

Revisiting Lorentz-factor constraints in blazars and gamma-ray bursts based on Polarization – dependent $\gamma\gamma$ absorption

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How the project connects: from polarization to $\delta_{\min}$

QUESTION • Does retaining the polarization dependence of $\gamma\gamma$ absorption change the inferred minimum Doppler factor ($\delta_{\min}$)?

1 MICROPHYSICS

$\sigma_{\perp}, \sigma_{\text{avg}}, \sigma_{\parallel}$

Polarization changes the pair-production probability

2 OPACITY

$\tau_{\parallel} < \tau_{\text{avg}} > \tau_{\perp}$

The difference survives photon-field integration

3 δ -CONSTRAINT

$\ell' \propto \delta^{-5}, \epsilon_{\gamma} \propto \delta^{-1}$

Solve $\tau_{\gamma\gamma}(\delta_{\min})=1$

4 RESULT

Bias = $100(\delta_{\text{avg}} - \delta_{\perp})/\delta_{\text{avg}}$

Quantify the correction

KEY EQUATIONS: $R' = c\delta t_{\text{var}} / (1+z) \bullet L' = L_{\text{obs}}/\delta^4 \bullet \epsilon'\gamma = (1+z)E_{\gamma,\text{obs}}/\delta \bullet \ell' \propto \delta^{-5} \bullet T_{\gamma\gamma}(\delta_{\min} = 1)$

Story: collision $\rightarrow$ opacity $\rightarrow$ polarization $\rightarrow$ validation $\rightarrow$ δ -dependent opacity $\rightarrow$ transparency boundary $\rightarrow$ bias $\rightarrow$ source-grid result

Cross Section

$$B = \sqrt{1 - \frac{1}{x}}$$

Average

$$\sigma^{avg}_{\gamma\gamma} = \frac{3}{16} \sigma_T (1 - \beta^2) \times \{(3 - \beta^4) \ln \left(\frac{1 + \beta}{1 - \beta} \right) - 2\beta(2 - \beta^2)\} \quad (1)$$

$$\sigma^{avg}_{\gamma\gamma} = \frac{\sigma^{avg}_{\gamma\gamma} + \sigma^{\perp}_{\gamma\gamma}}{2} \quad (2)$$

Parallel

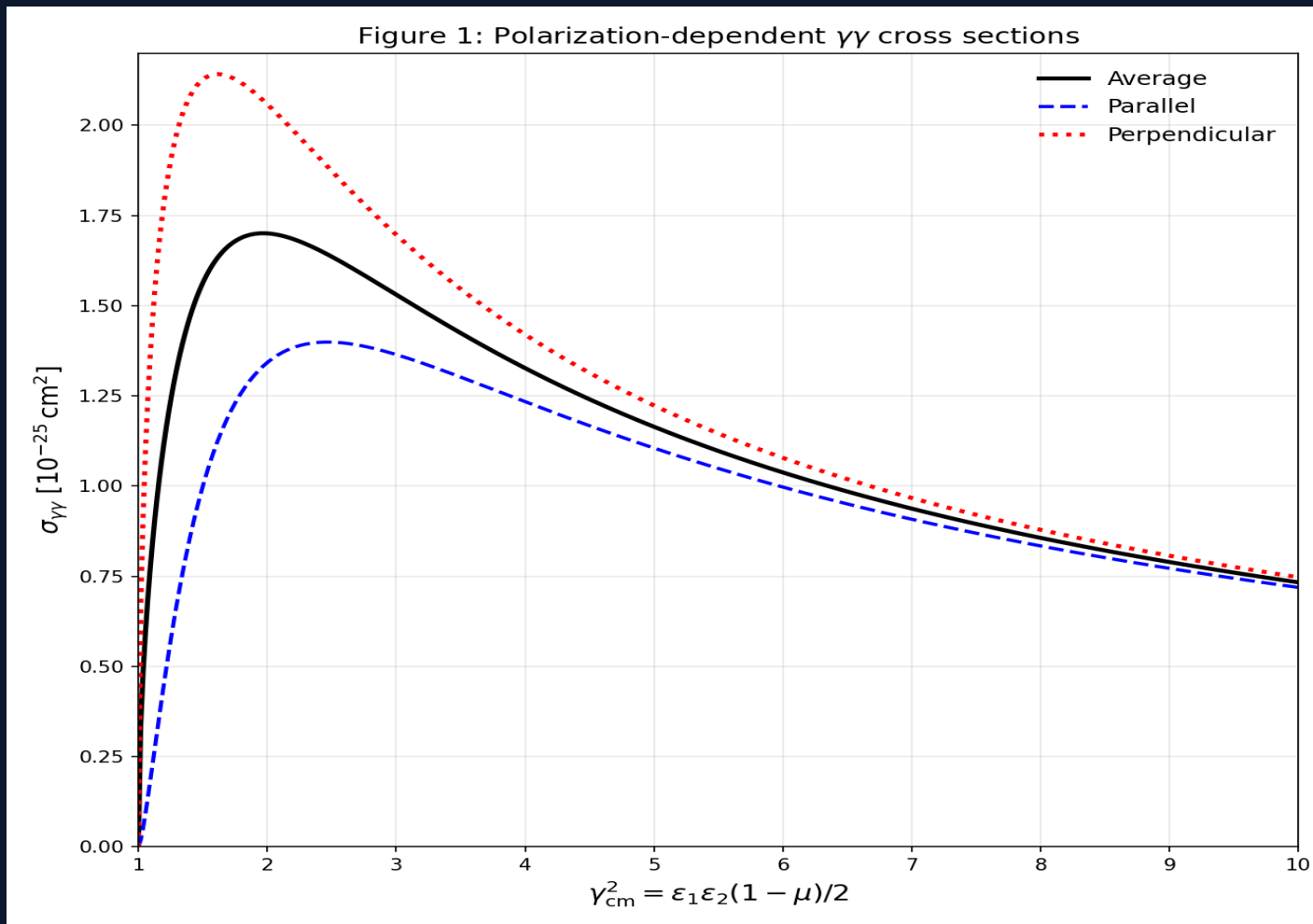
$$\sigma^{\parallel}_{\gamma\gamma} = \frac{3}{16} \sigma_T (1 - \beta^2) \times \left\{ \frac{7 - 2\beta^2 - \beta^4}{2} \ln \left(\frac{1 + \beta}{1 - \beta} \right) - \beta(5 - 3\beta^2) \right\} \quad (3)$$

Perpendicular

$$\sigma^{\perp}_{\gamma\gamma} = \frac{3}{16} \sigma_T (1 - \beta^2) \times \left\{ \frac{7 - 2\beta^2 - \beta^4}{2} \ln \left(\frac{1 + \beta}{1 - \beta} \right) - \beta(3 - \beta^2) \right\} \quad (4)$$

$x \downarrow \beta \downarrow$ cross section $\downarrow$ probability of $\gamma\gamma$ absorption

STEP 1 — THE COLLISION: polarization changes the $\gamma\gamma$ cross section



READ THE AXES

$$x: \frac{\epsilon_\gamma \epsilon_t (1 - \mu)}{2}$$

$$y: \sigma_{\gamma\gamma} \text{ in } 10^{-25} \text{ cm}^2$$

MAIN RESULT

$$\sigma_{\perp} > \sigma_{\text{avg}} > \sigma_{\parallel}$$

WHY THIS STARTS THE STORY

The two polarization states do not interact equally: in this regime, $\sigma_{\perp}$ is larger than σ_{avg} , so the perpendicular state is more strongly absorbed.

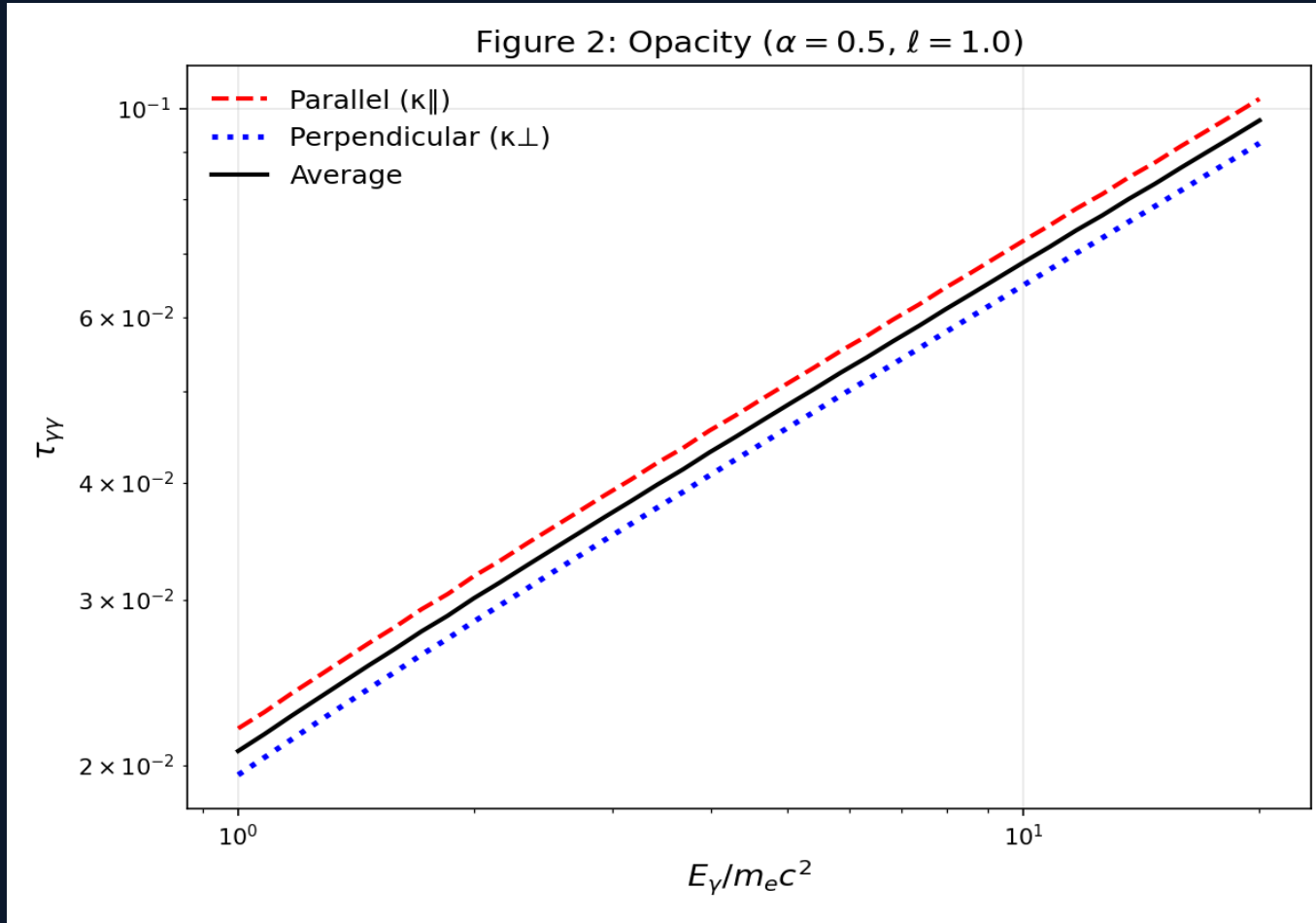
But one collision is not yet an astrophysical opacity. Does this difference survive the photon-field integration?"

$$\Pi = \frac{(\alpha+1)}{\alpha+\frac{5}{3}} \quad (5)$$

$$k_v^\perp = \int_0^\infty d\epsilon \int_{4\pi} d\Omega (1 - \mu) \{ \sigma_{\gamma\gamma}^\parallel (n^\perp[\epsilon, \Omega] \cos^2 + n^\parallel[\epsilon, \Omega] \sin^2) + \sigma_{\gamma\gamma}^\perp (n^\perp[\epsilon, \Omega] \sin^2 + n^\parallel[\epsilon, \Omega] \sin^2) \} \quad (6)$$

$$k_v^\parallel = \int_0^\infty d\epsilon \int_{4\pi} d\Omega (1 - \mu) \{ \sigma_{\gamma\gamma}^\parallel (n^\parallel[\epsilon, \Omega] \cos^2 + n^\perp[\epsilon, \Omega] \sin^2) + \sigma_{\gamma\gamma}^\perp (n^\perp[\epsilon, \Omega] \sin^2 + n^\parallel[\epsilon, \Omega] \sin^2) \} \quad (7)$$

STEP 2 — THE SOURCE: the cross-section difference becomes opacity



WHAT IS PLOTTED

$\tau_{\gamma\gamma}$ versus γ -ray energy, for $\alpha=0.5$ and $\ell=1$.

KEY ORDERING

$$T_{||} > \tau_{\text{avg}} > \tau_{\perp}$$

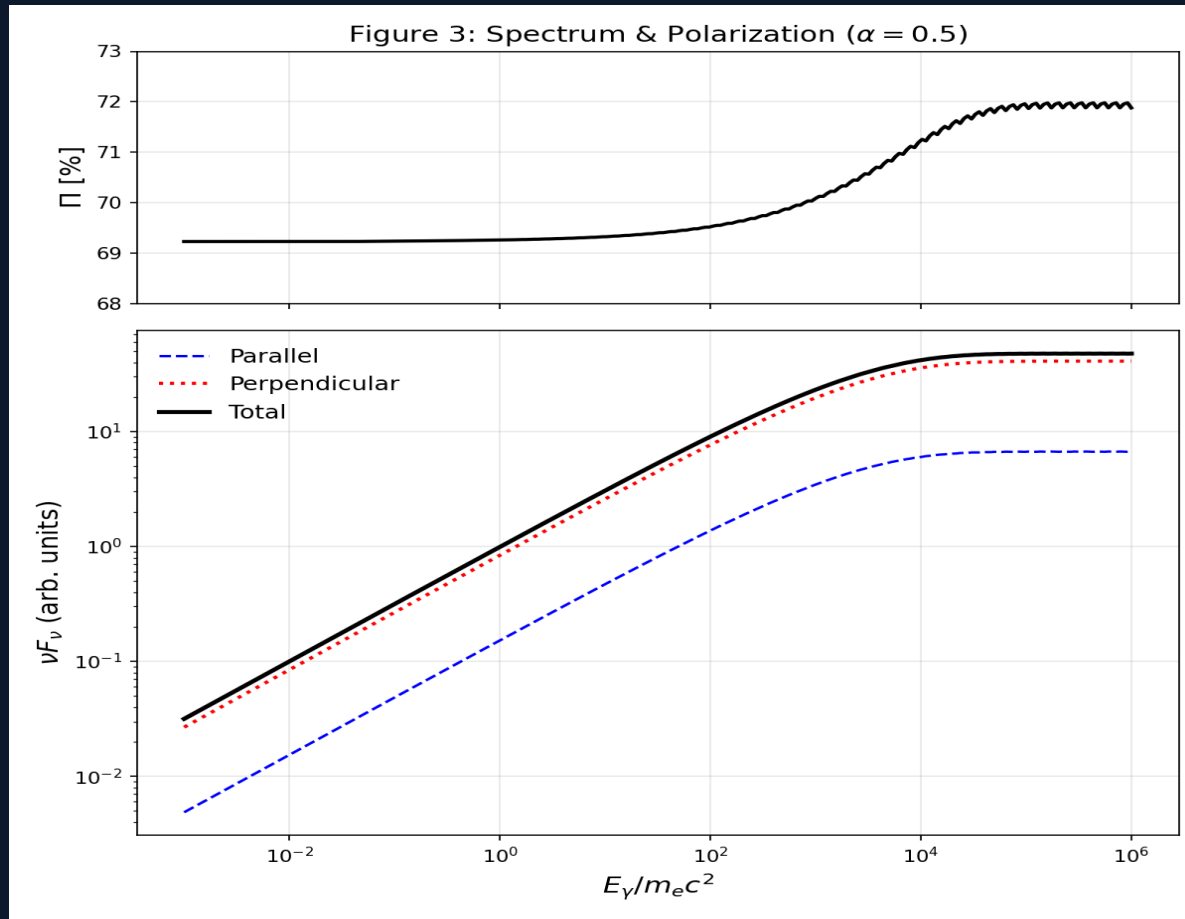
THE BRIDGE TO ASTROPHYSICS

The microscopic difference survives the integration over target photons and appears as different source transparency.

$$\Pi = f_B \frac{(\alpha+1)}{(\alpha+\frac{5}{3})} \quad (8)$$

If one polarization is preferentially absorbed, then the radiation that escapes should also change polarization.”

STEP 3 — THE OBSERVABLE: selective absorption changes polarization



TOP PANEL

Observed polarization rises from $\approx 69.2\%$ to $\approx 72\%$ as energy increases.

BOTTOM PANEL

The parallel component is absorbed more strongly; the surviving flux becomes increasingly dominated by the perpendicular component.

THE SURPRISING CONSEQUENCE

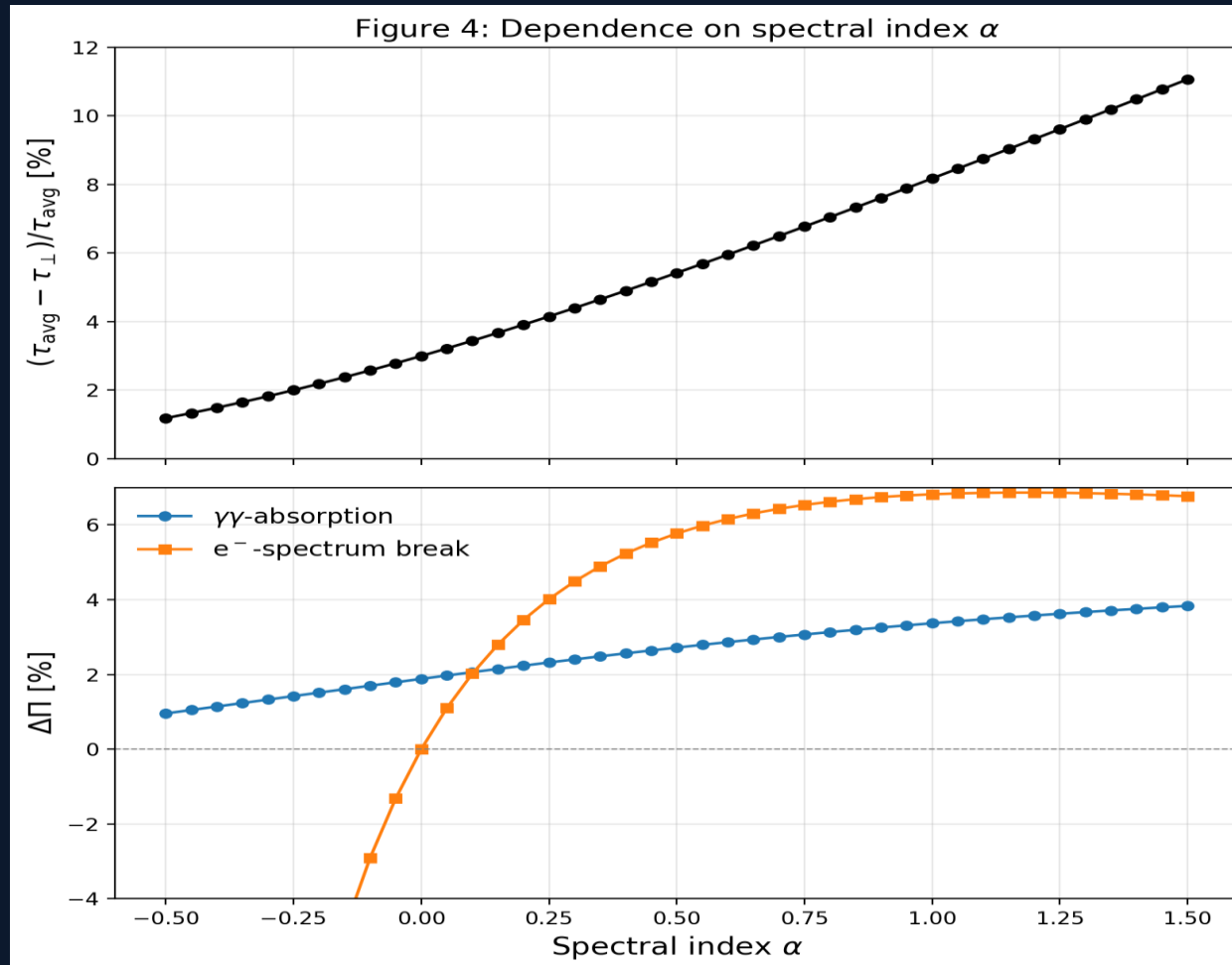
Absorption does not only dim the source—it changes its polarization.

$$F_{\nu}^{obs} = F_{\nu}^{int} \frac{1 - e^{-\tau}}{\tau_{\gamma\gamma}} \quad (9)$$

$$\Pi = \frac{F_{\perp} - F_{\parallel}}{F_{\perp} + F_{\parallel}} \quad (10)$$

Before using this machinery in a new Doppler-factor calculation, we need to check that the numerical implementation reproduces the established result.

STEP 4 — TRUST THE CALCULATION: validate before extending it



At $\alpha = 0.5$

$\gamma\gamma$ absorption: $\Delta \Pi_{\gamma\gamma} \approx 2.72\%$ | published $\approx 2.7\%$

ELECTRON-SPECTRUM BREAK

$\Delta \Pi_e \approx 5.77\%$ | published $\approx 5.8\%$

A break in the gamma-ray spectrum can have two different causes. Either $\gamma\gamma$ absorption removes photons on the way out, or the electrons themselves have a break, so the source emits a steeper spectrum. In both cases the spectrum steepens by the same amount, from $\alpha = 0.5$ to 1, so the spectrum alone can't tell them apart. But the polarization changes differently. For an electron break, it changes by about 5.8%. For absorption, only about 2.7%. My code gives 2.72% and 5.77%, matching the published values, so it is validated before I extend it

we can now make the opacity explicitly depend on δ

STEP 5 — THE NEW QUESTION: what happens when opacity depends on δ ?

This is the core extension. We convert the observed source quantities into the comoving frame using δ and redshift

SOURCE SIZE

$$R' = c \delta t_{\text{var}} / (1+z)$$

COMOVING LUMINOSITY

$$L' = L_{\text{obs}} / \delta^4$$

COMOVING γ -RAY ENERGY

$$\varepsilon_{\gamma} = (1+z) E_{\gamma, \text{obs}} / \delta$$

Therefore

$$\ell' \propto \delta^{-5} \quad \text{and} \quad \varepsilon_{\gamma} \propto \delta^{-1}$$

Higher $\delta \rightarrow$ lower photon compactness + lower comoving γ -ray energy $\rightarrow$ lower $\tau_{\gamma\gamma}$

If increasing δ makes the source more transparent, there must be a boundary that defines the minimum δ .”

STEP 6 — THE THRESHOLD: where does the gamma ray escape?

We define the minimum Doppler factor at the transparency boundary $\tau_{\gamma\gamma}(\delta_{\min})=1$. Below it the model is too opaque for the chosen gamma-ray energy; above it the source becomes progressively more transparent

$$\tau_{\gamma\gamma}(\delta_{\min}) = 1$$

$$\tau < 1$$

Mostly transparent

$$\tau = 1$$

Boundary $\rightarrow \delta_{\min}$

$$\tau > 1$$

Mostly opaque

The solver searches for the δ at which the gamma ray is just able to escape.

Now we have a conventional constraint and a polarization-dependent constraint. We need a single comparison number

STEP 7 — THE COMPARISON: how large is the polarization correction?

$$\text{Bias} = 100 \times (\delta_{\text{avg}} - \delta_{\perp}) / \delta_{\text{avg}}$$

CONVENTIONAL

δ_{avg} uses polarization-averaged opacity.

POLARIZATION-DEPENDENT

$\delta_{\perp}$ uses the perpendicular opacity.

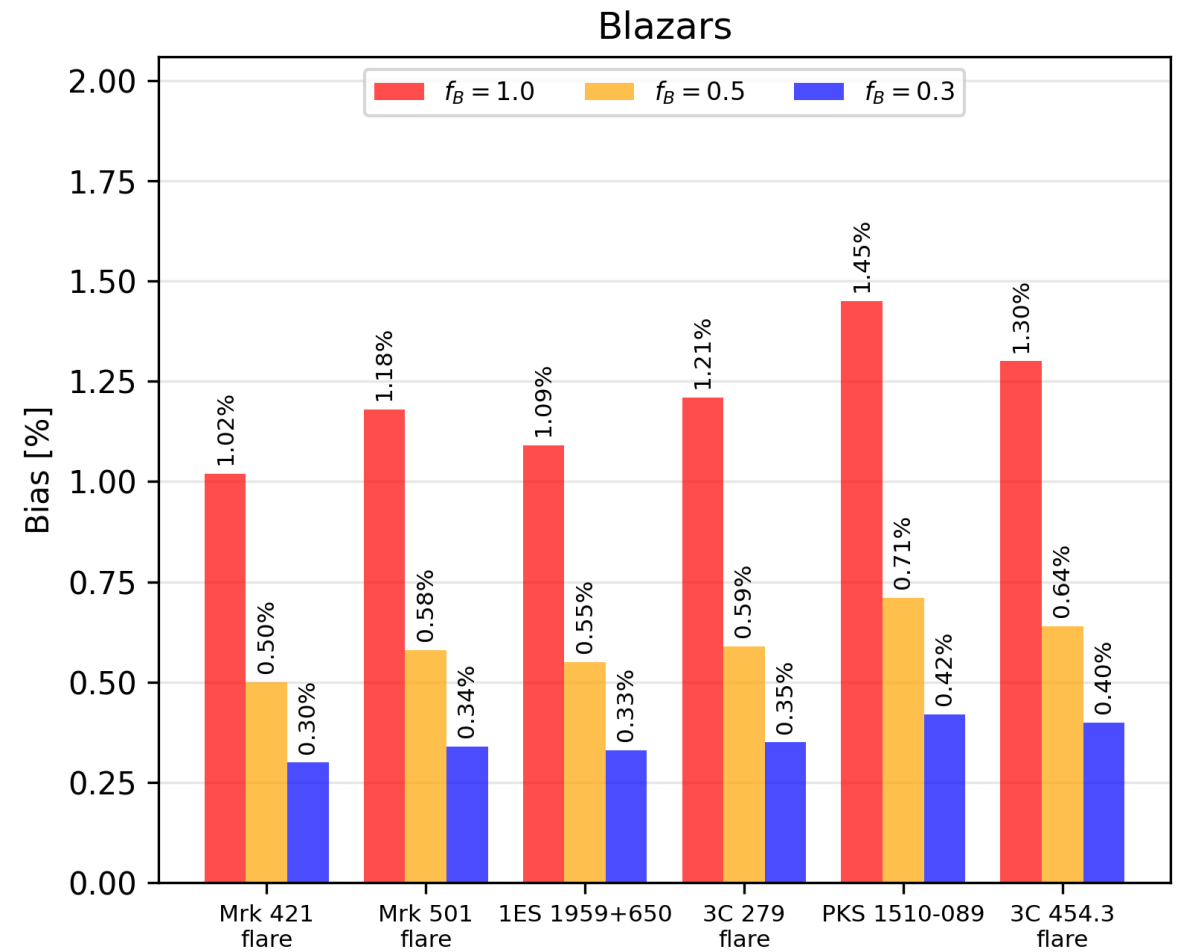
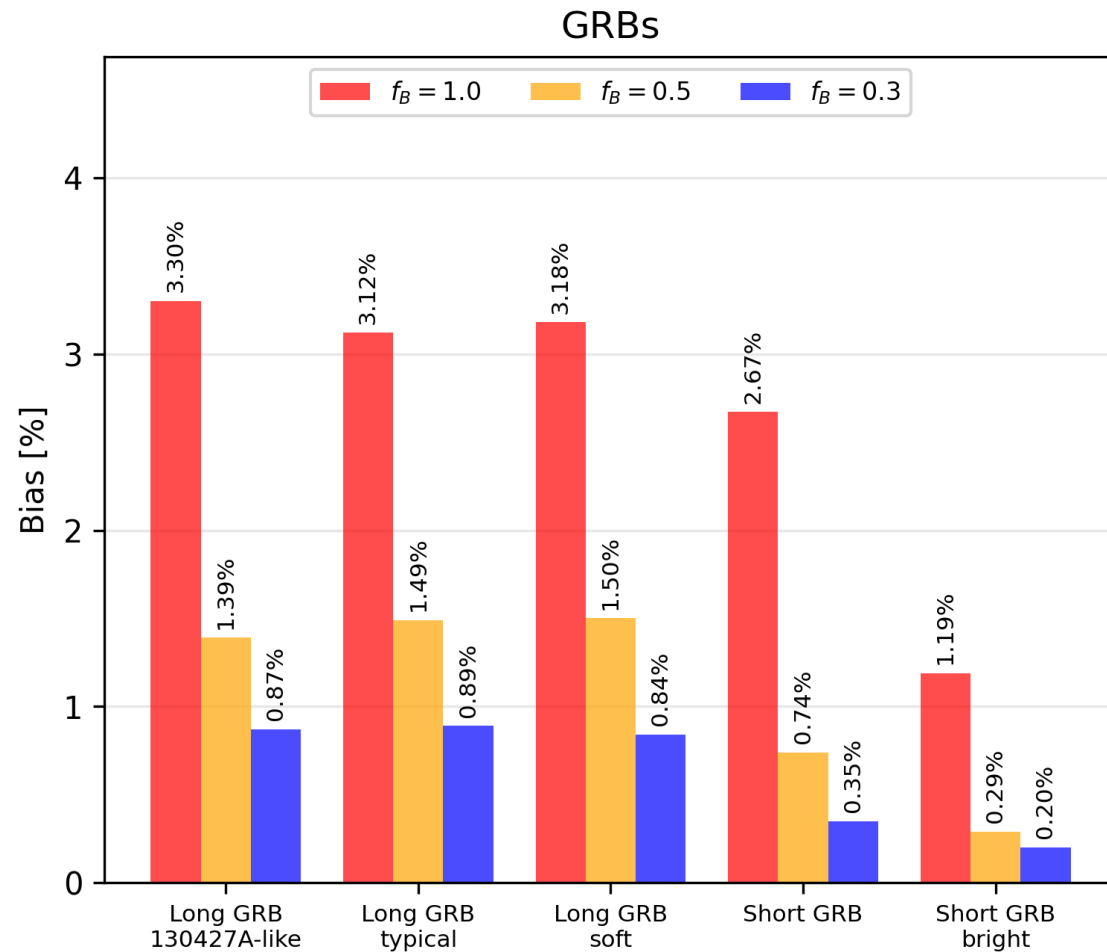
WHAT THE NUMBER MEANS

Bias measures how much the inferred δ changes when polarization is retained.

Example: $\delta_{\text{avg}}=100$, $\delta_{\perp}=97 \rightarrow \text{Bias} = 3\%$

The final question is how this correction behaves for representative blazars and GRBs

STEP 8 — THE RESULT: the correction depends on magnetic ordering



10 • CONTRIBUTION & CONCLUSION

- we implemented and validated the polarization-dependent $\gamma\gamma$ formalism.
- we made the compactness observable-based, so $\tau_{\gamma\gamma}$ becomes a function of δ .
- we solved $\tau_{\gamma\gamma}(\delta)=1$ to obtain polarization-dependent $\delta_{\min}$.
- The representative grid gives a percent-level correction that decreases as fB decreases.
- The implementation reproduces the published Böttcher benchmarks before the δ -dependent extension.

Bottom line: polarization produces a correction to the transparency constraint, not a replacement for the underlying $\gamma\gamma$ physics.

11. What's Next

- . Have a meeting with Prof after the talk

Thank you

APPENDIX

Minimum Doppler factors and bias (no cutoff)

Source	α	δ_{avg}	$f_B = 1.0$			$f_B = 0.5$		$f_B = 0.3$	
			$\delta_{\perp}$	bias [%]		$\delta_{\perp}$	bias [%]	$\delta_{\perp}$	bias [%]
HBL Mrk 421 flare	0.5	317.9	314.6	1.02		316.3	0.50	316.9	0.30
HBL Mrk 501 flare	0.7	184.0	181.9	1.18		183.0	0.58	183.4	0.34
HBL 1ES 1959+650	0.6	197.4	195.2	1.09		196.3	0.55	196.7	0.33
FSRQ 3C 279 flare	0.8	1408.5	1391.5	1.21		1400.2	0.59	1403.6	0.35
FSRQ PKS 1510-089	1.0	909.2	896.0	1.45		902.7	0.71	905.3	0.42
FSRQ 3C 454.3 flare	0.9	1512.2	1492.5	1.30		1502.5	0.64	1506.2	0.40
Long GRB 130427A-like	0.5	6833.1	6607.9	3.30		6738.3	1.39	6773.7	0.87
Long GRB typical	0.8	3892.3	3770.7	3.12		3834.2	1.49	3857.7	0.89
Long GRB soft	1.0	2019.0	1954.7	3.18		1988.8	1.50	2002.1	0.84
Short GRB	0.6	2543.7	2475.8	2.67		2524.9	0.74	2534.8	0.35
Short GRB bright	0.4	3690.9	3646.9	1.19		3680.0	0.29	3683.4	0.20

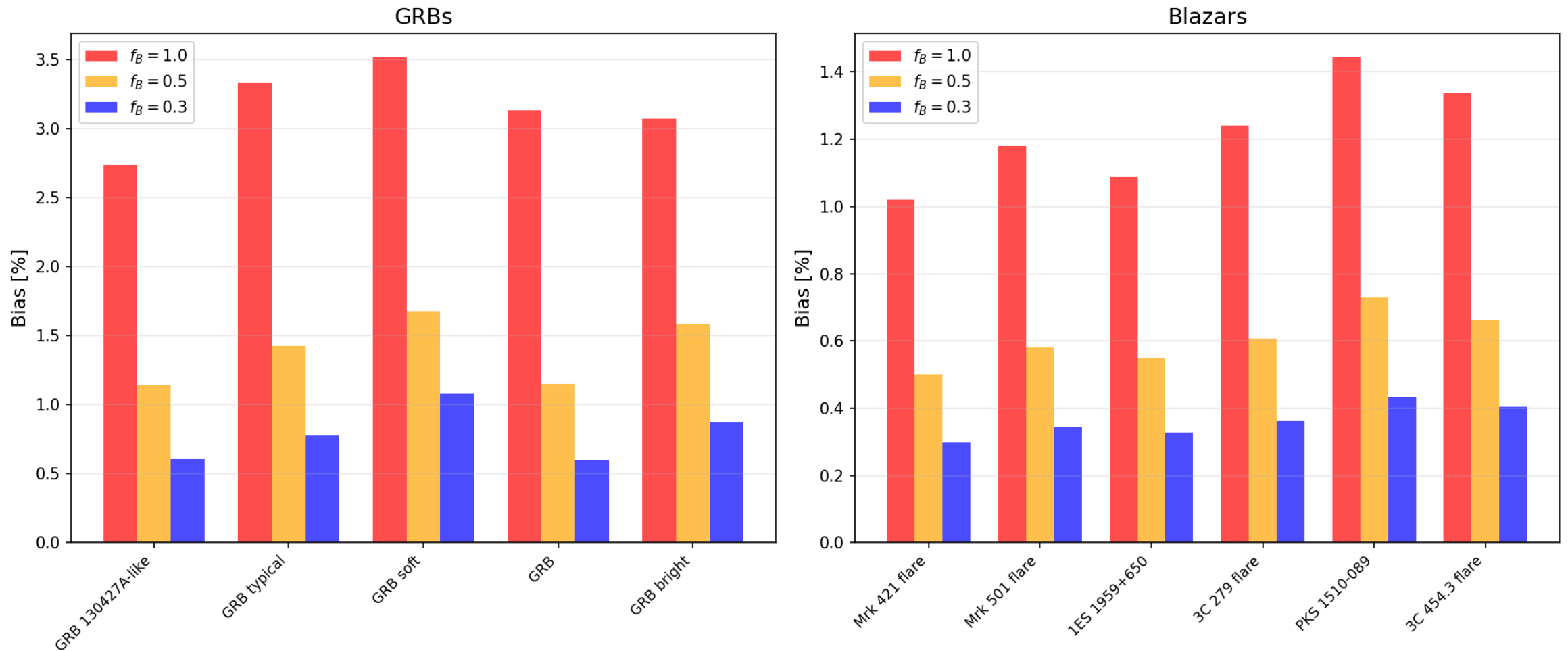
δ_{avg} (polarization-averaged opacity) does not depend on f_B , so it is listed once per source. Bias = $100 (\delta_{\text{avg}} - \delta_{\perp}) / \delta_{\text{avg}}$. α is the spectral index.

Minimum Doppler factors and bias ($\epsilon_{\text{cut}} = 1$)

Source	α	δ_{avg}	$f_B = 1.0$		$f_B = 0.5$		$f_B = 0.3$	
			$\delta_{\perp}$	bias [%]	$\delta_{\perp}$	bias [%]	$\delta_{\perp}$	bias [%]
HBL Mrk 421 flare	0.5	317.8	314.6	1.02	316.2	0.50	316.8	0.30
HBL Mrk 501 flare	0.7	184.0	181.8	1.18	182.9	0.58	183.4	0.34
HBL 1ES 1959+650	0.6	197.3	195.1	1.09	196.2	0.55	196.6	0.33
FSRQ 3C 279 flare	0.8	1402.4	1385.0	1.24	1393.8	0.61	1397.3	0.36
FSRQ PKS 1510-089	1.0	904.5	891.5	1.44	897.9	0.73	900.6	0.43
FSRQ 3C 454.3 flare	0.9	1487.9	1468.0	1.34	1478.0	0.66	1481.9	0.40
Long GRB 130427A-like	0.5	1815.3	1765.5	2.74	1794.4	1.15	1804.2	0.61
Long GRB typical	0.8	1128.9	1091.3	3.33	1112.8	1.43	1120.2	0.78
Long GRB soft	1.0	597.4	576.4	3.52	587.4	1.68	591.0	1.08
Short GRB	0.6	739.2	716.0	3.13	730.7	1.15	734.7	0.60
Short GRB bright	0.4	1088.7	1055.2	3.07	1071.4	1.59	1079.1	0.88

δ_{avg} (polarization-averaged opacity) does not depend on f_B , so it is listed once per source. Bias = $100 (\delta_{\text{avg}} - \delta_{\perp}) / \delta_{\text{avg}}$. α is the spectral index.

Cutoff test ($\epsilon_{\text{cut}} = 1$): bias by group



Cutoff test ($\epsilon_{\text{cut}} = 1$): bias by source

