CRISP

A python package for efficient UHECR stochastic analysis

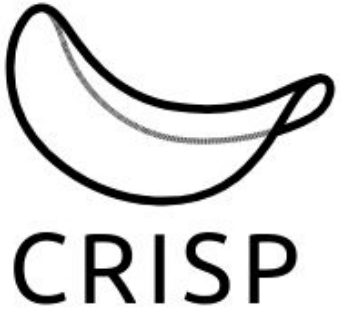
Leonel Morejon

Hands-on sessions
ACME

07.04.2026

ACME
Astrophysics Centre for Multimessenger studies in Europe

BERGISCHE UNIVERSITÄT WUPPERTAL



Cosmic **R**ay **S**tochastic **I**nteractions for **P**ropagation

https://github.com/mohller/crisp

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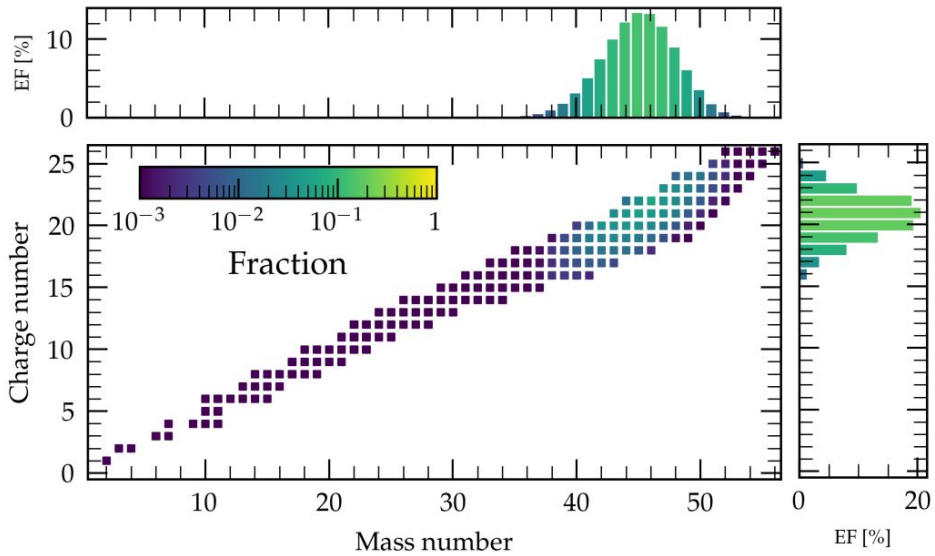
Describes **Nuclear Cascades** as a Markov Jump Process …



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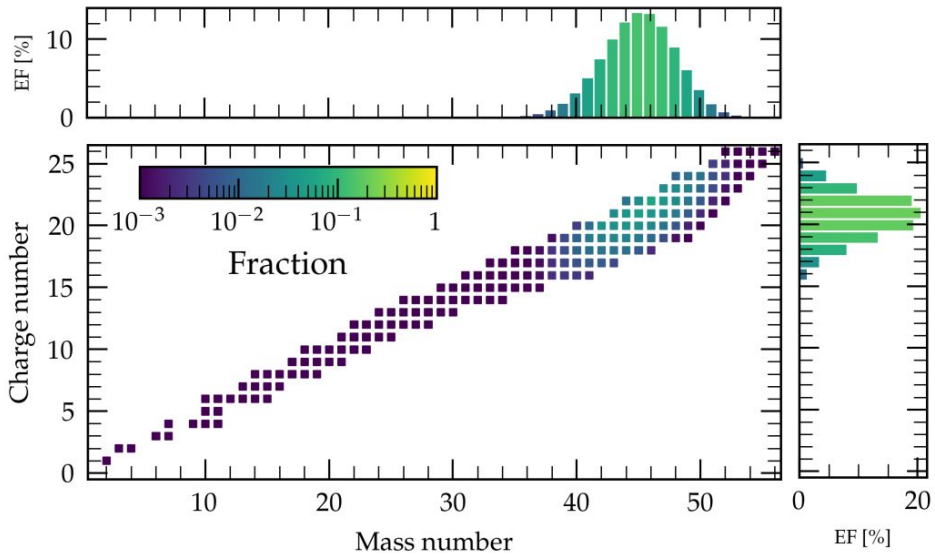
Describes **Nuclear Cascades** as a Markov Jump Process …

- **Probability distributions**



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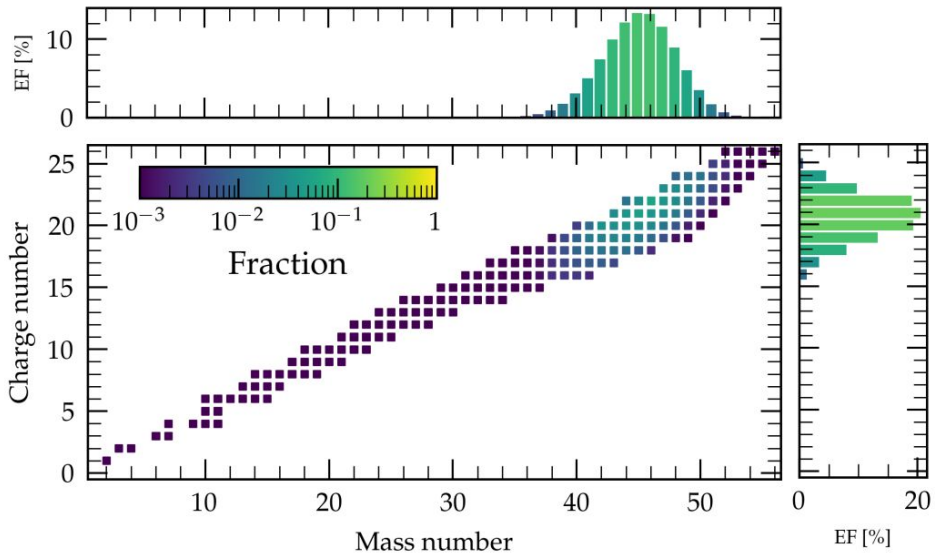
Describes **Nuclear Cascades** as a Markov Jump Process …

- **Probability distributions**
- **Analytic expressions**



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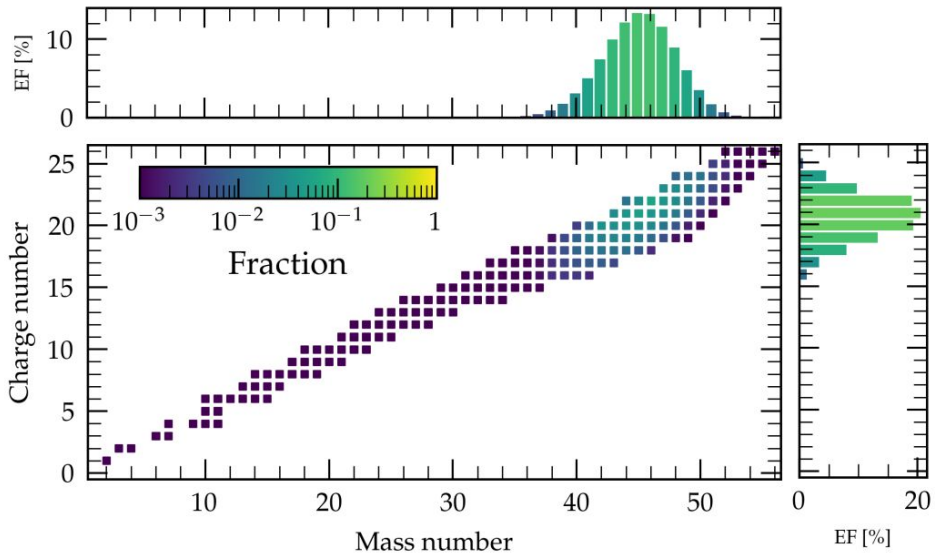
- **In-source & Propagation**



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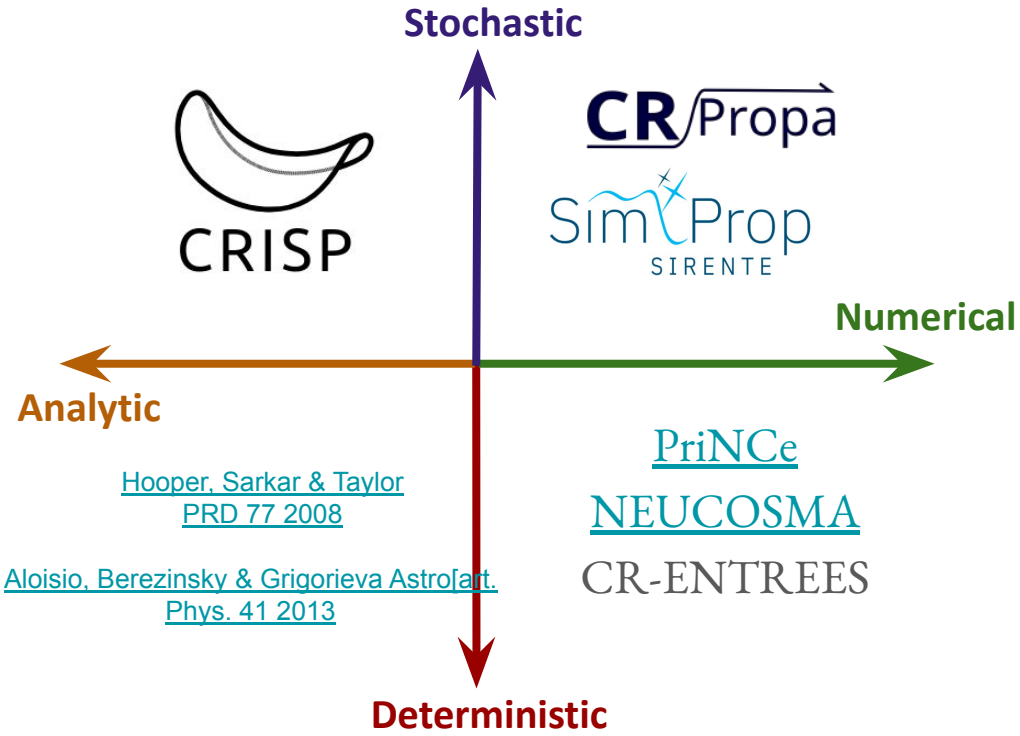
Comparison to other approaches



CRISP: Describes **Nuclear Cascades** as a Markov Jump Process …

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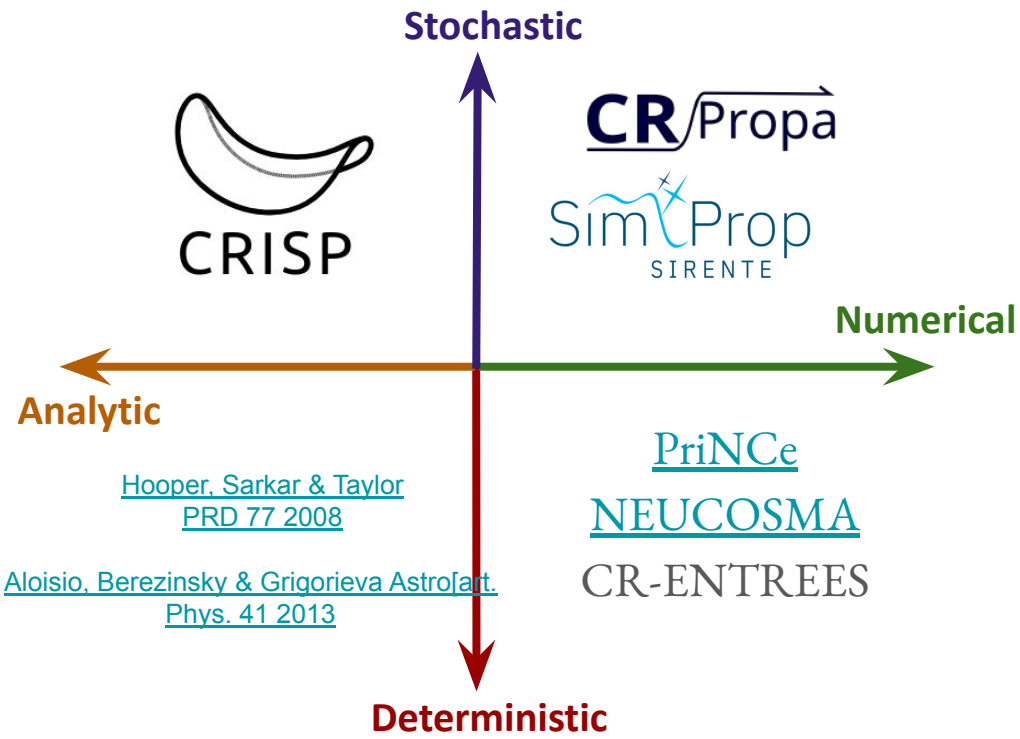
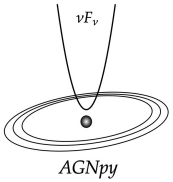
Comparison to other approaches

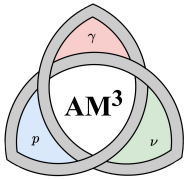


CRISP could be coupled to…

SOPRANO

OneHaLe

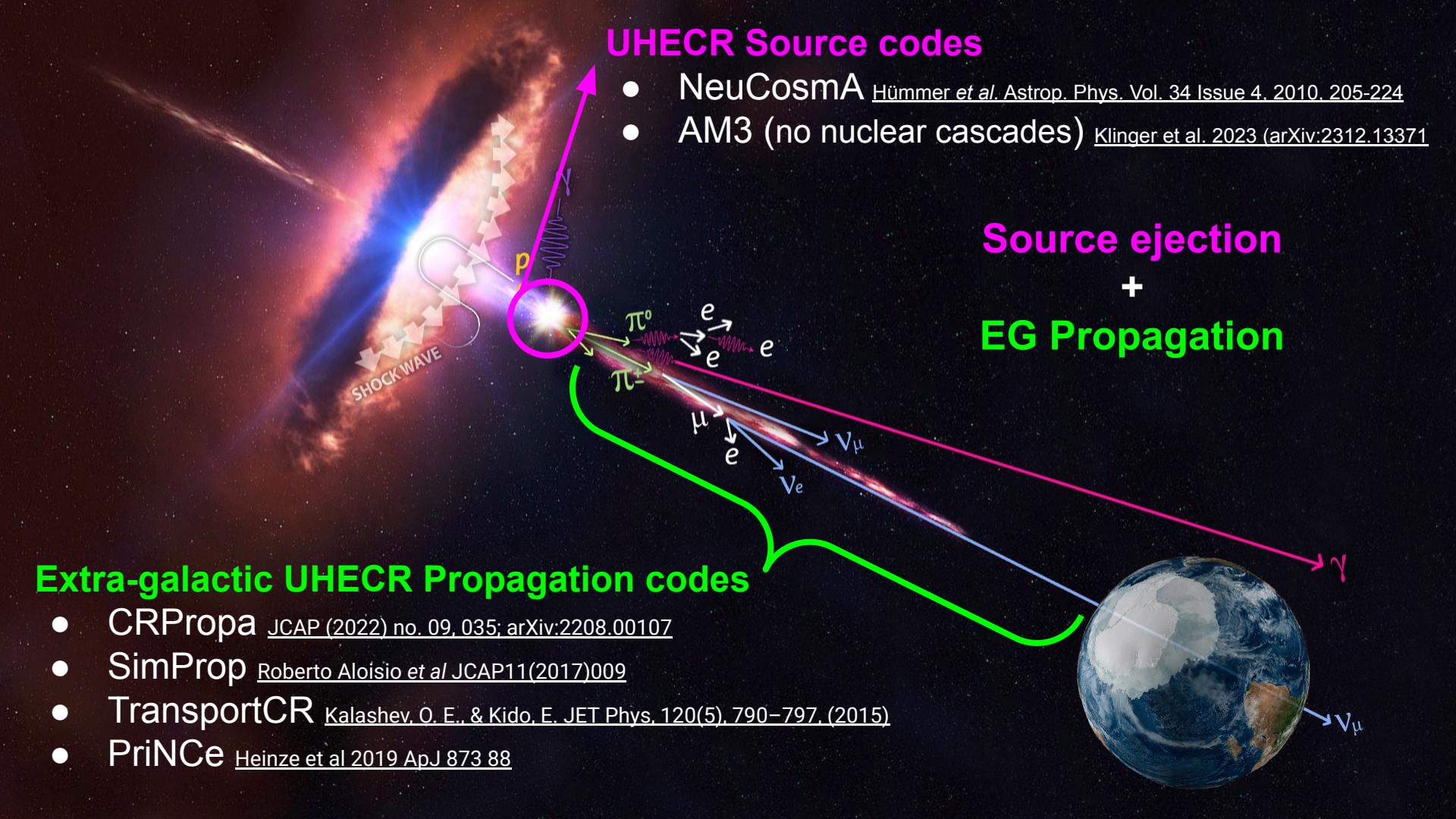




UHECR Source codes

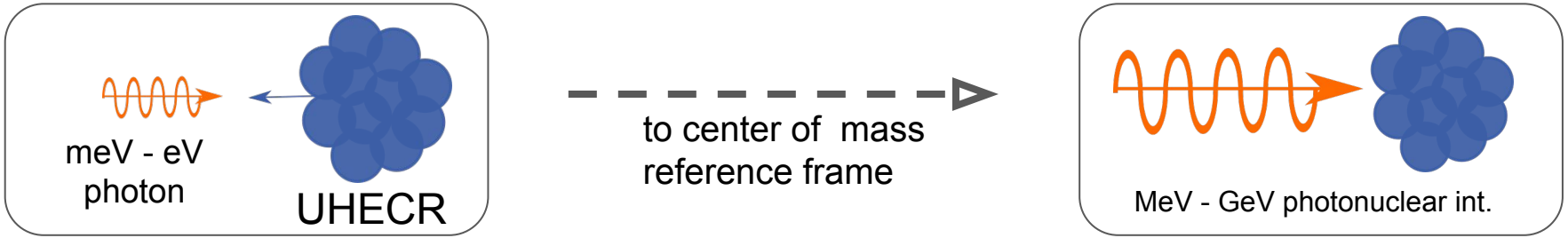
- NeuCosmA Hümmer *et al.* Astrop. Phys. Vol. 34 Issue 4, 2010, 205-224
- AM3 (no nuclear cascades) Klinger et al. 2023 (arXiv:2312.13371

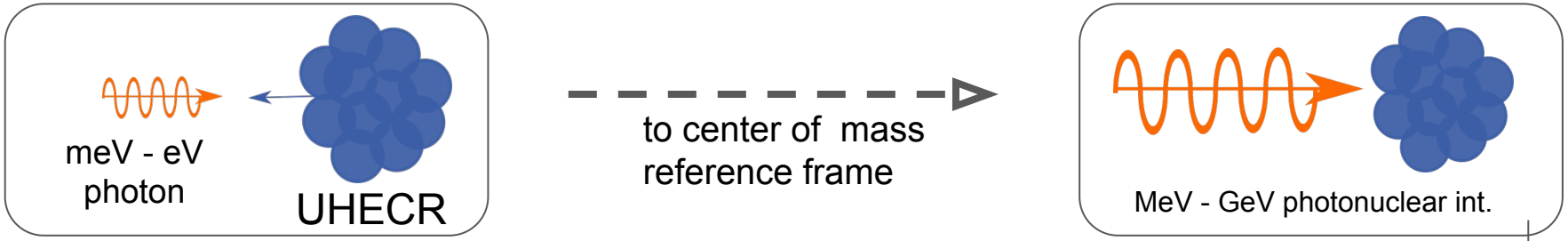
Source ejection
+
EG Propagation



Extra-galactic UHECR Propagation codes

- CRPropa JCAP (2022) no. 09, 035; arXiv:2208.00107
- SimProp Roberto Aloisio *et al* JCAP11(2017)009
- TransportCR Kalashev, O. E., & Kido, E. JET Phys, 120(5), 790–797, (2015)
- PriNCe Heinze et al 2019 ApJ 873 88


meV - eV
photon
UHECR
to center of mass
reference frame
MeV - GeV photonuclear int.



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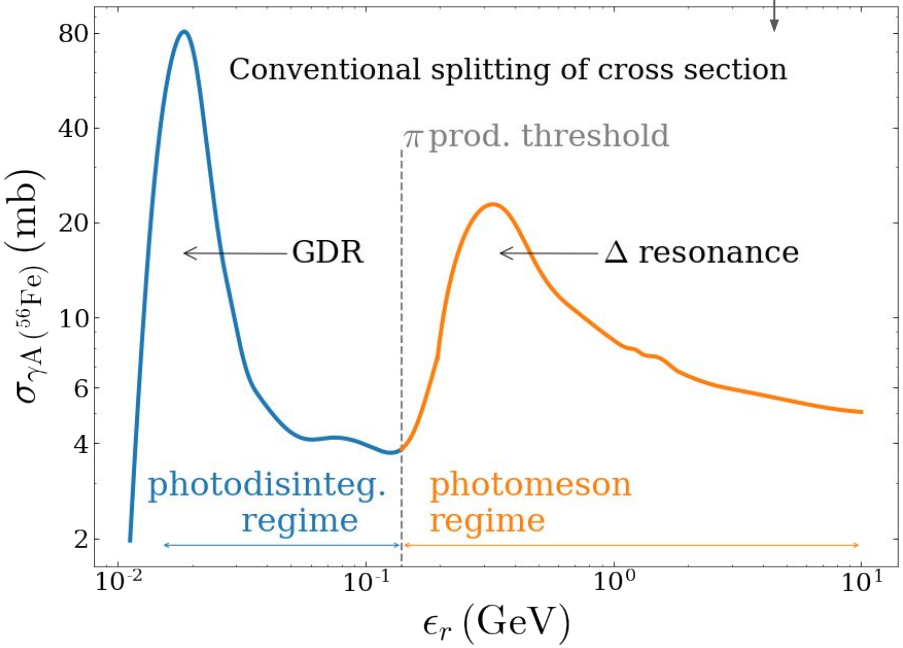
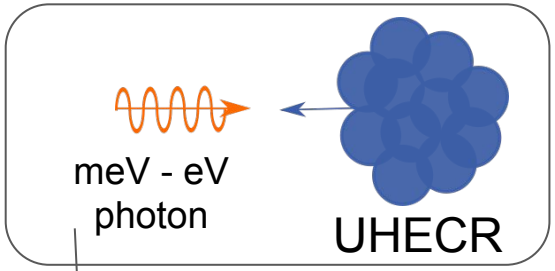
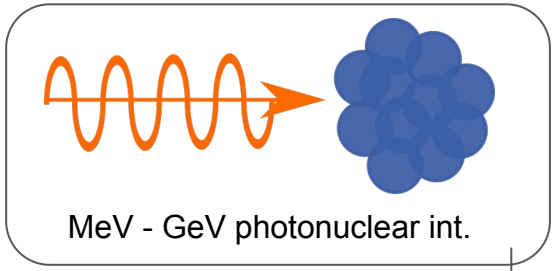
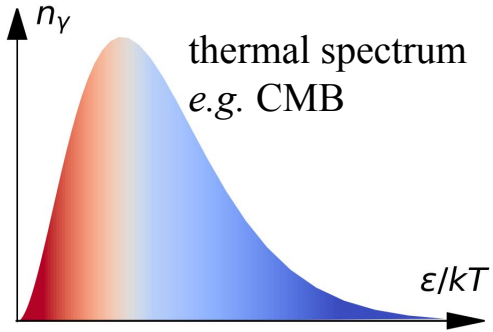




to center of mass reference frame



PROPAGATION



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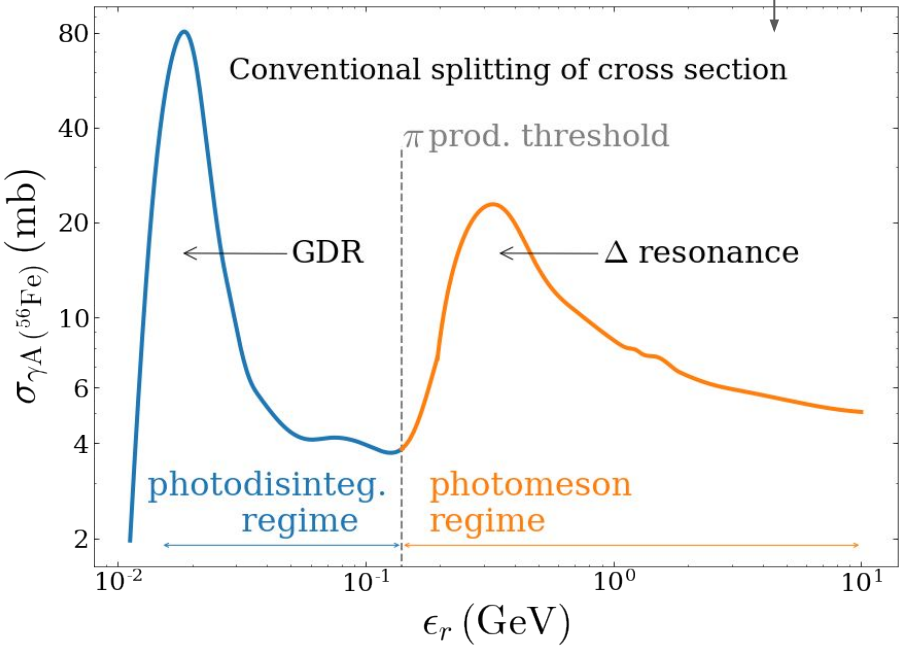

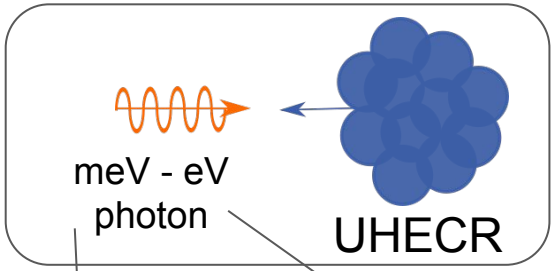

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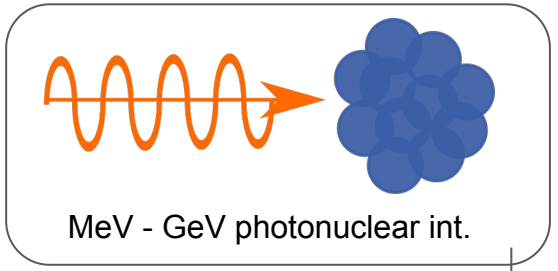

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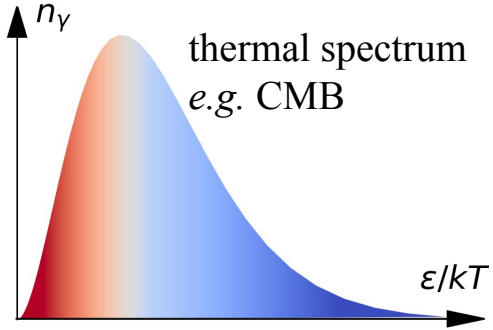
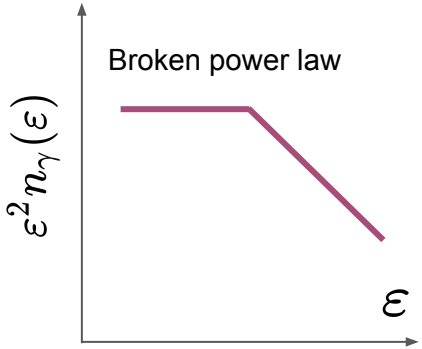
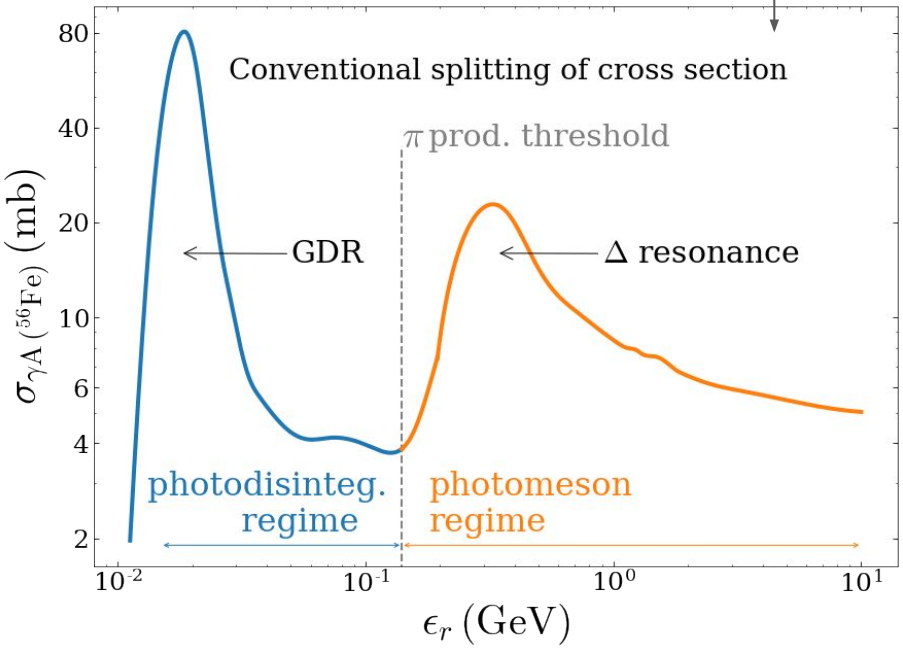
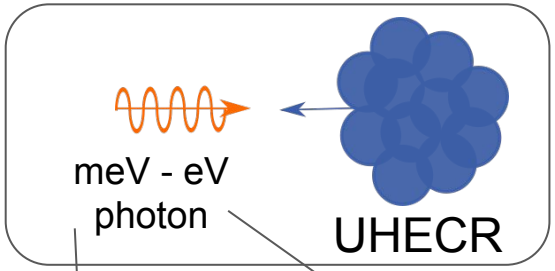


SOURCE







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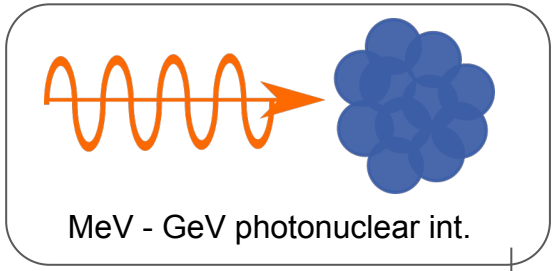
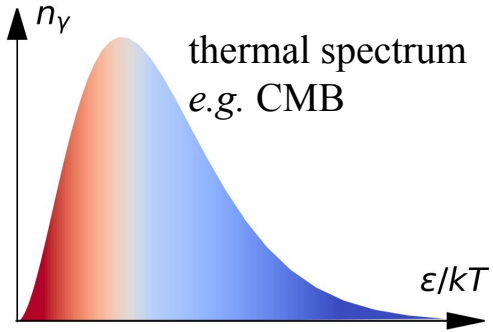
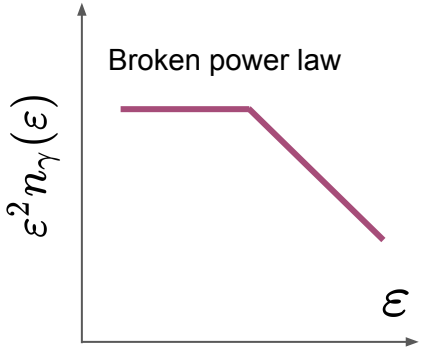


$$\lambda(\gamma) = \frac{1}{2\gamma^2}\int_0^\infty \frac{n(\epsilon)}{\epsilon^2} d\epsilon \int_0^{2\epsilon\gamma} \varepsilon\sigma(\varepsilon) d\varepsilon$$

PROPAGATION

SOURCE





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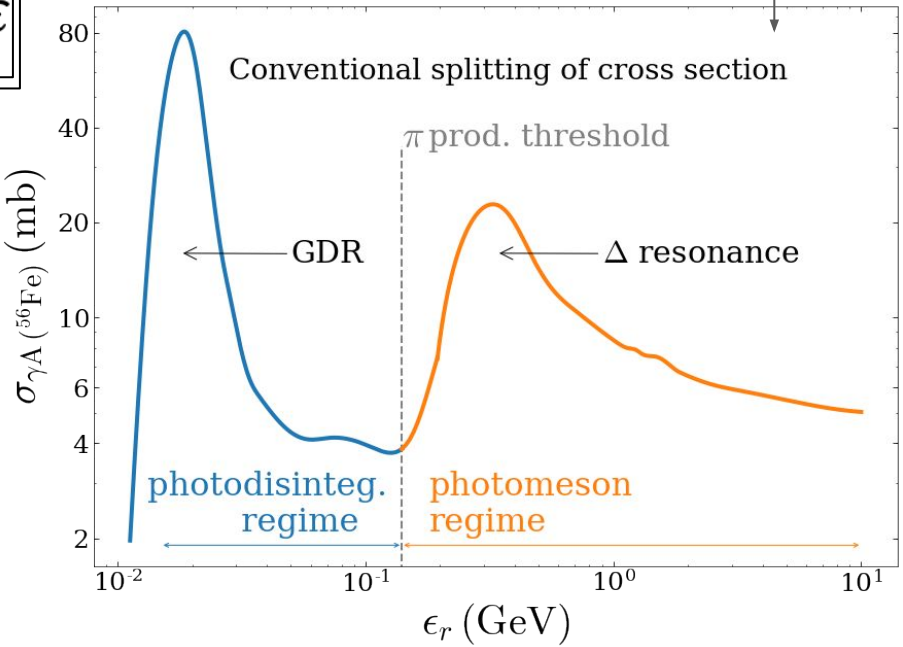
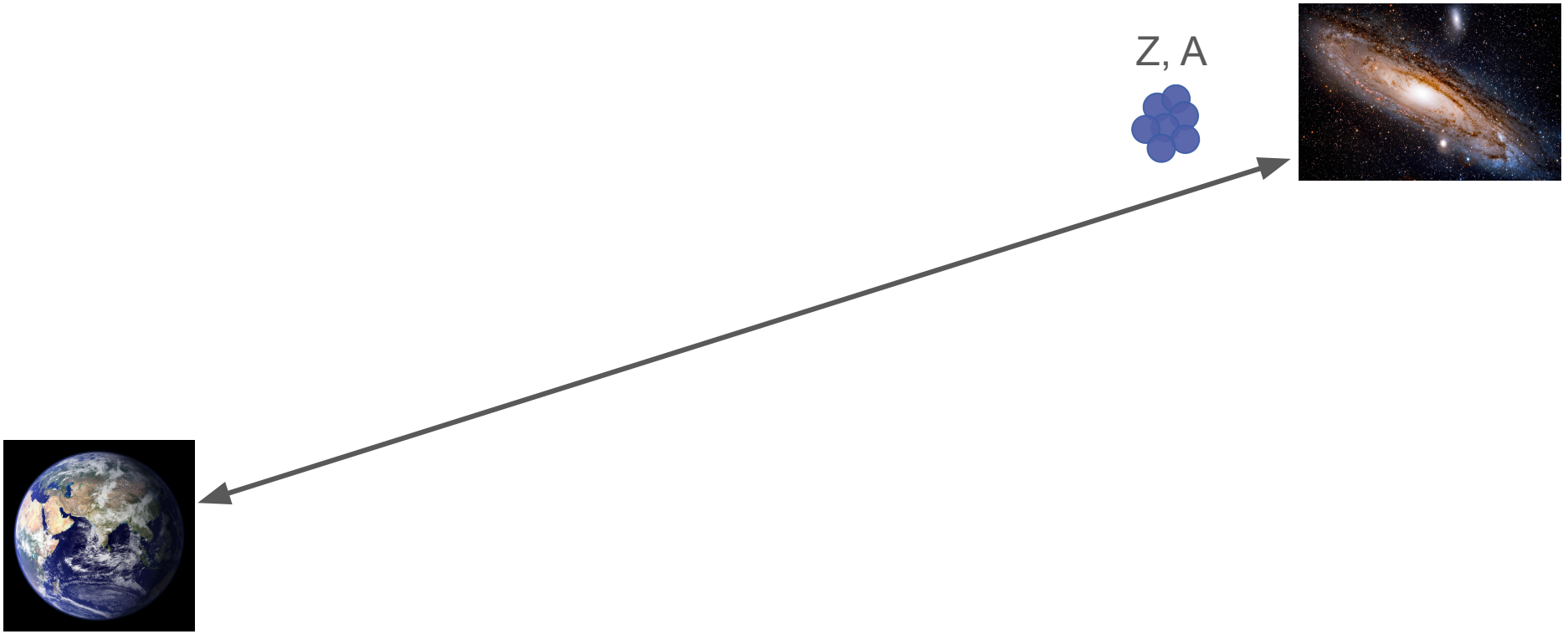
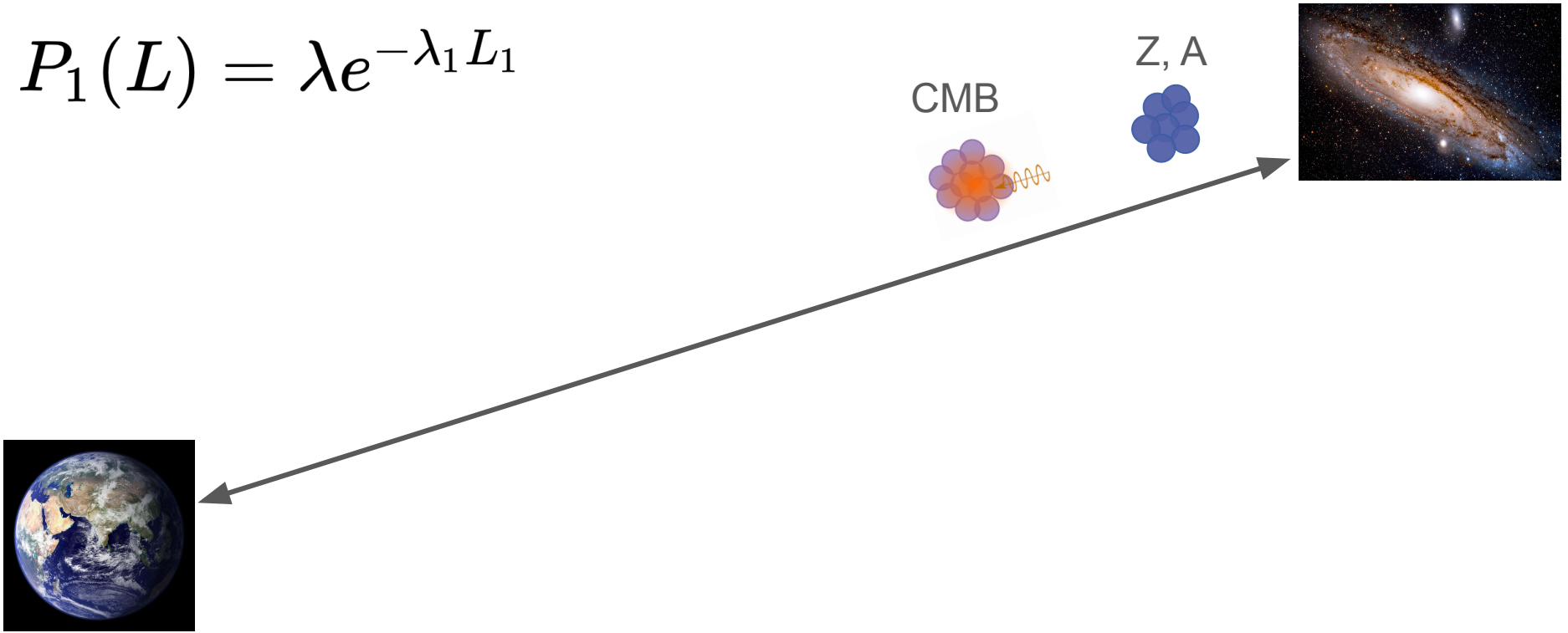


Nuclear interactions over cosmic distances



Nuclear interactions over cosmic distances

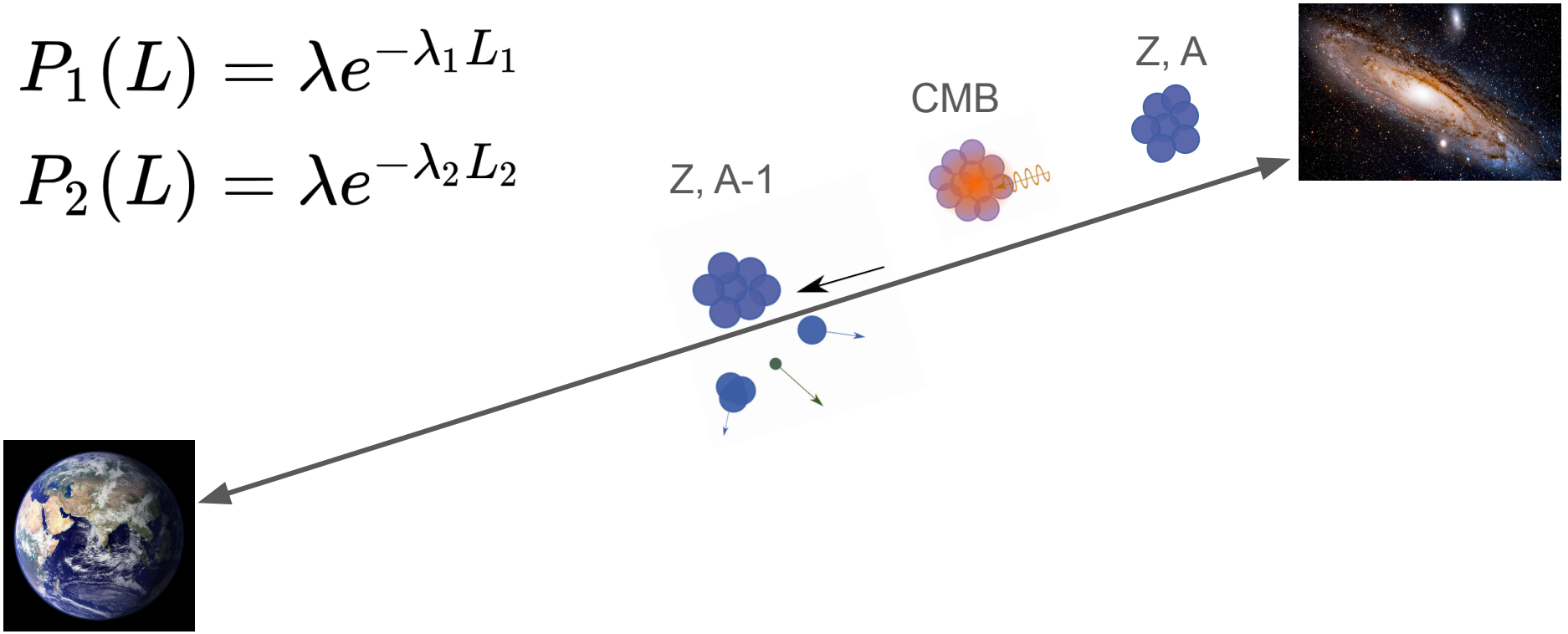
$$P_1(L) = \lambda e^{-\lambda_1 L_1}$$



Nuclear interactions over cosmic distances

$$P_1(L) = \lambda e^{-\lambda_1 L_1}$$

$$P_2(L) = \lambda e^{-\lambda_2 L_2}$$



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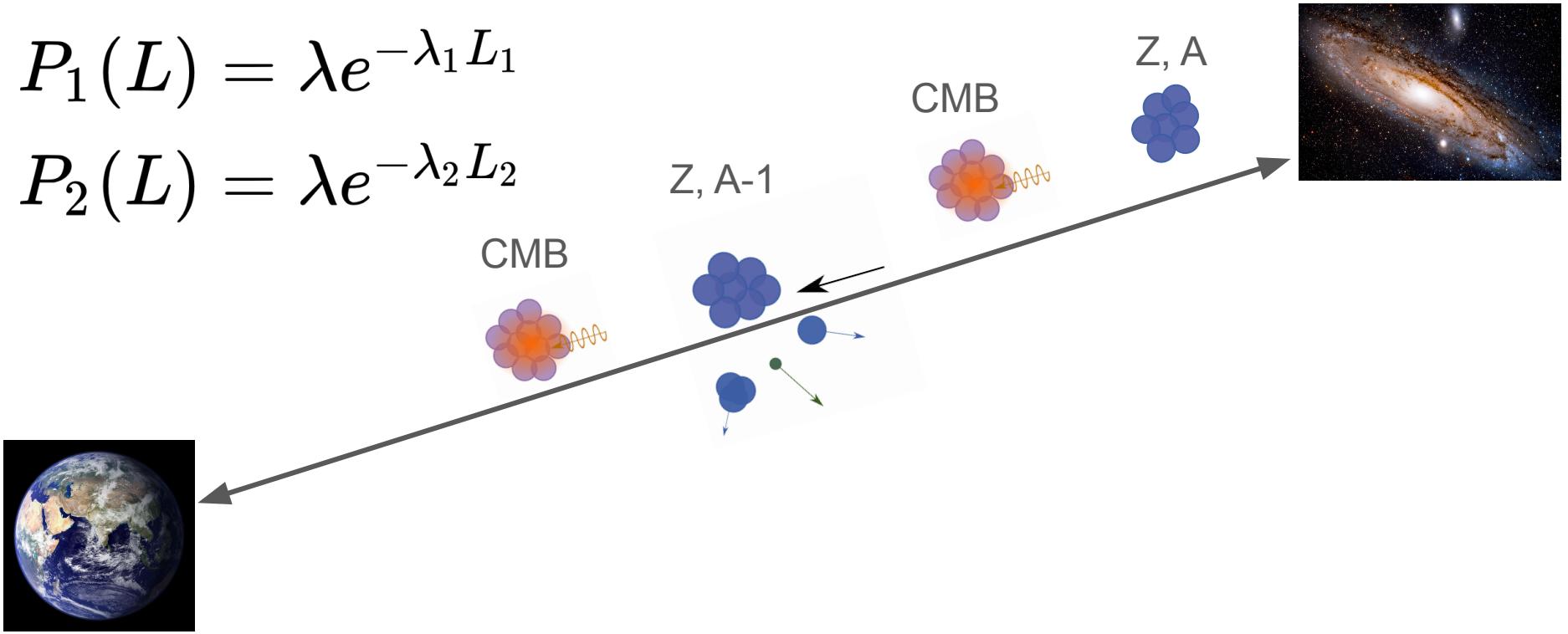

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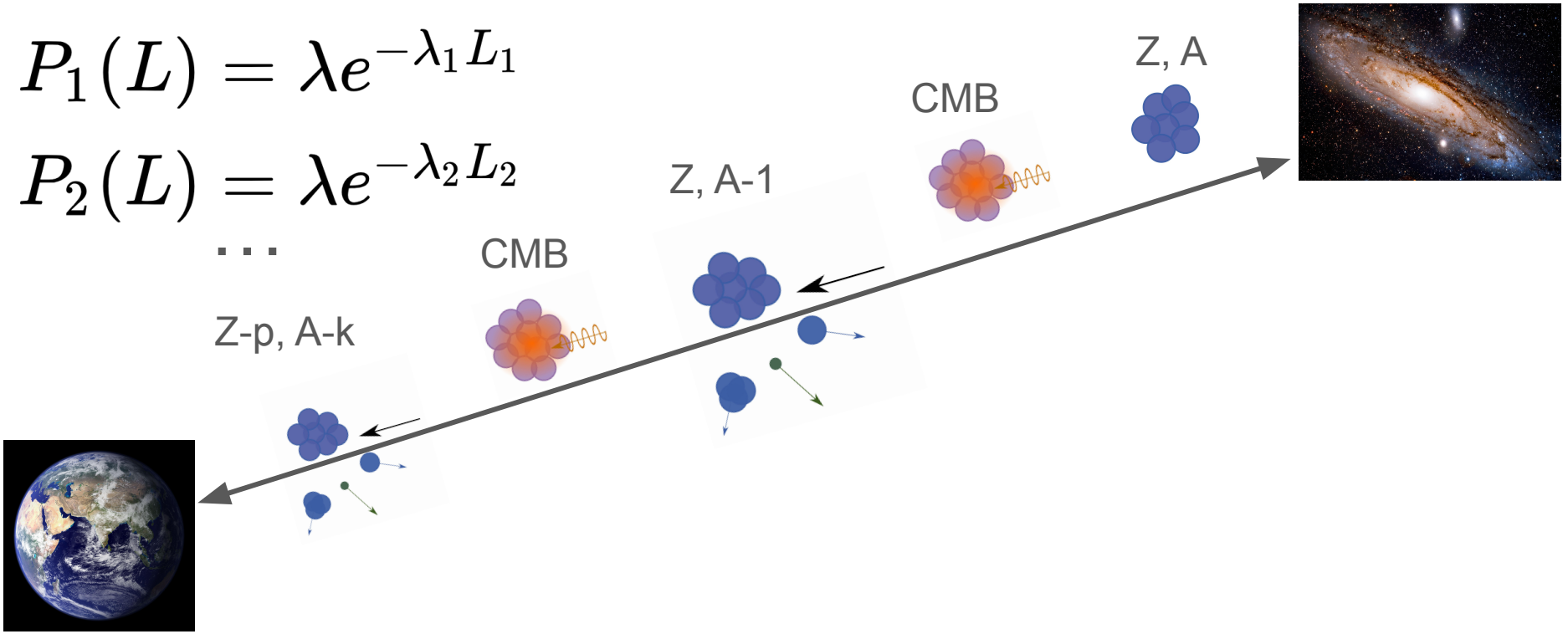

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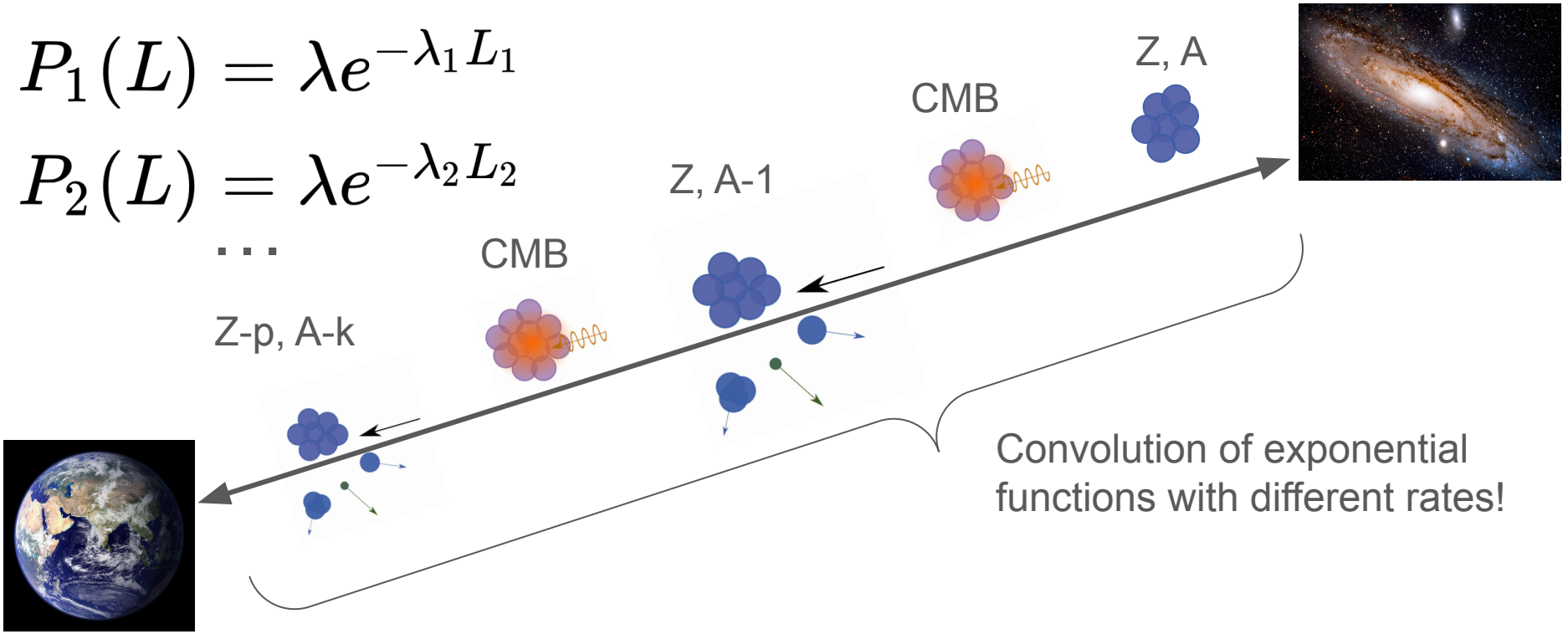

Convolution of exponential functions with different rates!

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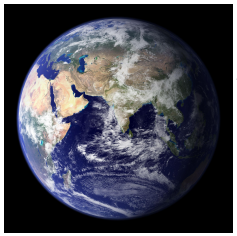

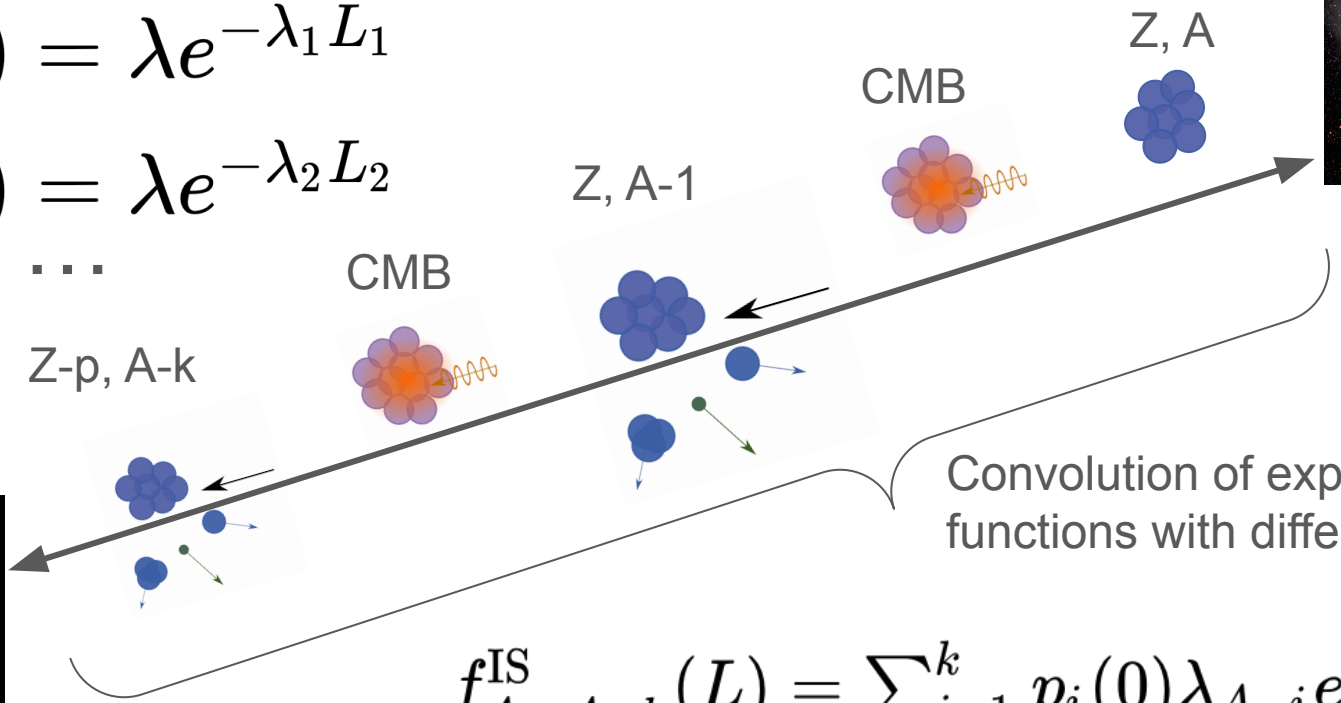

Convolution of exponential functions with different rates!

$$f^{\mathrm{IS}}_{A \to A-k}(L) = \sum_{i=1}^{k} p_i(0) \lambda_{A-i} e^{(-\lambda_{A-i} L)}$$

Markov Chain Description

Evolution of probability distribution vector

$$\pi(t) = \pi(0) e^{\mathbf{\Lambda}(\gamma) t}$$

$$\mathbf{\Lambda}(\gamma) = \begin{pmatrix} -\lambda_{S_1}^{\text{tot}} & \lambda_{S_1 \to S_2} & \lambda_{S_1 \to S_3} & \lambda_{S_1 \to S_4} & \lambda_{S_1 \to S_5} & \dots & \lambda_{S_1 \to S_N} \\ 0 & -\lambda_{S_2}^{\text{tot}} & \lambda_{S_2 \to S_2} & \lambda_{S_2 \to S_3} & \lambda_{S_2 \to S_4} & \dots & \lambda_{S_2 \to S_N} \\ 0 & 0 & -\lambda_{S_3}^{\text{tot}} & \lambda_{S_3 \to S_3} & \lambda_{S_3 \to S_3} & \dots & \lambda_{S_3 \to S_N} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & -\lambda_{S_N}^{\text{tot}} \end{pmatrix}$$

Evolution of probability vector

Initial probability vector

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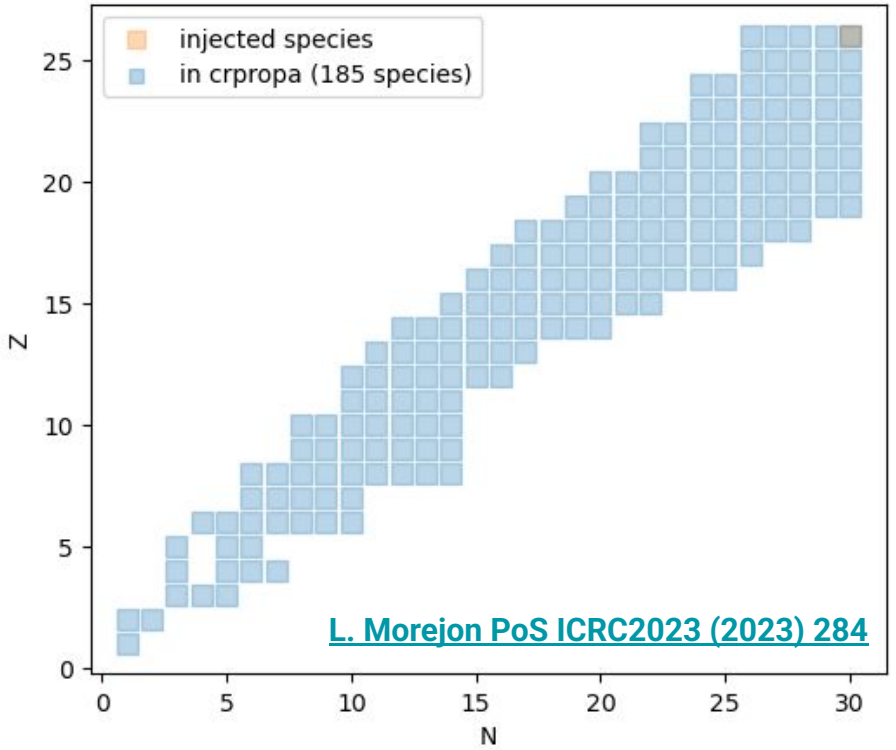

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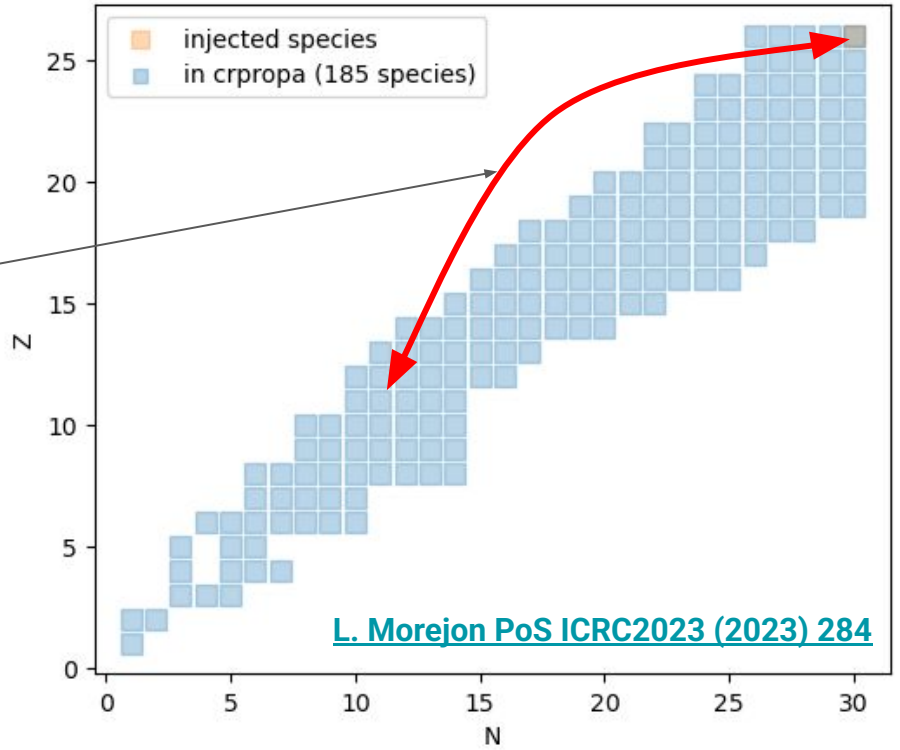

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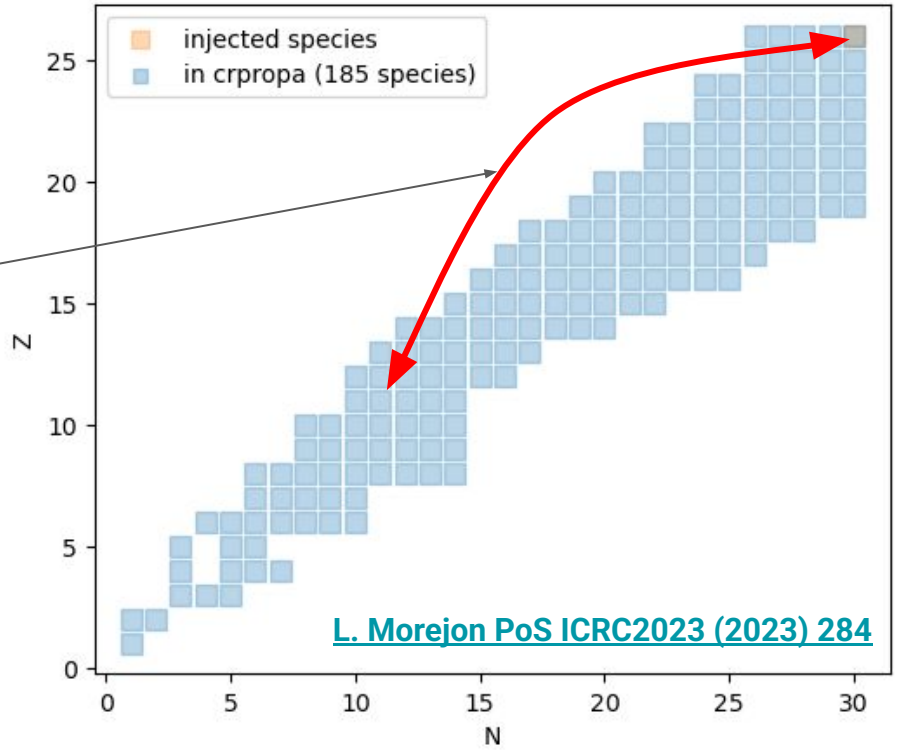
Total interaction rate



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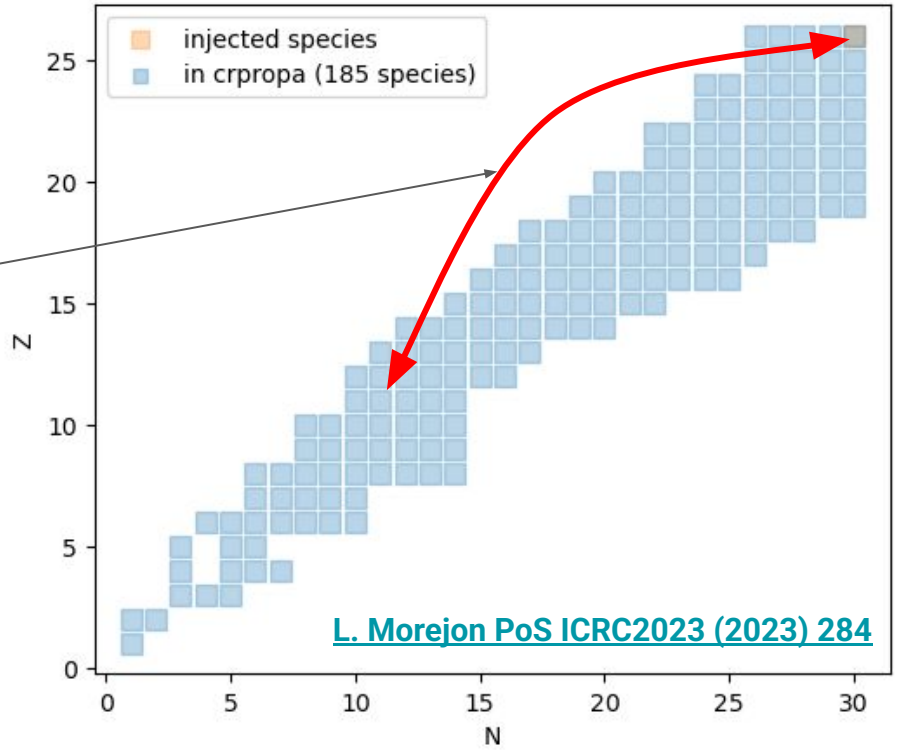
Total interaction rate

Lower triangular is null (mass increase not possible)



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Example distributions: Sequential vs Concurrent casca

Interaction matrix for SeCs is mostly empty

$$\mathbf{\Lambda}(\gamma) = \begin{pmatrix} -\lambda^{\mathrm{tot}}_{S_1} & \lambda^{\mathrm{tot}}_{S_1} & 0 & 0 & 0 & \dots & 0 \\ 0 & -\lambda^{\mathrm{tot}}_{S_2} & \lambda^{\mathrm{tot}}_{S_2} & 0 & 0 & \dots & 0 \\ 0 & 0 & -\lambda^{\mathrm{tot}}_{S_3} & \lambda^{\mathrm{tot}}_{S_3} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & -\lambda^{\mathrm{tot}}_{S_N} \end{pmatrix}$$

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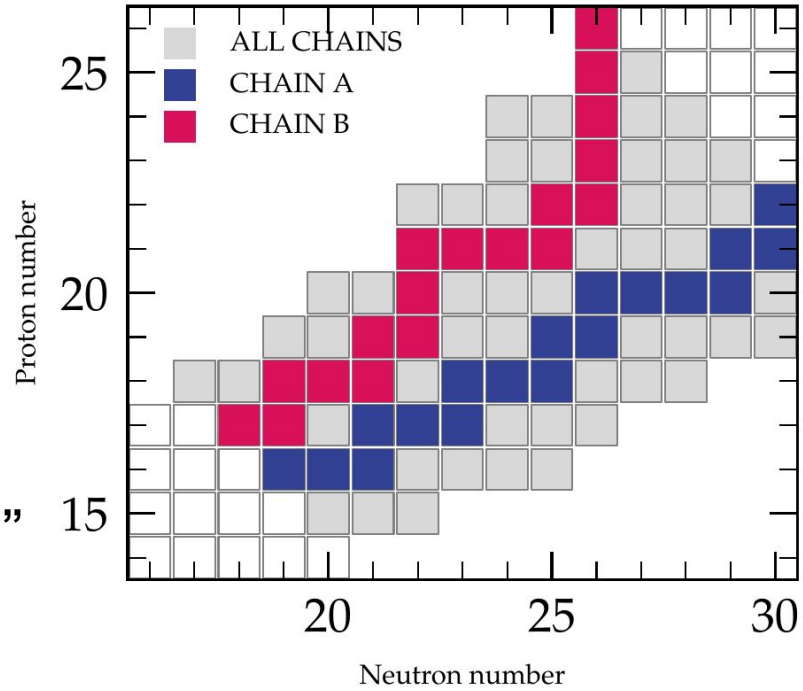
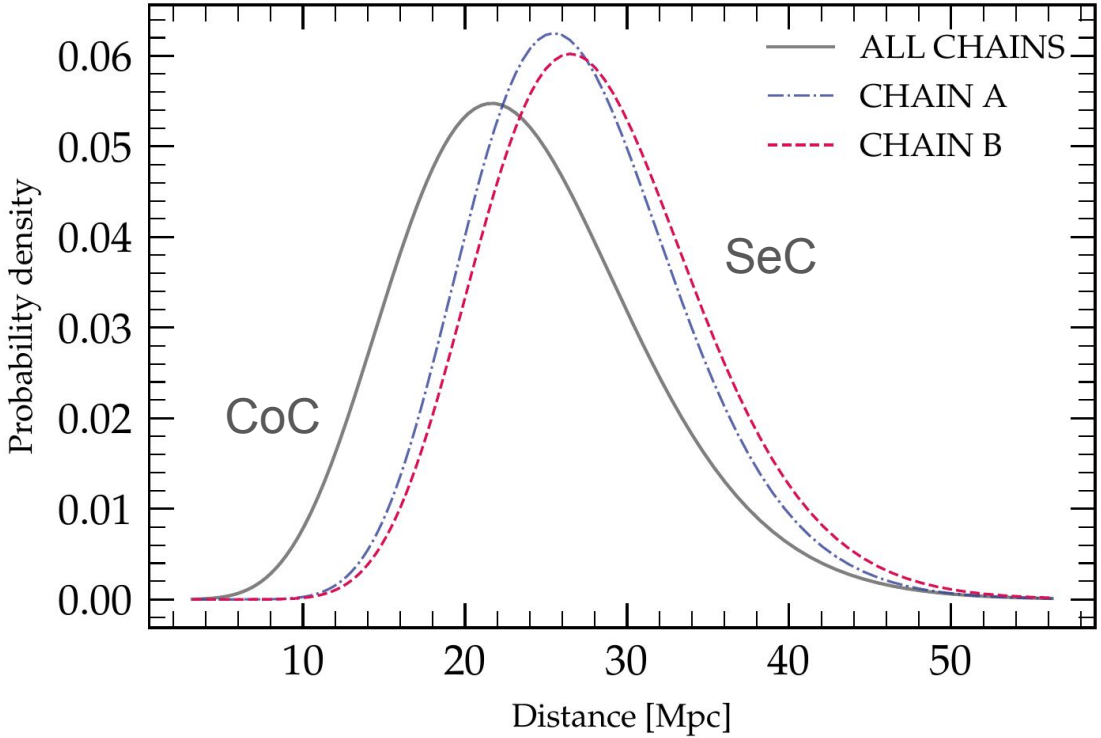


Probability density of distance until "absorption"

$$f(L) = \boldsymbol{\pi} \exp(\mathbf{\Lambda} L)\mathbf{\Lambda 1}$$

Example distributions: Sequential vs Concurrent casca



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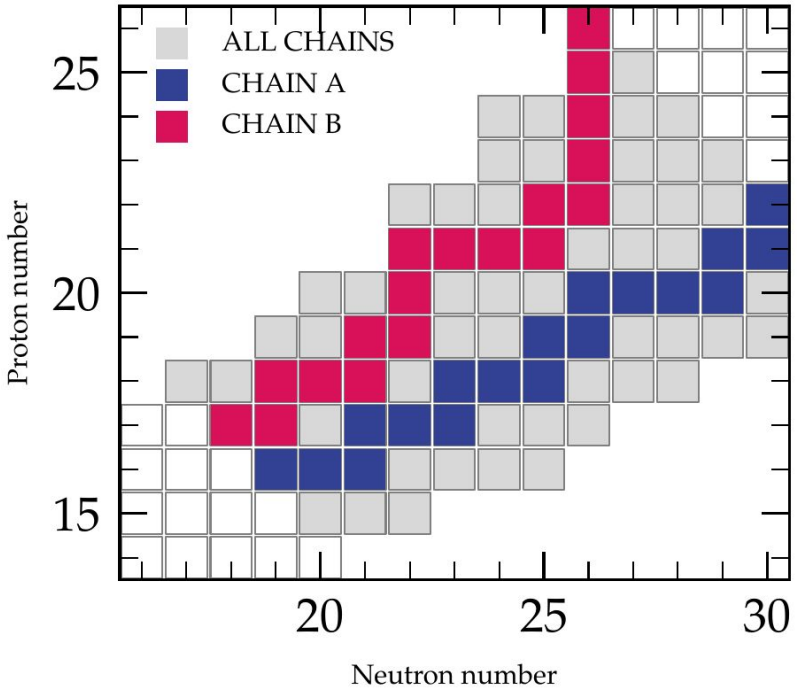


Impact of composition on the distributions

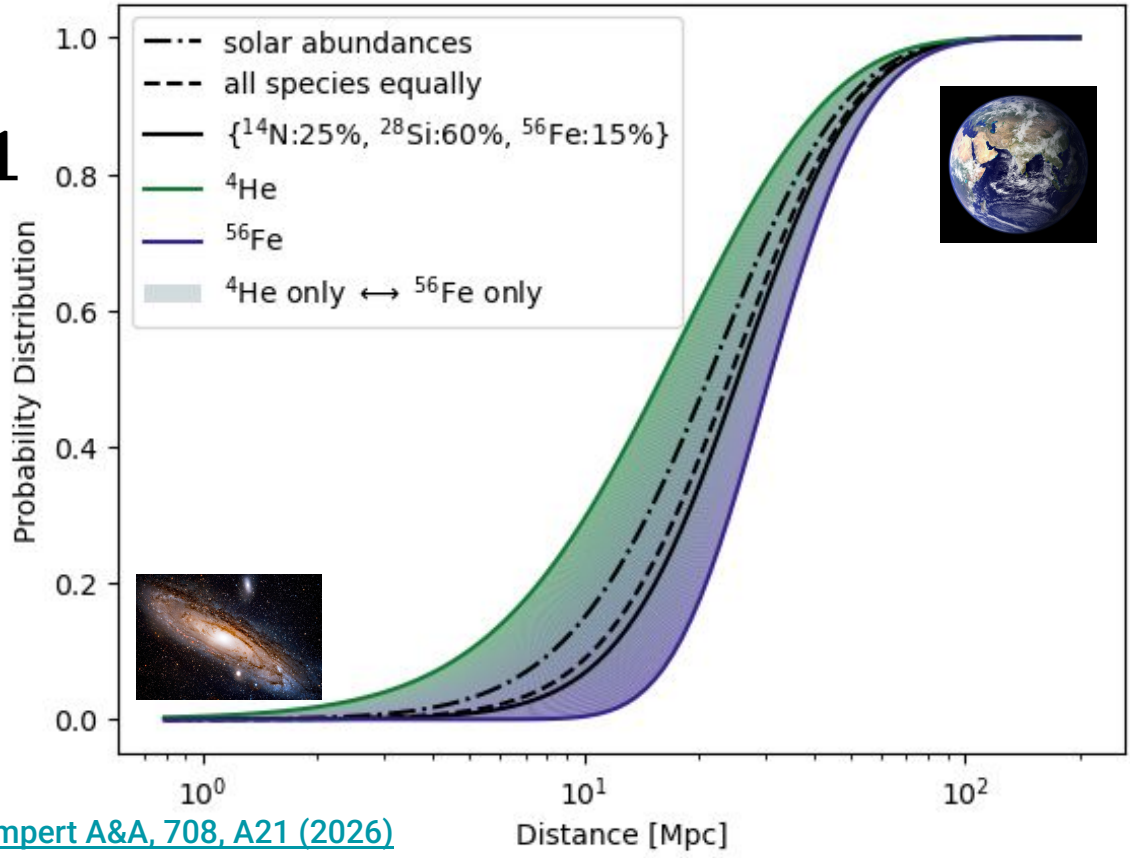
Cumulative distribution

$$F(L) = 1 - \boldsymbol{\pi} \exp(\boldsymbol{\Lambda} L)\mathbf{1}$$

Different injection compositions

- Solar abundances
- Equal share for all species
- Like UHECR-fit comp.

Lorentz boost 1E9



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Support & Feedback…

Feedback

Please use this link and provide feedback on this session…

https://www.acme-astro.eu/survey-hands-on-sessions-xwj4jm9/

Support

For questions, support and issues in the code you can open an issue on the github repository.

Additionally you can open a ticket in the ACME support website:

https://support.acme-astro.eu/

MICRO website

MICRO@github

CRISP

Cosmic Ray Stochastic Interactions for Propagation

Thanks!

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LM arxiv:2501.06677

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