

N. Emil J. Bjerrum-Bohr

General Relativity from Amplitudes and Fundamental Gravitational Interactions



Niels Bohr Institutet



General Relativity

Einstein's theory presents us with a beautiful theory of gravity and leave us with several exciting questions:

Quantum extensions

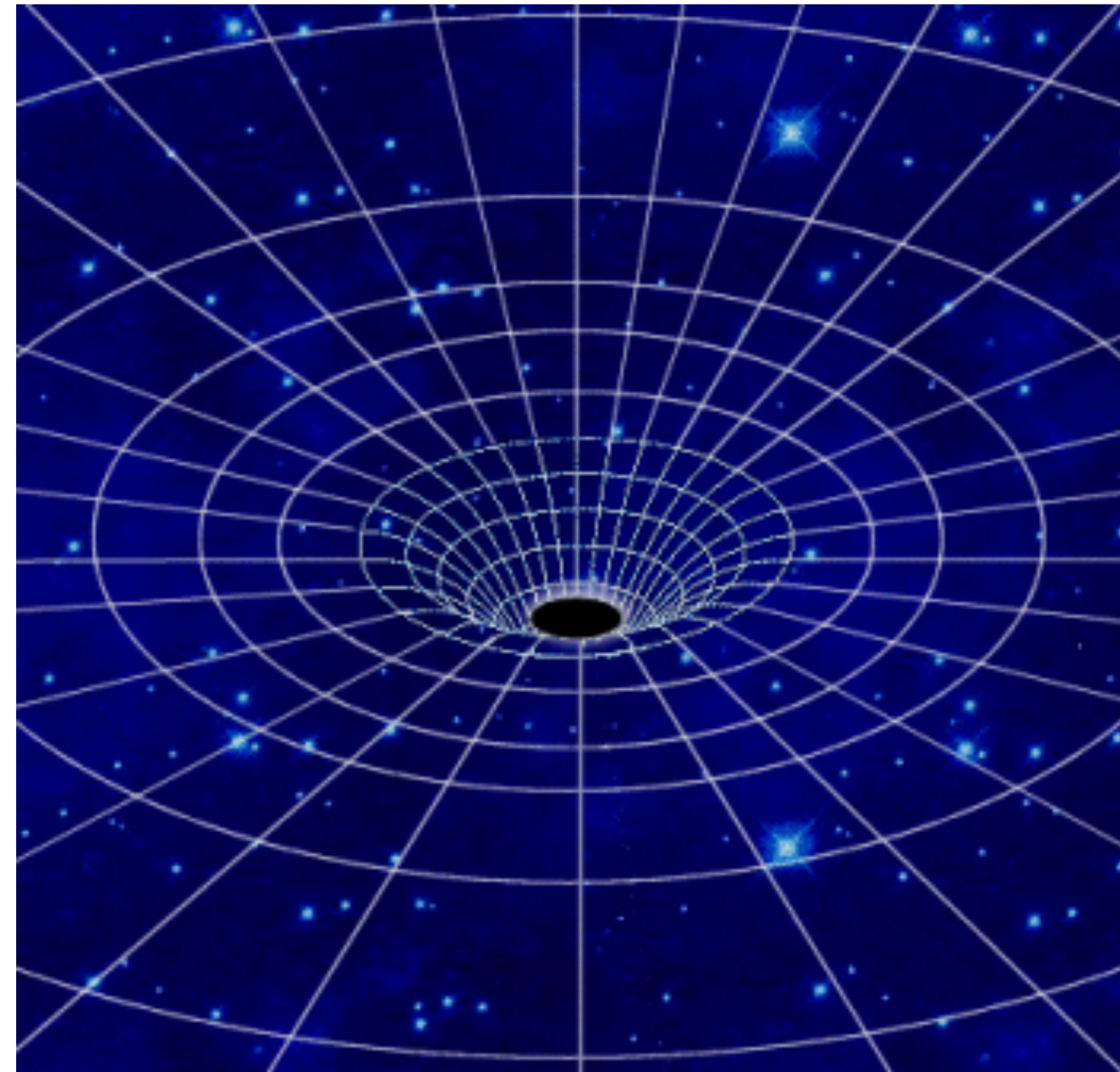
Geometrical description $\leftrightarrow$ QFT
(flat space / curved space)
formulations

Cosmology

Test of classical gravity principles

Test of extra dimensions / SUSY

Signs of string theory gravity...



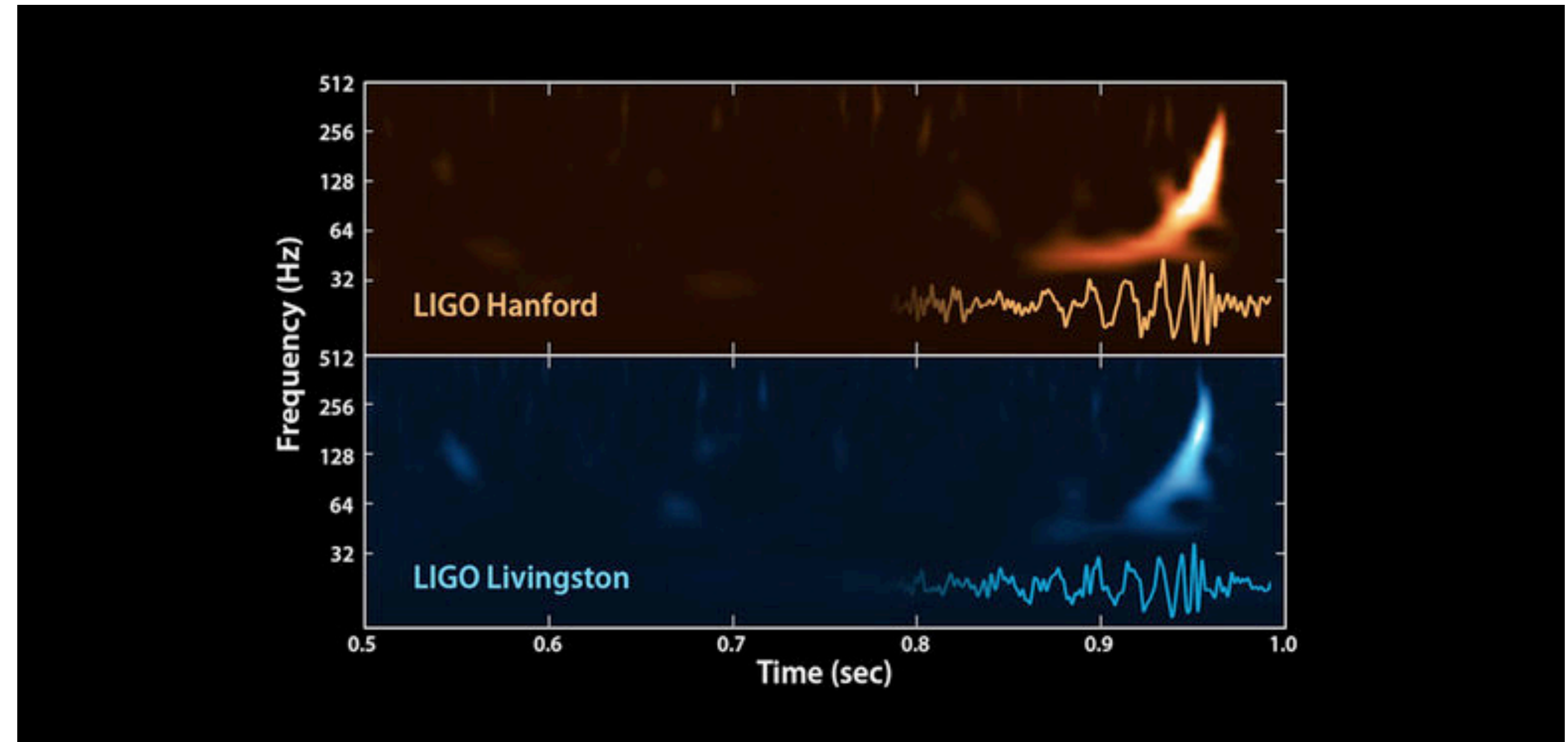
New observations

New data - new window

First direct observation of a binary merger of black holes

Direct access to gravitational interactions in the most extreme regimes

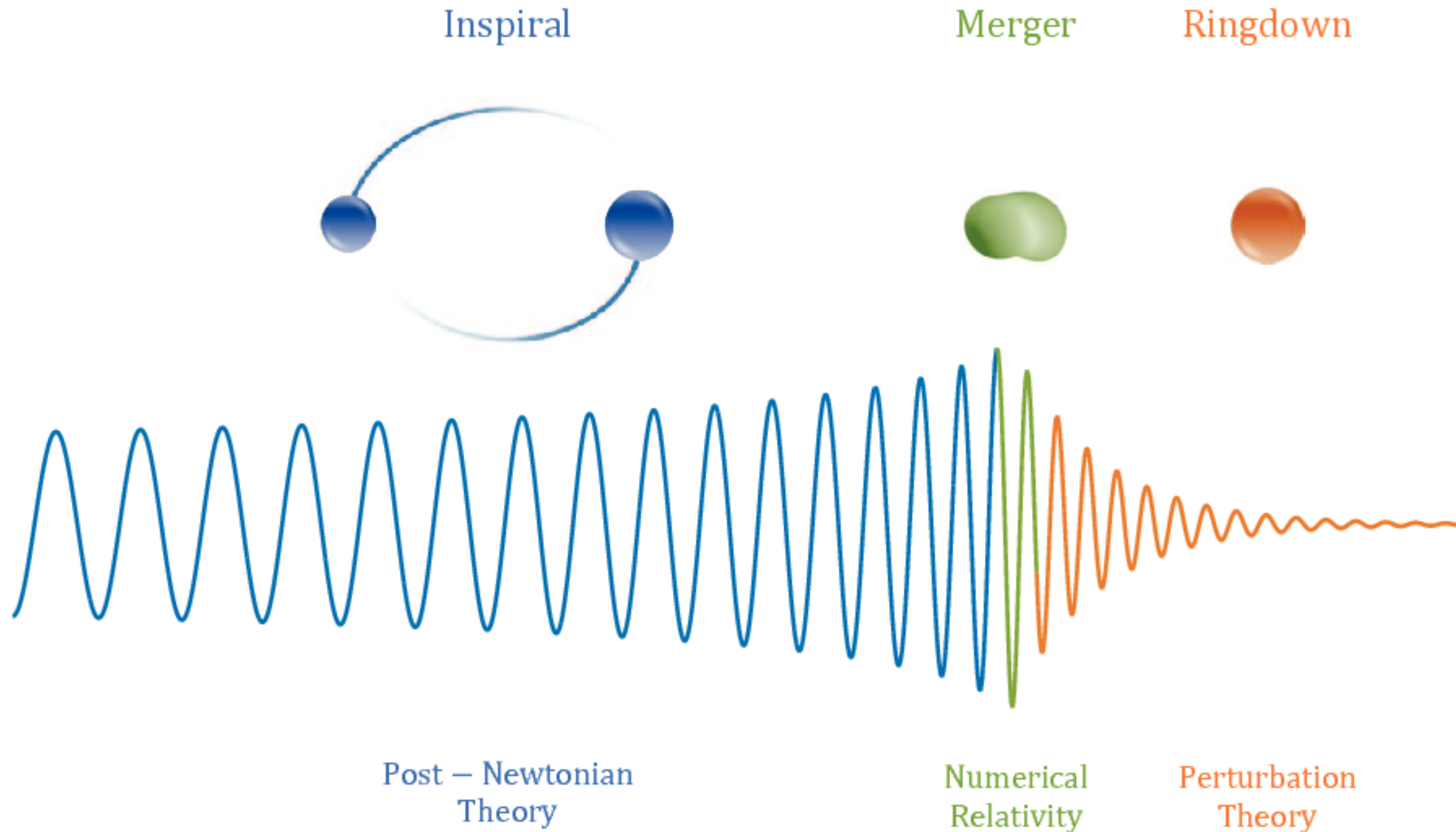
Need for theory to match observational progress & precision.



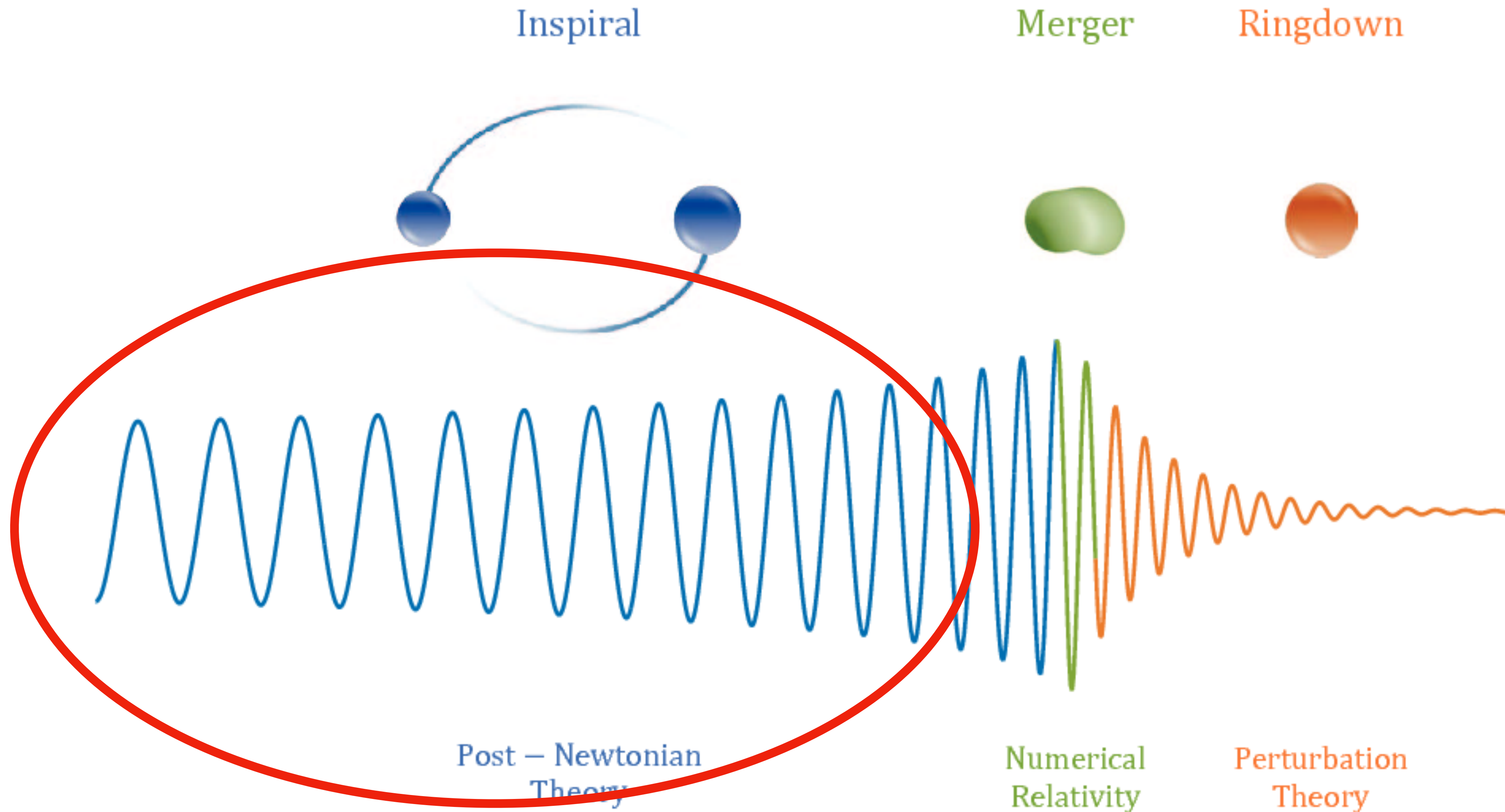
Amplitude methods facilitate refined computation and heightened precision, complementing traditional analysis.

A potential opportunity to advance new discoveries in the field of gravity.

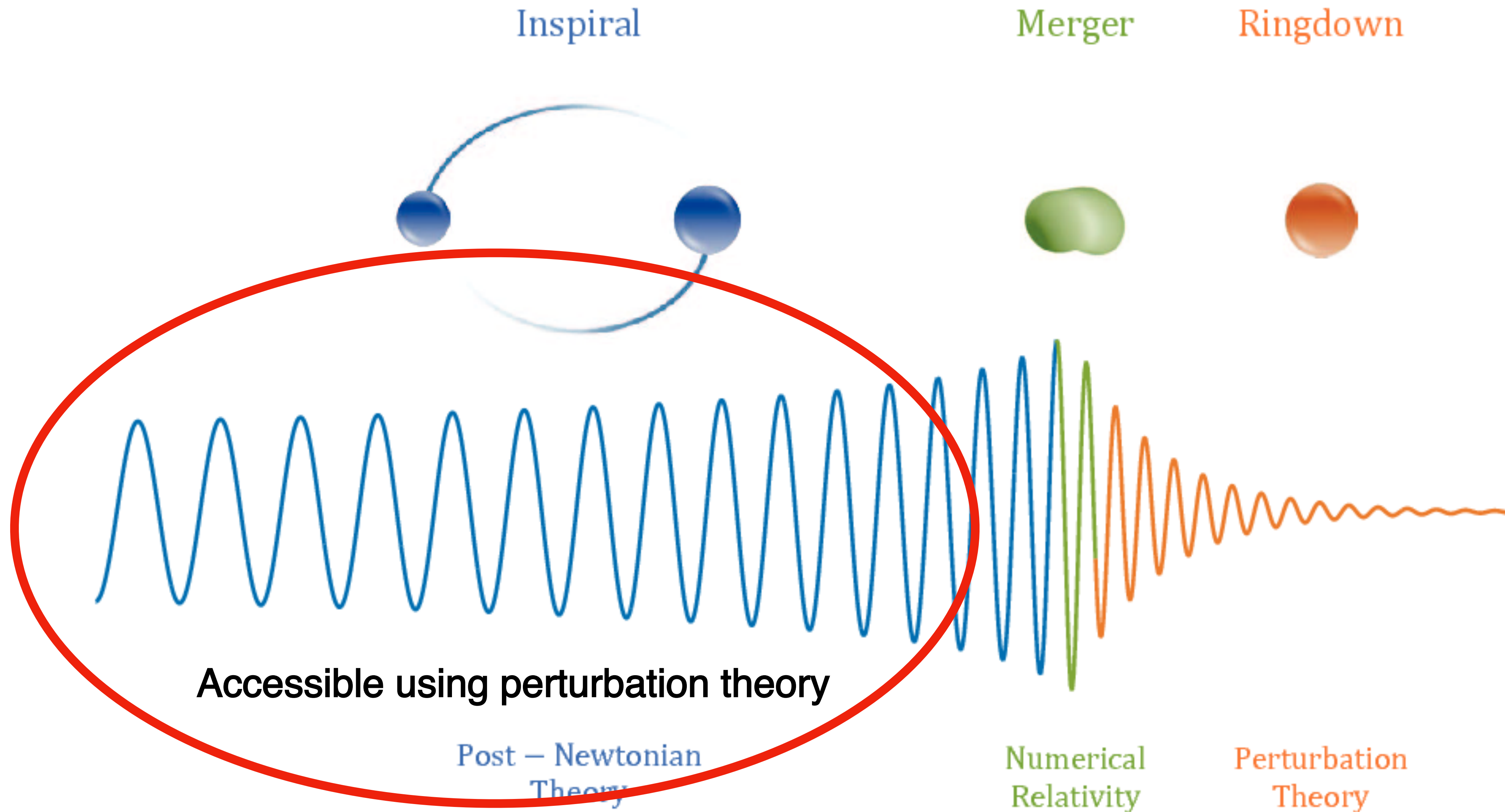
Stages of the gravitational merger of Black Holes



Stages of the gravitational merger of Black Holes



Stages of the gravitational merger of Black Holes



Precision Tests of Einstein's theory

Using numerical relativity, we can examine behavior during merger and ringdown phases

Numerical methods validate general relativity during inspiral but become costly in some regimes due to error accumulation

Precision Tests of Einstein's theory

A particle physics approach to gravity enables a particle physics methodology for studying quantum gravitational effects using on-shell S-matrix techniques

The validation of general relativity using quantum particle physics also offers new insights into the applicability of quantum physics concepts as an extension of the classical theory

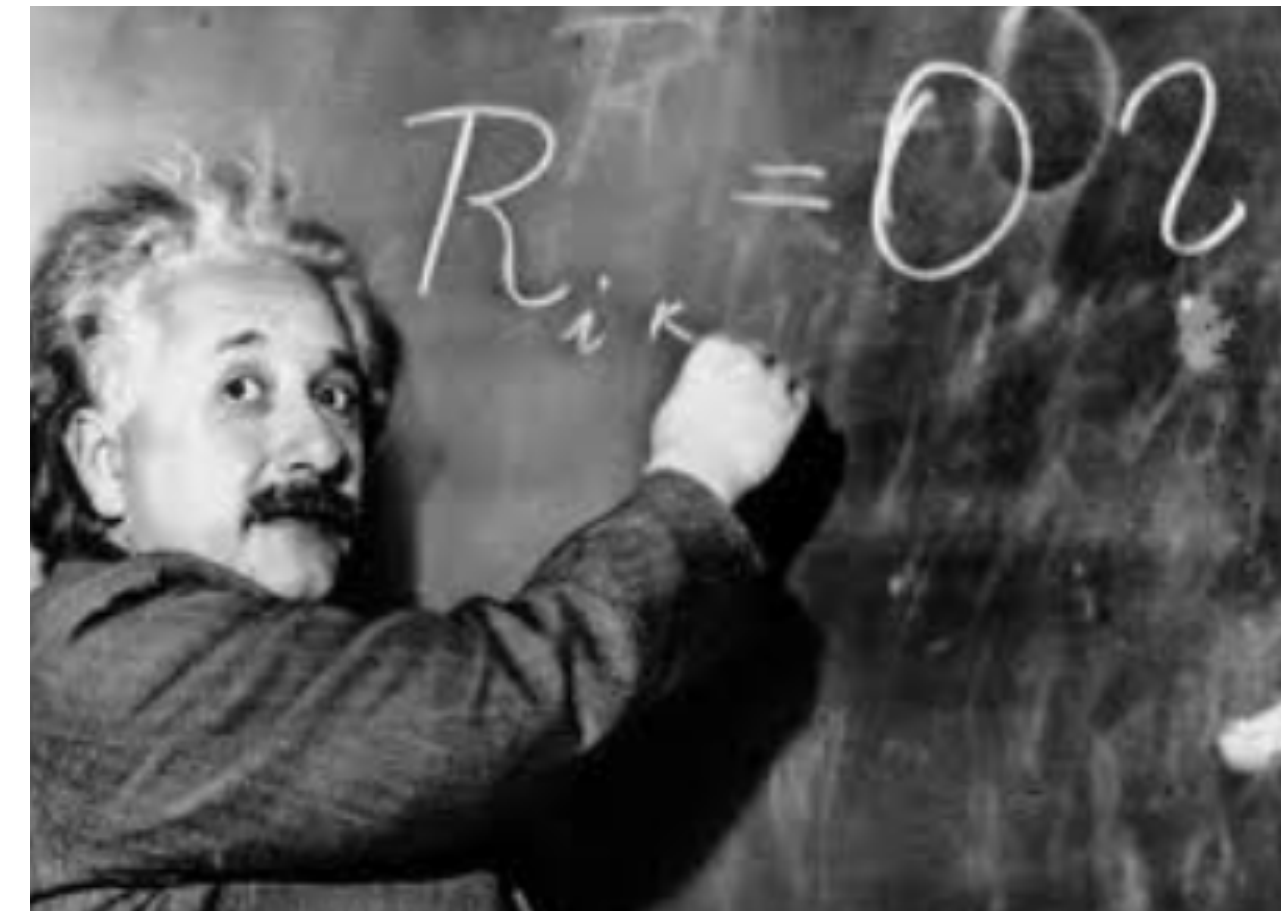
Gravity as a
particle theory?

Quantization of gravity?

Known since the 1960ties that a particle version of General Relativity can be derived from the Einstein-Hilbert Lagrangian (Feynman, DeWitt)

Expand the Einstein-Hilbert Lagrangian:

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$$



Derive vertices as in a particle theory - compute amplitudes as Feynman diagrams! (GW Kovacs and Thorne 1977)

Quantum gravity as a particle theory

Gravity as a theory with self-interactions

3pt, 4pt, ... n-pt self-interactions

Non-renormalizable theory! ('t Hooft and Veltman)

Dimensionful coupling: $G_N = 1/M_{\text{planck}}^2$

No known symmetry removes all UV divergences

String theory can, through the introduction of new length scales.

Quantum gravity? An effective field theory

A modern viewpoint (**Weinberg**) to view the quantization of general relativity from the viewpoint of effective field theory

$$\mathcal{L}_{\text{EH}} = \sqrt{-g} \left[\underbrace{\frac{R}{16\pi G_N}}_{\text{EFT-gravity}} + \underbrace{\mathcal{L}_{\text{matter}}}_{\text{EFT-matter}} \right]$$
$$\mathcal{L}_{\text{eff GR}} = \sqrt{-g} \left[\frac{2R}{16\pi G_N} + R^2 + R_{\mu\nu}^2 + \dots + \mathcal{L}_{\text{matter}} + \dots \right]$$

Consistent quantum gravity at low energies long-distance (**Donoghue; NEJBB, Donoghue, Holstein**)

Effective field theory gravity

At low energies, we assume the graviton is the only light particle in the gravitational theory

Next, we order the Lagrangians in an energy expansion in derivatives of the light graviton fields.

Effective field theory gravity

Pure gravity

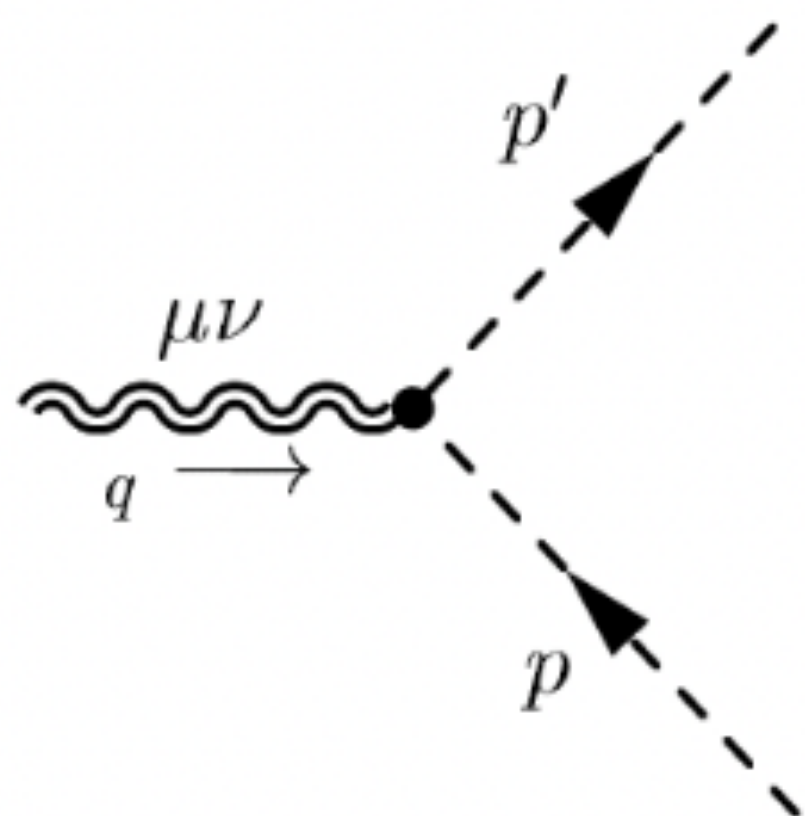
$$\begin{aligned}\mathcal{L}_{\text{gravitation}} &= \mathcal{L}_{\text{g}0} + \mathcal{L}_{\text{g}2} + \mathcal{L}_{\text{g}4} + \dots \\ \mathcal{L}_{\text{matter}} &= \mathcal{L}_{\text{m}0} + \mathcal{L}_{\text{m}2} + \mathcal{L}_{\text{m}4} + \dots\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\text{g}0} &= 0, \\ \mathcal{L}_{\text{g}2} &= \frac{2}{\kappa^2} R, \\ \mathcal{L}_{\text{g}4} &= c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} . \\ &\dots\end{aligned}$$

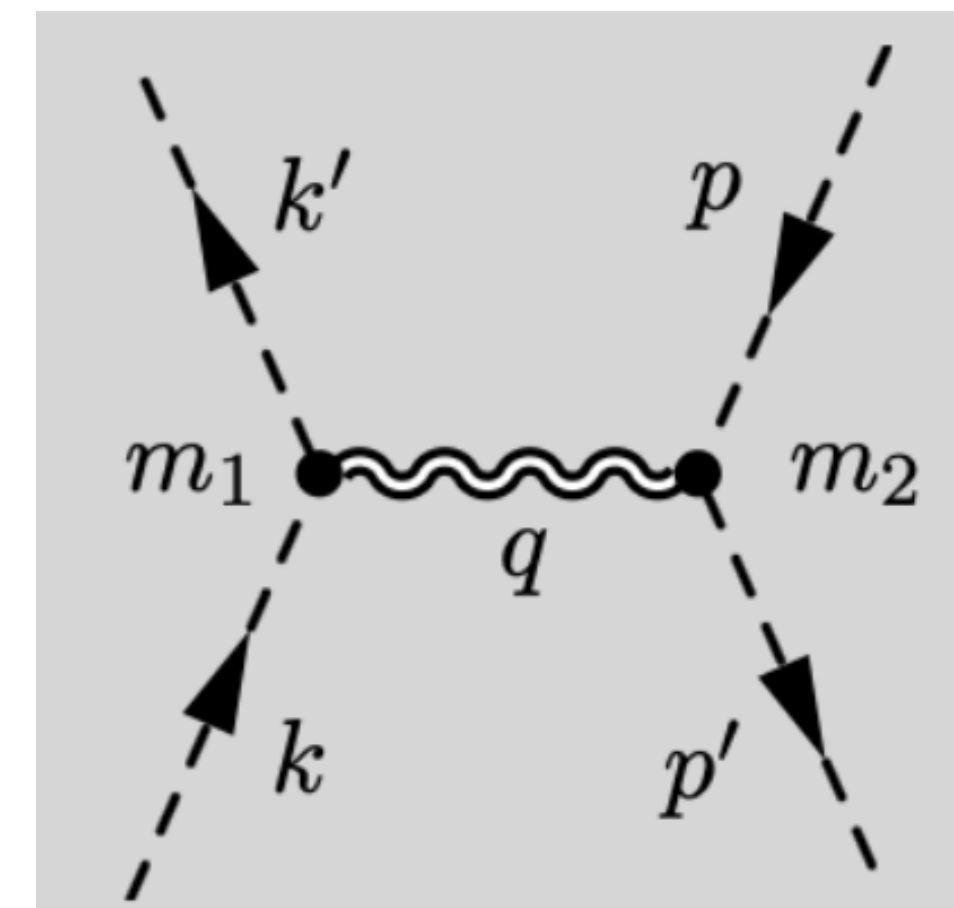
Scalar matter

$$\begin{aligned}\mathcal{L}_{\text{m}0} &= \frac{1}{2} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 \right] \\ \mathcal{L}_{\text{m}2} &= d_1 R^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + R \left(d_2 \partial_\mu \phi \partial^\mu \phi + d_3 m^2 \phi^2 \right) .\end{aligned}$$

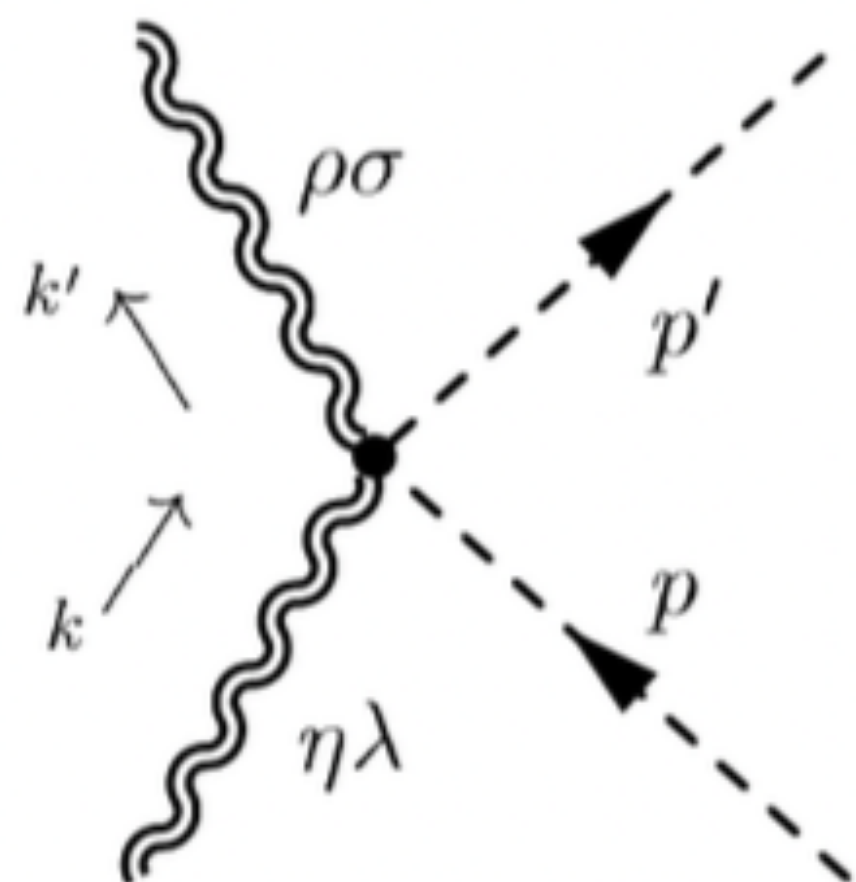
Scalar vertex rules and propagator



$$\tau_{\mu\nu}^{ssh}(p, p', m) = -\frac{i\kappa}{2} \left[p_\mu p'_\nu + p_\nu p'_\mu - \eta_{\mu\nu} [(p \cdot p') - m^2] \right]$$



$$P_{\alpha\beta, \gamma\delta}^D(q) \equiv -\frac{i}{2} \frac{\left[\eta_{\alpha\gamma} \eta_{\beta\delta} + \eta_{\alpha\delta} \eta_{\beta\gamma} - \frac{2}{D-2} \eta_{\alpha\beta} \eta_{\gamma\delta} \right]}{q^2 + i\epsilon}$$



$$\alpha\beta \quad \text{wavy line} \quad q \quad \gamma\delta$$

$$\frac{1}{2} \text{---} + \frac{1}{2} \text{---} - \frac{1}{D-2} \text{---}$$

$$\tau_{\eta\lambda\rho\sigma}^{ssgg}(p, p', m) = \frac{i\kappa^2}{2} \left[\left[I_{\eta\lambda\alpha\delta} I_{\rho\sigma\beta}{}^\delta - \frac{1}{4} \left[\eta_{\eta\lambda} I_{\rho\sigma\alpha\beta} + \eta_{\rho\sigma} I_{\eta\lambda\alpha\beta} \right] \right] (p^\alpha p'^\beta + p'^\alpha p^\beta) - \frac{1}{2} \left[I_{\eta\lambda\rho\sigma} - \frac{1}{2} \eta_{\eta\lambda} \eta_{\rho\sigma} \right] [(p \cdot p') - m^2] \right]$$

Gravity as EFT Quantum Gravity

General relativity as an effective field theory is a natural bottom-up generalization of Einstein's theory

Provides an ideal low-energy setup for testing quantum gravity effects

What we discover must align with predictions from a fundamental quantum theory

New computational
capacity

Concrete computation in gravity

Featuring a number of unpleasant features

Complicated Feynman rules (infinitely many vertices)

Numerous double contractions

Factorial growth in the number of legs

Feynman diagram topologies: no ordering!

Loop order: complicated tensor integrals

Example:

Scalar field theory: constant vertex (~ 1 term)

Gluons: momentum-dependent vertex (~ 3 terms)

Gravitons: momentum-dependent vertex (~ 100 terms)

Naïve basic 4pt diagram count (graviton exchange) :

$100 \times 100 \sim 10^4$ terms + index contractions (~ 36 pr diagram)

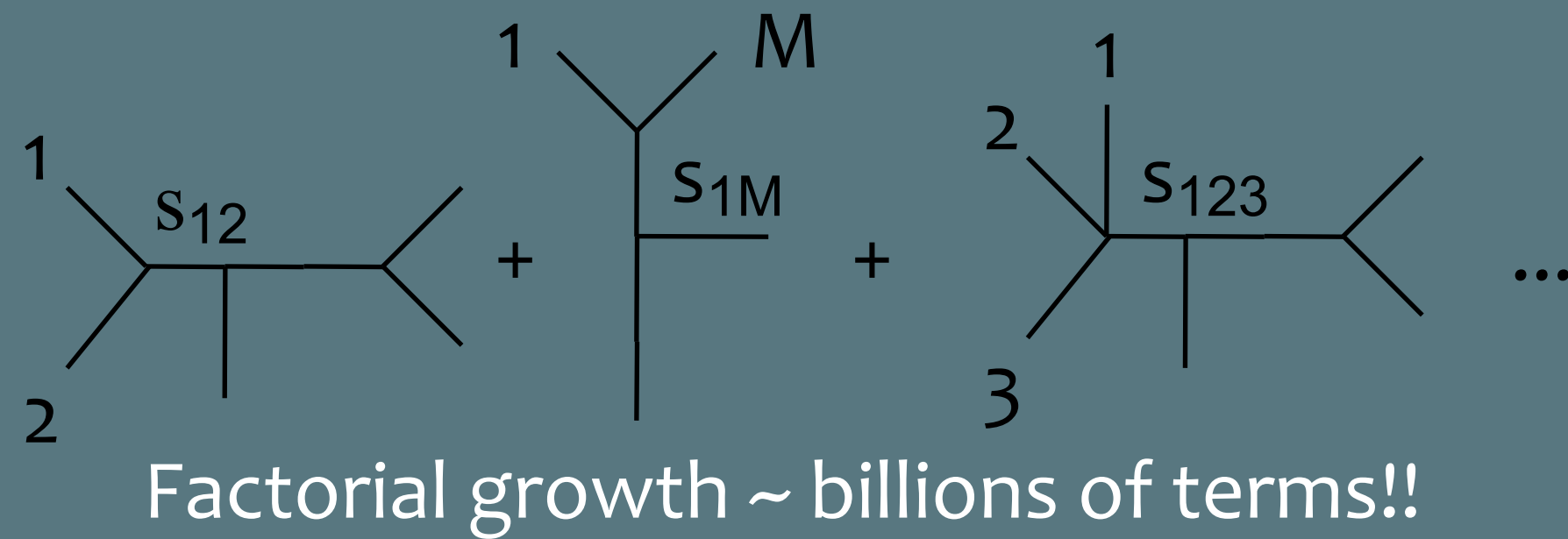
Number of diagrams:

4-point: ($\sim 4 !$) $\sim 10^5$ terms $\sim 10^6$ index contractions

n-point: ($\sim n !$) $\sim$ more atoms in your brain!

Too much off-shell (gauge-dependent) clutter.....

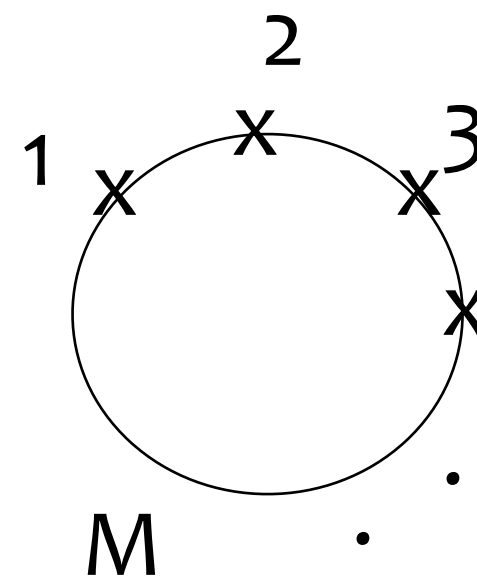
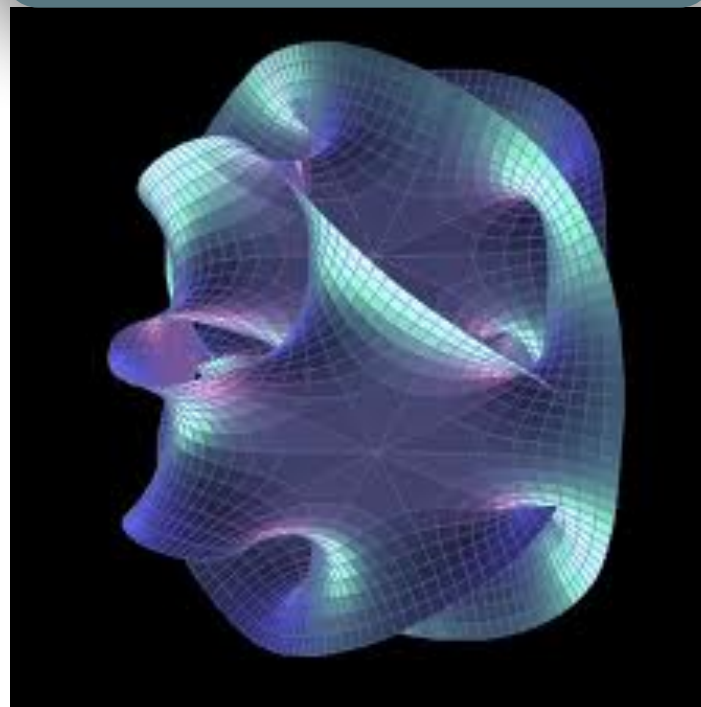
Tree amplitude revolution in QCD!



Rich hidden structure

Inspiration across fields

String Theory



New relations

Key: on-shell states formalism

Spinor products :

$$\langle i \ j \rangle = \epsilon^{mn} \lambda_m^i \lambda_n^j \quad [i \ j] = \epsilon^{\dot{m}\dot{n}} \tilde{\lambda}_{\dot{m}}^i \tilde{\lambda}_{\dot{n}}^j$$

Different representations of
the Lorentz group

$$p_{a\dot{a}} = \sigma_{a\dot{a}}^\mu p_\mu$$

$$p^\mu p_\mu = 0 \quad p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$$

Momentum parts of amplitudes:

$$q_{a\dot{a}} = \mu_a \tilde{\mu}_{\dot{a}} \quad p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} \quad 2(p \cdot q) = s_{ij} = -\langle \lambda, \mu \rangle [\tilde{\lambda}, \tilde{\mu}]$$

Spin-2 polarisation tensors in terms of helicities,
(squares of those of YM):

$$\varepsilon_{a\dot{a}}^- = \frac{\lambda_a \tilde{\mu}_{\dot{a}}}{[\tilde{\lambda}, \tilde{\mu}]} \quad \tilde{\varepsilon}_{a\dot{a}}^+ = \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu, \lambda \rangle} \quad \begin{matrix} \varepsilon^- & \varepsilon^- \\ \tilde{\varepsilon}^+ & \tilde{\varepsilon}^+ \end{matrix} \quad \text{(Xu, Zhang, Chang)}$$

Yang-Mills MHV-amplitudes

(n) same helicities vanishes

$$A^{\text{tree}}(1^+, 2^+, 3^+, 4^+, \dots) = 0$$

(n-1) same helicities vanishes

$$A^{\text{tree}}(1^+, 2^+, \dots, j^-, \dots) = 0$$

(n-2) same helicities:

$$A^{\text{tree}}(1^+, 2^+, \dots, j^-, \dots, k^-, \dots)$$

$A^{\text{tree MHV}}$ Given by the formula
(Parke and Taylor) and proven
by (Berends and Giele)

First non-trivial example,
(M)aximally
(H)elicity (V)iolating
(MHV) amplitudes

One single term!!

$$i \frac{\langle j k \rangle^4}{\langle 1 2 \rangle \langle 2 3 \rangle \cdots \langle n 1 \rangle}$$

Simplifications from Spinor-Helicity

Huge simplifications

$$s_{ij} = -\langle \lambda, \mu \rangle [\tilde{\lambda}, \tilde{\mu}]$$

$$V_{\mu\alpha,\nu\beta,\sigma\gamma}^{(3)}(k_1, k_2, k_3) = \kappa \text{sym} \left[-\frac{1}{2} P_3(k_1 \cdot k_2 \eta_{\mu\alpha} \eta_{\nu\beta} \eta_{\sigma\gamma}) - \frac{1}{2} P_6(k_{1\nu} k_{1\beta} \eta_{\mu\alpha} \eta_{\sigma\gamma}) \right. \\ \left. + \frac{1}{2} P_3(k_1 \cdot k_2 \eta_{\mu\nu} \eta_{\alpha\beta} \eta_{\sigma\gamma}) + P_6(k_1 \cdot k_2 \eta_{\mu\alpha} \eta_{\nu\sigma} \eta_{\beta\gamma}) + 2P_3(k_{1\nu} k_{1\gamma} \eta_{\mu\alpha} \eta_{\beta\sigma}) \right. \\ \left. - P_3(k_{1\beta} k_{2\mu} \eta_{\alpha\nu} \eta_{\sigma\gamma}) + P_3(k_{1\sigma} k_{2\gamma} \eta_{\mu\nu} \eta_{\alpha\beta}) + P_6(k_{1\sigma} k_{1\gamma} \eta_{\mu\nu} \eta_{\alpha\beta}) \right. \\ \left. + 2P_6(k_{1\nu} k_{2\gamma} \eta_{\beta\mu} \eta_{\alpha\sigma}) + 2P_3(k_{1\nu} k_{2\mu} \eta_{\beta\sigma} \eta_{\gamma\alpha}) - 2P_3(k_1 \cdot k_2 \eta_{\alpha\nu} \eta_{\beta\sigma} \eta_{\gamma\mu}) \right],$$

45 terms
+ sym



Polarisations in Yang-Mills theory

$$(\epsilon_i^+)_{\mu}(p_i, p_{\text{ref},i}) \equiv \frac{\langle p_{\text{ref},i} | \gamma_{\mu} | p_i \rangle}{\sqrt{2} \langle p_{\text{ref},i} p_i \rangle}, \quad (\epsilon_i^-)_{\mu}(p_i, p_{\text{ref},i}) \equiv \frac{-[p_{\text{ref},i} | \gamma_{\mu} | p_i \rangle]}{\sqrt{2} [p_{\text{ref},i} p_i]}$$

Gravity: $A_3(1^-, 2^-, 3^+)$

$$\begin{array}{cc} \epsilon^- & \epsilon^- \\ \tilde{\epsilon}^+ & \tilde{\epsilon}^+ \end{array} \quad \parallel \quad -i \frac{\langle 1 2 \rangle^6}{\langle 2 3 \rangle \langle 3 1 \rangle}$$

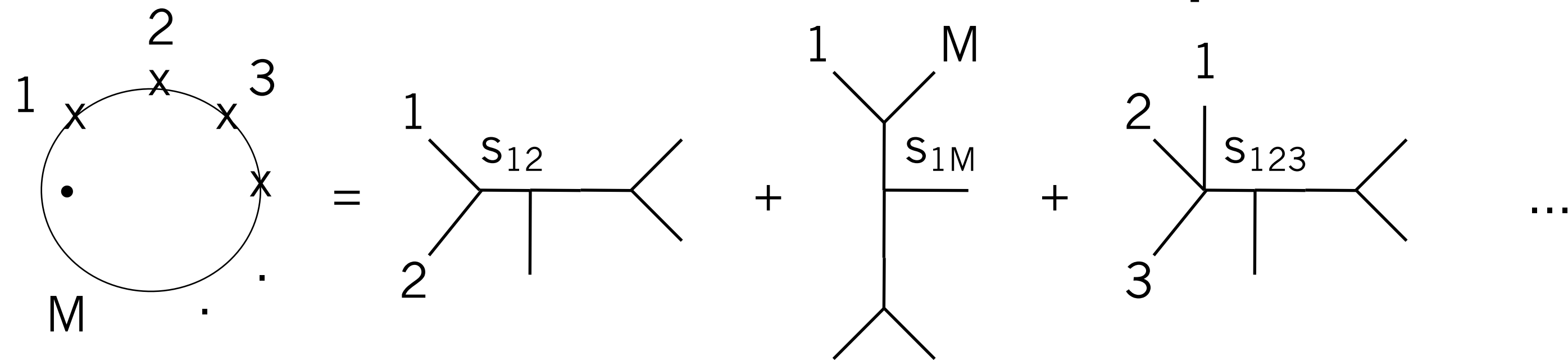
String theory inspiration

Different form for amplitude

String theory
adds
channels up..

$\leftrightarrow$

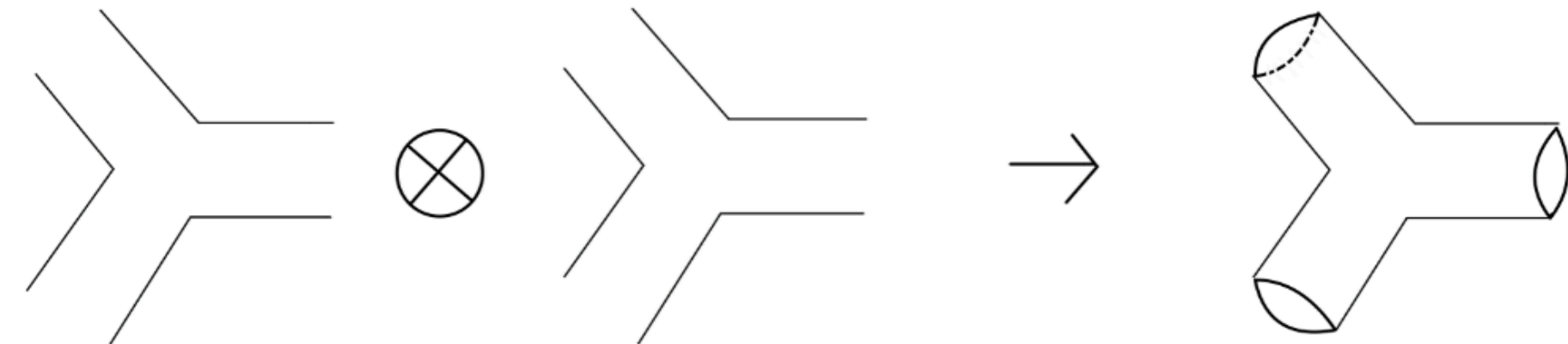
Feynman
diagrams
sums
separate
kinematic
poles



Squaring relation for gravity

Gravity from (Yang-Mills)² (Kawai, Lewellen, Tye)

Natural from the decomposition of closed strings into open.



Gives a smart way to recycle Yang-Mills results into gravity results.
(Bern et. al.)

$$\left[(\equiv)_{\mu\mu'\nu\nu'\beta\beta'} \right] = \left[(\sim)_{\mu\nu\beta}^L \right] \otimes \left[(\sim)_{\mu'\nu'\beta'}^R \right]$$

Gravity amplitudes from Yang-Mills

A practical way to compute gravity amplitudes (without having to use vertex rules) from amplitudes in Yang-Mills theory

$$M_3^{\text{tree}}(p_1, p_2, p_3) = -i A_3^{\text{tree}}(p_1, p_2, p_3) A_3^{\text{tree}}(p_1, p_2, p_3),$$

$$M_4^{\text{tree}}(p_1, p_2, p_3, p_4) = -i(p_1 \cdot p_2) A_4^{\text{tree}}(p_1, p_2, p_3, p_4) A_4^{\text{tree}}(p_1, p_2, p_4, p_3),$$

$$\begin{aligned} M_5^{\text{tree}}(p_1, p_2, p_3, p_4, p_5) &= i(p_1 \cdot p_2)(p_1 \cdot p_2) A_5^{\text{tree}}(p_1, p_2, p_3, p_4, p_5) A_5^{\text{tree}}(2, 1, 4, 3, 5) \\ &\quad + i(p_1 \cdot p_3)(p_2 \cdot p_4) A_5^{\text{tree}}(p_1, p_3, p_2, p_4, p_5) A_5^{\text{tree}}(p_3, p_1, p_4, p_2, p_5), \end{aligned}$$

$$\begin{aligned} M_6^{\text{tree}}(p_1, p_2, p_3, p_4, p_5, p_6) &= -i(p_1 \cdot p_2)(p_4 \cdot p_5) A_6^{\text{tree}}(p_1, p_2, p_3, p_4, p_5, p_6) ((p_3 \cdot p_5) A_6^{\text{tree}}(p_2, p_1, p_5, p_3, p_4, p_6) \\ &\quad + ((p_3 \cdot (p_4 + p_5)) A_6^{\text{tree}}(p_2, p_1, p_5, p_4, p_3, p_6)) + \mathcal{P}(2, 3, 4), \end{aligned}$$

$$i M^{\text{tree}}(1, 2, \dots, n) = \sum_{\tau, \gamma \in \mathcal{S}_{n-3}} \mathcal{S}[\tau(2, \dots, n-2) | \tau(2, \dots, n-2)]_{p_1} \times A_n^{\text{tree}}(1, \tau(2, \dots, n-2), n-1, n) A_n^{\text{tree}}(n, n-1, \sigma(2, \dots, n-2), 1)$$

$$\mathcal{S}[i_1, \dots, i_r | j_1, \dots, j_r]_p = \prod_{t=1}^r (p \cdot p_{i_r} + \sum_{s>t}^r \theta(i_r, i_s) p_{i_r} \cdot p_{i_s}).$$

S kernel

Gravity MHV amplitudes

Can be generated from KLT via YM MHV amplitudes.

$$M_4^{\text{tree}}(1^-, 2^-, 3^+, 4^+) = i \langle 1 2 \rangle^8 \frac{[1 2]}{\langle 3 4 \rangle N(4)}$$

$$M_5^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+) = i \langle 1 2 \rangle^8 \frac{\varepsilon(1, 2, 3, 4)}{N(5)}$$

Anti holomorphic
Contributions
– feature in gravity

(Berends-Giele-Kuijf) recursion formula

$$M_n^{\text{tree}}(1^-, 2^-, 3^+, \dots, n^+)$$

$$= -i \langle 1 2 \rangle^8 \times \left[\frac{[1 2] [n - 2 \ n - 1]}{\langle 1 \ n - 1 \rangle N(n)} \left(\prod_{i=1}^{n-3} \prod_{j=i+2}^{n-1} \langle i \ j \rangle \right) \prod_{l=3}^{n-3} (-[n | K_{l+1, n-1} | l \rangle) \right. \\ \left. + \mathcal{P}(2, 3, \dots, n - 2) \right]$$

Gravity amplitudes from Yang-Mills

Can also build scalar graviton amplitudes from KLT via
YM MHV amplitudes.

$$iM_0^{\text{tree}}(p_1, p'_1, p_1^+, p_2^+) = \frac{\kappa_{(4)}^2}{16} \frac{1}{(p_1 \cdot p_2)} \frac{m^4 [p_1 p_2]^4}{(p_1 \cdot p_1)(p_1 \cdot p'_1)}, \quad iM_0^{\text{tree}}(p_1, p'_1, p_1^-, p_2^+) = \frac{\kappa_{(4)}^2}{16} \frac{1}{(p_1 \cdot p_2)} \frac{\langle p_1 | p_1 | p_2 \rangle^2 \langle p_1 | p'_1 | p_2 \rangle^2}{(p_1 \cdot p_1)(p_1 \cdot p'_1)}$$

$$i\mathcal{M}_0^{\text{tree}}(p_1, p'_1, p_1^+, p_2^+, p_3^+) = \left(- \frac{(8\pi G_N)^{\frac{3}{2}} m_1^4}{\langle p_1 p_2 \rangle^2 \langle p_1 p_3 \rangle^2 \langle p_2 p_3 \rangle^2} \sum_{1 \leq i \neq j \neq k \leq 3} \frac{(p_i \cdot p_j)(p_j \cdot p_k) \text{tr}_+[p_k, p_1, p'_1, p_i]}{(p_1 \cdot p_k)(p'_1 \cdot p_i)} \right),$$

Testing general
relativity

Tests of general relativity

- Post-Minkowskian expansion of Einstein's general theory of relativity is useful for connecting with scattering amplitudes
- Results valid for all energy regimes ideal for precision predictions - both quantum and classical results are important for physical consistency
- We work with heavy fields: Black holes as point particles in quantum field theory

Current 'classical gravity' applications

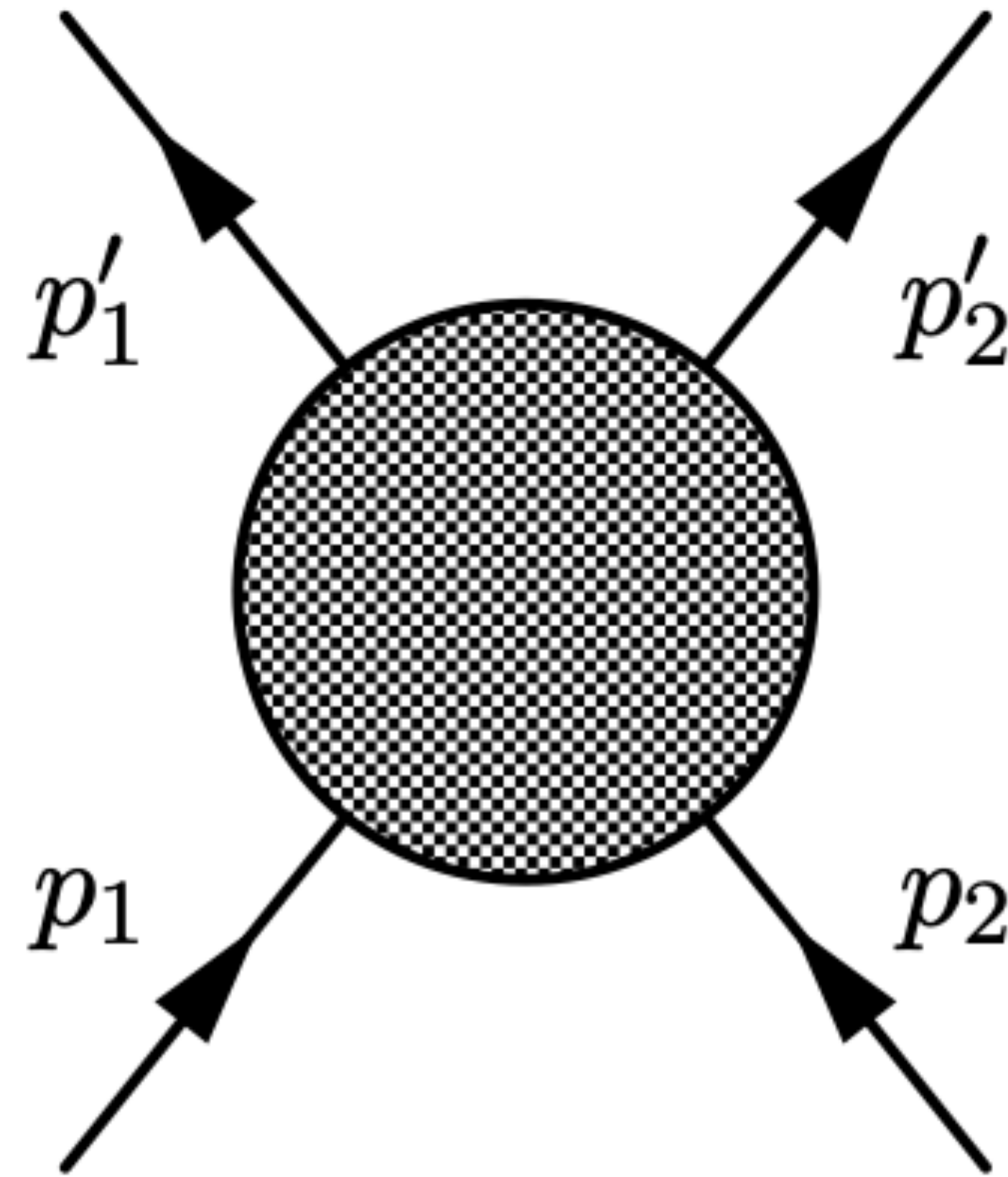
The particle physics approach to gravity is an ideal perturbative setup for analyzing black hole binary mergers

Such methods complement solving Einstein's field equations - and can even be more efficient!!

Example: Spinless gravity from quantum field theory

For analysis of the merger we consider a $2 \rightarrow 2$ process from the path integral (taking black holes as scalars)

$$= \sum_{L=0}^{\infty} \mathcal{M}_L(p_1, p_2, p'_1, p'_2) =$$



$$\varphi_1(p_1, m_1) + \varphi_2(p_2, m_2) \rightarrow \varphi_1(p'_1, m_1) + \varphi_2(p'_2, m_2)$$

Gravity from quantum field theory

We start with the leading effective Lagrangian term

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + g^{\mu\nu} T_{\mu\nu} \right]$$

where the minimal ‘energy-momentum’ tensor for spinless fields is

$$T_{\mu\nu} \equiv \partial_\mu \varphi \partial_\nu \varphi - \frac{\eta_{\mu\nu}}{2} \left[\partial^\alpha \varphi \partial_\alpha \varphi - m^2 \varphi^2 \right]$$

Gives vertices, propagators $\rightarrow$ Feynman rules (general setup for any type of external matter) or use on-shell techniques

Tree level — two ‘heavy’-masses

- γ^2 factor resum velocity and generate covariant expressions

$$\mathcal{M}_0(\gamma, \underline{q}^2, \hbar) = \begin{array}{c} p'_1 \quad p'_2 \\ \diagdown \quad \diagup \\ \text{---} \text{wavy line} \text{---} \\ \diagup \quad \diagdown \\ p_1 \quad p_2 \end{array} = \hbar \frac{2\pi m_1^2 m_2^2 G_N (2\gamma^2 - 1)}{|\underline{q}|^2} + O(\hbar^0)$$

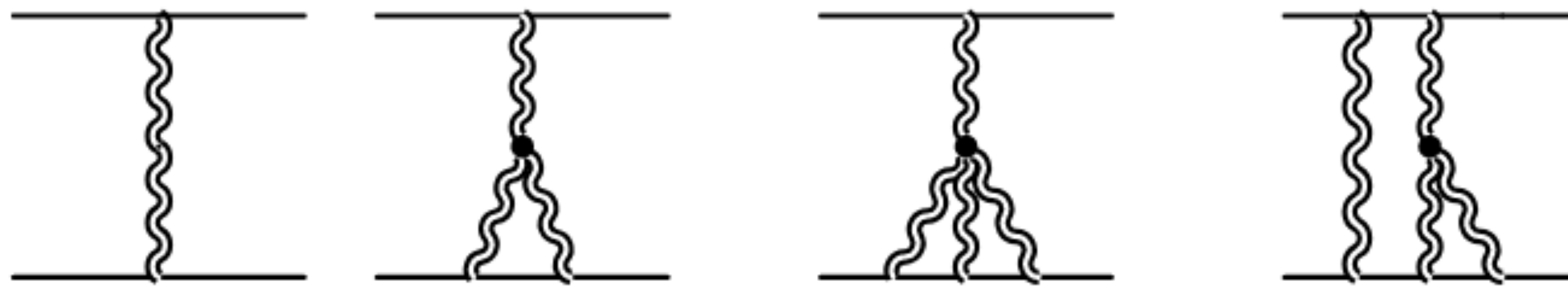
Newton's law through Fourier transform

$$V(r) = - \frac{G m_1 m_2}{r}$$

- but does **not** give higher G correction effects

Classical gravity from particle scattering

- Surprise: Non-linear (classical) corrections from loop diagrams!
- Can consider the various exchanges



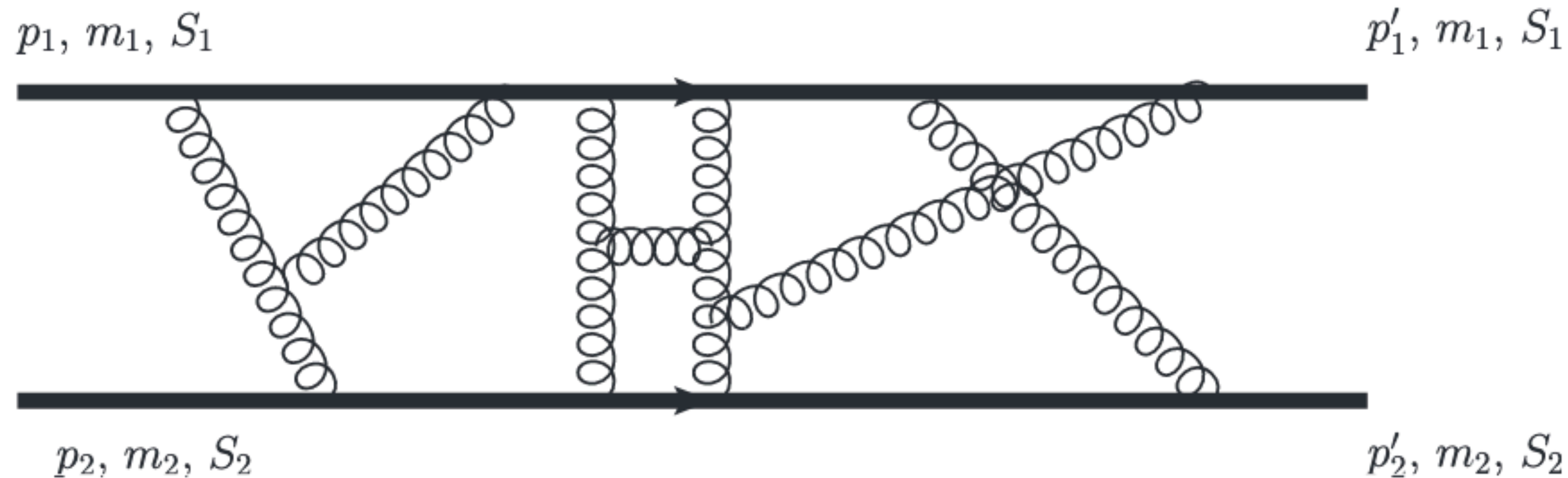
Normally we work
with expressions
where factors of $\hbar$ are
hidden ($\hbar = c = 1$)

- Define transfer momentum, CM energy

$$q^2 \equiv (p_1 - p'_1)^2 \quad \gamma \equiv \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$\mathcal{E}_{CM}^2 \equiv (p_1 + p_2)^2 \equiv (p'_1 + p'_2)^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma$$

Classical gravitational scattering from quantum field theory



Reinstating $\hbar$
there is a
difference
between
massive and
massless
propagation

Classical limit: we keep wave number fixed and take Planck's constant to zero, leads to the following Laurant expansion (quantum / classical / superclassical terms)

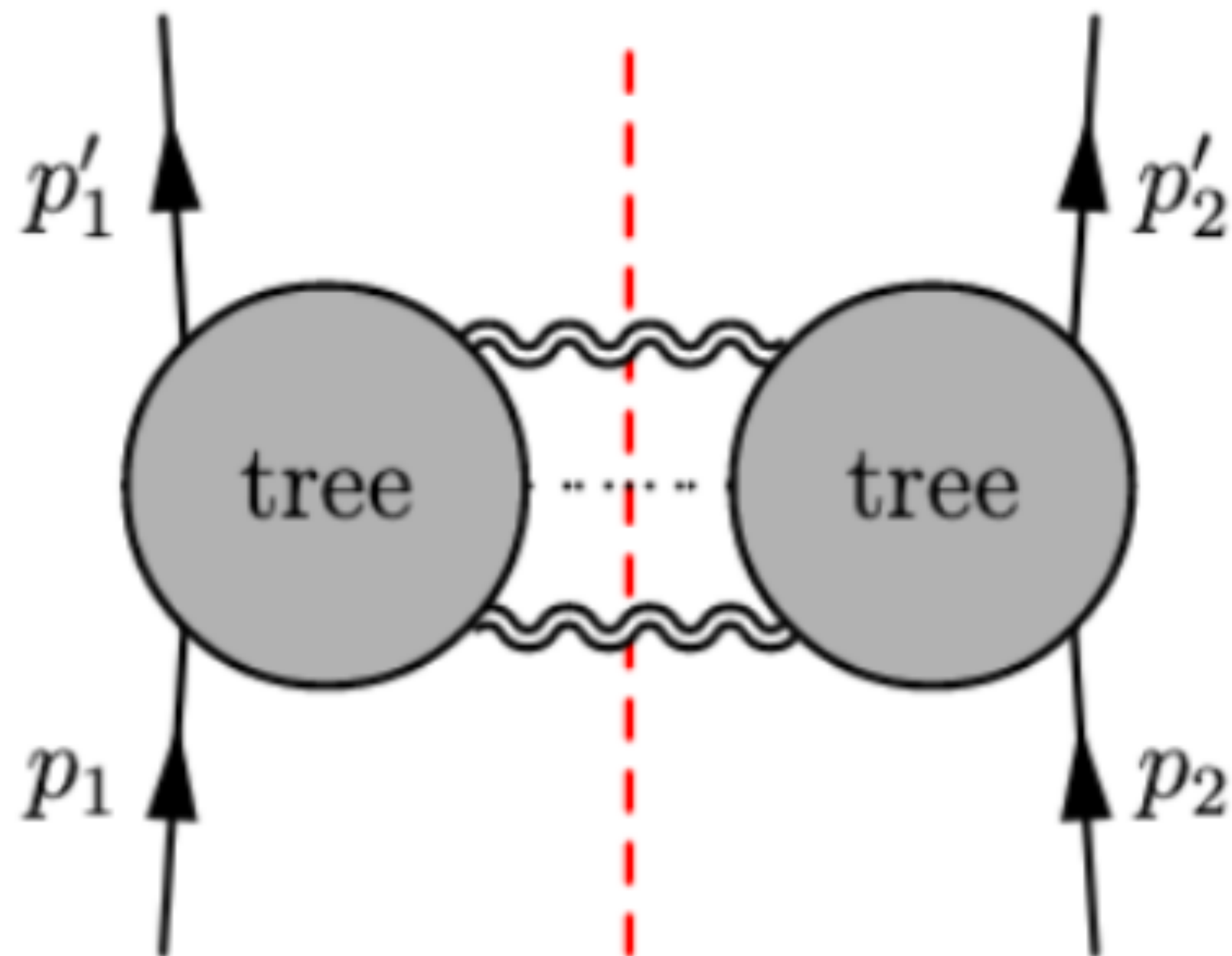
$$\mathcal{M}_L(\gamma, \underline{q}^2, \hbar) = \frac{\mathcal{M}_L^{(-L-1)}(\gamma, \underline{q}^2)}{\hbar^{L+1} |\underline{q}|^{\frac{L(4-D)}{2} + 2}} + \dots + \frac{\mathcal{M}_L^{(-1)}(\gamma, \underline{q}^2)}{\hbar |\underline{q}|^{\frac{L(4-D)}{2} + 2 - L}} + O(\hbar^0) \quad 1/\hbar$$

Important simplification

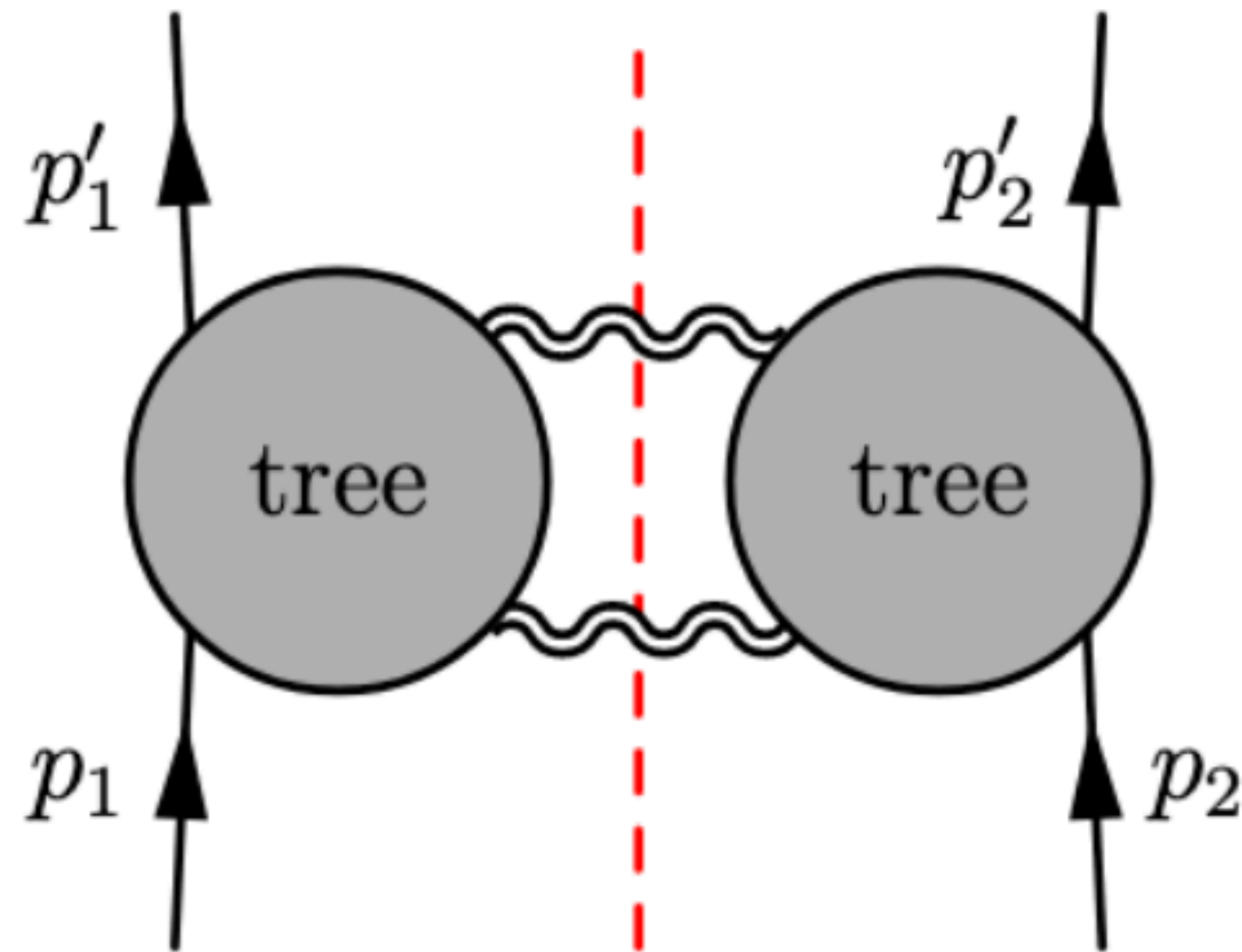
Important point: Long range behaviour can be captured from **unitarity cuts**

$$C_{i,\dots,j} = \text{Im}_{K_{i,\dots,j} > 0} M^{1\text{-loop}}$$

Using on-shell
amplitude techniques
(Neill, Rothstein;
NEJBB, Donoghue,
Vanhove)



Example: One-loop amplitude potential



- Reduce to scalar integral basis
 - Isolate coefficients
- (NEJB, Donoghue, Vanhove)
(See also Cachazo and Guevara;
Bern, Cheung Roiban, Shen, Solon,
Zeng)

$$\mathcal{M}_1(\gamma, \underline{q}^2, \hbar) = \mathcal{M}_1^\square + \mathcal{M}_1^\triangleright + \mathcal{M}_1^\triangleleft + \mathcal{M}_1^\circ$$

Provide long-range behaviour

Example: One-loop amplitude potential

The amplitude has a
Laurent expansion

$$\mathcal{M}_1(\gamma, \underline{q}^2, \hbar) = \frac{1}{|\underline{q}|^{4-D}} \left(\frac{\mathcal{M}_1^{(-2)}(\gamma, \underline{q}^2)}{\hbar^2} + \frac{\mathcal{M}_1^{(-1)}(\gamma, \underline{q}^2)}{\hbar} + \mathcal{M}_1^{(0)}(\gamma, \underline{q}^2) + \mathcal{O}(\hbar) \right)$$

$$\mathcal{M}_1^{(-2)}(\gamma, \underline{q}^2) = \mathcal{M}_1^{\square(-2)}(\gamma, \underline{q}^2),$$

$$\mathcal{M}_1^{(-1)}(\gamma, \underline{q}^2) = \mathcal{M}_1^{\square(-1)}(\gamma, \underline{q}^2) + \mathcal{M}_1^{\triangleright(-1)}(\gamma, \underline{q}^2) + \mathcal{M}_1^{\triangleleft(-1)}(\gamma, \underline{q}^2),$$

$$\mathcal{M}_1^{(0)}(\gamma, \underline{q}^2) = \mathcal{M}_1^{\square(0)}(\gamma, \underline{q}^2) + \mathcal{M}_1^{\triangleright(0)}(\gamma, \underline{q}^2) + \mathcal{M}_1^{\triangleleft(0)}(\gamma, \underline{q}^2) + \mathcal{M}_1^{\circ(0)}(\gamma, \underline{q}^2)$$

Organise order by order in Planck's constant

PM potential one-loop amplitude

$$\mathcal{M}^{1\text{-loop}} = \frac{i16\pi^2 G_N^2}{E_a E_b} \left(c_{\square} \mathcal{I}_{\square} + c_{\boxtimes} \mathcal{I}_{\boxtimes} + c_{\triangleright} \mathcal{I}_{\triangleright} + c_{\triangleleft} \mathcal{I}_{\triangleleft} + \dots \right)$$

$$\mathcal{I}_{\square} = \int \frac{d^{d+1}\ell}{(2\pi)^{d+1}} \frac{1}{((\ell + p_1)^2 - m_a^2 + i\varepsilon)((\ell - p_3)^2 - m_b^2 + i\varepsilon)(\ell^2 + i\varepsilon)((\ell + q)^2 + i\varepsilon)}$$

$$\mathcal{I}_{\boxtimes} = \int \frac{d^{d+1}\ell}{(2\pi)^{d+1}} \frac{1}{((\ell + p_1)^2 - m_a^2 + i\varepsilon)((\ell + p_4)^2 - m_b^2 + i\varepsilon)(\ell^2 + i\varepsilon)((\ell + q)^2 + i\varepsilon)}$$

$$\mathcal{I}_{\triangleright} = \int \frac{d^{d+1}\ell}{(2\pi)^{d+1}} \frac{1}{((\ell + q)^2 + i\varepsilon)(\ell^2 + i\varepsilon)((\ell + p_1)^2 - m_a^2 + i\varepsilon)}$$

$$\mathcal{I}_{\triangleleft} = \int \frac{d^{d+1}\ell}{(2\pi)^{d+1}} \frac{1}{((\ell - q)^2 + i\varepsilon)(\ell^2 + i\varepsilon)((\ell - p_3)^2 - m_b^2 + i\varepsilon)}$$

Putting it all together

phase

$$\mathcal{M}^{1\text{-loop}} = \frac{i16\pi^2 G_N^2}{E_a E_b} \left(c_{\square} \mathcal{I}_{\square} + c_{\boxtimes} \mathcal{I}_{\boxtimes} + c_{\triangleright} \mathcal{I}_{\triangleright} + c_{\triangleleft} \mathcal{I}_{\triangleleft} + \dots \right)$$

$$\mathcal{I}_{\square} = -\frac{i}{16\pi^2 |\vec{q}|^2} \left(-\frac{1}{m_a m_b} + \frac{m_a(m_a - m_b)}{3m_a^2 m_b^2} + \frac{i\pi}{|p|E_p} \right) \left(\frac{2}{3-d} - \log |\vec{q}|^2 \right) + \dots$$

$$\mathcal{I}_{\boxtimes} = -\frac{i}{16\pi^2 |\vec{q}|^2} \left(\frac{1}{m_a m_b} - \frac{m_a(m_a - m_b)}{3m_a^2 m_b^2} \right) \left(\frac{2}{3-d} - \log |\vec{q}|^2 \right) + \dots$$

$$\mathcal{I}_{\triangleright} = -\frac{i}{32m_a} \frac{1}{|\vec{q}|} + \dots$$

$$\mathcal{I}_{\triangleleft} = -\frac{i}{32m_b} \frac{1}{|\vec{q}|} + \dots$$

$$c_{\square} = c_{\boxtimes} = 16m_1^4 m_2^4 \frac{(1 - (D-2)\sigma^2)^2}{(D-2)^2},$$

$$c_{\triangleright} = \frac{4m_1^4 m_2^2 (D-7 + (D(4D-17) + 19)\sigma^2)}{(D-2)^2},$$

$$c_{\triangleleft} = \frac{4m_1^2 m_2^4 (D-7 + (D(4D-17) + 19)\sigma^2)}{(D-2)^2}$$

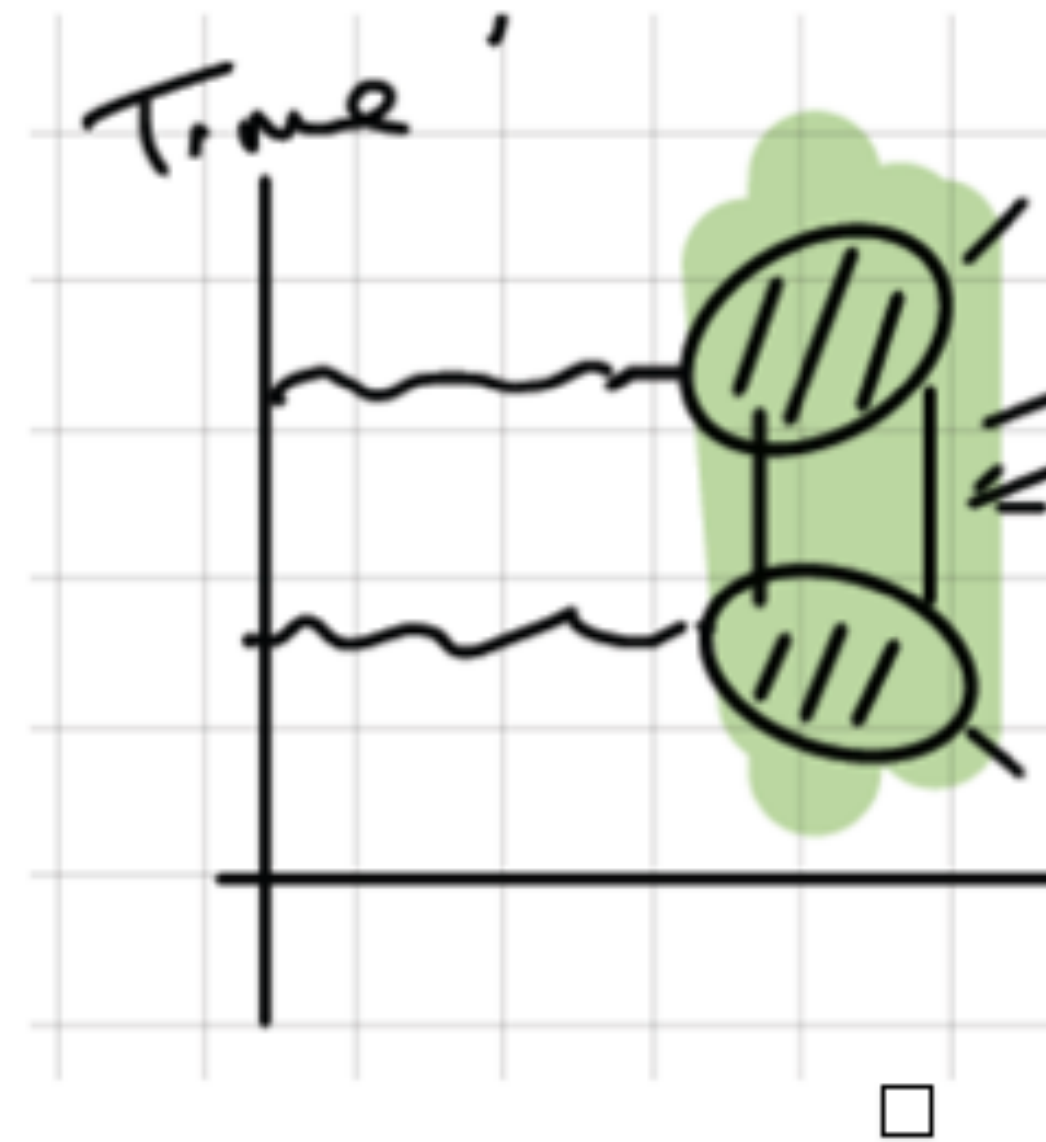
Link to Einstein's theory

- Problem in scattering theory to relate a scattering loop amplitude M to an interaction potential V .
- In post-Newtonian computations, we consider non-relativistic quantum mechanics, and this can be generalized to the relativistic case.

Classical potential from a Lippmann-Schwinger equation

- Non-relativistic limit, the tree classical potential is simply equal to the amplitude after a Fourier transform:

$$\mathcal{V}(r, p) = \int \frac{d^3 q}{(2\pi)^3} e^{iq \cdot r} \mathcal{V}(p, q) = \int \frac{d^3 q}{(2\pi)^3} e^{iq \cdot r} \tilde{\mathcal{M}}(p, q)$$



- Extension is given by Lippmann-Schwinger eq. (involves iterations/subtractions)

$$\tilde{\mathcal{M}}(p, p') = \mathcal{V}(p, p') + \int \frac{d^3 k}{(2\pi)^3} \frac{\mathcal{V}(p, k) \mathcal{M}(k, p')}{E_p - E_k + i\epsilon}$$

One-loop

Subtraction important to make contact with classical physics potential

$$\mathcal{M}^{\text{Iterated}} = \frac{i\pi G_N^2 4c_1^2 (\log |\vec{q}|^2 - \frac{2}{3-d})}{E_p^3 \xi |\vec{p}| |\vec{q}|^2} + \frac{2\pi^2 G_N^2}{E_p^3 \xi^2 |\vec{q}|} \left(\frac{c_1^2 (\xi - 1)}{2E_p^2 \xi} - 4c_1 p_1 \cdot p_3 \right)$$

$$\mathcal{M}^{1\text{-loop}} = \frac{\pi^2 G_N^2}{E_p^2 \xi} \left[\frac{1}{2|\vec{q}|} \left(\frac{c_{\triangleright}}{m_a} + \frac{c_{\triangleleft}}{m_b} \right) + \frac{i}{E_p |\vec{p}|} \frac{c_{\square} (\frac{2}{3-d} - \log |\vec{q}|^2)}{\pi |\vec{q}|^2} \right]$$

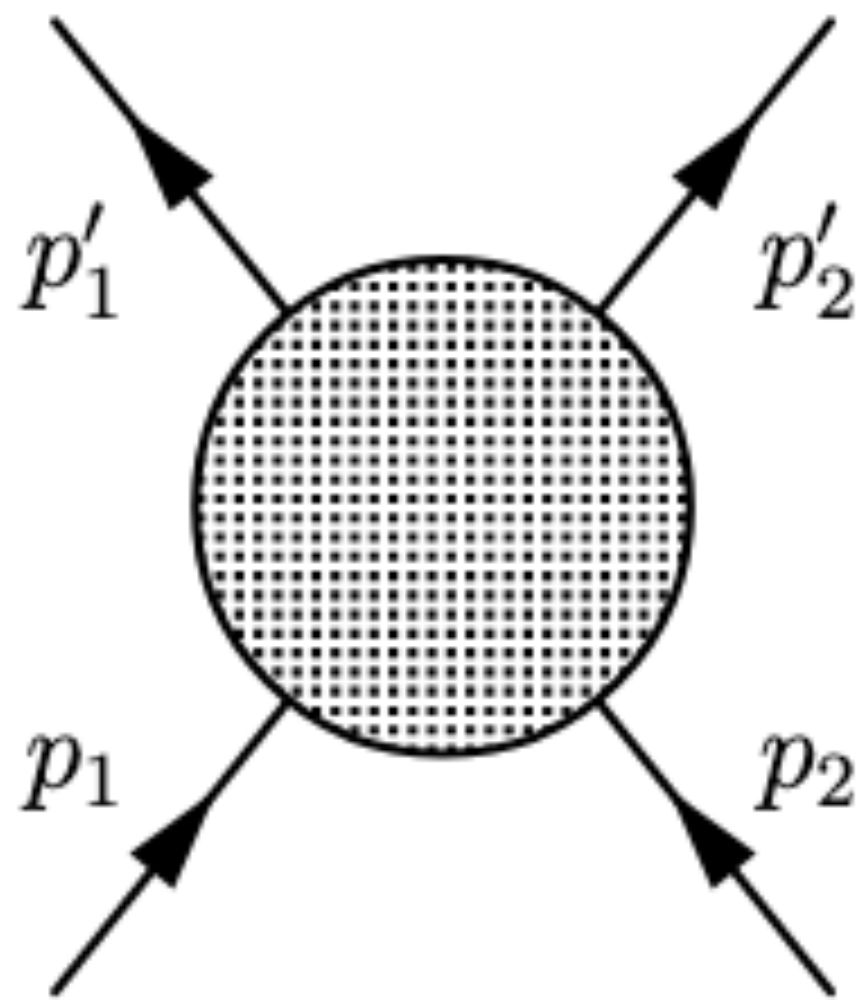
$$V_{2\text{PM}}(p, q) = \mathcal{M}^{1\text{-loop}} + \mathcal{M}^{\text{Iterated}} = \frac{\pi^2 G_N^2}{E_p^2 \xi |\vec{q}|} \left[\frac{1}{2} \left(\frac{c_{\triangleright}}{m_a} + \frac{c_{\triangleleft}}{m_b} \right) + \frac{2}{E_p \xi} \left(\frac{c_1^2 (\xi - 1)}{2E_p^2 \xi} - 4c_1 p_1 \cdot p_3 \right) \right]$$

(NEJBB, Cristofoli, Damgaard, Vanhove)

Thus follows from the Lippmann-Schwinger subtraction

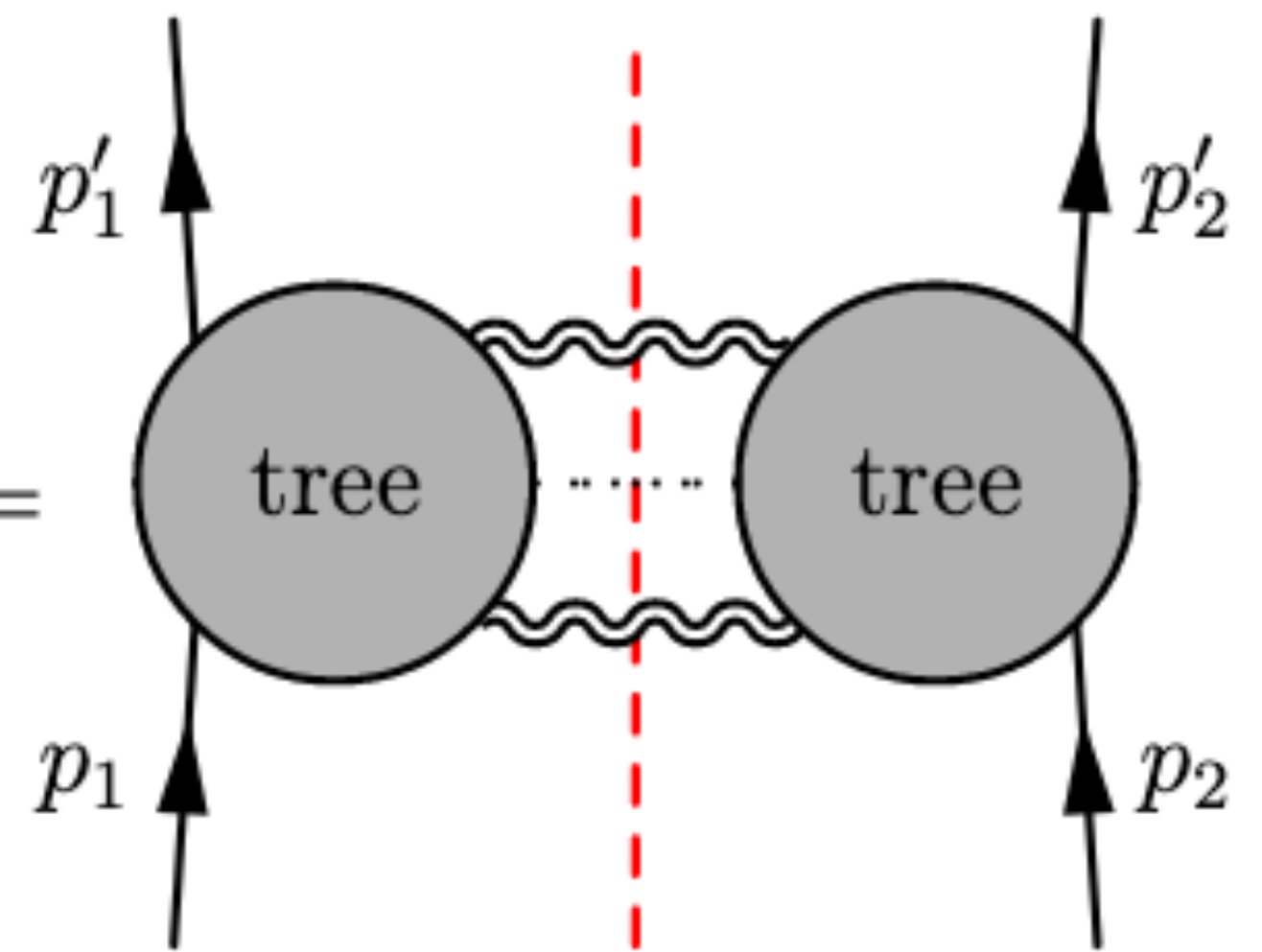
Classical gravitational scattering: Generic loop level

Extraction of integrand similar to QCD
Spinor-helicity and D-dimension
covariant tree
amplitudes can be used in cuts



$$= \mathcal{M}(\gamma, q^2) = \sum_{L=0}^{\infty} \mathcal{M}_L(\gamma, q^2).$$

$$\mathcal{M}_{L+1}^{\text{cut}}(\gamma, q^2) =$$



Einstein gravity at two-loop order

New integrals

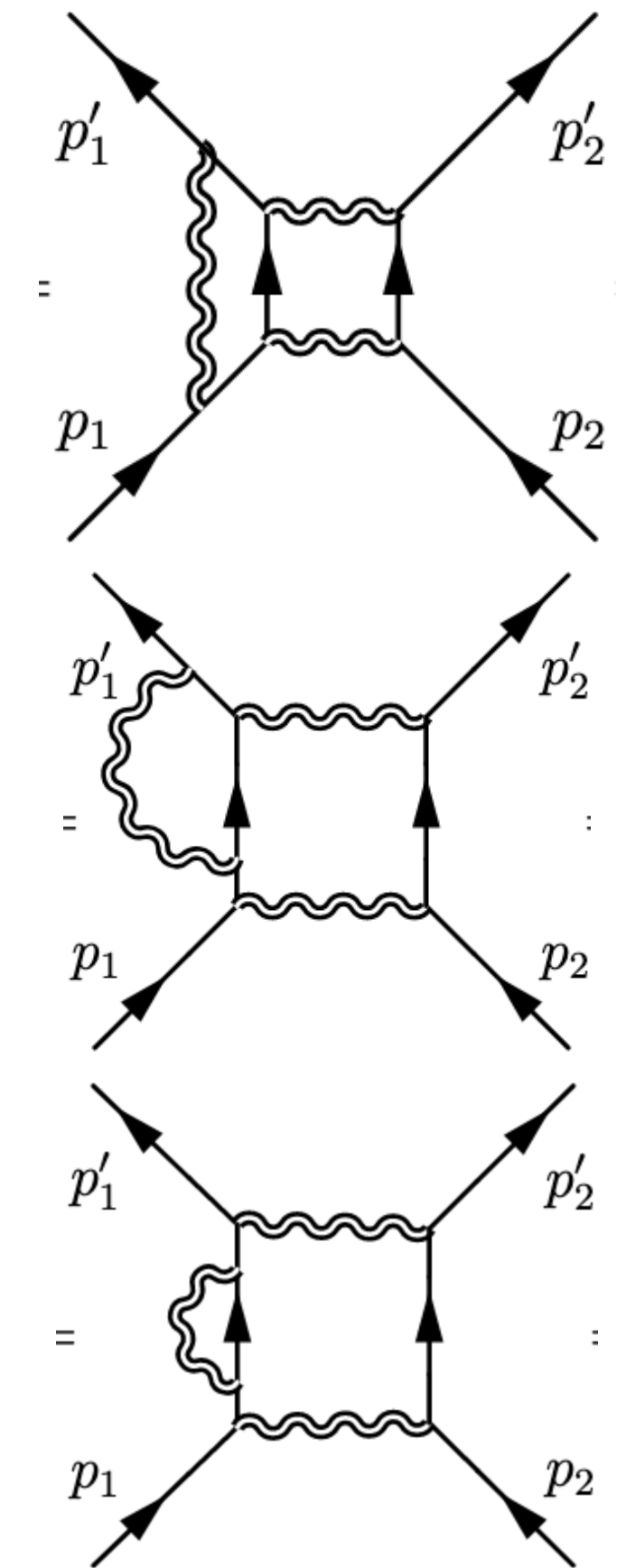
$$\mathcal{M}_2^{3\text{-cut}}(\sigma, q^2) = \mathcal{M}_2^{\square\square} + \mathcal{M}_2^{\triangleleft\square} + \mathcal{M}_2^{\square\triangleright} + \mathcal{M}_2^{\triangleleft\triangleleft} + \mathcal{M}_2^{\triangleright\triangleright} + \mathcal{M}_2^H + \mathcal{M}_2^{\square\circ}$$

We use unitarity cut to fix coefficients in front of master-integrals. The full result can be written

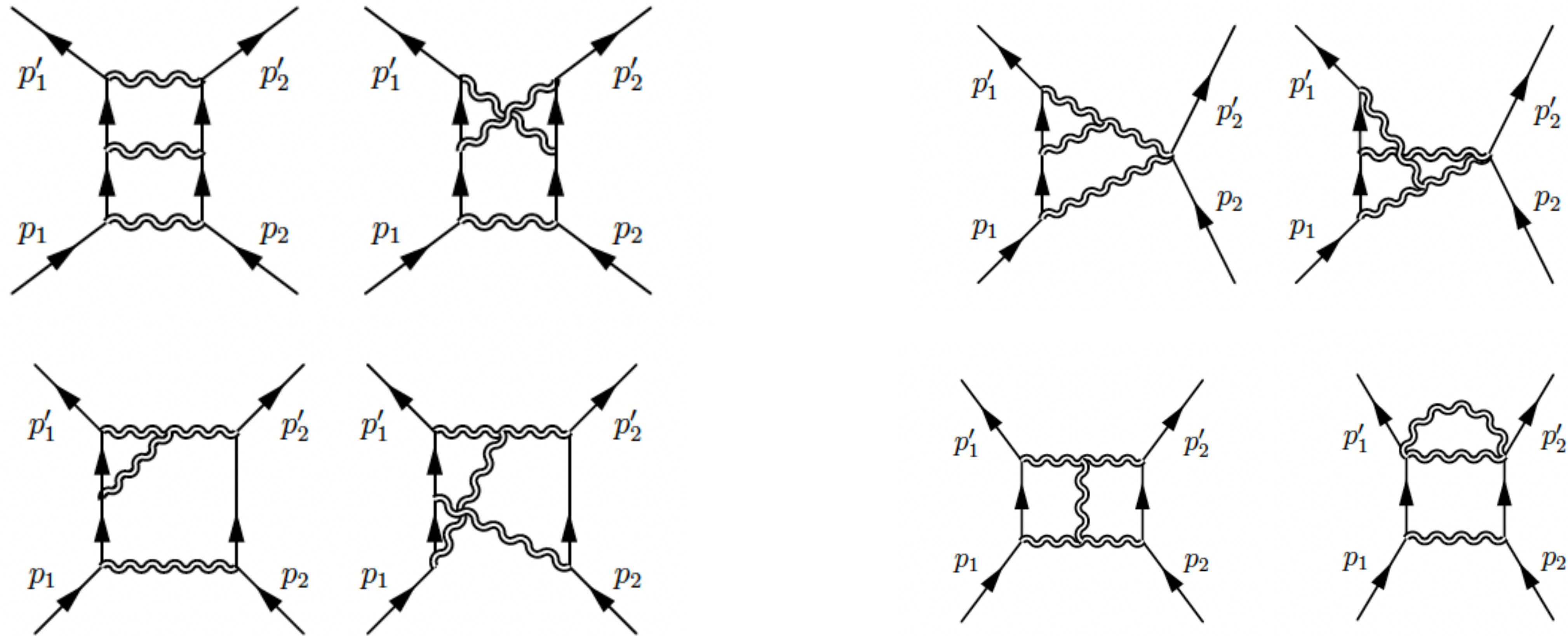
$$\mathcal{M}_2(\gamma, q^2) = \mathcal{M}_2^{3\text{-cut}}(\gamma, q^2) + \mathcal{M}_2^{\text{SE}}(\gamma, q^2)$$

Where the SE contribution is

$$\mathcal{M}_2^{\text{self-energy}}(\gamma, \underline{q}^2) = -4(16\pi G_N)^3 \sum_{i=I}^{IV} (J_{SE}^{i,s} + J_{SE}^{i,u}) + (m_1 \leftrightarrow m_2)$$



Einstein gravity at two-loop order



Needed master integrals at two-loops for the conservative part of the amplitude - determined by LiteRed/FIRE6/KIRA etc.

Gravity amplitude in powers of $\hbar$

$$\mathcal{M}_2(\sigma, |\underline{q}|) = \frac{1}{|\underline{q}|^{4\epsilon}} \left(\mathcal{M}_2^{(-3)}(\sigma, |\underline{q}|) + \mathcal{M}_2^{(-2)}(\sigma, |\underline{q}|) + \mathcal{M}_2^{(-1)}(\sigma, |\underline{q}|) + \mathcal{O}(\hbar^0) \right)$$

Laurent expansion in Planck's constant imaginary contribution cancelled by radiative contributions

(Di Vecchia, Heissenberg, Russo, Veneziano)

Interesting exponentiation in b-space

$$\widetilde{\mathcal{M}}_2(\sigma, b) = \frac{1}{4E_{\text{c.m.}}P} \int_{\mathbb{R}^{D-2}} \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} \mathcal{M}_2(p_1, p_2, p'_1, p'_2) e^{i\vec{q}\cdot\vec{b}}$$

$$\begin{aligned} \widetilde{\mathcal{M}}_2(\sigma, b) = & -\frac{1}{6} \left(\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b) \right)^3 + i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b) \left(\widetilde{\mathcal{M}}_1^{\text{Cl.}}(\sigma, b) + \widetilde{\mathcal{M}}_1^{\text{Qt.}}(\sigma, b) \right) \\ & + \widetilde{\mathcal{M}}_2^{\text{Cl.}}(\sigma, b) + \mathcal{O}(\hbar^0). \end{aligned}$$

Again iterative structure like one-loop, part of a bigger scheme..

$$\widetilde{\mathcal{M}}_2^{\square\square(-3)}(\sigma, b) = -\frac{1}{6} \left(\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b) \right)^3,$$

$$\widetilde{\mathcal{M}}_2^{\square\square(-2)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\widetilde{\mathcal{M}}_1^{\square(-1)}(\sigma, b),$$

$$\widetilde{\mathcal{M}}_2^{\triangleleft\square(-2)}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\square\triangleright(-2)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b) \left(\widetilde{\mathcal{M}}_1^{\triangleleft(-1)}(\sigma, b) + \widetilde{\mathcal{M}}_1^{\triangleright(-1)}(\sigma, b) \right)$$

$$\widetilde{\mathcal{M}}_2^{\square\square(-1)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\widetilde{\mathcal{M}}_1^{\square(0)}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\square\square \text{ Cl.}}(\sigma, b),$$

$$\begin{aligned} \widetilde{\mathcal{M}}_2^{\triangleleft\square(-1)}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\square\triangleright(-1)}(\sigma, b) = & i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b) \left(\widetilde{\mathcal{M}}_1^{\triangleleft(0)}(\sigma, b) + \widetilde{\mathcal{M}}_1^{\triangleright(0)}(\sigma, b) \right) \\ & + \widetilde{\mathcal{M}}_2^{\triangleleft\square \text{ Cl.}}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\square\triangleright \text{ Cl.}}(\sigma, b), \end{aligned}$$

$$\widetilde{\mathcal{M}}_2^{\square\circ(-1)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\widetilde{\mathcal{M}}_1^{\circ(0)}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\square\circ \text{ Cl.}}(\sigma, b),$$

Seen after Fourier transform to b space

Scattering angle from QFT computation

$$1 + i \sum_{L \geq 0} \widetilde{\mathcal{M}}_L(\sigma, b) = (1 + 2i\Delta(\sigma, b)) \exp \left(\frac{2i}{\hbar} \sum_{L \geq 0} \delta_L(\sigma, b) \right)$$

Gravity eikonal

$$\delta_0(\sigma, b) = -\frac{G_N m_1 m_2 (2\sigma^2 - 1)}{2\epsilon \sqrt{\sigma^2 - 1}} (\pi b^2 e^{\gamma_E})^\epsilon + \mathcal{O}(\epsilon),$$

$$\delta_1(\sigma, b) = \frac{3\pi G_N^2 (m_1 + m_2) m_1 m_2 (5\sigma^2 - 1)}{8b \sqrt{\sigma^2 - 1}} (\pi b^2 e^{\gamma_E})^{2\epsilon}.$$

Scattering angle from QFT computation

$$1 + i \sum_{L \geq 0} \widetilde{\mathcal{M}}_L(\sigma, b) = (1 + 2i\Delta(\sigma, b)) \exp \left(\frac{2i}{\hbar} \sum_{L \geq 0} \delta_L(\sigma, b) \right)$$

Gravity eikonal

$$2\Delta_1 = \widetilde{\mathcal{M}}_1^{\text{Qt.}}(\sigma, b)$$

$$\begin{aligned} \delta_2(\sigma, b) = & \frac{G_N^3 m_1 m_2 (\pi b^2 e^{\gamma_E})^{3\epsilon}}{2b^2 \sqrt{\sigma^2 - 1}} \left(\frac{2s(12\sigma^4 - 10\sigma^2 + 1)}{\sigma^2 - 1} \right. \\ & - \frac{4m_1 m_2 \sigma}{3} (25 + 14\sigma^2) + \frac{4m_1 m_2 (3 + 12\sigma^2 - 4\sigma^4) \operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}} \\ & \left. + \frac{2m_1 m_2 (2\sigma^2 - 1)^2}{\sqrt{\sigma^2 - 1}} \frac{1}{(4(\sigma^2 - 1))^\epsilon} \left(-\frac{11}{3} + \frac{d}{d\sigma} \left(\frac{(2\sigma^2 - 1) \operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}} \right) \right) \right). \end{aligned}$$

Scattering angle from QFT computation

$$\sin\left(\frac{\chi}{2}\right)\Big|_{3PM} = -\frac{\sqrt{s}}{m_1 m_2 \sqrt{\sigma^2 - 1}} \frac{\partial \delta_2(\sigma, b)}{\partial b}$$

$$J = \frac{m_1 m_2 \sqrt{\sigma^2 - 1}}{\sqrt{s}} b \cos\left(\frac{\chi}{2}\right)$$

$$\chi_{1PM} = \frac{2G_N m_1 m_2 (2\sigma^2 - 1)}{J \sqrt{\sigma^2 - 1}},$$

$$\chi_{2PM} = \frac{3\pi G_N^2 m_1^2 m_2^2 (m_1 + m_2) (5\sigma^2 - 1)}{4J^2 \sqrt{s}};$$

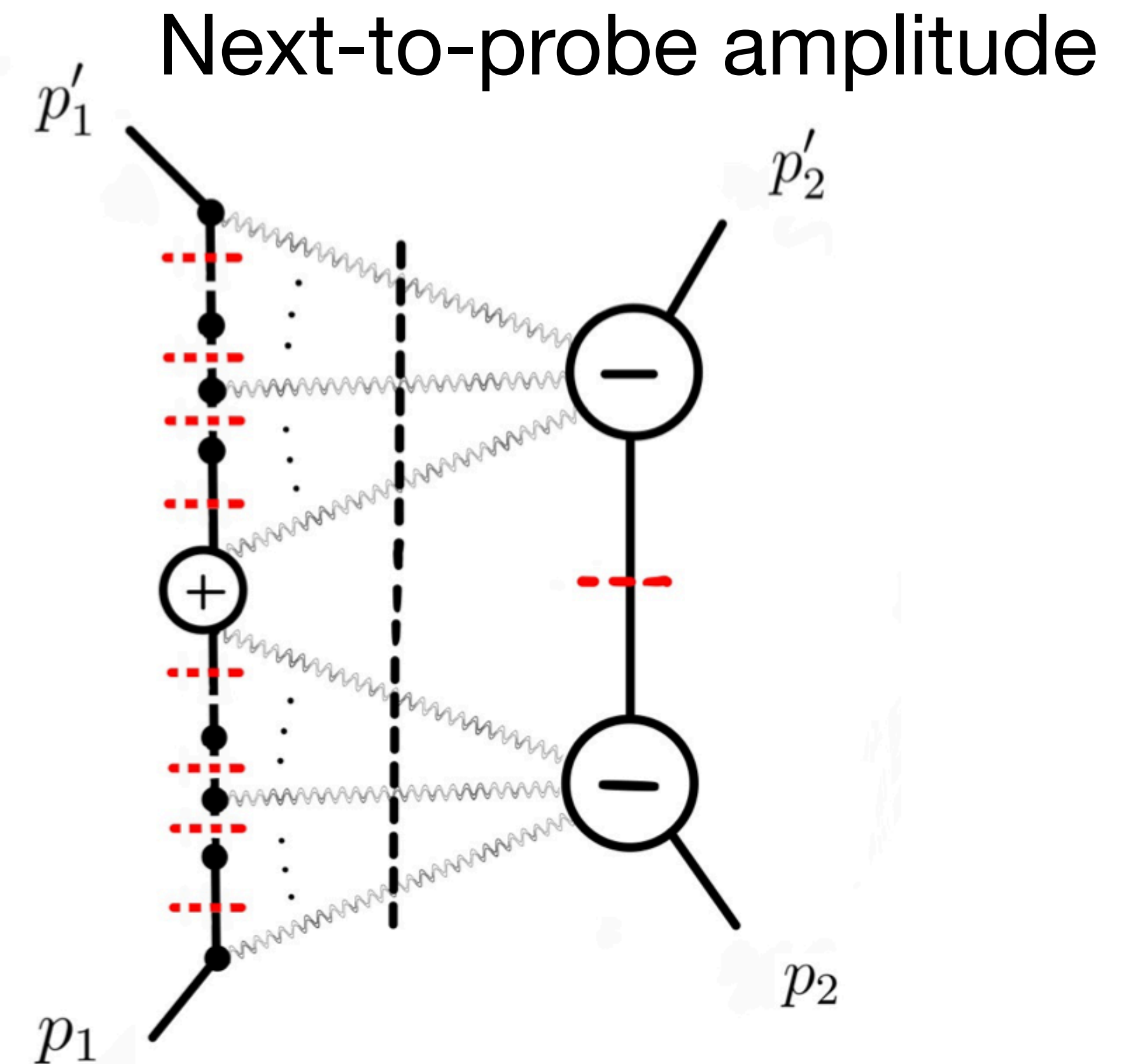
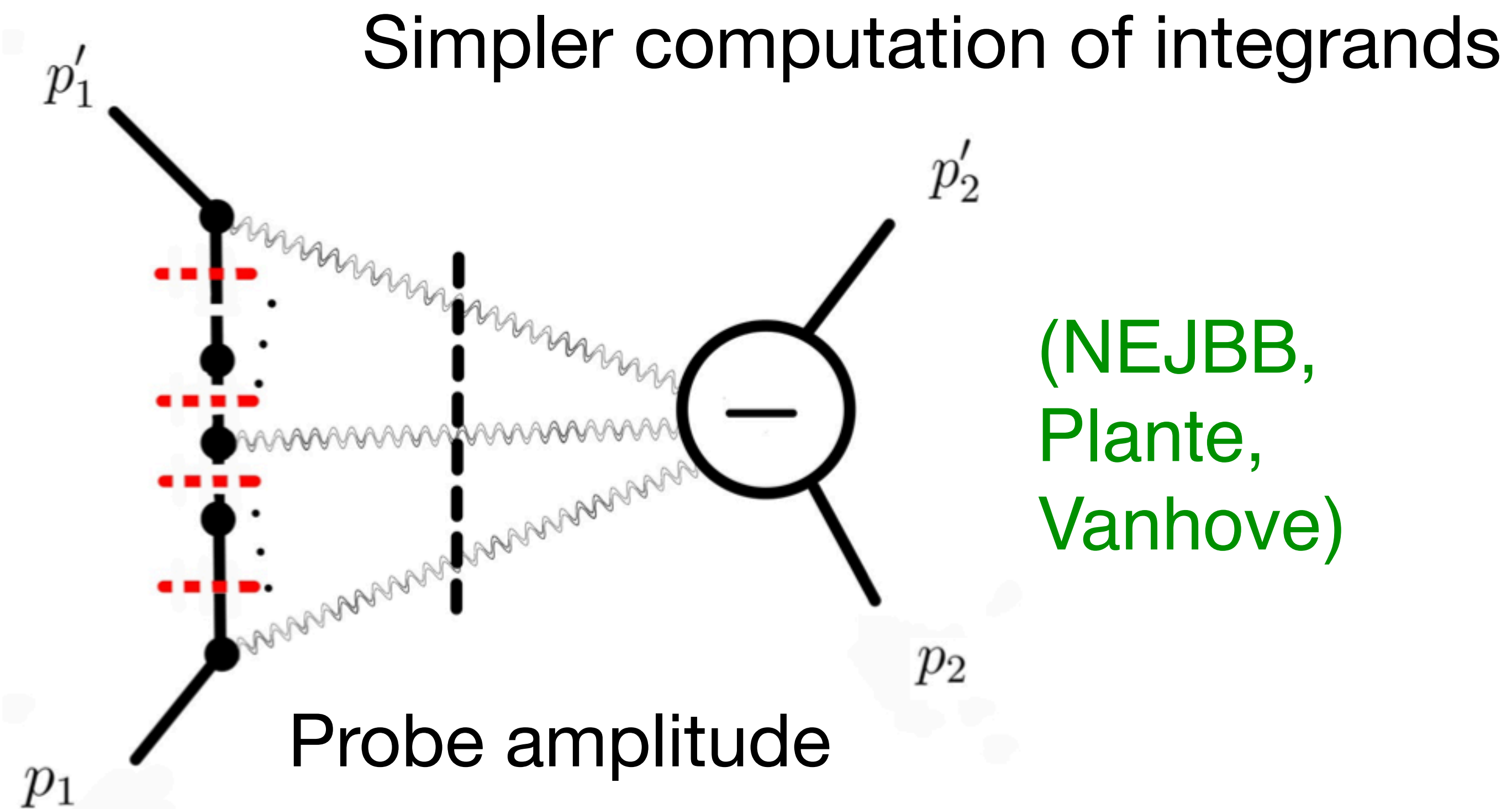
Scattering angle from QFT computation

$$\hat{\chi}_{3PM} = \frac{2G_N^3 m_1^3 m_2^3 (64\sigma^6 - 120\sigma^4 + 60\sigma^2 - 5)}{3J^3 (\sigma^2 - 1)^{\frac{3}{2}}} + \frac{8G_N^3 m_1^4 m_2^4 \sqrt{\sigma^2 - 1}}{3J^3 s} \left(\sigma(-25 - 14\sigma^2) + \frac{3(3 + 12\sigma^2 - 4\sigma^4) \operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}} \right)$$
$$\chi_{3PM}^{\text{Rad.}} = \frac{4G_N^3 m_1^4 m_2^4 (2\sigma^2 - 1)^2}{J^3 s} \frac{1}{(4(\sigma^2 - 1))^\epsilon} \left(-\frac{11}{3} + \frac{d}{d\sigma} \left(\frac{(2\sigma^2 - 1) \operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}} \right) \right)$$

Match with (Damour; Di Vecchia et al; Hermann et al)

(NEJB,
Damgaard,
Plante,
Vanhove)

Simpler integrand - velocity cuts tree topologies!



Heavy-quark—EFT inspiration:

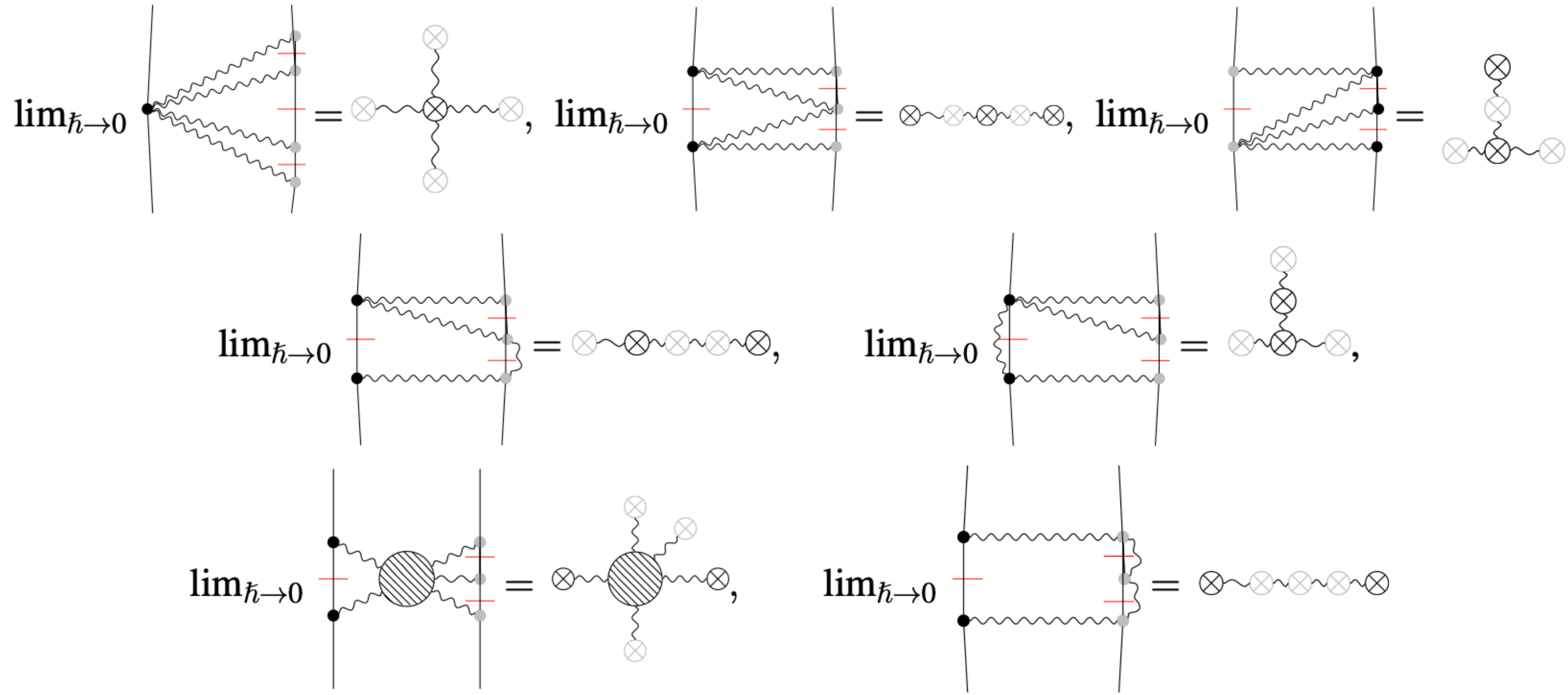
(Damgaard, Haddad, Helset)

- heavy mass vs small $|q|$ expansion?
- some similarities / some differences

(Brandhuber, Chen, Travaglini, Wen)

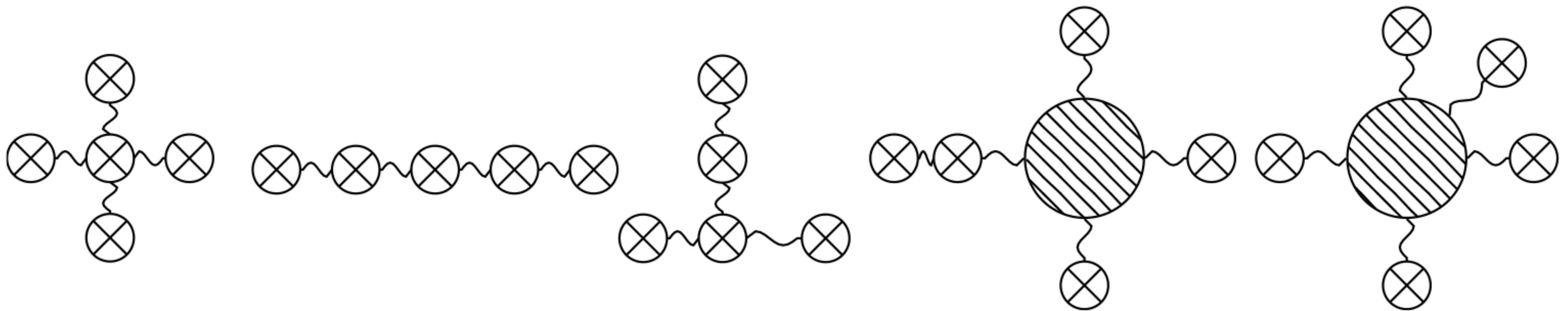
Interesting stuff to investigate

Enables a simple extension to fourth order in Newton's constant



Extension to fourth order in Newton's constant

Only five integrand topologies have to be considered

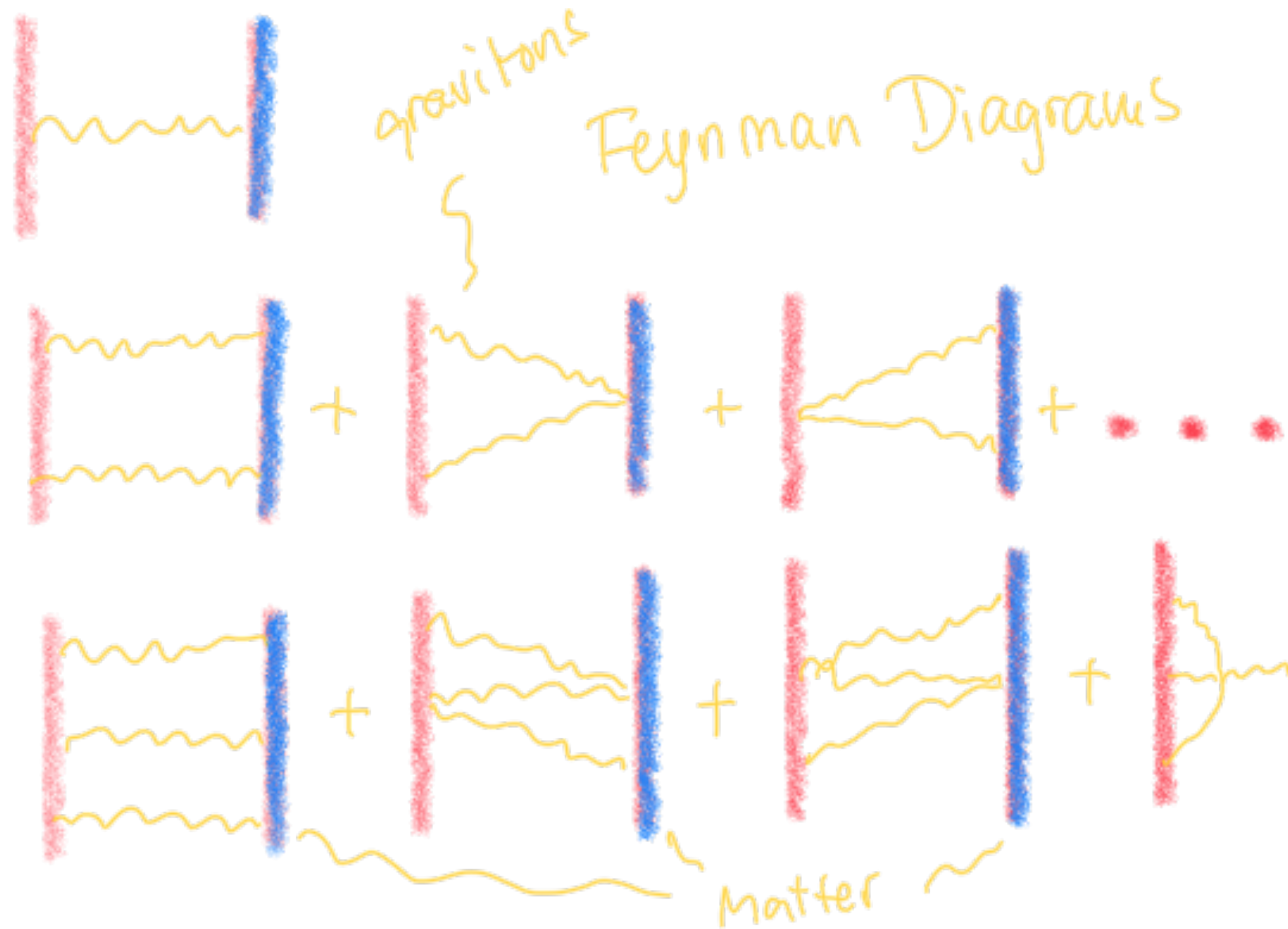


(NEJBB, Plante, Vanhove)

Computational framework

- We need all ‘perturbative orders’ to fully compute gravitational effects, but we can work loop order by loop order.
- For tests of Einstein’s theory, we only need to retain leading classical terms (often with simplifications beyond expectations)
- For ‘real’ quantum effects, one needs to include subleading terms as well (possible - but a much harder game...).

Quantum mechanical description of black hole scattering?



- ⑤ Classical potential from quantum mechanical propagation via quantum / classical correspondence principle.

Large quantum numbers (angular impulse enormous) -> Classical physics (gravity) (essence of Bohr's correspondence principle)

- ⑤ Although we are not directly dealing with quantum gravity, we are still learning a dual way to think of gravity that is consistent with quantum mechanics.

Classical gravity from amplitude questions

Amplitude techniques are surprisingly effective for Post-Minkowskian gravity calculations and for bridging the gap to current data.

Developing these has required new insights alongside brute-force computations; however, each new loop order introduces new challenges.

Current bottlenecks:

- Identifying IBP relations,

- Solving the differential equations/integrals.

- New effects (tidal) have to be introduced (as well as spin)

Quantum Gravity

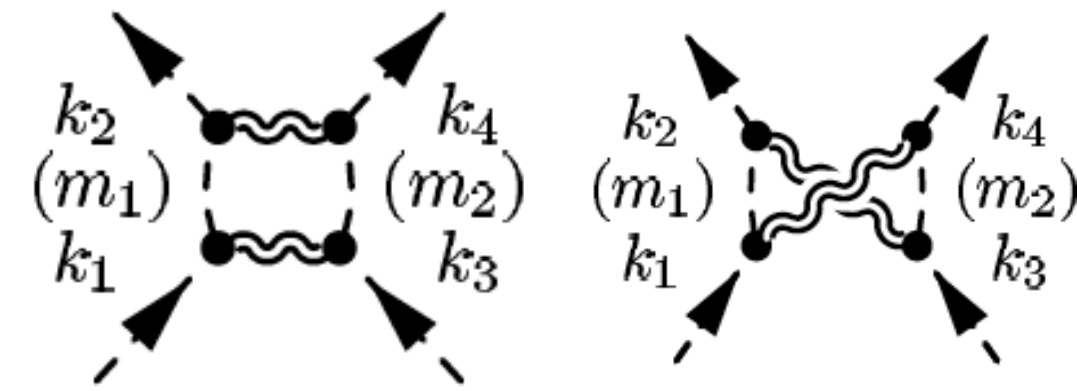
Quantum gravity

At very tiny scales, we expect quantum gravitational interactions to be important - quantum gravity — 40 orders of magnitude smaller — than classical effects

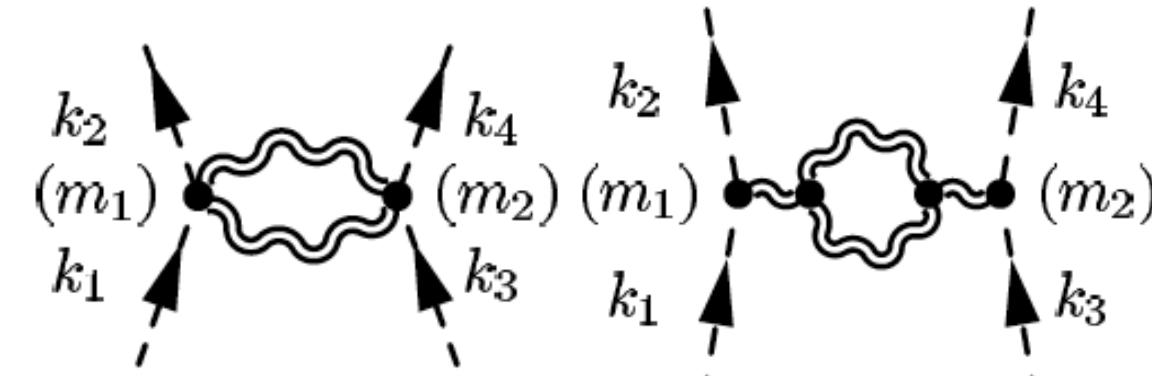
The methods used to extract classical physics from field theory can also extract quantum terms, but with more computational effort.

Example: One-loop gravity computation

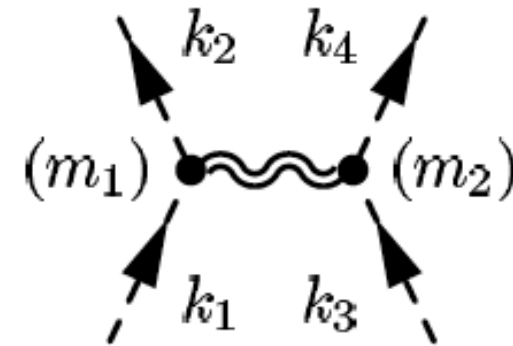
(NEJB, Donoghue, Holstein)



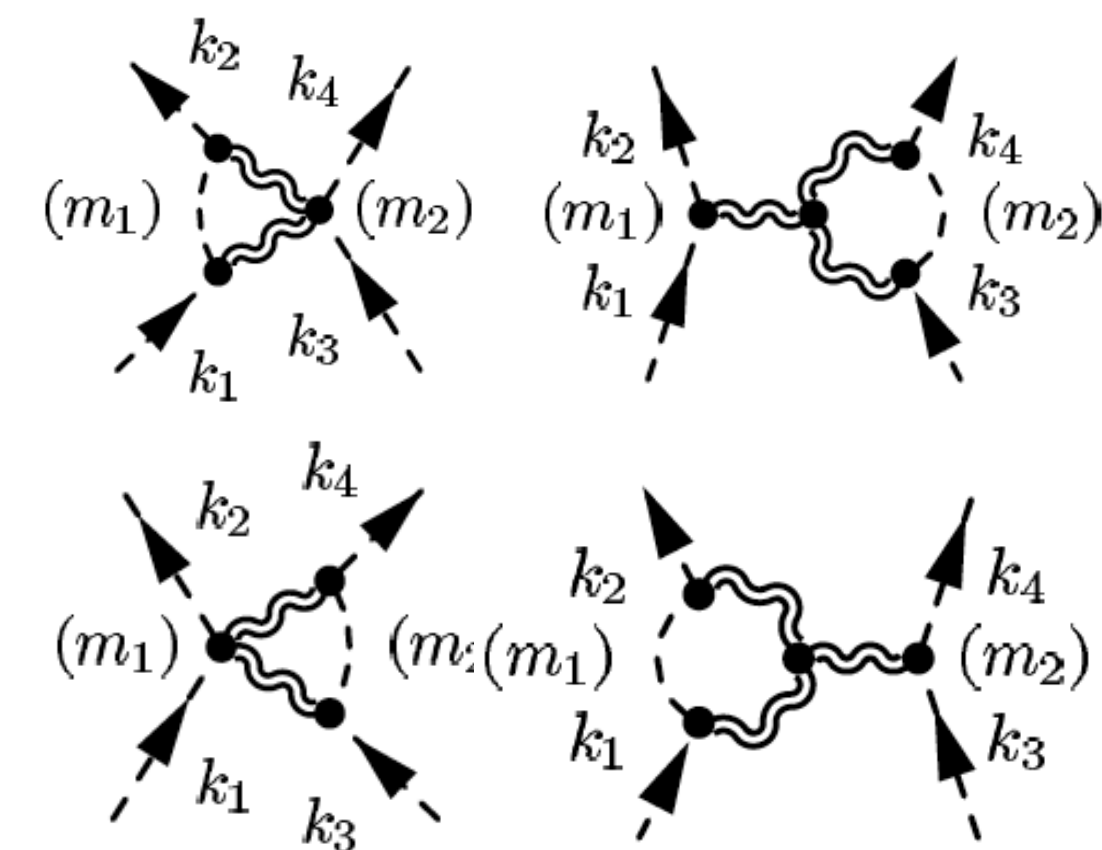
Boxes (hyperclassical/classical/quantum)



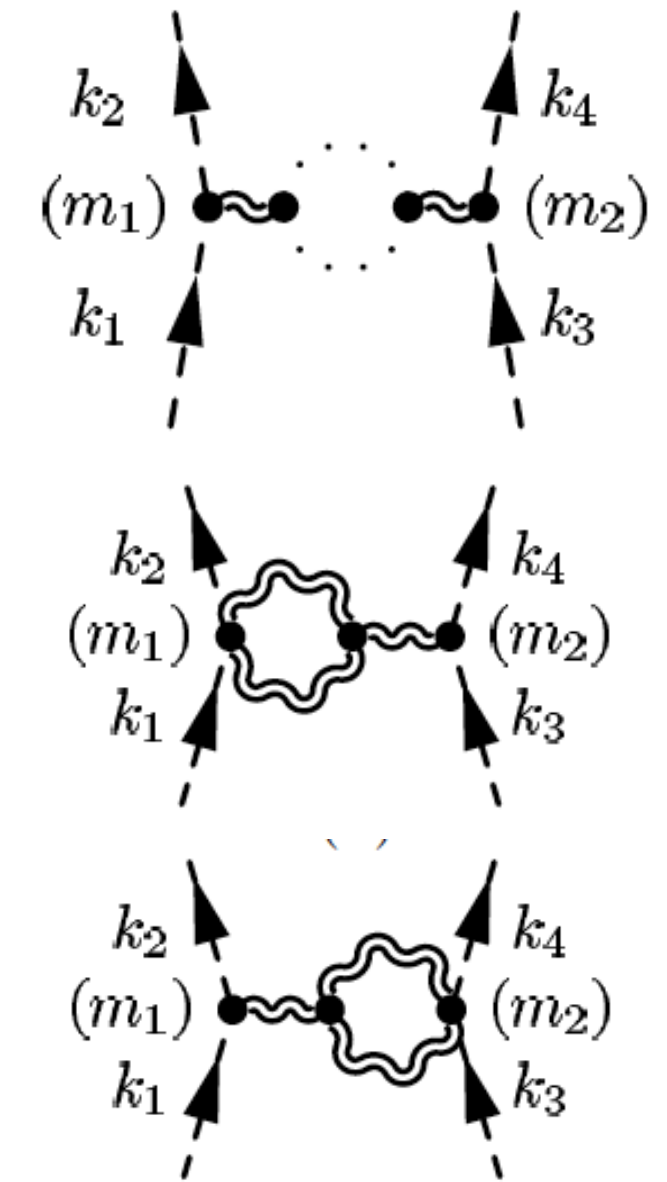
Bubbles (quantum)



Triangles (classical / quantum)



Tree



One-loop result for gravity

The result for the amplitude (in coordinate space) after summing all diagrams is:

$$-\frac{Gm_1m_2}{r} \left[1 + 3 \frac{G(m_1 + m_2)}{r} + \frac{41}{10\pi} \frac{G\hbar}{r^2} \right]$$

(NEJB, Donoghue,
Holstein)



Post-Newtonian



Quantum

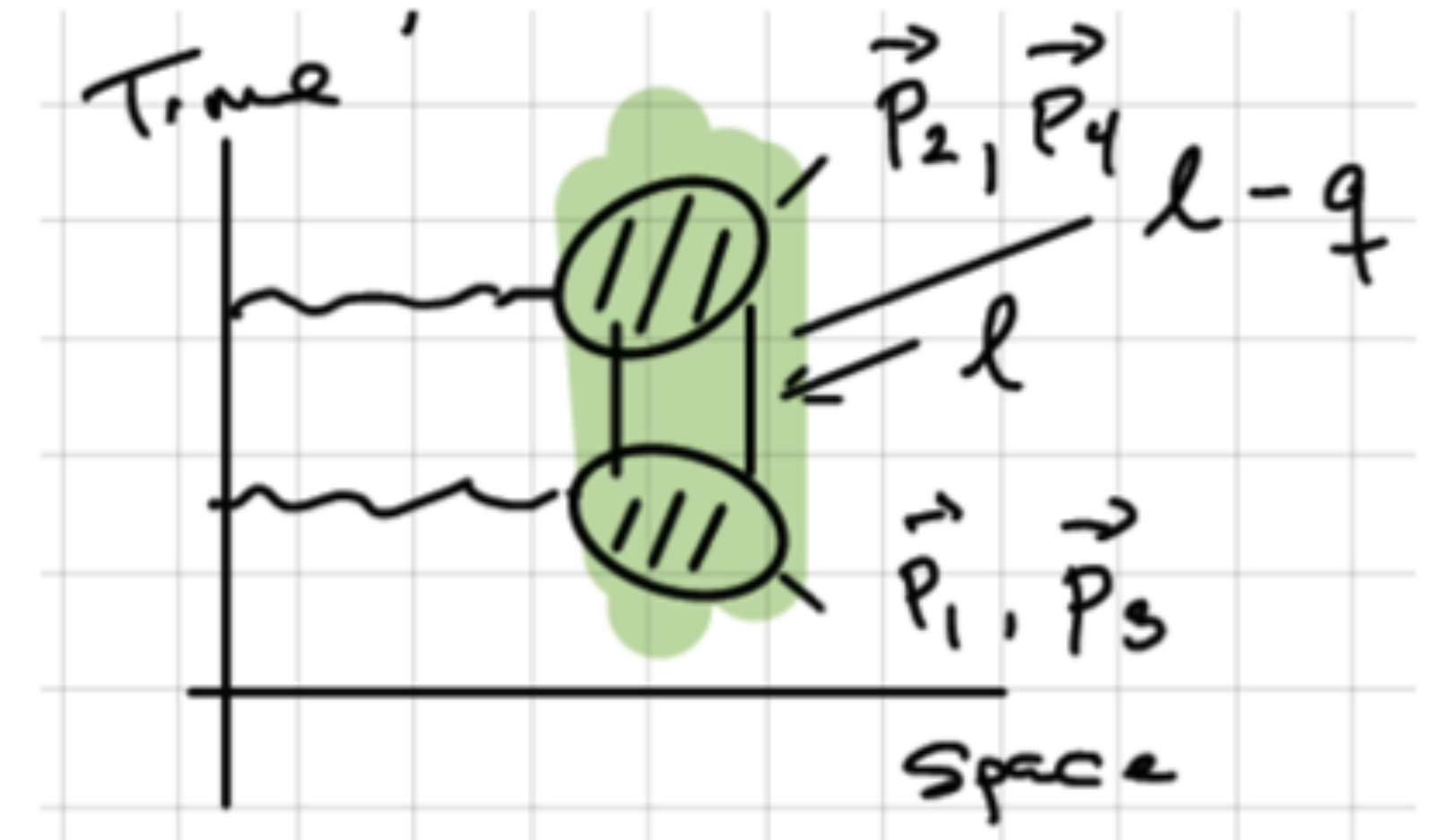
Post-Newtonian term in complete accordance with general relativity: (Holstein and Ross, Neill and Rothstein, NEJB, Damgaard, Festuccia, Plante, Vanhove) (can also be done in post-Minkowskian formalism — fully relativistic result (NEJB, Damgaard, Plante, Vanhove))

Post-Newtonian interaction potential

$$\begin{aligned}
 H = & \frac{\vec{p}_1^2}{2m_1} + \frac{\vec{p}_4^2}{2m_2} - \frac{\vec{p}_1^4}{8m_1^3} - \frac{\vec{p}_4^4}{8m_2^3} \\
 & - \frac{Gm_1m_2}{r} - \frac{G^2m_1m_2(m_1 + m_2)}{2r^3} \\
 & - \frac{Gm_1m_2}{2r} \left(\frac{3\vec{p}_1^2}{m_1^2} + \frac{3\vec{p}_4^2}{m_2^2} - \frac{7\vec{p}_1 \cdot \vec{p}_4}{m_1m_2} - \frac{(\vec{p}_1 \cdot \vec{r})(\vec{p}_4 \cdot \vec{r})}{m_1m_2r^2} \right)
 \end{aligned}$$

(Einstein-Infeld-Hoffman)

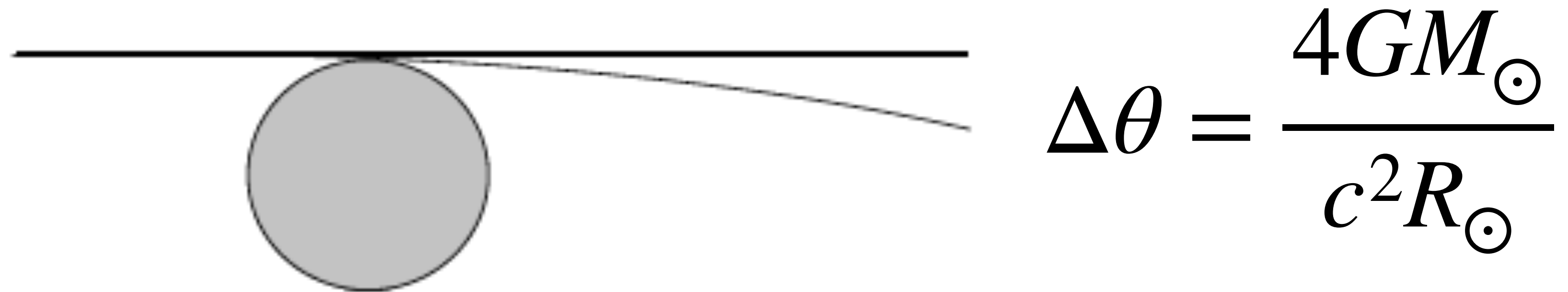
Important: subtraction of the tree-level Born term in order to get the correct classical potential (consistency of the classical limit)



Example: Einstein Light
bending including
quantum effects

New results: massless matter

- As an example we will consider scattering of massless matter



- Bending of light/massless matter around the Sun (famous Einstein test of general relativity)
- New features: mass-less external fields $\leadsto$ IR singularities (related to intricate QFT consistency properties)
- New test of universality of matter (quantum effects?)

Setting up the computation

We use the EFT framework starting with the Lagrangian

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} \mathcal{R} + S_{\text{model}} + S_{\text{EF}} \right]$$

where

$$\mathcal{S}_{\text{scalar}} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} \left((\partial_\mu \Phi)^2 - M^2 \Phi^2 \right) \right)$$

$$\mathcal{S}_{\text{fermion}} = \frac{i}{2} \int d^4x \sqrt{-g} \bar{\chi} \not{D} \chi,$$

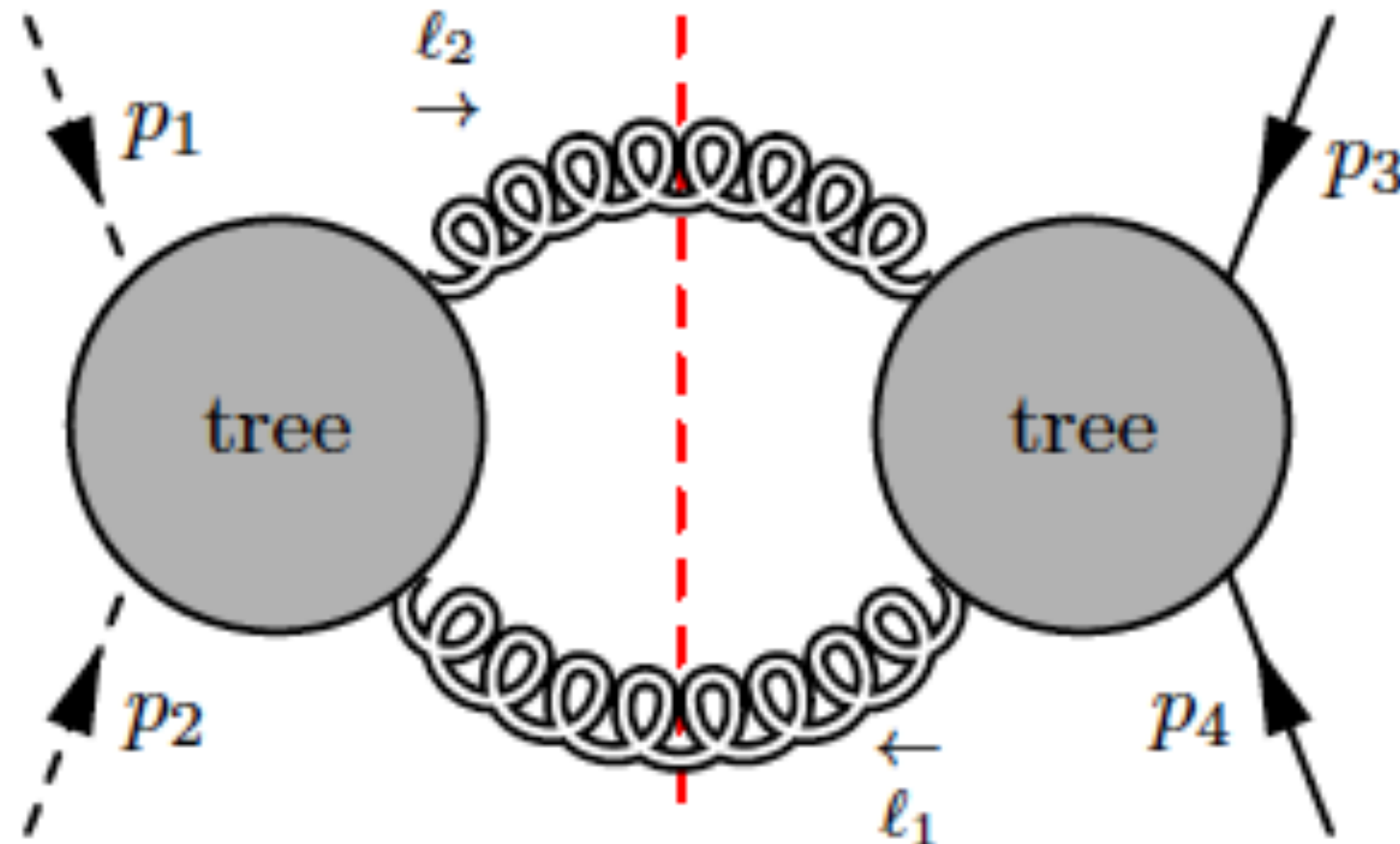
$$\mathcal{S}_{\text{QED}} = -\frac{1}{4} \int d^4x \sqrt{-g} (\nabla_\mu A_\nu - \nabla_\nu A_\mu)^2$$

Trees and the cut

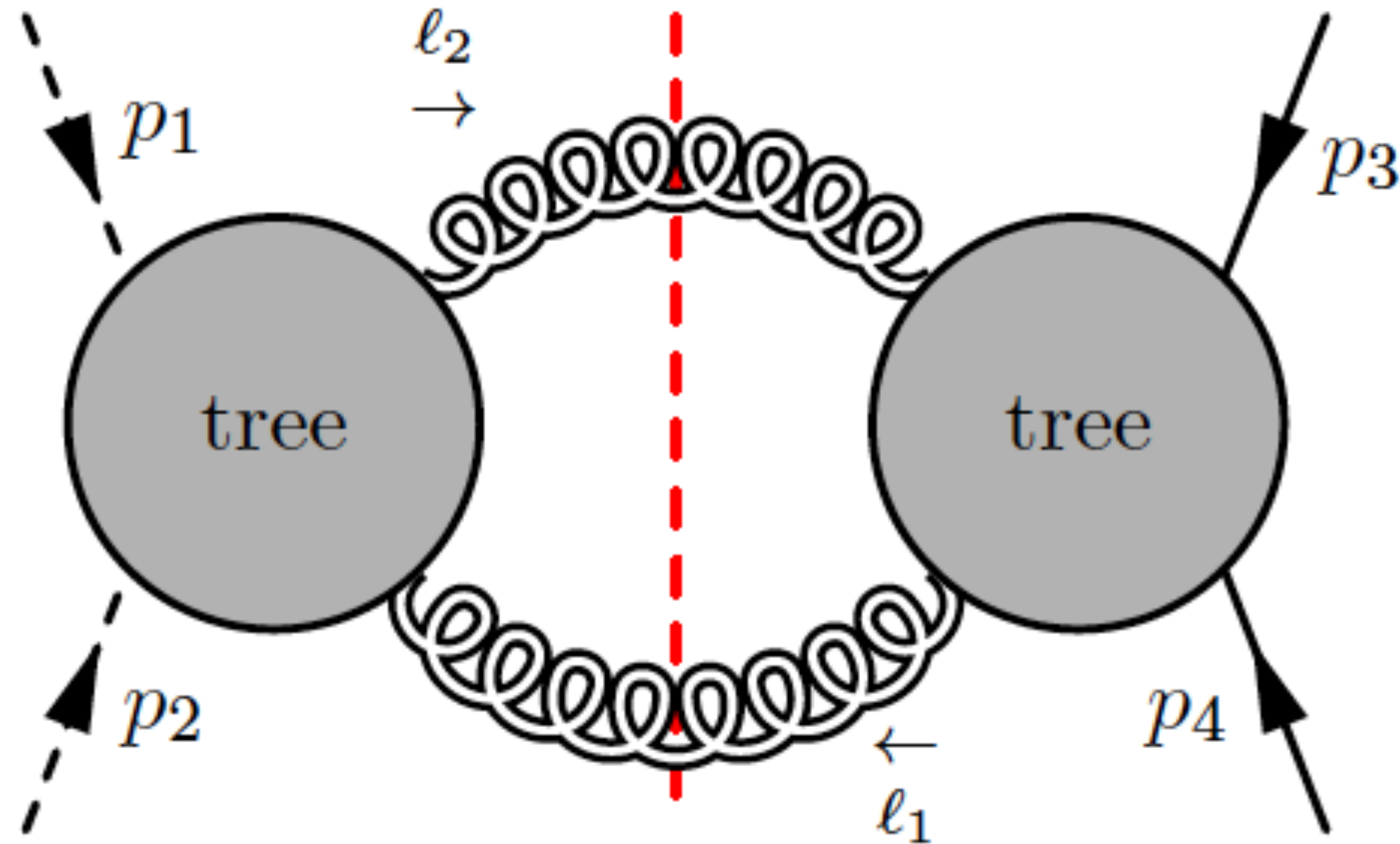
We have the Lagrangian

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} \mathcal{R} + S_{\text{model}} + S_{\text{EF}} \right]$$

We want to compute the cut

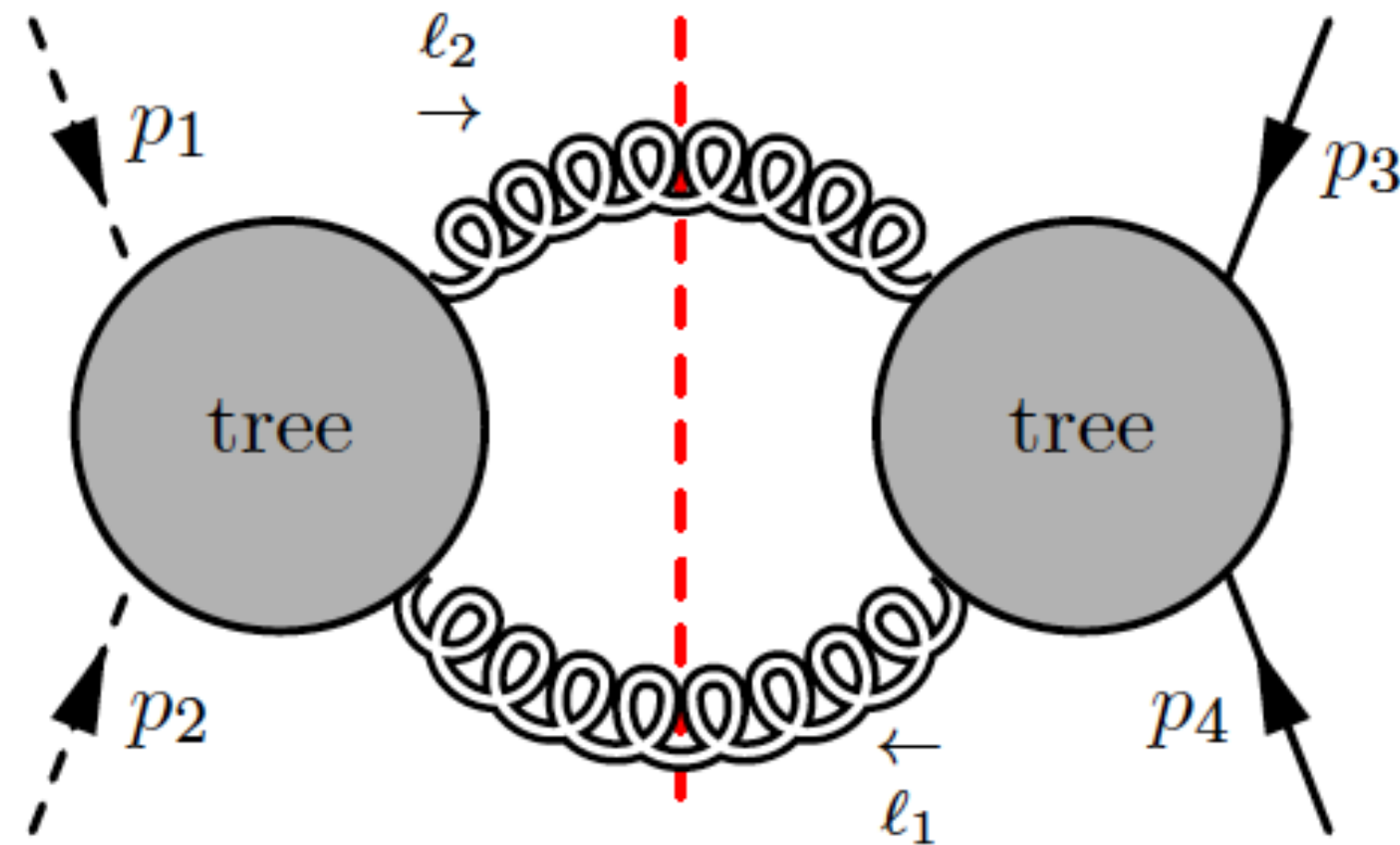


Result for the amplitude



$$bo^S(t, s) I_4(t, s) + bo^S(t, u) I_4(t, u) \\ + t_{12}^S(t) I_3(t, 0) + t_{34}^S(t) I_3(t, M^2) + bu^S(t, 0) I_2(t, 0)$$

Result for the amplitude

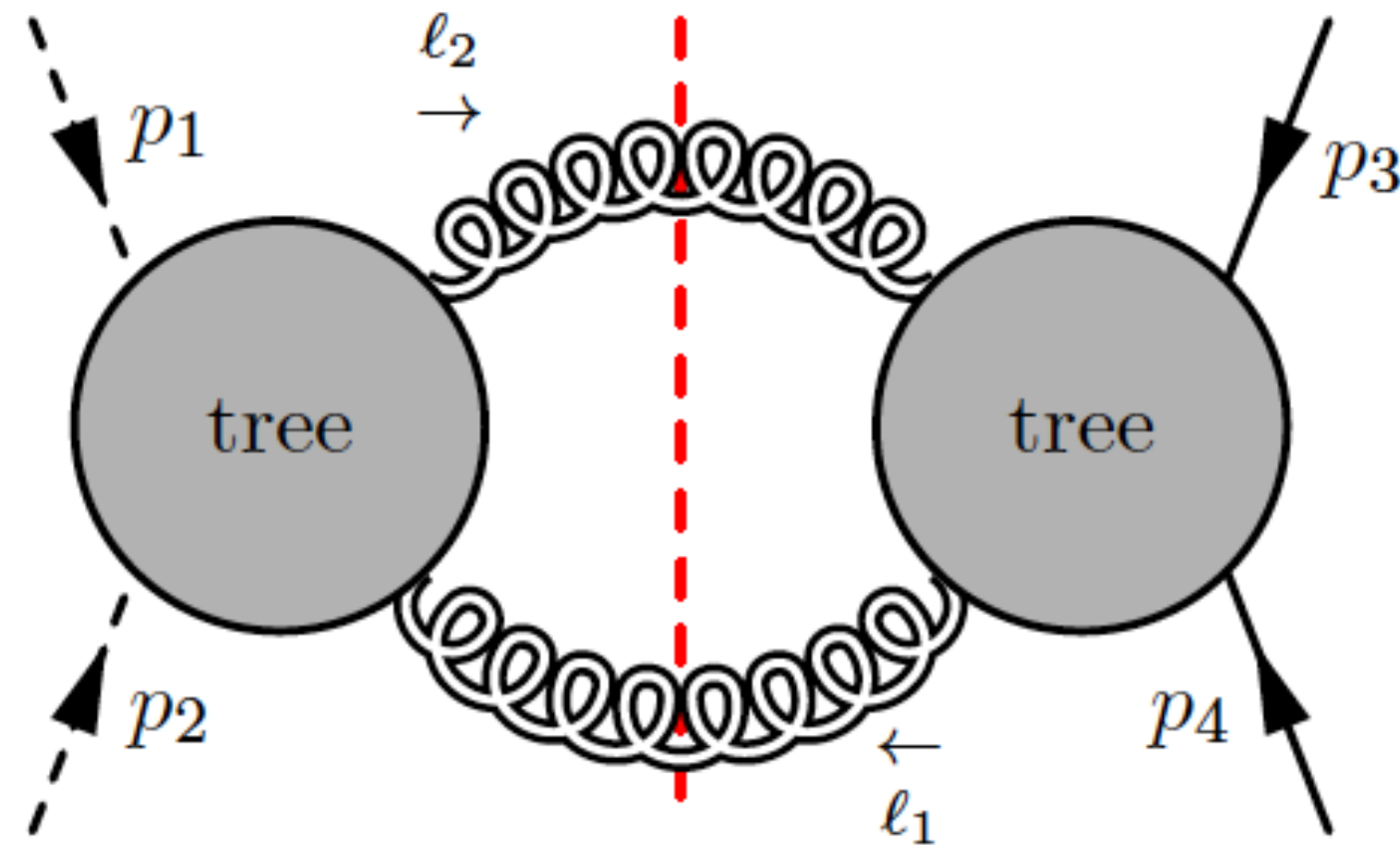


$$bu^\varphi = \frac{3}{40} \quad bu^\gamma = -\frac{161}{120}$$

$$bu^X = -\frac{31}{30}$$

$$-\frac{\mathcal{N}^X}{\hbar} \left[\hbar \frac{\kappa^4}{4} \left(4(M\omega)^4 (I_4(t, u) + I_4(t, s)) + 3(M\omega)^2 t I_3(t) \right. \right. \\ \left. \left. - 15(M^2\omega)^2 I_3(t, M) + bu^X (M\omega)^2 I_2(t) \right) \right]$$

Result for the amplitude



$$bu^{\varphi} = \frac{3}{40} \quad bu^{\gamma} = -\frac{161}{120}$$

$$bu^{\chi} = -\frac{31}{30}$$

Taking the post-Newtonian
non-relativistic low energy limit

$$\frac{\mathcal{N}^X}{\hbar} (M\omega)^2 \left[-\kappa^4 \frac{15}{512} \frac{M}{|\mathbf{q}|} - \hbar \kappa^4 \frac{15}{512\pi^2} \log \left(\frac{\mathbf{q}^2}{M^2} \right) + \hbar \kappa^4 \frac{bu^X}{(8\pi)^2} \log \left(\frac{\mathbf{q}^2}{\mu^2} \right) \right. \\ \left. - \hbar \kappa^4 \frac{3}{128\pi^2} \log^2 \left(\frac{\mathbf{q}^2}{\mu^2} \right) + \kappa^4 \frac{M\omega}{8\pi} \frac{i}{\mathbf{q}^2} \log \left(\frac{\mathbf{q}^2}{M^2} \right) \right]$$

(NEJB, Donoghue,
Holstein,
Plante, Vanhove)

Bending of light

Interpreted as a bending angle (eikonal approximation) we have:

$$\theta_{\eta} \simeq \frac{4GM}{b} + \frac{15}{4} \frac{G^2 M^2 \pi}{b^2}$$

plus a quantum effect of the order of magnitude:

$$+ \frac{8bu^{\eta} + 9 + 48 \log \frac{b}{2r_o}}{\pi} \frac{G^2 \hbar M}{b^3}$$

We see universality among scalars, fermions, and photons only for the ‘Newton’ and ‘post-Newtonian’ contributions.

What is still needed is an interpretation of what “quantum” means as a physical concept in terms of wave mechanics - but we see a unique effect and observe a ‘difference’ that does not match classical universality.

Caveat: bending of light is a very difficult effect to measure directly...

Quantum effects in observations

Quantum gravity likely introduces an uncertainty $x \sim \hbar$ that affects, e.g., the concept of distance

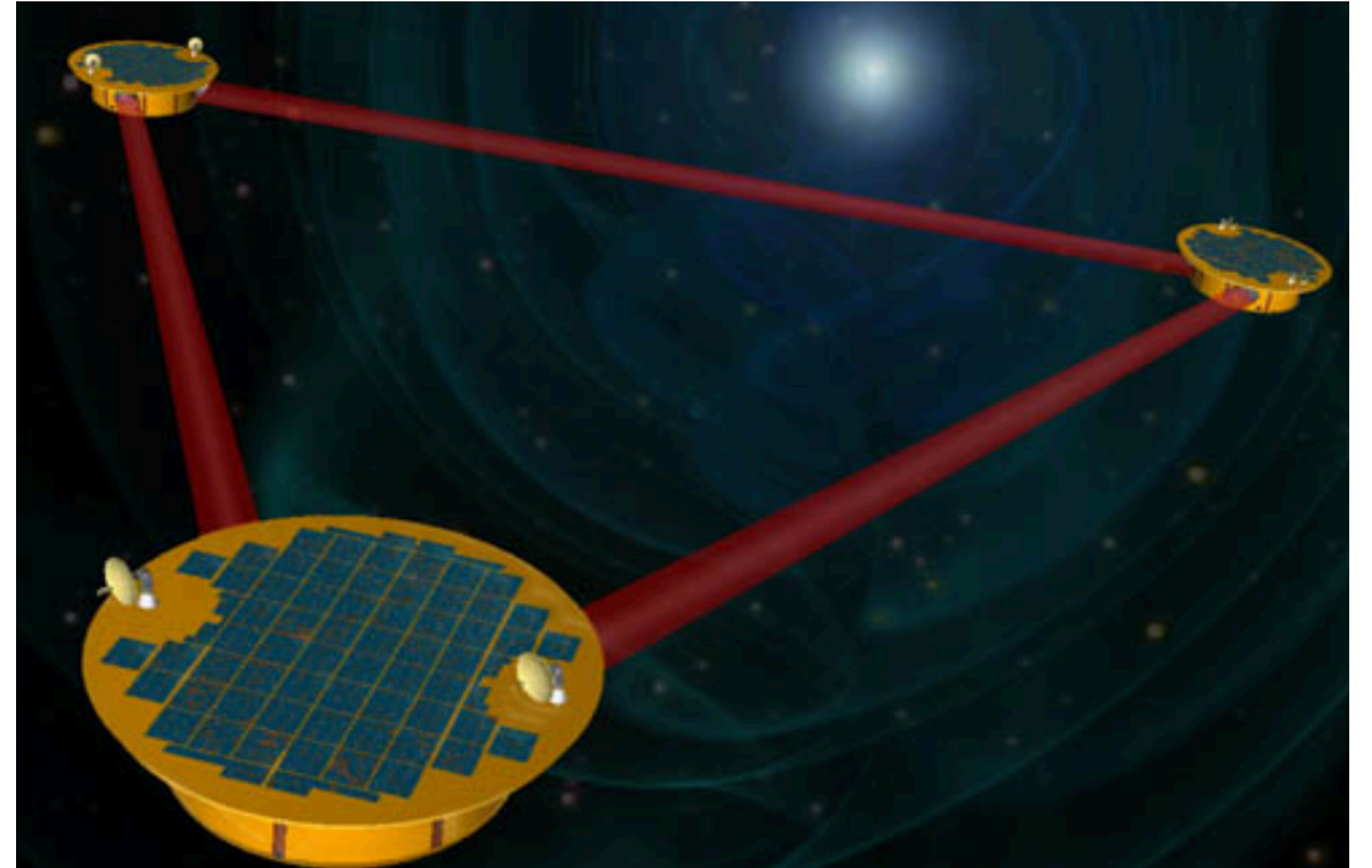
$$\frac{1}{r+x} \sim \frac{1}{r} \left(1 - \frac{x}{r} + \frac{x^2}{r^2} + \dots \right)$$


Effect: too small to probe in current observations..

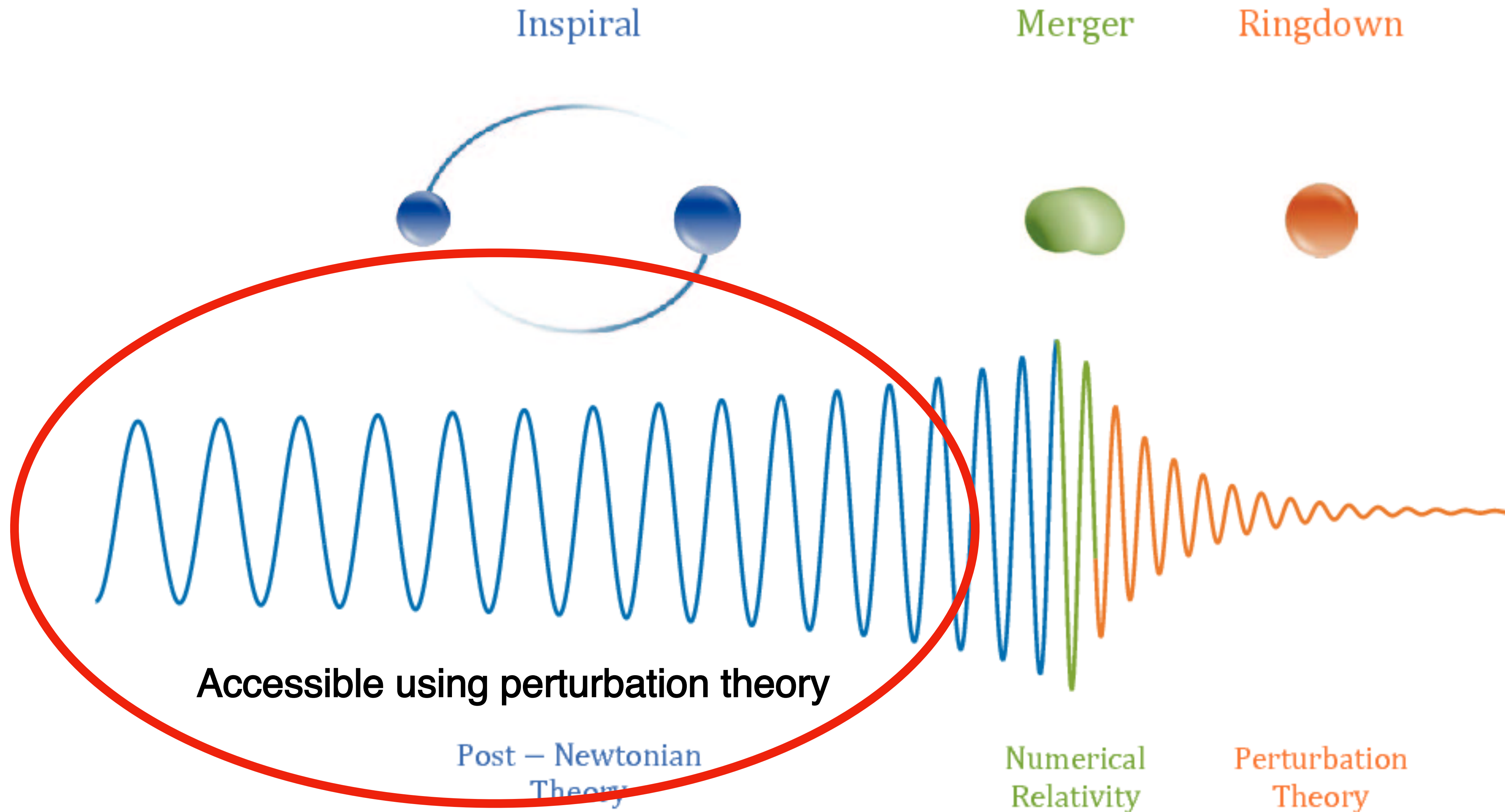
Future measurements

Verification of the effective field theory framework — by detection of higher-derivative operators

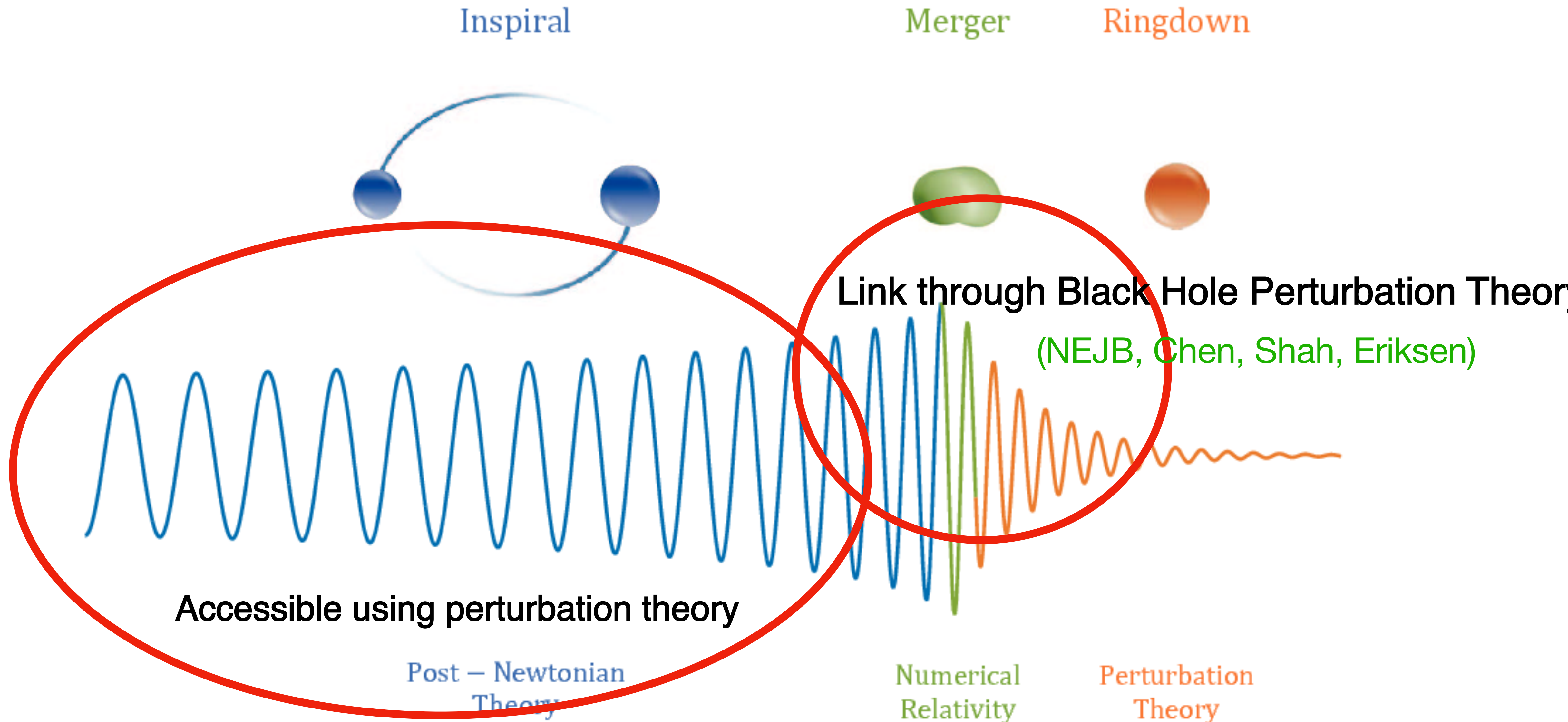
New investigations of other quantum effects — perhaps accumulated effects (resummed / non-perturbative) — that could make them easier to detect?



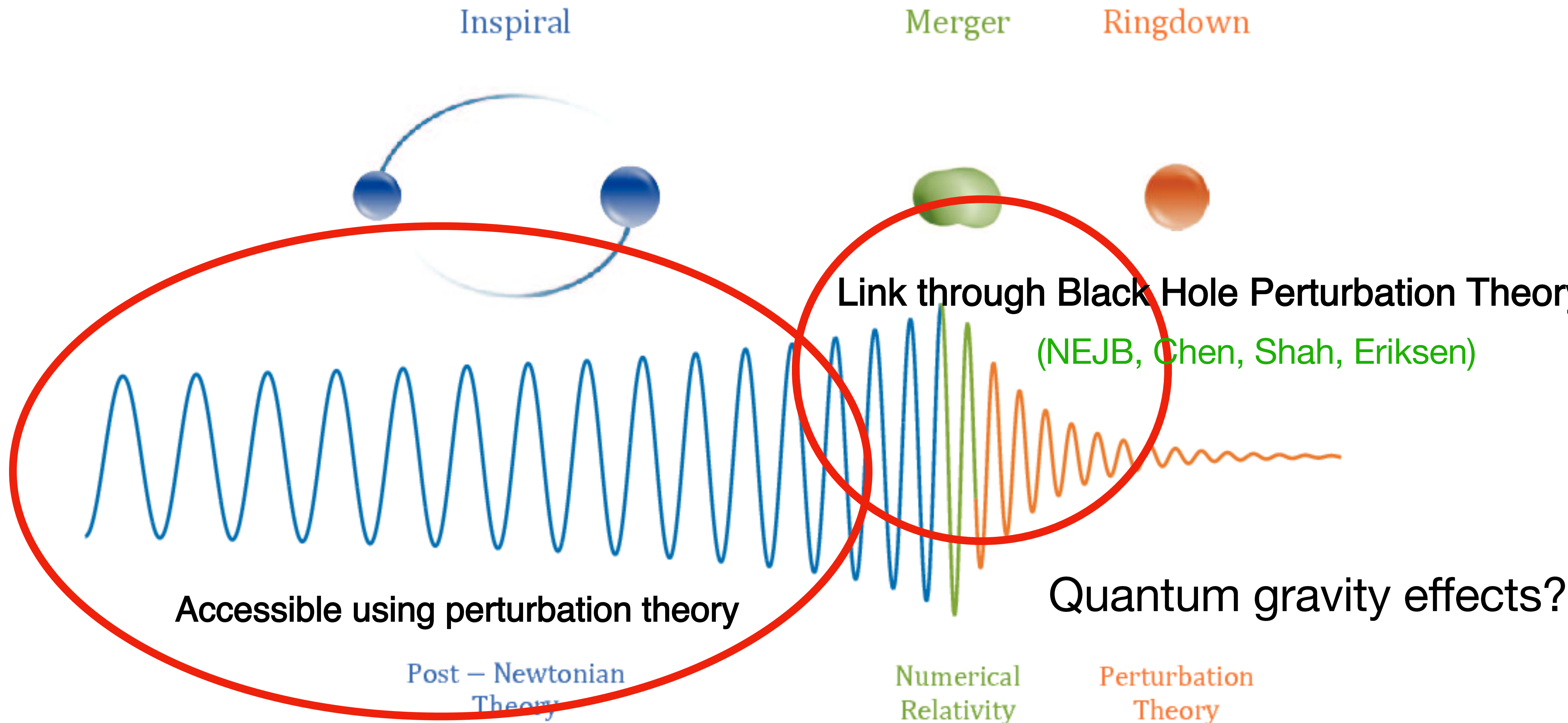
Stages of the gravitational merger of Black Holes



Stages of the gravitational merger of Black Holes



Stages of the gravitational merger of Black Holes



Quantum effects in observations

An interesting new theoretical result by (Jones, Pajanjape, Skowronek) investigates bound states in EFT gravity and the conservation of the Laplace-Runge-Lenz vector, which supergravity can restore (quantum mechanically).

Similar observations regarding universality in the quantum light-bending result were also made in (NEJB, Donoghue, Holstein, Plante, Vanhove)

Supersymmetry and consistent EFT-Quantum Gravity??

Outlook

Amplitude toolbox for computations has already provided many new, efficient methods for gravity computation

- Amplitude tools very useful
 - Double-copy/KLT
 - Unitarity
 - Spinor-helicity
 - CHY formalism
 - Low energy limits of string theory
- Identifying IBP-relations solving DE equations/integral
- Recycling tools from QCD computations
- Numerical programs for amplitude computation

Conclusion

Endless tasks ahead

- radiation/validity of exponentiation/**validity of perturbative amplitudes at high-energy scattering** (open question: classical physics questions...)
- quantum terms?? high-order curvature terms / finite-size effects
many new computations to be done...
- String theory amplitudes useful (**quantum?**)? Gravity from amplitude geometry/twistors, etc.?
- Classical spin black hole effects (a current hot topic, very recent papers) (NEJB, Chen, Skowronek; NEJB, Chen, Su, Wang; NEJB, Chen, Papadimos, Zhang; NEJB, Chen, Papadimos, Shah, Su) (**quantum effects??**)

Conclusion

- Interesting perspectives in the analysis of gravity in the context of general relativity vs. a quantum field theory framework
- The effective field theory treatment of gravity allows for a direct particle physics treatment of gravitational effects using on-shell S-matrix methods
- Give clearer physical meaning to ‘quantum effects’
- Use consistency of perturbative quantization and match of classical limit with general relativity to (perhaps) gain new quantum gravity understanding?

THANKS!!!