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Calibrating GRB Phenomenological relations with Type Ia Supernovae data

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Amati Relation

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**Astronomy
&
Astrophysics**

Intrinsic spectra and energetics of BeppoSAX Gamma-Ray Bursts with known redshifts

L. Amati¹, F. Frontera^{1,2}, M. Tavani³, J. J. M. in 't Zand⁴, A. Antonelli⁵, E. Costa⁶, M. Feroci⁶, C. Guidorzi², J. Heise⁴, N. Masetti¹, E. Montanari², L. Nicastro⁷, E. Palazzi¹, E. Pian⁸, L. Piro⁶, and P. Soffitta⁶

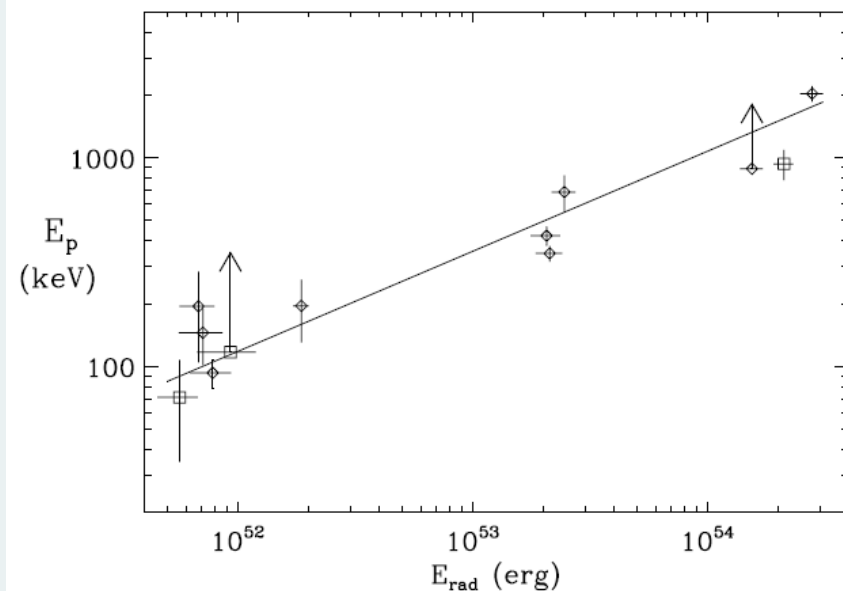
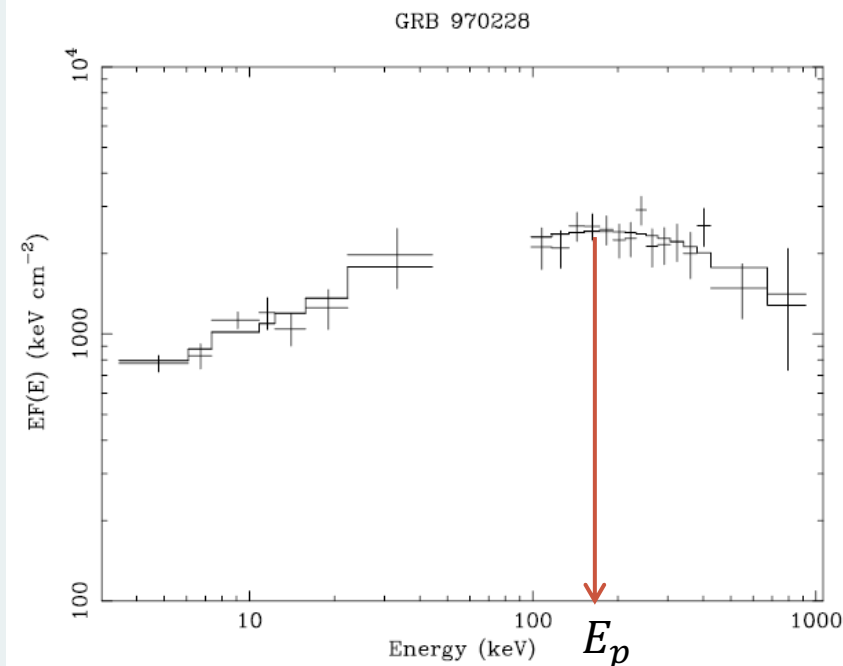
$$E_p \propto E_{\text{rad}}^{0.52 \pm 0.06}$$

$$E_{\text{rad}} = \frac{\int_1^{10000} EN(E, \alpha, E_0, \beta, A) dE \times 4\pi D_L^2}{(1+z)^2}.$$

Flat LCDM cosmology

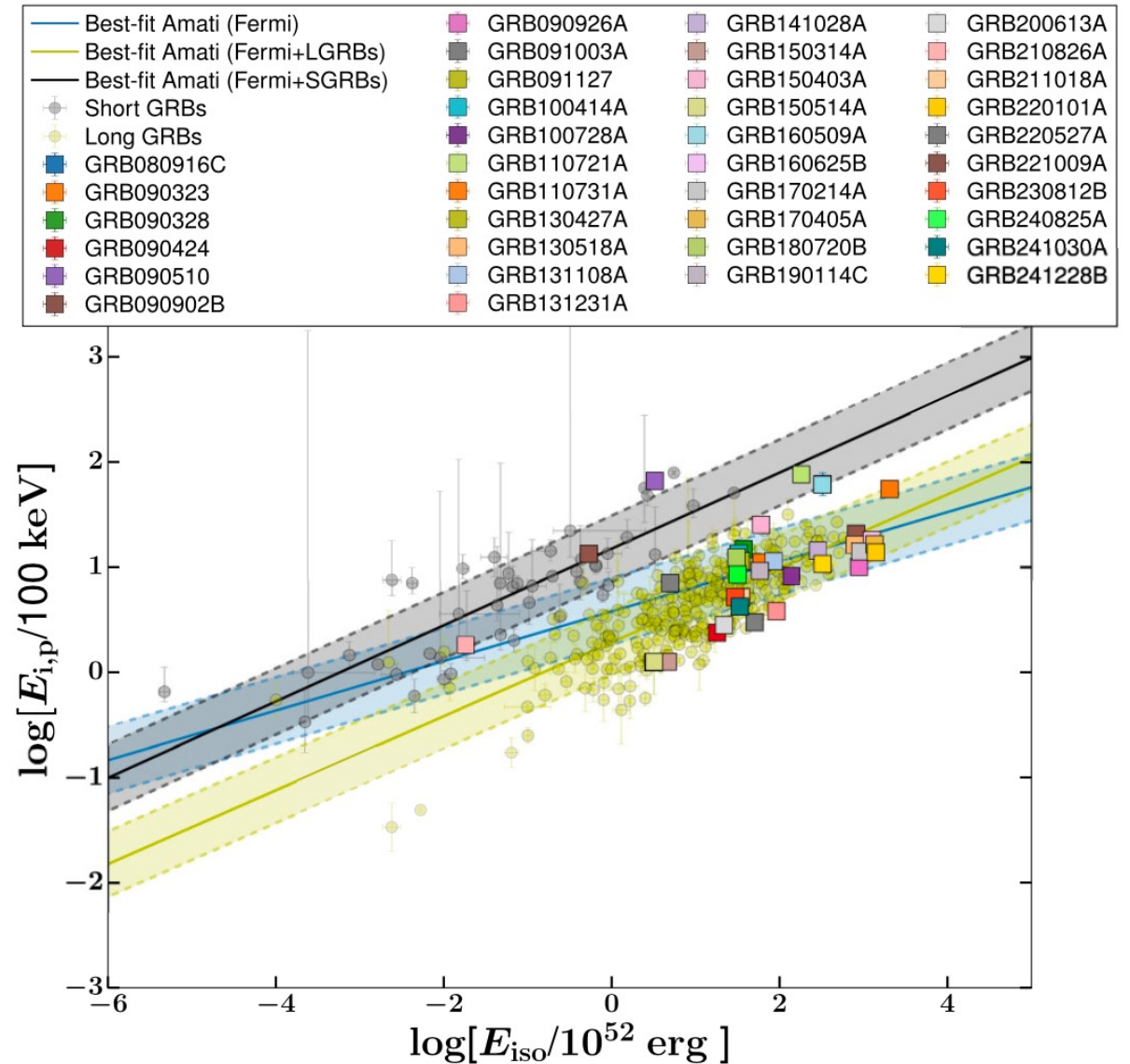
$$H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\Omega_M = 0.3, \Omega_\Lambda = 0.7$$



Amati Relation

- Many iterations by different authors with different datasets
- Slope m in $E_{i,p} \propto E_{\text{iso}}^m$ different for short- and long duration GRBs
- Large variation of the slope: $m \sim 0.1 - 1$
- Mostly flat LCDM cosmology used but also with different cosmology (*Demianski et al. 2017*)
- No confirmed physical origin of the relation



How do the
phenomenological
relations change with
cosmological models?



Luminosity Distances in Cosmological Models

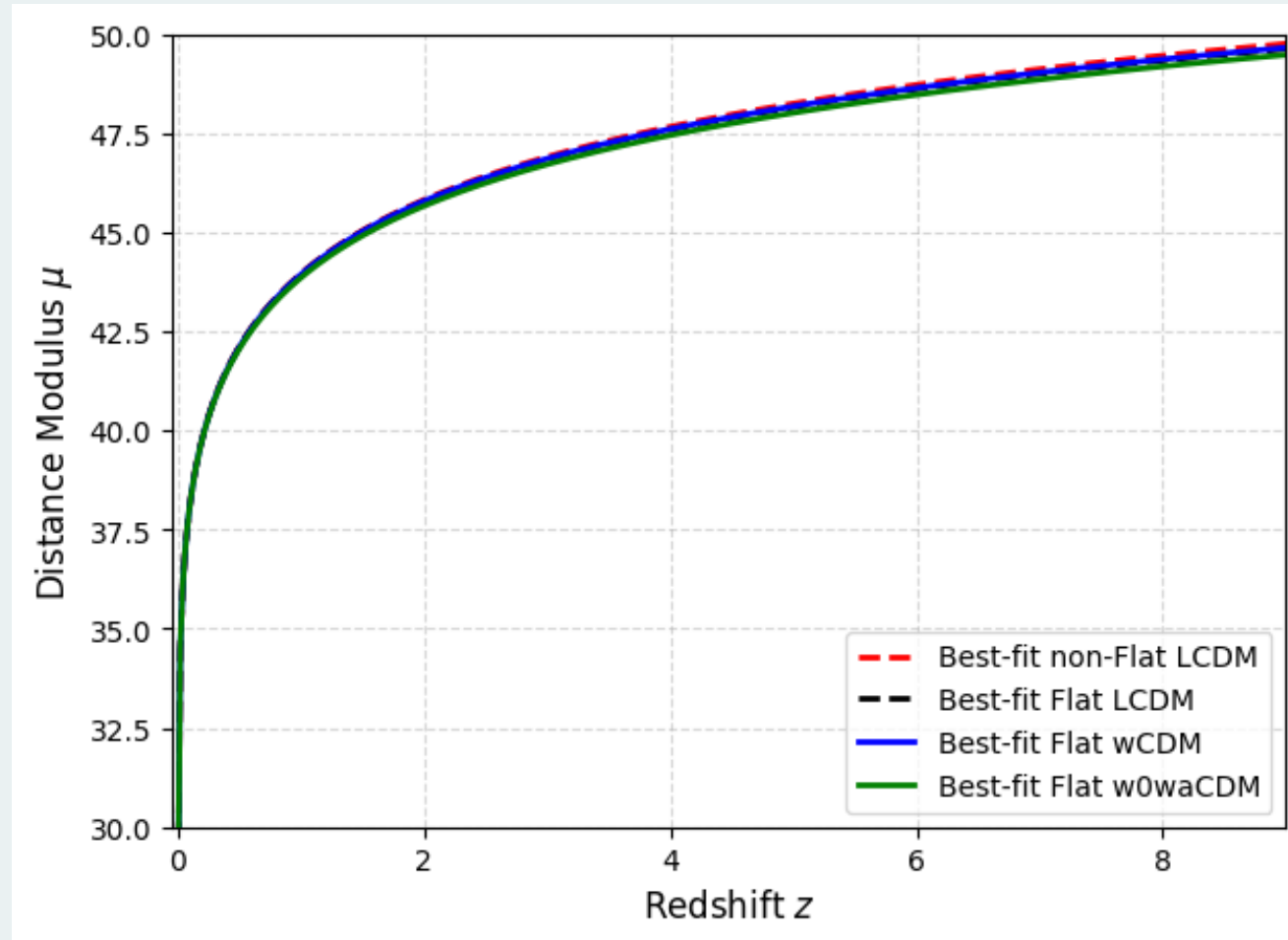
$$d_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{H_0}{H(z')} dz' \quad \text{Spatially flat case } (\Omega_k = 0)$$

$$d_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{1}{\sqrt{\Omega_M (1+z')^3 + \Omega_\Lambda}} dz' \quad \Lambda\text{CDM}$$

$$d_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{1}{\sqrt{\Omega_M (1+z')^3 + (1 - \Omega_M)(1+z')^{3(1+w)}}} dz' \quad \text{Flat } \omega\text{CDM}$$

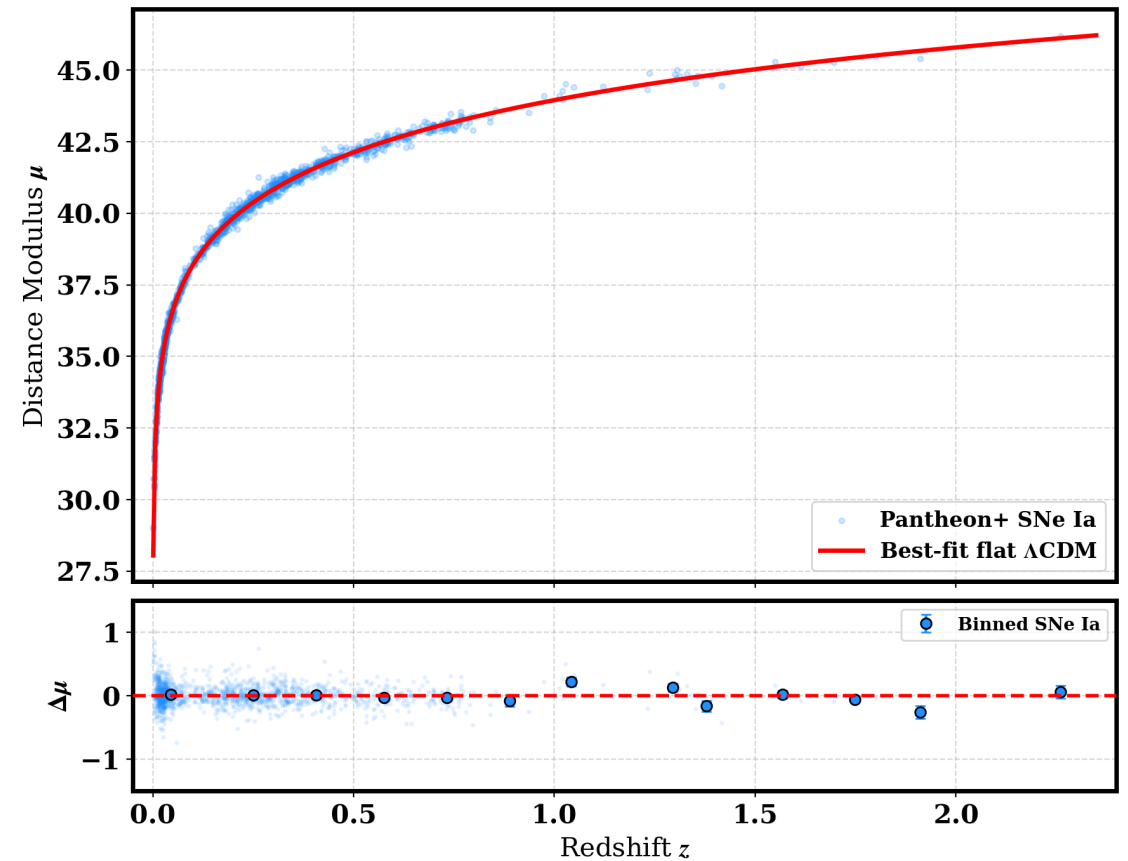
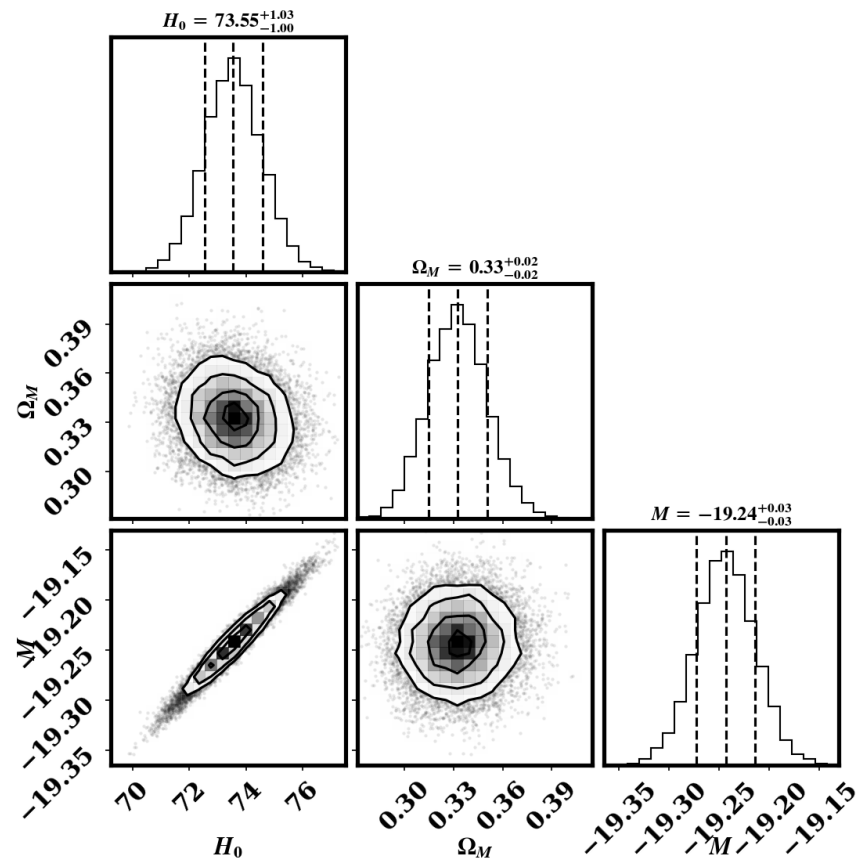
$$d_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{1}{\sqrt{\Omega_M (1+z')^3 + (1 - \Omega_M)(1+z')^{3(1+w_0+w_a)} \exp\left(-3w_a \frac{z'}{1+z'}\right)}} dz' \quad \text{Flat } \omega_0\omega_a\text{CDM}$$

Luminosity Distances in Cosmological Models



Pantheon+ Type Ia Supernovae data

1701 data points, 1657 after calibrated with Cepheids from SHOES



Reproducing Pantheon+ & SHOES Results

Best-fit parameters from *Brout, Scolnic, Popovic et al. 2022*

Model	H_0	Ω_M	Ω_Λ	ω/ω_0	ω_a	BIC
Flat Λ CDM	73.6 ± 1.1	0.334 ± 0.018	0.666 ± 0.018			
(Our fit)	$73.55^{+1.03}_{-1.00}$	$0.33^{+0.02}_{-0.02}$				1474.26
Non-flat Λ CDM	73.4 ± 1.1	0.306 ± 0.057	0.625 ± 0.084			
(Our fit)	$73.42^{+1.02}_{-1.00}$	$0.30^{+0.05}_{-0.05}$	$0.62^{+0.08}_{-0.08}$			1481.37
Flat w CDM	73.5 ± 1.1	$0.309^{+0.063}_{-0.069}$	$0.691^{+0.069}_{-0.063}$	-0.90 ± 0.14		
(Our fit)	$73.47^{+1.02}_{-1.02}$		$0.70^{+0.07}_{-0.06}$	$-0.92^{+0.14}_{-0.15}$		1481.36
Flat $w_0 w_a$ CDM	73.3 ± 1.1	$0.403^{+0.054}_{-0.098}$	$0.597^{+0.098}_{-0.054}$	-0.93 ± 0.15	$-0.1^{+0.9}_{-2.0}$	
(Our fit)	$73.34^{+1.04}_{-1.03}$		$0.63^{+0.12}_{-0.07}$	$-0.92^{+0.13}_{-0.15}$	$-0.96^{+1.39}_{-2.20}$	1488.79

Amati Relation applied to GRB data

312 long-duration GRBs with measured redshift

$$\frac{E_{i,p}}{100 \text{ keV}} = 10^k \left(\frac{E_{iso}}{10^{52} \text{ erg}} \right)^m \Rightarrow y = mx + k ; \quad y = \log_{10} \left(\frac{E_{i,p}}{100 \text{ keV}} \right), x = \log_{10} \left(\frac{E_{iso}}{10^{52} \text{ erg}} \right)$$

Luminosity distance using Amati relation

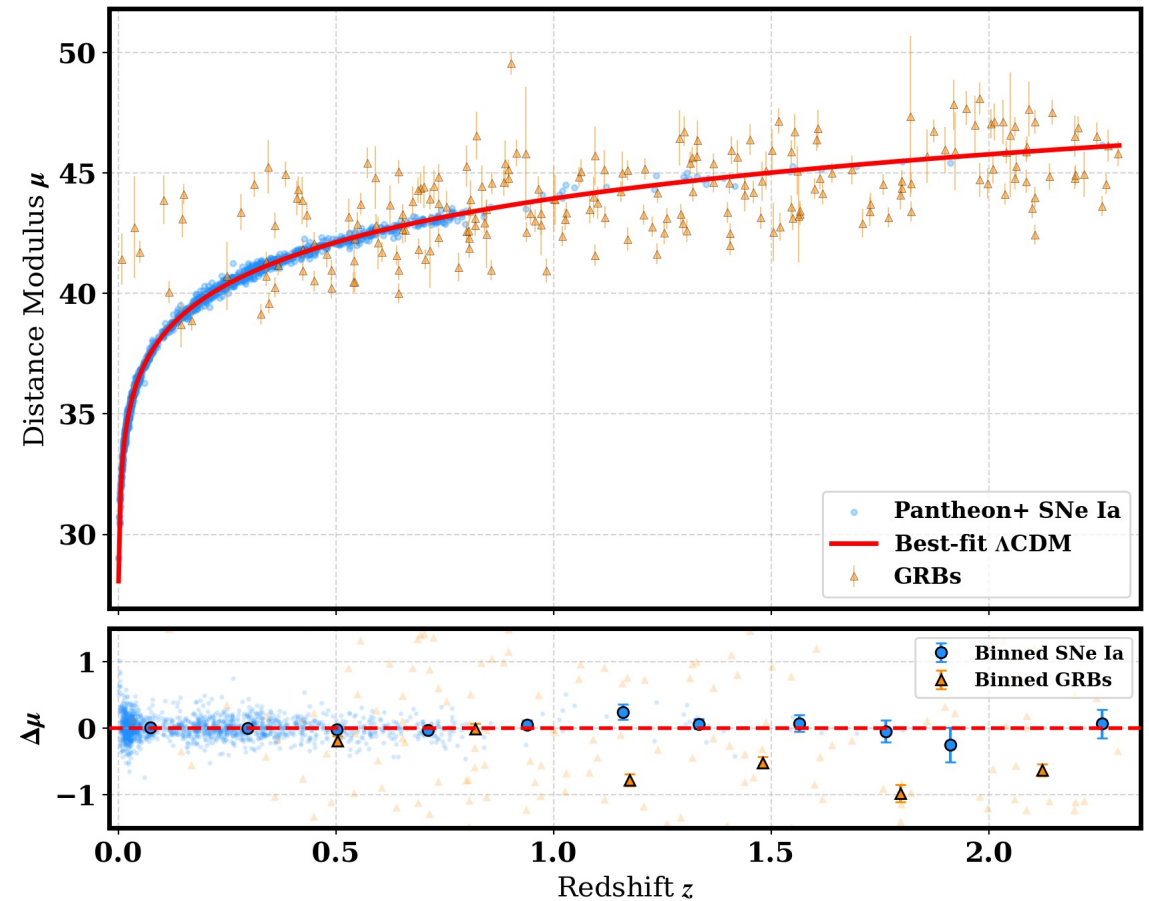
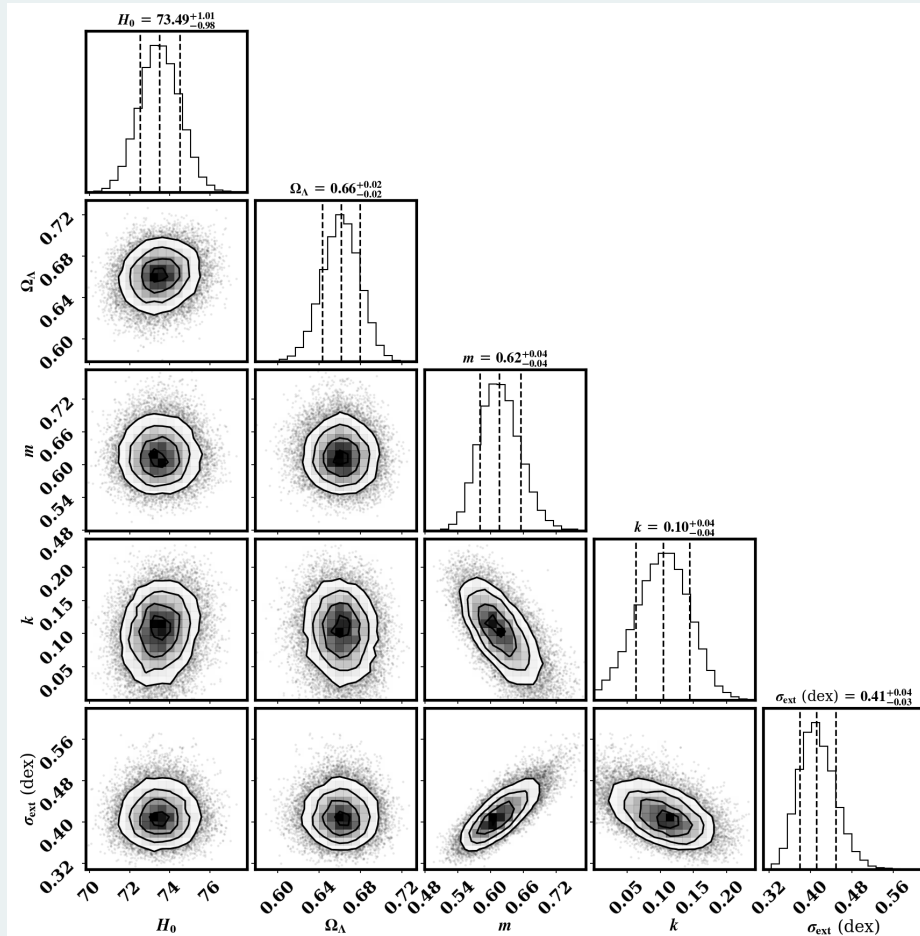
$$\log_{10} d_L = \frac{1}{2} \log_{10} \left(\frac{1+z}{4\pi S_{bol}} \right) + \frac{1}{2m} \log_{10} \left(\frac{E_p(1+z)}{100 \text{ keV}} \right) - \frac{k}{2m} + \frac{52}{2}, \quad S_{bol} : \text{Bolometric fluence (erg/cm}^2\text{) in 1 keV – 10 MeV}$$

Distance modulus $\mu_{GRB}(z) = 5 \log_{10} \left(\frac{d_L}{1 \text{ Mpc}} \right) + 25$ Compare with theoretical predictions

Maximize the log likelihood $\ln L(m, k, \sigma_{ext}, \hat{\theta}) = \ln L_{GRB}(m, k, \sigma_{ext}, \hat{\theta}) + \ln L_{SNe}(\hat{\theta})$

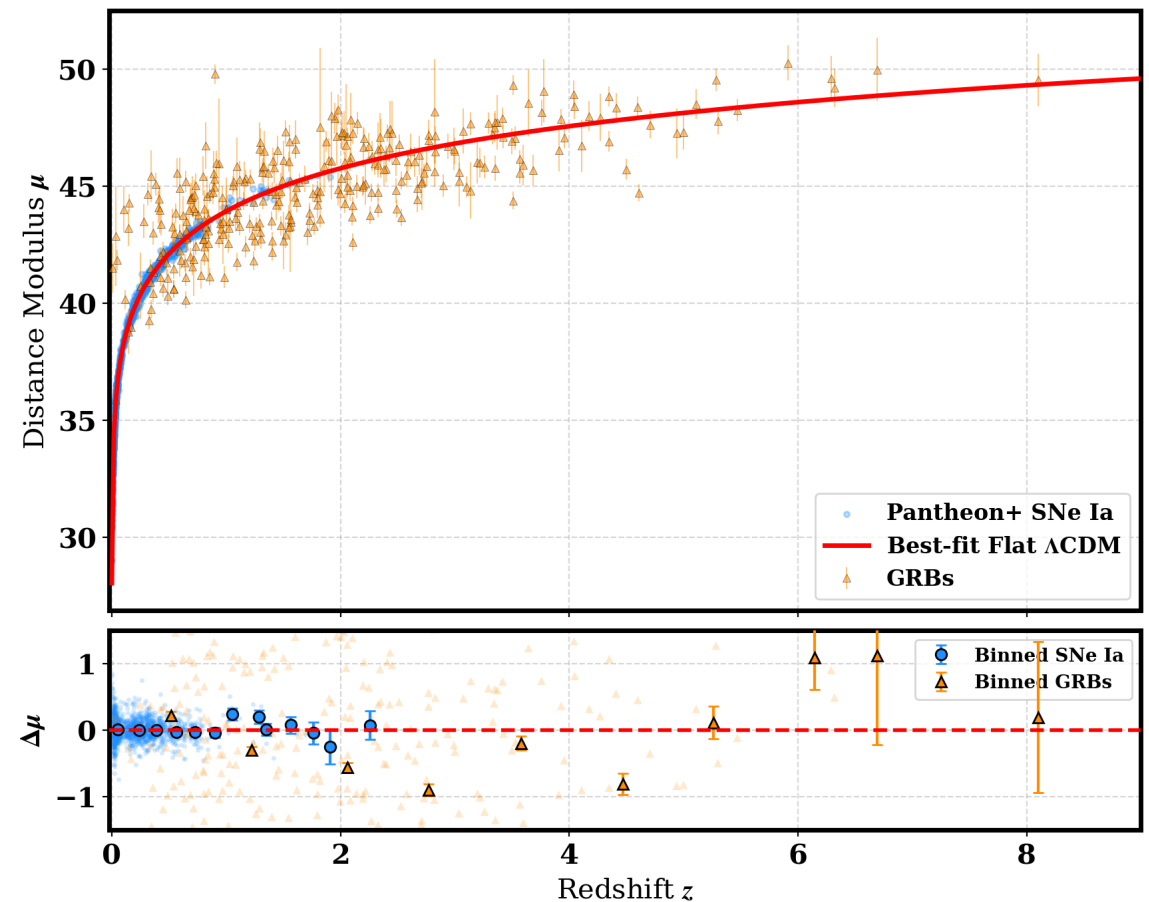
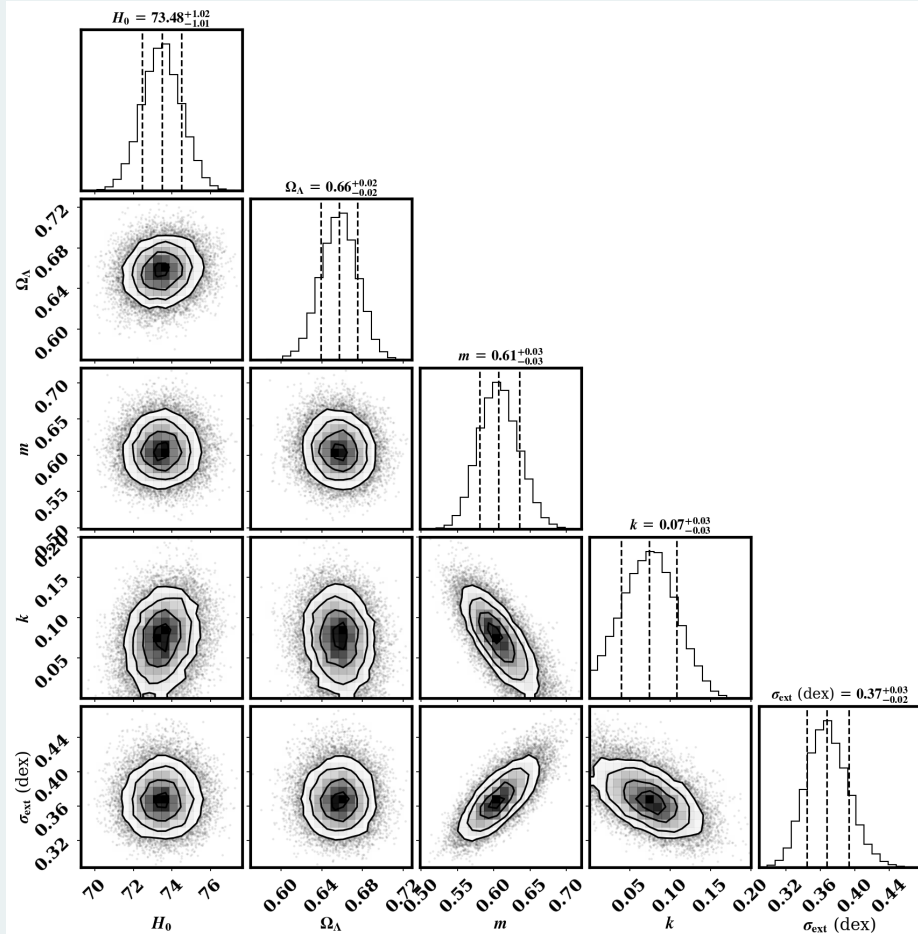
Pantheon+ & SHOES and GRBs

1657 SNe (after Cepheid calibration) and 218 long GRBs ($z < 2.3$)



Pantheon+ & SHOES and GRBs

1657 SNe (after Cepheid calibration) and 312 long GRBs ($z < 9.0$)

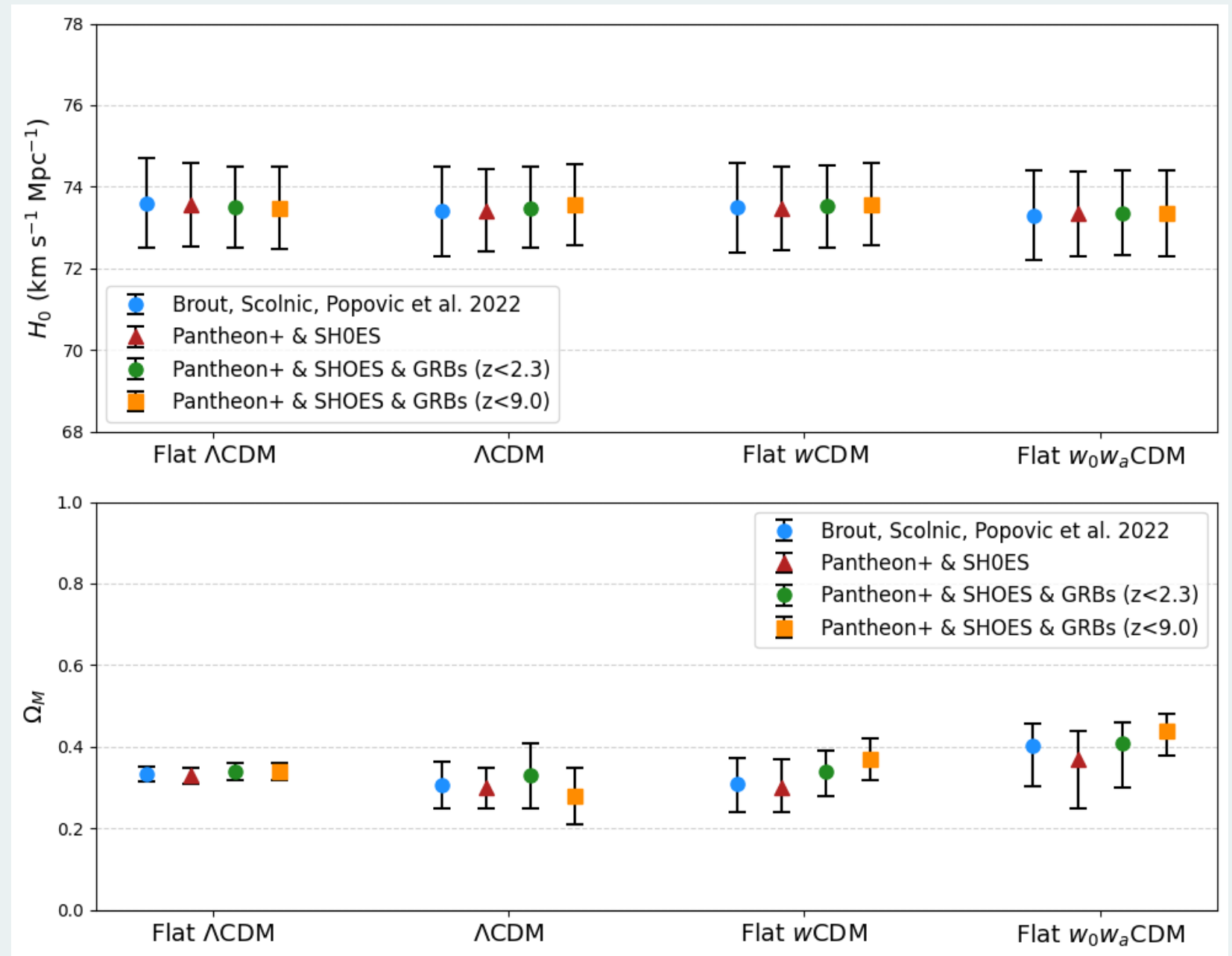


Fits to different cosmological models

Pantheon+ with SHOES and GRBs

Model ($z < 2.3$)	H_0	Ω_M	Ω_Λ	ω/ω_0	ω_a	m	k	σ_{ext}
Flat Λ CDM	$73.49^{+1.01}_{-0.98}$		$0.66^{+0.02}_{-0.02}$			$0.62^{+0.04}_{-0.04}$	$0.10^{+0.04}_{-0.04}$	$0.41^{+0.04}_{-0.03}$
Non-flat Λ CDM	$73.48^{+1.03}_{-0.97}$	$0.35^{+0.05}_{-0.05}$	$0.67^{+0.08}_{-0.08}$			$0.62^{+0.04}_{-0.04}$	$0.10^{+0.04}_{-0.04}$	$0.41^{+0.04}_{-0.03}$
Flat w CDM	$73.52^{+1.01}_{-1.01}$		$0.66^{+0.06}_{-0.05}$	$-1.01^{+0.15}_{-0.16}$		$0.62^{+0.04}_{-0.04}$	$0.10^{+0.04}_{-0.04}$	$0.41^{+0.04}_{-0.03}$
Flat $w_0 w_a$ CDM	$73.36^{+1.05}_{-1.03}$		$0.59^{+0.11}_{-0.05}$	$-0.98^{+0.14}_{-0.16}$	$-1.60^{+2.00}_{-2.18}$	$0.62^{+0.04}_{-0.04}$	$0.11^{+0.04}_{-0.04}$	$0.41^{+0.04}_{-0.03}$
Model ($z < 9.0$)	H_0	Ω_M	Ω_Λ	ω/ω_0	ω_a	m	k	σ_{ext}
Flat Λ CDM	$73.48^{+1.02}_{-1.01}$		$0.66^{+0.02}_{-0.02}$			$0.61^{+0.03}_{-0.03}$	$0.07^{+0.03}_{-0.03}$	$0.37^{+0.03}_{-0.02}$
Non-flat Λ CDM	$73.55^{+1.02}_{-0.99}$	$0.38^{+0.05}_{-0.05}$	$0.72^{+0.07}_{-0.07}$			$0.61^{+0.03}_{-0.03}$	$0.08^{+0.04}_{-0.04}$	$0.37^{+0.03}_{-0.02}$
Flat w CDM	$73.57^{+1.02}_{-1.00}$		$0.63^{+0.05}_{-0.05}$	$-1.10^{+0.15}_{-0.17}$		$0.61^{+0.03}_{-0.03}$	$0.08^{+0.03}_{-0.03}$	$0.37^{+0.03}_{-0.02}$
Flat $w_0 w_a$ CDM	$73.34^{+1.06}_{-1.03}$		$0.56^{+0.06}_{-0.04}$	$-1.03^{+0.15}_{-0.17}$	$-2.59^{+2.14}_{-1.65}$	$0.61^{+0.03}_{-0.03}$	$0.08^{+0.03}_{-0.03}$	$0.37^{+0.03}_{-0.02}$

Variation of Hubble constant and matter density parameter



Take home points

- Amati relation ($E_{i,p} \propto E_{\text{iso}}^m$) slope parameter, that is stable against cosmological model variations is $m = 0.62 \pm 0.04$.
- The unknown systematic uncertainty is quite large: $\sigma_{\text{ext}} \approx 0.4$, plausibly hindering variation of m with cosmology.
- No significant change in cosmological model parameters after including GRB data.

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Thank
you