

Core and cone radio beam geometry for γ -ray pulsars under a force-free magnetosphere

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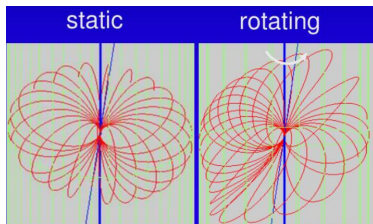
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Pulsar magnetosphere models: RVD vs FF

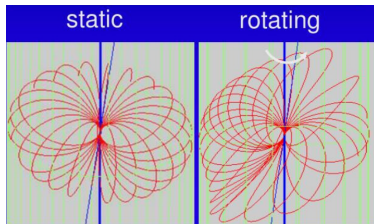
- **Rotating vacuum dipole (RVD):** empty magnetosphere, no charges, no currents. Unrealistic.



Credit: H. Wang

Pulsar magnetosphere models: RVD vs FF

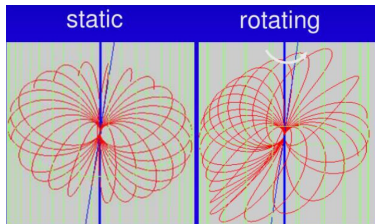
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- In reality, the strong $\mathbf{E}$ field strips charges from the NS surface, filling the magnetosphere with plasma.



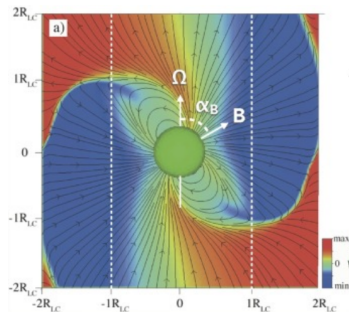
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Pulsar magnetosphere models: RVD vs FF

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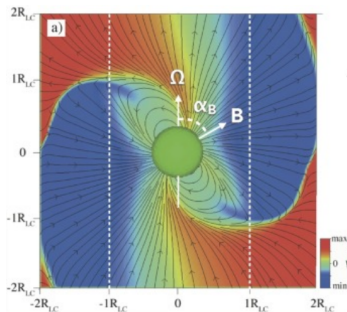


Spitkovsky (2006)

Force-Free Magnetosphere cont.

- Force-free condition: Net electromagnetic force vanishes:

$$\rho_e \mathbf{E} + \mathbf{J} \times \mathbf{B} = 0$$

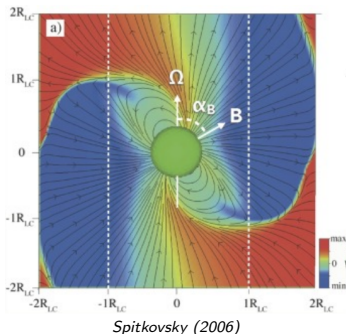


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Force-Free Magnetosphere cont.

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- Consequences:
 - The electric field has no component along $\mathbf{B}$: $\mathbf{E} \cdot \mathbf{B} = 0$
 - Particles are confined to field lines and co-rotate with the star out to the light cylinder R_{LC}

- The geometry of γ -ray pulsar magnetospheres determines observed emission patterns.
- Force-free (FF) electrodynamics provides an alternative description of the magnetosphere.
- Skymaps encode how emission varies with observer angle ζ and rotational phase ϕ .
- Understanding skymaps is the key to interpreting pulse profiles and constraining emission geometry.

Motivation

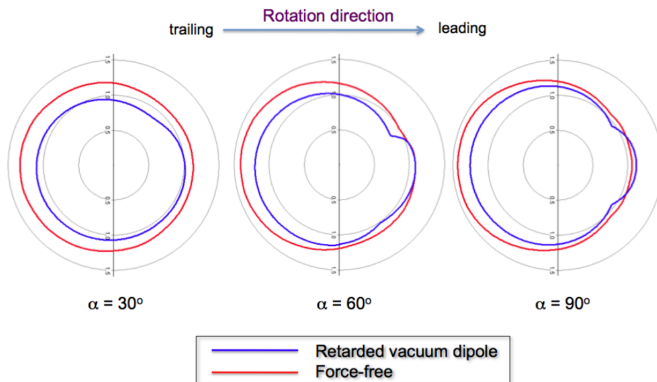
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This talk

Implement a core + cone beam model within a FF magnetosphere and generate skymaps as a function of observer colatitude ζ and rotational phase ϕ .

Polar cap size

$$r_{pc} = R_{ns} \sqrt{\frac{R_{ns}}{R_{lc}}}$$

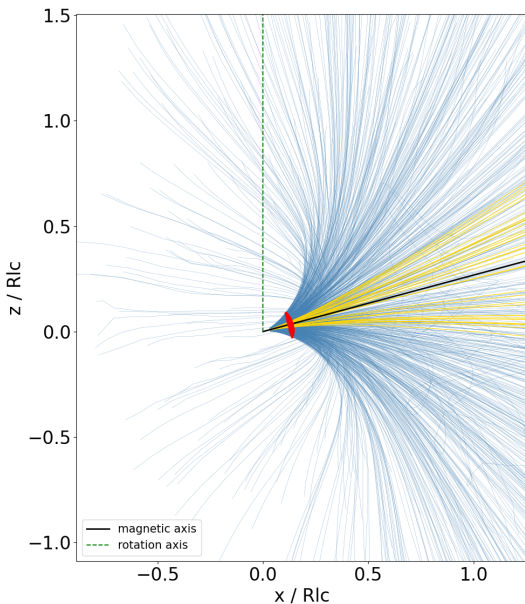


Harding et al. 2011

The volume of open fields is larger in FF, hence bigger FF polar caps

Ray tracing

- Electrons trace the magnetic field lines
- Yellow **B** lines: core
- Blue **B** lines: cone
- Red disk: Spatial distribution of emitting electrons within the polar cap



Cone + Core Radio Beam Geometry

The FF magnetosphere provides the magnetic geometry onto which this emission model is mapped.

Radii:

$$\rho_{\text{cone}} = 1.1^\circ \left(\frac{r_{\text{ph0}}}{R_{\text{ns}}} \right) P^{-0.5}, \quad \rho_{\text{core}} = 1.1^\circ P^{-0.5}$$

Cone width: Story et al. 2007

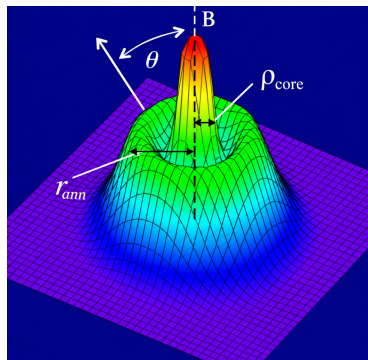
$$w_e = \delta_w \rho_{\text{cone}}, \quad \delta_w = 0.18$$

Cone position: Story et al. 2007

$$r_{\text{ann}} = (1.0 - 2.63 \delta_w) \rho_{\text{cone}}$$

Emission altitude: Kijak et al. 2003

$$r_{\text{ph0,cone}} = 40 R_{\text{ns}} \nu_{\text{GHz}}^{-0.26} (\dot{P}_{-15})^{0.07} P^{0.30}, \quad r_{\text{ph0,core}} = 1.1 R_{\text{ns}}$$



Credit: A. Harding

Radio Geometric Model

Core beam — centred on the magnetic axis:

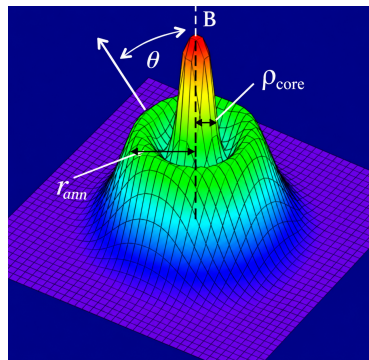
$$f_{\text{core}}(\theta) = \frac{1}{\Omega_{\text{core}}} \exp\left(-\frac{\theta^2}{\rho_{\text{core}}^2}\right)$$

Cone beam — Gaussian ring offset from the axis:

$$f_{\text{cone}}(\theta) = \frac{1}{\Omega_{\text{cone}}} \exp\left(-\frac{(\theta - r_{\text{ann}})^2}{\rho_{\text{cone}}^2}\right)$$

Flux (core or cone):

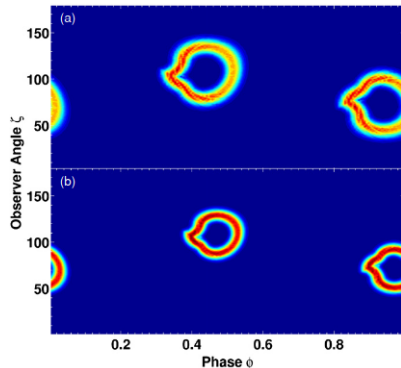
$$S_X(\theta) = -\frac{(\alpha_X + 1)}{\nu} \left(\frac{\nu}{50}\right)^{\alpha_X + 1} \frac{f_X(\theta) L_X}{d^2}, \quad X \in \{\text{core, cone}\}$$



Credit: A. Harding

Simulation Parameters

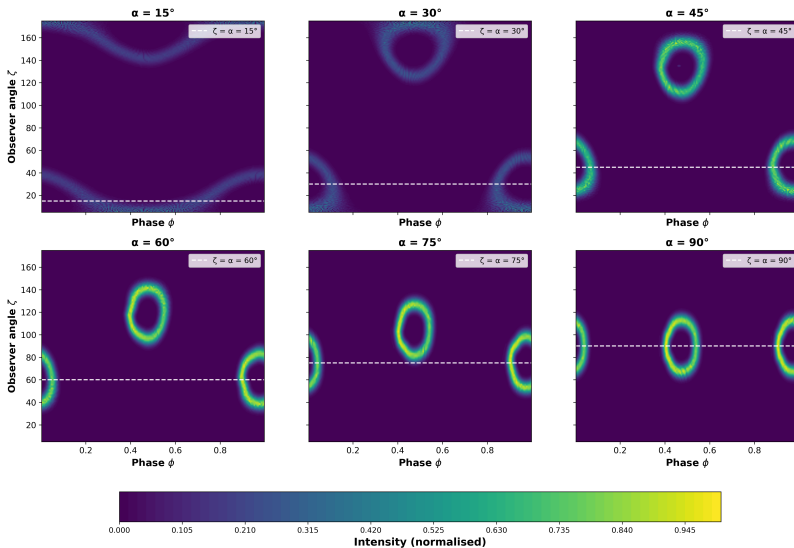
Parameter	Value
Spin period P	0.005 s
Period derivative $\dot{P}$	1.00×10^{-20} s/s
Frequency	1400 MHz
Inclination angle α	$\in [15^\circ, 90^\circ]$
Emission model	Core + cone



Venter et al. 2009

RVD cone skymaps for $\alpha = 70^\circ$, with $P = 2$ ms (top panel) and $P = 5$ ms (bottom panel).

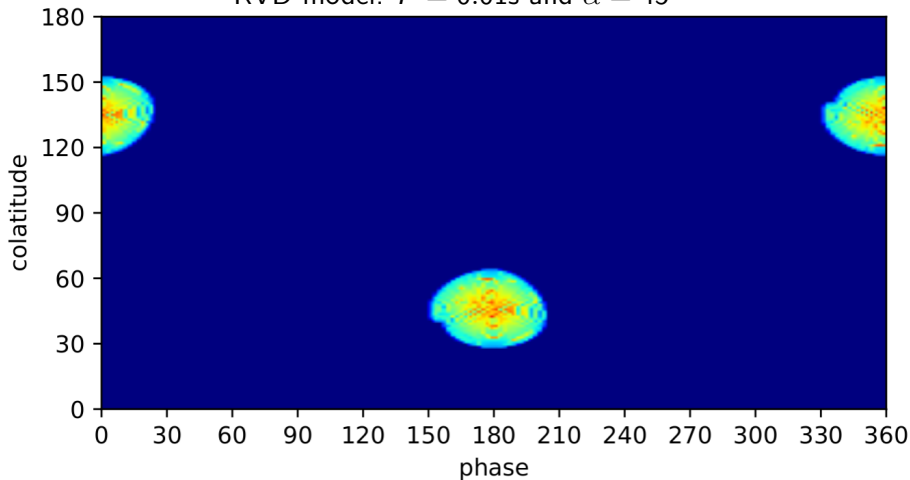
FF phase plot: cone only



Intensity as a function of rotational phase ϕ and observer colatitude ζ for $P = 5\text{ms}$.

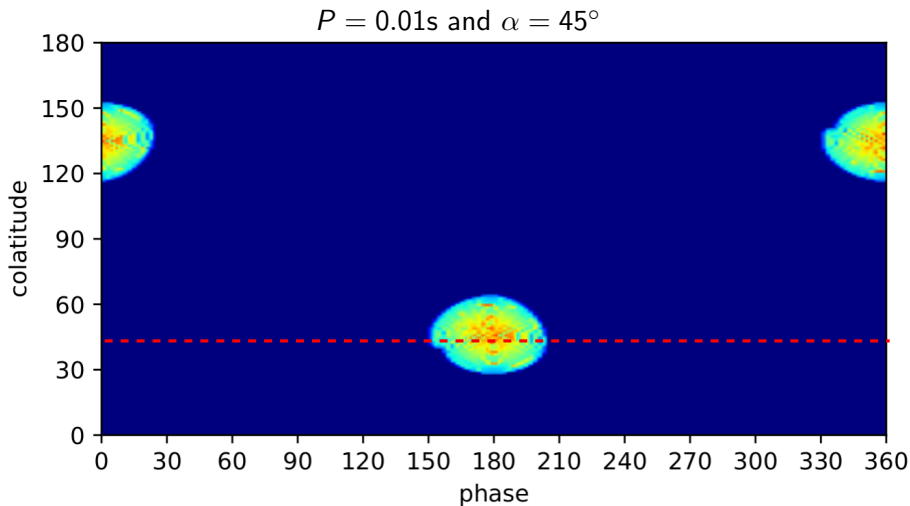
Expected phase plot: Core + cone

RVD model: $P = 0.01\text{s}$ and $\alpha = 45^\circ$

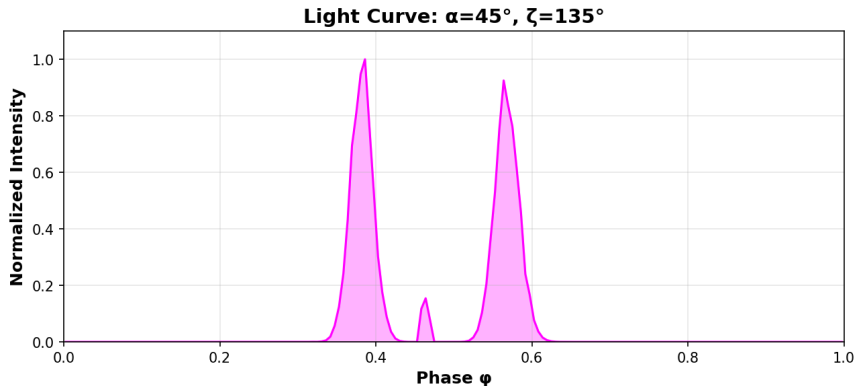


Credit: A. Harding

Expected phase plot: Core + cone

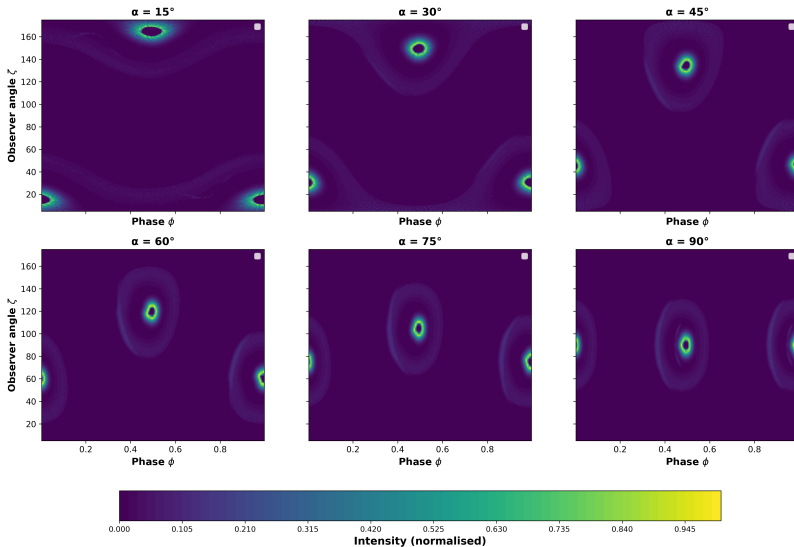


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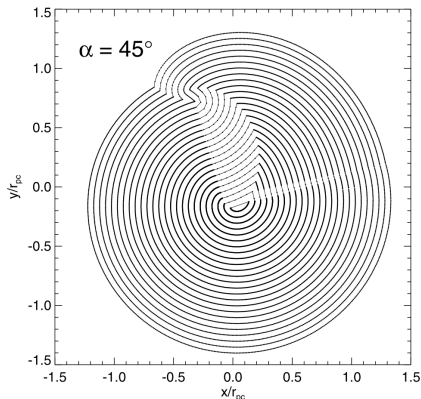
Pulse phase shifts are caused by aberration, time-of-flight delays, and retardation in the rotating FF magnetosphere.

FF Core + cone: what we get, **empty core**



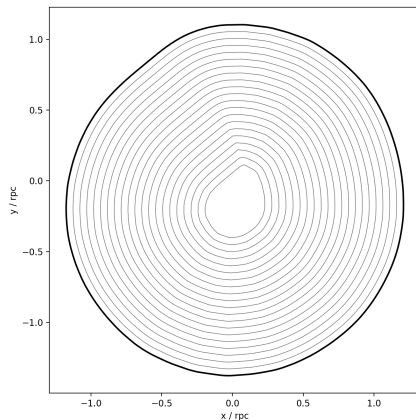
Empty core?

FF rings don't reach the magnetic axis, hence no core emission.



Dyks et al. 2004

RVD polar cap rings.



FF polar cap rings.

Conclusion

- Revisited the radio geometric model (core + cone) within a FF magnetosphere framework, as a foundation for a physically-anchored beam geometry.
- Generated two-pole radio skymaps for $\alpha \in [15^\circ, 90^\circ]$ and a light curve for $\alpha = 45^\circ$.
- Cone-only skymaps reproduce the expected FF morphology
- Ring footprints don't reach the magnetic axis, so the core region is unpopulated

Next steps

- Include electrons from the innermost field lines to generate emission close to the magnetic axis.
- Include multi-conal phase plots.
- Extract pulse profiles to constrain geometric parameters against observed data.
- Create a core + cone light curve atlas across (α, ζ) space.
- Include the FF current structure along each field line.

Thank You!

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