detection.

- We have computed from first principle the gravitational noise in QFT due to gravitational back reaction.

$$S(\omega) = \frac{1}{L^2} \int_{-\infty}^{\infty} dT e^{-i\omega T} \langle \delta\tau(T) \delta\tau(0) \rangle$$

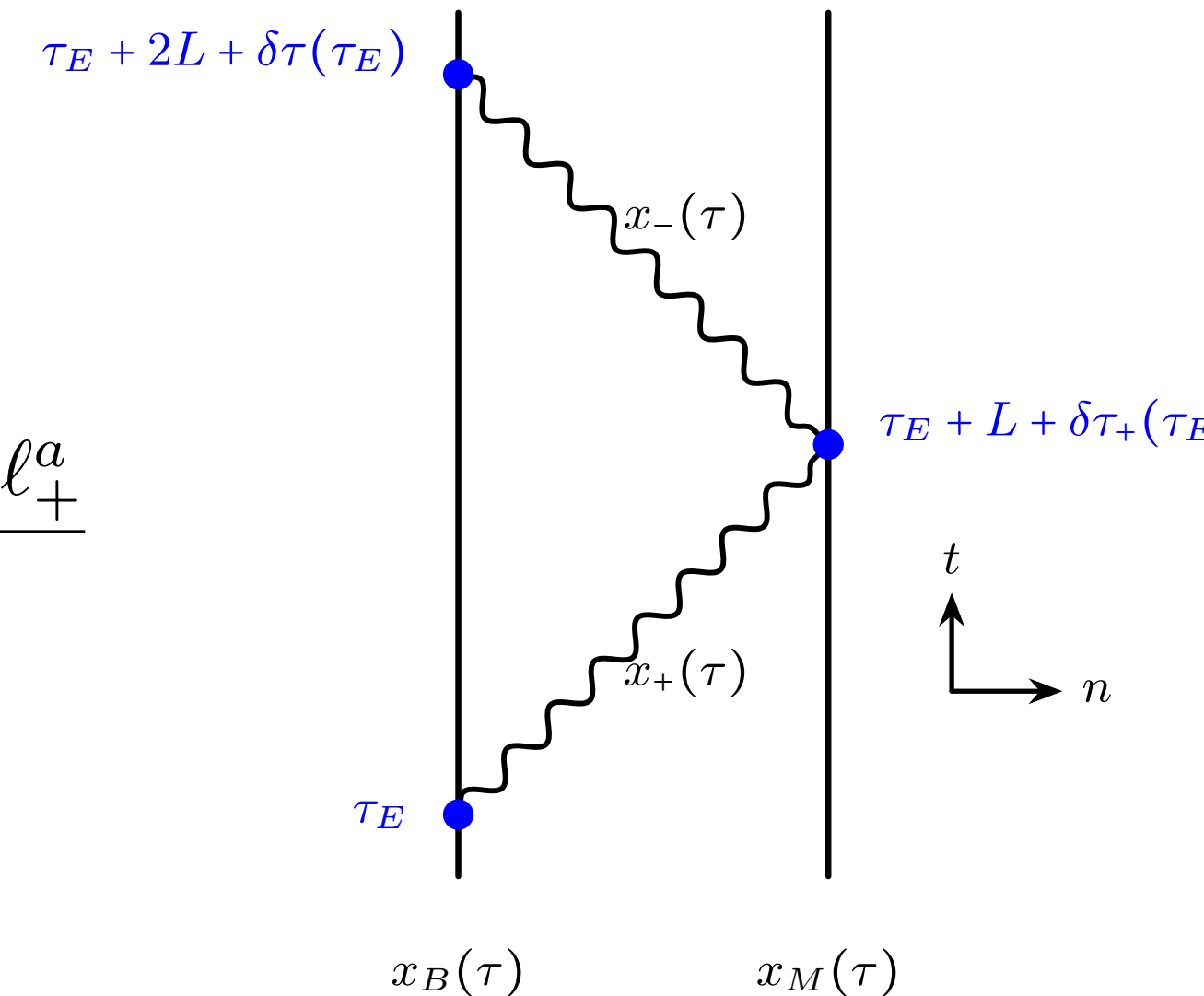
$$\sigma_n^a(p) := \frac{(p \cdot \ell_+) \ell_-^a - (p \cdot \ell_-) \ell_+^a}{(p \cdot t)}$$

$$= \int \frac{d^4 p}{(2\pi)^4} |R_L(\omega, \vec{p})|^2 \langle \tilde{h}_{\sigma_n \sigma_n} \tilde{h}_{\sigma_n \sigma_n} \rangle(\omega, \vec{p})$$

Interferometer
Response function

Metric fluctuations
Vacuum f + Energy f

$$\langle h_{ab} h_{cd} \rangle(p) = \frac{(8\pi \hbar G)^2}{(p^2)^2} \langle \bar{T}_{ab} \bar{T}_{cd} \rangle(p)$$



- In QFT (massless back reaction) no measurable effect yet beyond gravitational squeezing
- We need to evaluate the effect of the QRF fluctuations to see if we can infer UV-IR mixing.

Carney, Sivaramakrishnan 24
Oberfrank, LF 26

Zurek verlinde 19

Open Questions

- The presence of the central charge for frame reorientations creates new fluctuations between different time framed vacua proportional to $1/c$

$$|\langle F|G\rangle| = e^{-\frac{c}{24}K(F,G)} \quad F = \exp(f\partial_v)$$

- Are these extra vacuum/ QRF fluctuations observables in principle in interferometry noise experiments? Carney et al. '25, LF-Oberfrank '26

- The connection between **double smeared energy** bounds suggest that finite c corresponds to a finite spacetime uncertainty of t'Hooft type Flanagan 97
Freivogel, Fliss '21
Freivogel, Verlinde '26

$$(\Delta u \Delta v) \geq \frac{\ell_P^2}{\sqrt{c}}$$

- More generally Cardinal renormalization leads to a modification of EFT that resolves the cc problem. What its phenomenology? Reuter-Becker 21.
Leigh-Minic-Kowalski-G, LF 22
Percacci-Fererro 24
Branchina et al 25

