



G. DALL'AGATA (U. PADOVA)

QUANTUM VACUA OF STRING THEORY ON RIEMANN FLAT MANIFOLDS

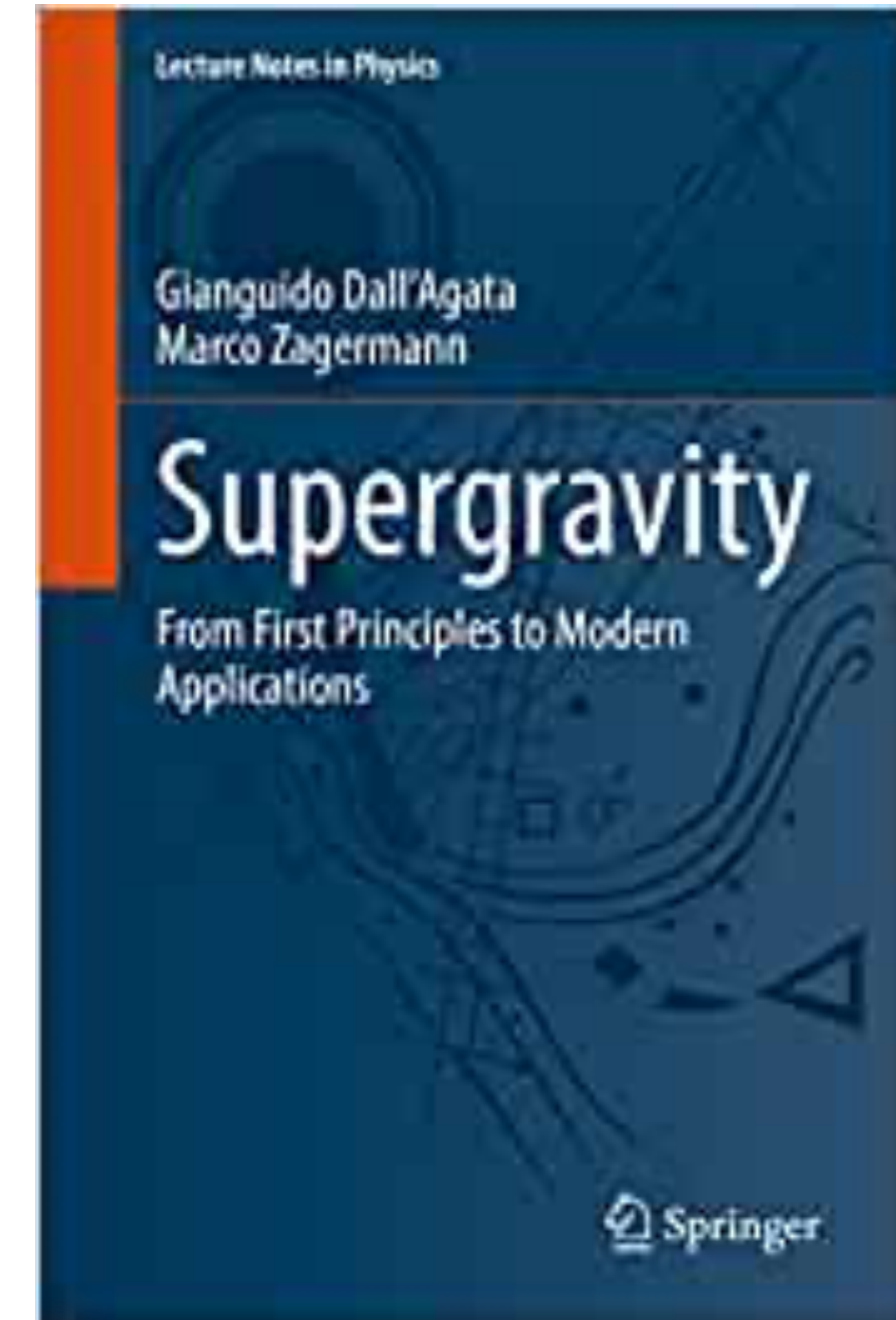
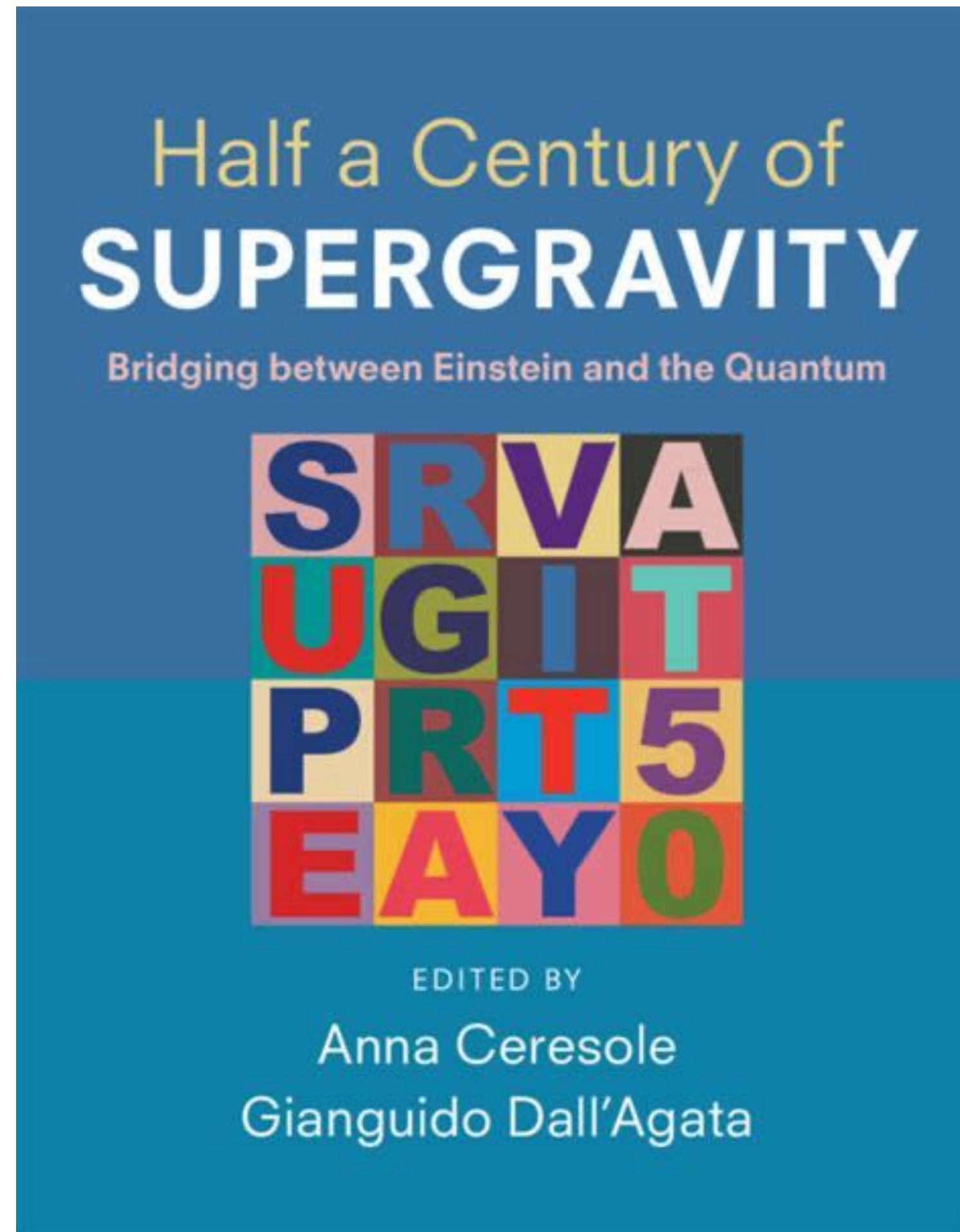
INTRODUCTION

- ▶ Aspirational goal: **susy breaking to address hierarchy problems**

$$G_F^{-1/2} \sim 10^{-16} M_P \qquad \rho \sim (10^{-30} M_P)^4$$

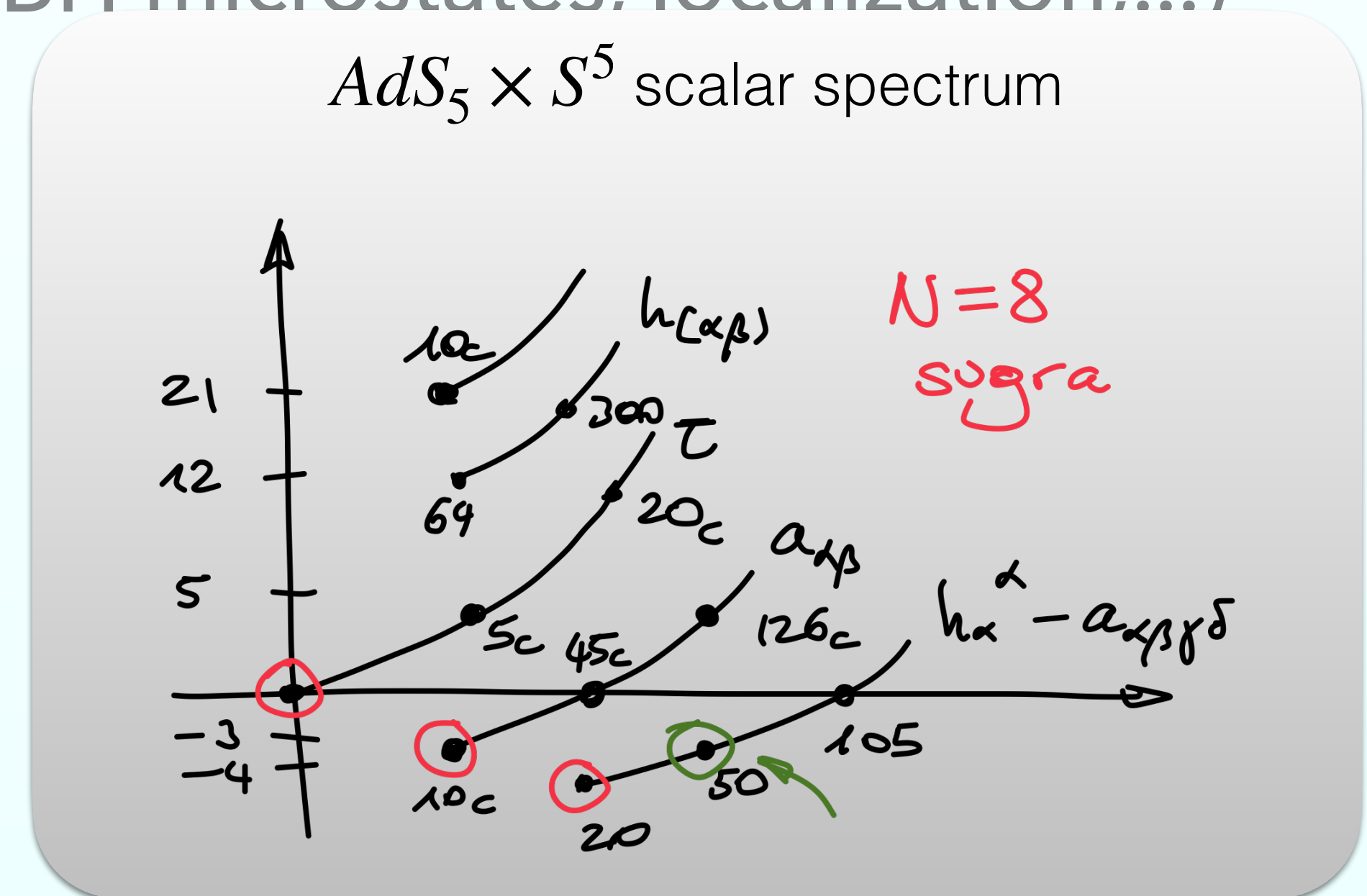
- ▶ May be related by susy breaking, though
 - ▶ *No susy at the Fermi scale*
 - ▶ *Generic quantum corrections give very large contributions*
- ▶ Direct string theory attempts complicated and often result in anecdotal knowledge

50 YEARS OF SUPERGRAVITY!!!



INTRODUCTION

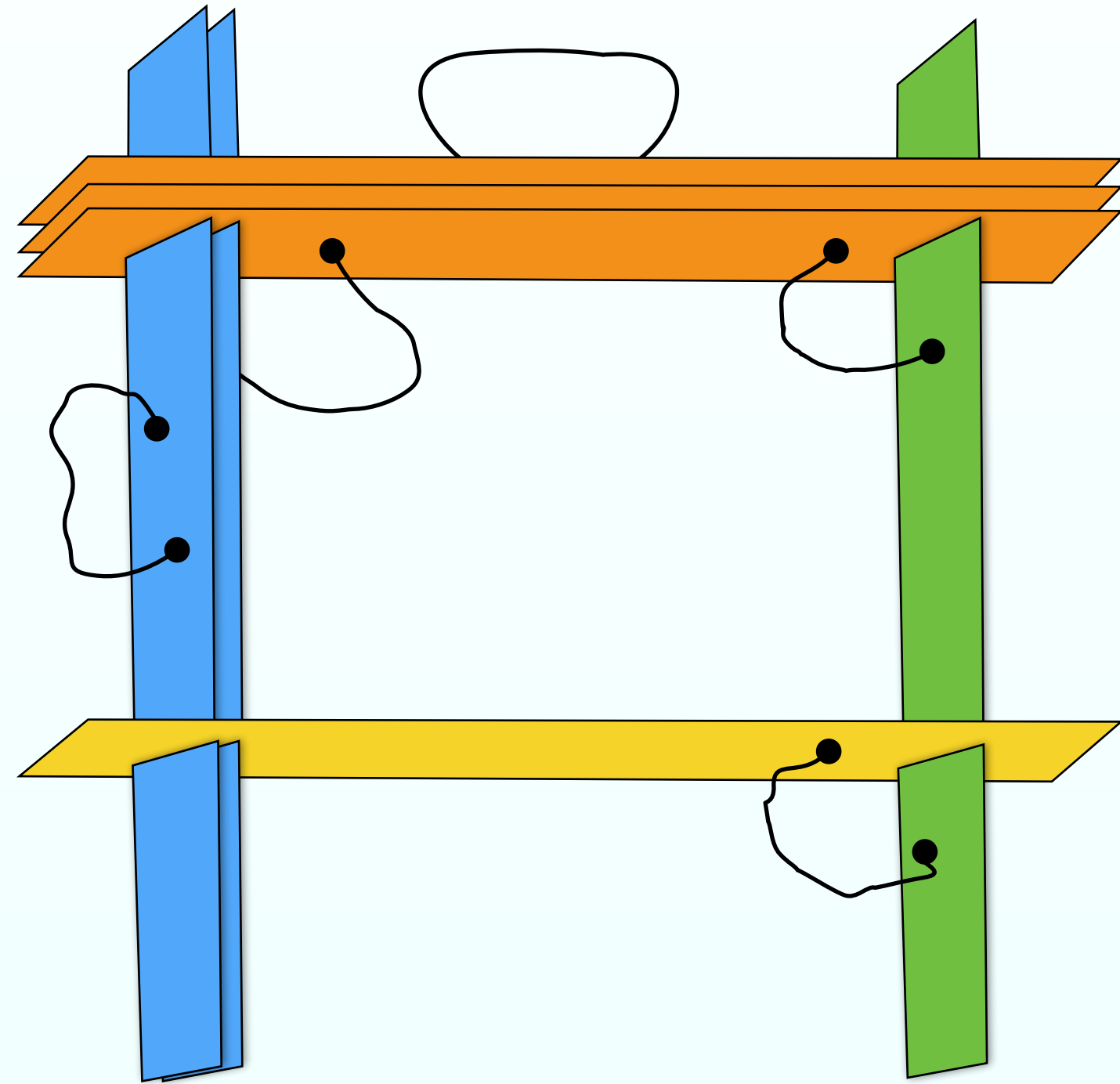
- ▶ Standard lore views **Supergravity as good EFTs for String Theory**
 - ▶ **Dualities** mostly understood thanks to Supergravity
 - ▶ Incredibly **efficient** also for **Quantum Gravity** (BH microstates, localization....)
 - ▶ Tool to study string solutions
- ▶ However
- ▶ **When** do we really use it as an EFT?
- ▶ Can we use it **more broadly** to learn about QG?



INTRODUCTION

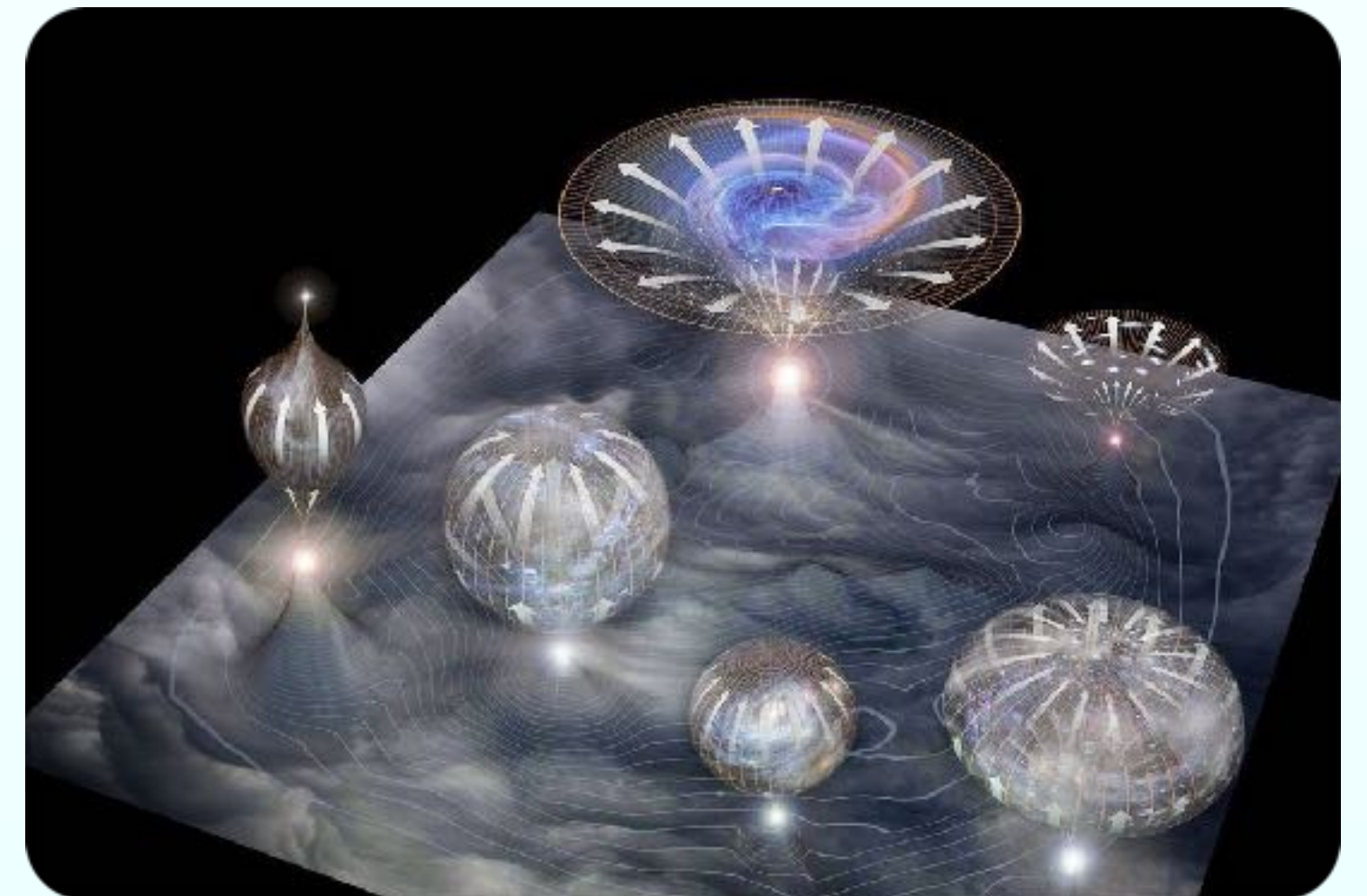
- ▶ Also:
- ▶ The **landscape** of supergravities seems **much larger than ST**
 - ▶ Example *infinite family of $SO(8)$ $N=8$ supergravities*
- ▶ Simple models are reductions on tori and **twisted tori**
 - ▶ Still many things to understand
 - ▶ **More theories in the swampland...**

STRING PHENOMENOLOGY

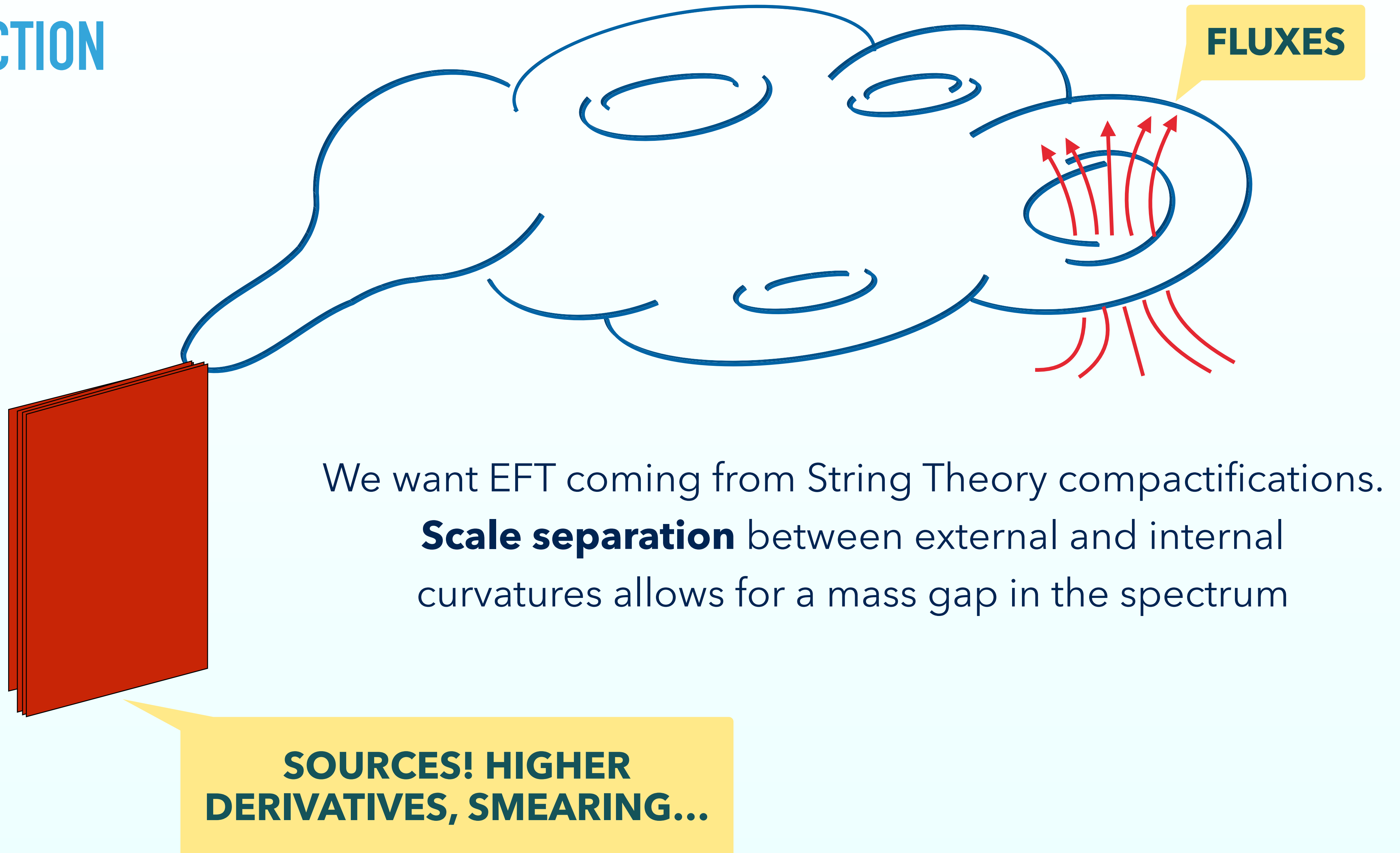


From constructive scenarios...

*... to the swampland
program*



INTRODUCTION



We want EFT coming from String Theory compactifications.

Scale separation between external and internal curvatures allows for a mass gap in the spectrum

CONSTRUCTING SUPERGRAVITY MODELS

Modern view: ***Embedding Tensor***

Any gauged supergravity is specified by two ingredients

Frame + $\ominus$

CONSTRUCTING SUPERGRAVITY MODELS

In 4d we have **electric** & **magnetic** vector fields, which are Hodge-dual to each other

$$\begin{array}{l} \text{Equations of} \\ \text{motion +} \\ \text{Bianchi Identities} \end{array} \quad \left\{ \begin{array}{l} dF^\Lambda = 0 \\ dG_\Lambda = d \left(\frac{\partial \mathcal{L}}{\partial F^\Lambda} \right) = 0 \end{array} \right.$$

The sum of **eoms+BI** is **invariant under duality** transformations, the Lagrangian is not!

Consistency of the dual field-strength definition constrains **S** to be a *Symplectic Transformation*

$$\begin{array}{c} \text{invariance consistency} \\ \left(\begin{array}{c} F' \\ G' \end{array} \right) = S \left(\begin{array}{c} F \\ G \end{array} \right) \quad S \in U \subset Sp(2n_V, \mathbb{R}) \end{array}$$

CONSTRUCTING SUPERGRAVITY MODELS

► *Gauging* a subgroup of dualities:

► Make $t_\alpha \in \mathfrak{g}_D$ **local** using A_μ^M

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - g A_\mu^M X_M$$

► Gauge algebra $\mathfrak{g}$ generated by the **embedding tensor** $X_M = \Theta_M^\alpha t_\alpha$

closure: $[X_M, X_N] = -X_{MN}{}^P X_P$

locality: $\Theta_M^\alpha \Theta^{M\beta} = 0$

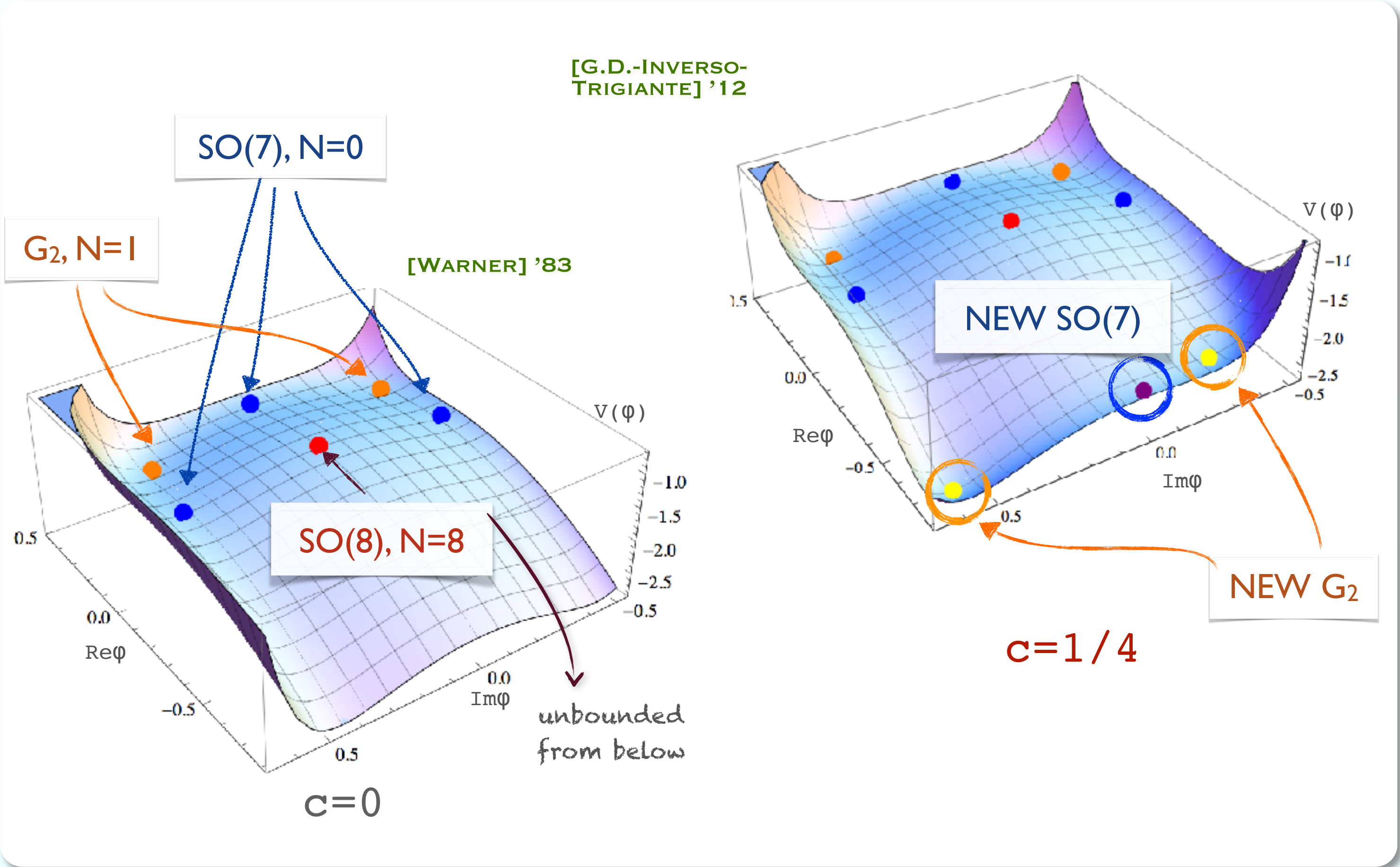
susy: $\Theta \in \mathbf{912}$ of $E_{7(7)}$

NICOLAI-SAMTLEBEN.
DE WIT-SAMTLEBEN-
TRIGIANTE

CONSTRUCTING SUPERGRAVITY MODELS

Many new theories and vacua!!

[GD,INVERSO,TRIGIANTE]
[BORGHESE,GUARINO,ROEST]
[BORGHESE,DIBITETTO,GUARINO,ROEST,VARELA]



INTRODUCTION (AGAIN!)

- ▶ Aspirational goal: **susy breaking to address hierarchy problems**
 - ▶ Riemann flat manifolds offer a fully controlled setup:
 - ▶ No branes, no orientifolds, no higher corrections, no warping
 - ▶ Clear perturbative analysis
 - ▶ Full analytic or numeric computations available
- ▶ Results in G.D.-Zwirner 2507.02339, Bento-Montero 2507.02037, plus work in progress with Inverso–Porrati–Takor

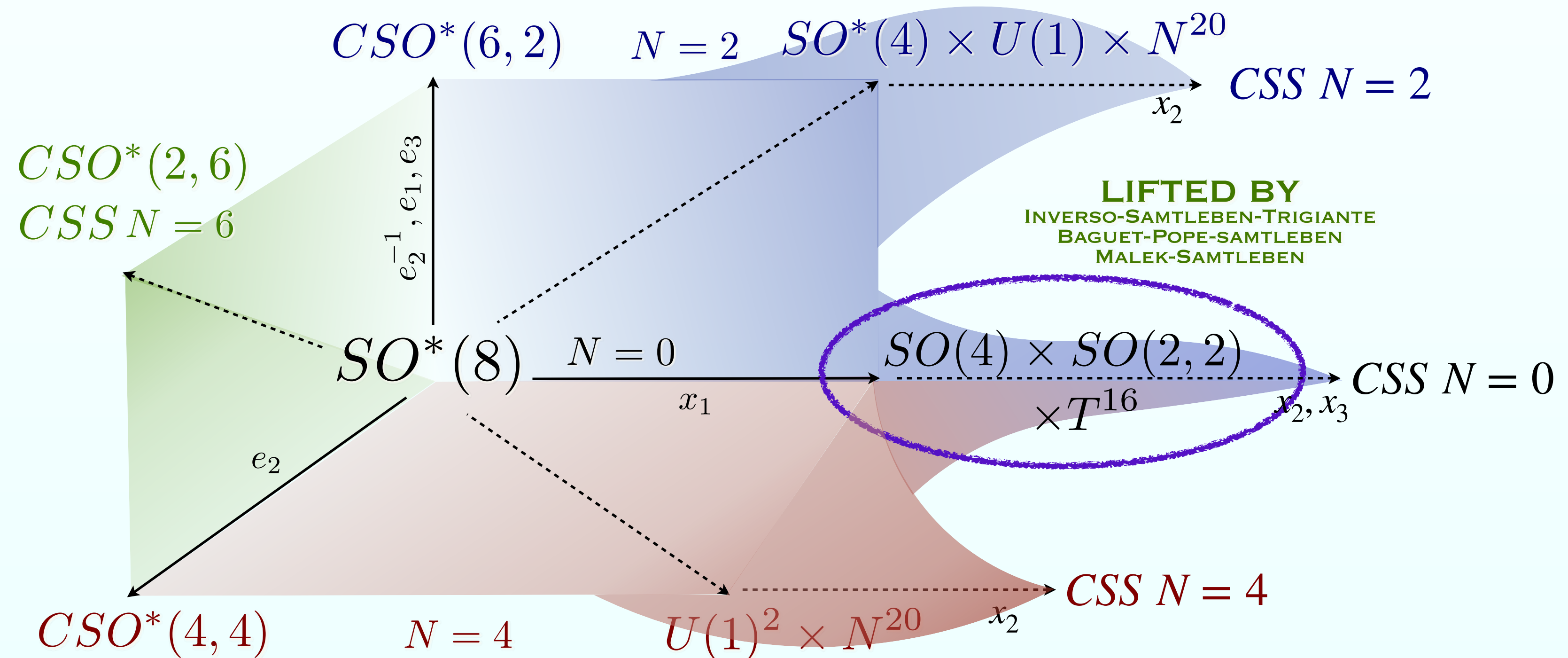
INTRODUCTION

- ▶ For EFTs: Minkowski vacua of supergravities and their moduli space
 - ▶ Simplified setup: maximal SUSY at Lagrangian level
 - ▶ **How much Lagrangian SUSY protects quantum corrections?**
 - ▶ Simple models are reductions on tori and twisted tori
 - ▶ Intricate web of vacua connected by moduli fields
 - ▶ Sliding susy breaking scale (at the classical level)
- ▶ **Finite quantum effects lifting vacuum energy and degeneration?**



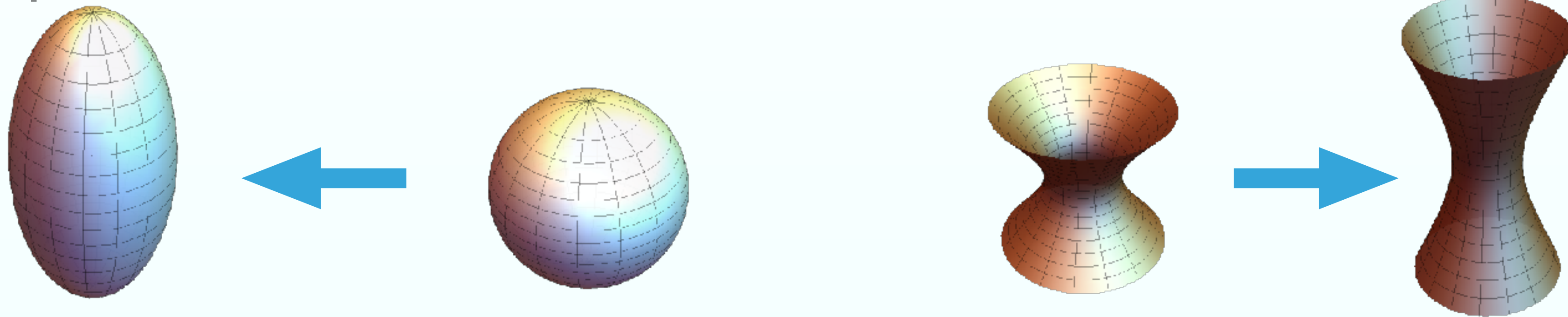
STRING VS SUPERGRAVITY LANDSCAPE

- GD-INVERSO
CATINO-GD-INVERSO-ZWIRNER



MAXIMAL SUPERGRAVITY & FLAT FOLDS

- ▶ The uplift uses as internal manifold $S^3 \times H^{2,2}$



- ▶ Trick: CSS reduction of DFT ALDAZABAL-BARON-MARQUÉS-NÚÑEZ
GEISSBÜHLER

$$g_{\mu\nu} = e^{4\gamma\varphi(x)} g_{\mu\nu}(x) \quad e^\phi = \rho^2(y) e^{\varphi(x)}$$

$$\mathcal{H}_{MN} = U_M^A(y) M_{AB}(x) U_N^B(y)$$

$$\mathcal{A}_\mu^M = U^{-1}{}_A{}^M(y) A_\mu^A(x)$$

$$M_{AB}(x) \in \frac{\text{SO}(6,6)}{\text{SO}(6) \times \text{SO}(6)}$$

MAXIMAL SUPERGRAVITY & FLAT FOLDS

GD-INVERSO-SPEZZATI

- ▶ The result is a non-geometric space with **T-duality patching**

- ▶ Example:

$$ds^2 = dx_1^2 + dx_2^2 + d\theta^2 + dy_1^2 + dy_2^2 + d\psi^2$$

$$\theta \sim \theta + \alpha$$

$$z_L \sim e^{-i\alpha} z_L$$

$$z_R \sim e^{i\alpha} z_R$$

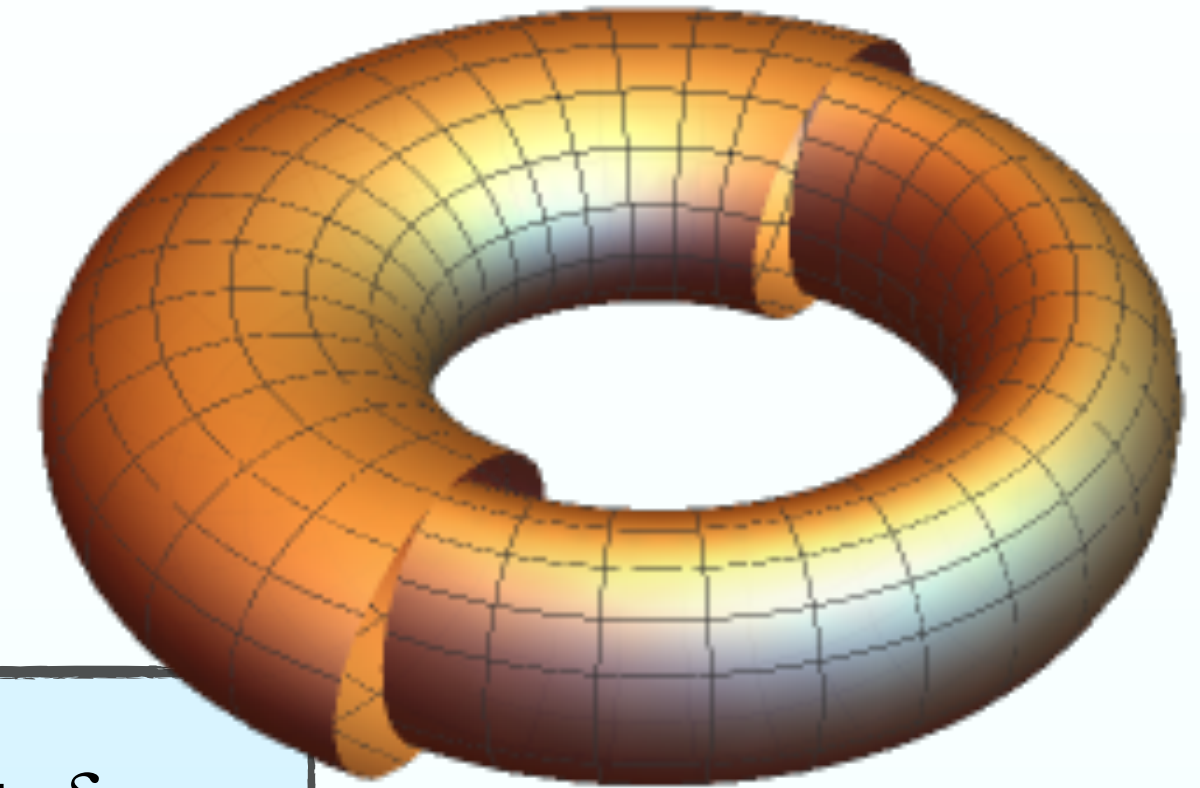
$$w \sim e^{i\alpha} w$$

$$\psi \sim \psi + \delta$$

$$w_L \sim i e^{-i\delta} w_L$$

$$w_R \sim -i e^{i\delta} w_R$$

$$z \sim e^{i\delta} z$$



- ▶ Still, 4d supergravities from non-geometric backgrounds

SUPERGRAVITY VACUUM STABILITY AGAINST RADIATIVE CORRECTIONS

- ▶ *Ungauged sugra is finite up to 4 loops (possibly 7)*
- ▶ Gauging = new couplings
- ▶ One-loop divergencies governed by super traces

$$\text{Str} \left(\mathcal{M}^{2k} \right) = \sum_J (-1)^{2J} (2J + 1) \text{tr}(\mathcal{M}_J)^{2k}$$

- ▶ Example: 1-loop potential

$$\begin{aligned} V_{eff} &= \frac{1}{64\pi^2} \text{Str} \mathcal{M}^0 \Lambda^4 \log \frac{\Lambda^2}{\mu^2} + \frac{1}{32\pi^2} \text{Str} \mathcal{M}^2 \Lambda^2 - \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \log \Lambda^2 \\ &+ \frac{1}{64\pi^2} \text{Str} \left(\mathcal{M}^4 \log \mathcal{M}^2 \right) \end{aligned}$$

SUPERGRAVITY VACUUM STABILITY AGAINST RADIATIVE CORRECTIONS

- ▶ Using only **general identities** and the vacuum condition GD-ZWIRNER

$$\text{Str}(\mathcal{M}^2) = \text{Str}(\mathcal{M}^4) = \text{Str}(\mathcal{M}^6) = 0$$

- ▶ **1-loop finiteness**

- ▶ 1-loop potential

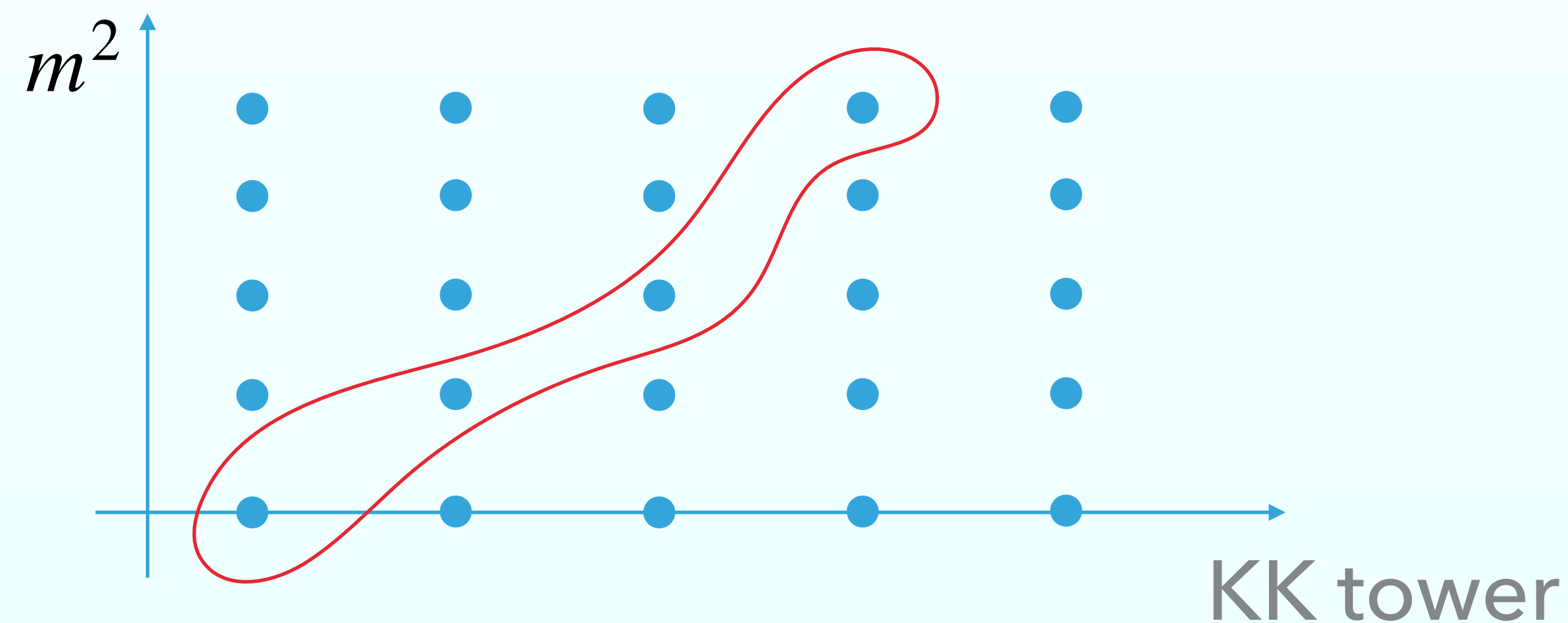
$$V = \frac{1}{64\pi^2} \text{Str}(\mathcal{M}^4 \log \mathcal{M}^2) < 0$$

- ▶ Runaway...

ARE WE REALLY USING AN EFT?

GD-PREZAS
GRAÑA-MINASIAN-TRIENDL-VAN RIET

- ▶ Wolf: *Any Riemannian homogeneous flat space is the direct product of the Euclidean plane with the torus*
- ▶ Consistent truncations vs EFT



Can we deal with the full KK spectrum and check quantum corrections with susy breaking at the compactification scale?



**RIEMANN-FLAT MANIFOLDS AKA
BIEBERBACH MANIFOLDS**

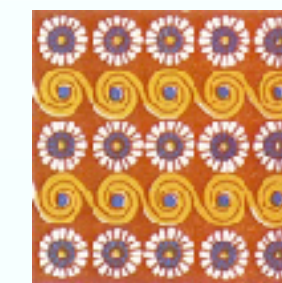
WHAT ABOUT KK STATES? GD-ZWIRNER

- ▶ Higher-dimensional twisted tori related to Bieberbach manifolds

- ▶ Bieberbach = smooth flat quotients: $\mathbb{R}^d/\Gamma$

- ▶ $\mathbb{R}^6/\Gamma$: CID-SCHULZ
LUTOWSKY-PUTRYCZ

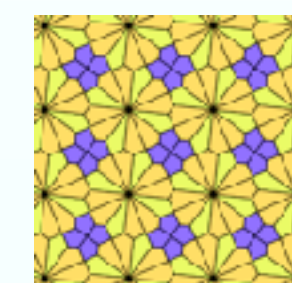
$$\Gamma \subset \mathbb{E}_2 = U(1) \ltimes \mathbb{R}^2$$



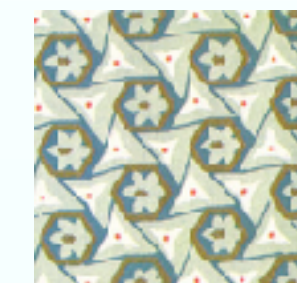
p2



p3



p4



p6

- ▶ 28927992 orbifolds (crystals)
- ▶ 38746 flat manifolds (Bieberbach)
- ▶ 3314 orientable
- ▶ 760 spin!*

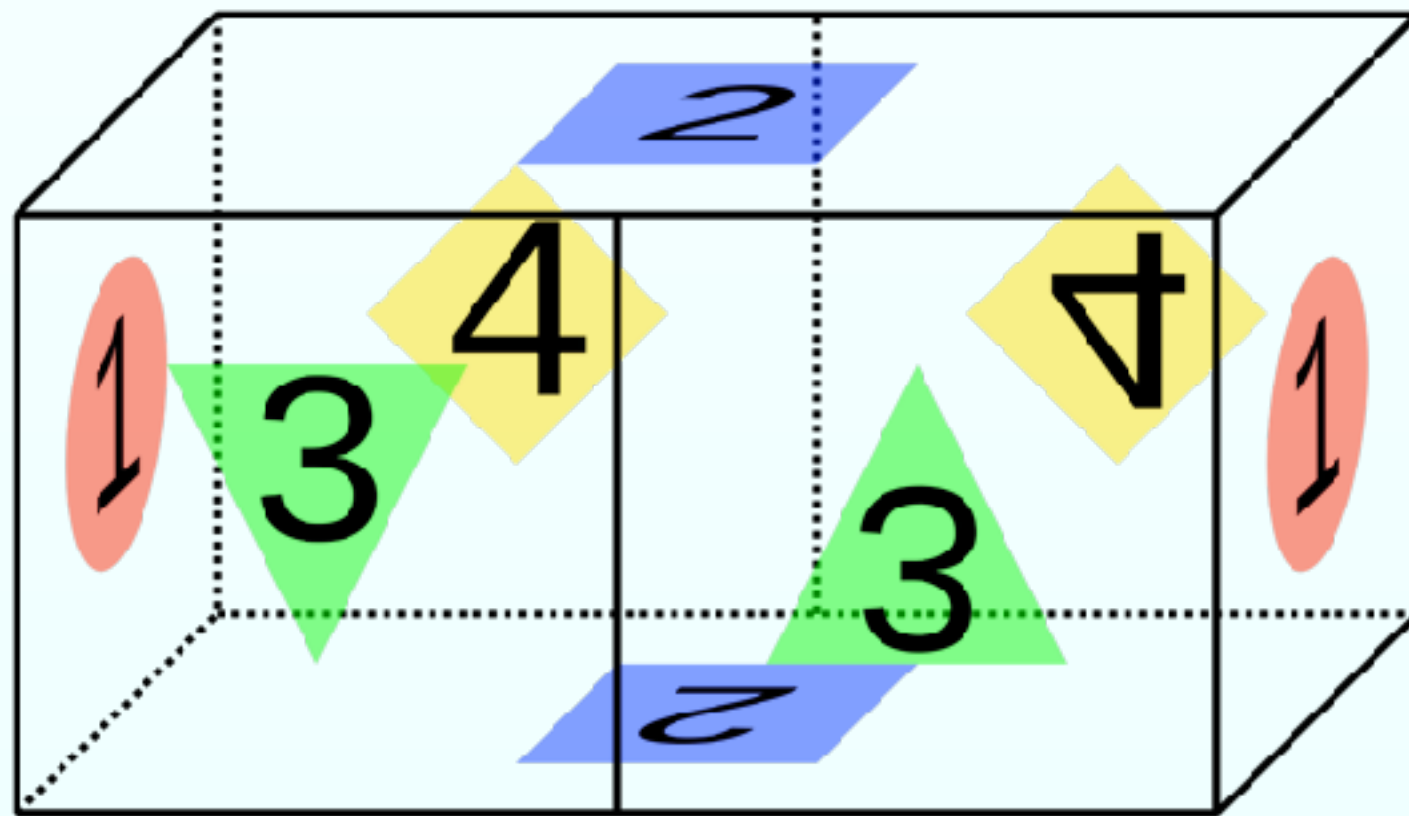
***Single cases analyzed in ST,
but no general lesson***

*Mathematicians' classification revised after our works!

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- Presentations (Hantzsche-Wendt 3-manifold):

$$O_6^3 : \Gamma = \langle a, b, c, r, \sigma \mid r^2 = c, \sigma^2 = a, \sigma r \sigma^{-1} = a b r^{-1}, r a r^{-1} = a^{-1}, \\ r b r^{-1} = b^{-1}, \sigma a \sigma^{-1} = a^{-1}, \sigma c \sigma^{-1} = c^{-1} \rangle.$$



*Klein bottle (non orientable)
fibred over
an interval (singular)
= Hantzsche-Wendt (smooth and spin!)*

BIEBERBACH MANIFOLDS GD-ZWIRNER

$$Z_{\Lambda}[s, \vec{a}, \vec{b}^{\star}] = \sum_{\vec{z} \in \Lambda} \frac{e^{-2\pi i \vec{z} \cdot \vec{b}^{\star}}}{\|\vec{z} + \vec{a}\|^s}$$

- General expression for the 1-loop potential

$$V_1 = \frac{1}{2} \int \frac{d^D p}{(2\pi)^D} \sum_{n=-\infty}^{+\infty} \sum_I (-1)^{F_I} N_I \sum_{\vec{k}_I} \log \left(p^2 + m_{\vec{k}_I}^2 \right)$$

- Can be expressed exactly in terms of the Epstein Zeta function

$$V_1 = - \frac{\sqrt{\Delta} \Gamma \left(\frac{D+d}{2} \right)}{2\pi^{\frac{D+d}{2}}} \sum_I (-1)^{F_I} \frac{N_I}{|r(\Gamma_I)|} Z_{\Lambda_I}(D+d, 0, \vec{a}_I^{\star})$$

$$V_1 = - \frac{\pi^{D/2}}{2} \Gamma(-D/2) \sum_I (-1)^{F_I} \frac{N_I}{|r(\Gamma_I)|} Z_{\Lambda_I^{\star}}(-D, \vec{a}_I^{\star}, 0),$$

BIEBERBACH MANIFOLDS GD-ZWIRNER

- ▶ Vanishing supertraces order by order (after suitable relabeling of the states)

$$\text{Str}(\mathcal{M}^2) = \text{Str}(\mathcal{M}^4) = \text{Str}(\mathcal{M}^6) = 0$$

- ▶ **Asymmetric spectrum can be realigned**
 - ▶ Signal of spontaneous susy breaking!
 - ▶ Negative 1-loop correction (boson zero modes > fermion zero modes)

BIEBERBACH MANIFOLDS GD-ZWIRNER

- ▶ Analytic expressions for the potentials:

- ▶ **Z3 trivial structure:** $V_1 = -\frac{162}{\pi^4 L_3^7} (3^8 - 1) \zeta(8) = -\frac{3936}{35} \frac{\pi^4}{L_3^7} < 0$

- ▶ **Z3 non-trivial structure:**

- ▶ $V_1 = -\frac{\sqrt{3}}{2} L^2 L_3 \frac{\Gamma(5)}{2\pi^5} \frac{128}{3} \left[Z_\Lambda(10, \vec{0}, \vec{0}) - Z_\Lambda(10, \vec{0}, (0, 0, 1/(2L_3))) - \frac{2}{L_3^{10}} (\zeta(10) + \eta(10)) \right] - \frac{197}{105} \frac{\pi^4}{L_3^7} < 0$

SUMMARY

- ▶ Technical progress: *We provided full consistency conditions, KK spectrum and 1-loop pot. for Riemann flat manifolds*
 - ▶ *Equivalent results by Bento-Montero (interesting interpretation in terms of Casimir branes)*
 - ▶ Clarified supergravity reductions on twisted tori **EFT vs consistent truncations**
- ▶ **Physics: Casimir finite but negative definite**
 - ▶ **Supertrace cancellations order by order! (REALIGNED SUSY)**
- ▶ **Quantum suppression?** $V_1 \sim O(\hbar^2) M_{KK}^4$

MORE GENERAL LESSONS

- ▶ Supergravity's landscape far from being fully explored
 - ▶ Susy breaking patterns, vacuum structure... *old problems, but new tools*
- ▶ String lifts desirable (*EFT vs. consistent truncations – scale separation problem*)
- ▶ **Quantum corrections** give *new avenues* and new interesting selection rules
- ▶ *Pheno and Black hole physics applications require a more solid framework!*