Cosmological Model with Nonminimal Coupling Between Nonlinear Electrodynamics (NLED) and Gravity

Chilwatun Nasiroh[*a], Ramy Fitrah Izzah[b], Mhd Yasir[b], Fiki Taufik Akbar[b], Bobby Eka Gunara[b]

[a]Universitas Jember, Indonesia
[b]Institut Teknologi Bandung, Indonesia

* chilwa@unej.ac.id

The Institute for Fundamental Study, Naresuan University, Thailand | 13–20 August, 2026

Gravity and NLED with nonminimal coupling

We shortly discuss a gravity and nonlinear electrodynamics with nonminimal coupling whose action is given by

$$S = \int \sqrt{-g} \left\{ \frac{1}{2\kappa^2} (R - 2\Lambda) + (1 - \xi R)\mathcal{G}(\mathcal{F}) \right\} \mathrm{d}^4 x \,, \tag{1}$$

we particularly take the form of nonlinear electrodynamics model which has been studied by Kruglov [1]:

$$\mathcal{G}(\mathcal{F}) = -\frac{\mathcal{F}}{(1 + \beta\mathcal{F})^2} \,, \tag{2}$$

where β is a positive real parameter, $\mathcal{F} \equiv \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor of gauge field A_μ.

Table: List of some nonlinear electrodynamics models

Model	Lagrangian $\mathcal{L}$	Maxwell's Limit
Novello's Model [2]	$-\mathcal{F} + \frac{\gamma}{4\mathcal{F}} + 16\alpha^2\mathcal{F}^2$	$\alpha \longrightarrow 0, \gamma \longrightarrow 0$
Kruglov's model A [1]	$-\frac{\mathcal{F}}{(\beta\mathcal{F}+1)^2}$	$\beta \longrightarrow 0$
Kruglov's model B [3]	$-\mathcal{F} + \frac{\alpha\mathcal{F}}{2\beta\mathcal{F}+1}$	$\alpha \longrightarrow 0$
Kruglov's model C [4]	$-\frac{\mathcal{F}}{2\beta\mathcal{F}+1}$	$\beta \longrightarrow 0$
Ovgun's model [5]	$\frac{-\mathcal{F}e^{-\alpha\mathcal{F}}}{\alpha\mathcal{F}+\beta}$	$\alpha \longrightarrow 0, \beta \longrightarrow 0$

The equations of motions of gravity-NLED system are respectively given by

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu} \ , \tag{3}$$

and

$$\partial_\mu \left(\frac{\sqrt{-g}(1-\xi R)(1-\beta\mathcal{F})}{(1+\beta\mathcal{F})^3} F^{\mu\nu} \right) = 0 \ , \tag{4}$$

We introduced the decomposition,

$$T_{\mu\nu} = T^{M}_{\mu\nu} + \xi T^{(\xi)}_{\mu\nu} \tag{5}$$

where $T^{M}_{\mu\nu}$ is the energy-momentum tensor of the matter (NLED) term and $T^{\xi}_{\mu\nu}$ is the energy-momentum tensor of due to the nonminimal coupling which can be expressed as follows

$$T^{(M)}_{\mu\nu} = \frac{(1-\beta\mathcal{F})}{(1+\beta\mathcal{F})^3} F_{\mu\rho}F^{\rho}_{\nu} + g_{\mu\nu}\mathcal{G}(\mathcal{F}), \tag{6}$$

$$T^{(\xi)}_{\mu\nu} = -\frac{(1-\beta\mathcal{F})R}{(1+\beta\mathcal{F})^3} F_{\mu\rho}F^{\rho}_{\nu} - 2\left(G_{\mu\nu} - \nabla_\mu\nabla_\nu + g_{\mu\nu}\nabla^2 \right)\mathcal{G}(\mathcal{F}), \tag{7}$$

The energy-momentum tensor of the NLED field has non-zero trace in which is different from the standard Maxwell theory,

$$\mathcal{T}^{(M)} = -\frac{8\beta\mathcal{F}^2}{(1+\beta\mathcal{F})^3}, \tag{8}$$

and the trace of the energy momentum tensor due to nonminimal coupling is

$$\mathcal{T}^{(\xi)} = -\frac{4\mathcal{F}R(1-\beta\mathcal{F})}{(1+\beta\mathcal{F})^3} + 2\left(R - 3g^{\mu\nu}\nabla_\mu\nabla_\nu\right)\mathcal{G}(\mathcal{F}). \tag{9}$$

A non-zero trace means the electromagnetic field is no longer a passive. It acts as an active energy-density source, enabling an energy exchange with spacetime geometry.

We will consider the gravity-NLED system on a homogeneous and isotropic spatially flat universe described by the Friedmann-Robertson-Walker (FRW) metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \tag{10}$$

where $a(t)$ is the scale factor as a function of the cosmic time. Then, we add an assumption that in the cosmic background, there exists a dominant, stochastic magnetic field. The wavelength of these random magnetic field fluctuations is much smaller than the universe's spacetime curvature scale. Because the magnetic field fluctuates randomly on small scales, we perform a spatial averaging that satisfy the following conditions:

$$\begin{aligned} \langle E \rangle = \langle B \rangle = 0, &\qquad \langle E_i B_j \rangle = 0, \\ \langle E_i E_j \rangle = 0, &\qquad \langle B_i B_j \rangle = \frac{1}{3} B^2 g_{ij}. \end{aligned} \tag{11}$$

Therefore, the energy-momentum tensor in Eq. (5) can be written in the perfect fluid form,

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + g_{\mu\nu}p, \tag{12}$$

where U^μ is the 4-velocity of the fluid rest frame, ρ and p are the energy density and the pressure, respectively which given by

$$\rho = \frac{2B^2}{(2+\beta B^2)^2} - \frac{12\xi B^2}{(2+\beta B^2)^2}\left(\frac{\dot{a}^2}{a^2}\right) - \frac{24\xi B\dot{B}(2-\beta B^2)}{(2+\beta B^2)^3}\left(\frac{\dot{a}}{a}\right), \tag{13}$$

$$\begin{aligned} p = {} & \frac{2B^2(2-7\beta B^2)}{3(2+\beta B^2)^3} - \frac{8\xi B^2(2-3\beta B^2)}{(2+\beta B^2)^3}\left(\frac{\ddot{a}}{a}\right) - \frac{4\xi B^2(6-5\beta B^2)}{(2+\beta B^2)^3}\left(\frac{\dot{a}^2}{a^2}\right) \\ & + \frac{16\xi B\dot{B}(2-\beta B^2)}{(2+\beta B^2)^3}\left(\frac{\dot{a}}{a}\right) + 8\xi\left(\frac{B\ddot{B}(4-\beta^2B^4) + \dot{B}^2(3\beta^2B^4 - 16\beta B^2 + 4)}{(2+\beta B^2)^4}\right), \end{aligned} \tag{14}$$

The conservation of the energy-momentum tensor, $\nabla^\mu T_{\mu\nu} = 0$, gives the relation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0\ . \tag{15}$$

Substituting equation (13) and equation (14) into equation (15) yields

$$\left(\dot{B} + 2B\frac{\dot{a}}{a}\right)\left[1 - 6\xi\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\right] = 0\ , \tag{16}$$

which implies,

$$B(t) = \frac{B_0}{a^2(t)}\ , \tag{17}$$

where $B_0 = B(t_0)$ is the value of magnetic field when the scale factor $a(t_0) = 1$.

NLED and the Einstein Tensor coupling

We consider a modified gravitational framework in which the Einstein tensor is nonminimally coupled to the energy–momentum tensor of matter fields. The total action of the model is given by

$$S = \int \sqrt{-g}\, d^4x \left(\frac{1}{2\kappa^2}(R - 2\Lambda) + \xi G_{\mu\nu} T^{\mu\nu} + \mathcal{L}_{NL} \right), \tag{18}$$

The specific form of the NLED Lagrangian is chosen as

$$\mathcal{L}_{NL} = -\frac{\mathcal{F} e^{-\gamma \mathcal{F}}}{\beta \mathcal{F} + \alpha}, \tag{19}$$

where, α, β, and γ are free nonlinear parameters. The specific form of action can generate general field equations that do not contain higher-order derivatives, and thus the theory does not suffer from ghost instabilities [6].

Considering the $(\mu, \nu) = (0, 0)$ component of the field equations, we obtain the modified first Friedmann equation

$$3\frac{\dot{a}^2}{a^2} - \Lambda = \kappa^2(\rho_{NL} + \xi\rho_{NM}). \tag{20}$$

$$3\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} - \Lambda = -\kappa^2\left(p_{NL} + \xi p_{NM}\right). \tag{21}$$

$$\rho_{NL} = \frac{B^2 e^{-\gamma B^2/2}}{(\beta B^2 + 2\alpha)}. \tag{22}$$

$$\begin{aligned}\rho_{NM} =& \frac{2Be^{-\frac{1}{2}\gamma B^2}}{(\beta B^2 + 2\alpha)^3}\dot{B}\left(\frac{\dot{a}}{a}\right)\Big(8\alpha^2 + (-36\alpha^2\gamma - 28\alpha\beta)B^2 + (8\alpha^2\gamma^2 - 20\alpha\beta\gamma)B^4 \\ &+ (8\alpha\beta\gamma^2 - \beta^2\gamma)B^6 + 2\beta^2\gamma^2 B^8\Big) + \frac{2B^2 e^{-\frac{1}{2}\gamma B^2}}{(\beta B^2 + 2\alpha)^3}\left(\frac{\dot{a}^2}{a^2}\right)\times \\ &+ \Big(12\alpha^2 - 12\alpha^2\gamma B^2 + (-12\alpha\beta\gamma - 3\beta^2)B^4 - 3\beta^2\gamma B^6\Big).\end{aligned} \tag{23}$$

$$p_{NL} = \frac{B^2 e^{-\gamma B^2/2}}{3(\beta B^2 + 2\alpha)^2}\Big(2\alpha - (4\alpha\gamma + 3\beta)B^2 - 2\beta\gamma B^4\Big). \tag{24}$$

$$
\begin{aligned}
p_{NM} =& \frac{2B^2 e^{-\frac{1}{2}\gamma B^2}}{3(\beta B^2+2\alpha)^4}\left(\frac{\dot{a}^2}{a^2}\right)\times\Big(-56\alpha^3 + (-108\alpha^2\beta - 24\alpha^3\gamma)B^2 + (-34\alpha\beta^2 + 16\alpha^3\gamma^2 - 4\alpha^2\beta\gamma)B^4 \\
&+ (3\beta^3 + 24\alpha^2\beta\gamma^2 + 14\alpha\beta^2\gamma)B^6 + (12\alpha\beta^2\gamma^2 + 5\beta^3\gamma)B^8 + 2\beta^3\gamma^2 B^{10}\Big) \\
&+ \frac{2B e^{-\frac{1}{2}\gamma B^2}}{3(\beta B^2+2\alpha)^4}\dot{B}\left(\frac{\dot{a}}{a}\right)\times\Big(-96\alpha^3 + (96\alpha^2\beta + 240\alpha^3\gamma)B^2 + (72\alpha\beta^2 - 48\alpha^3\gamma^2 + 264\alpha^2\beta\gamma)B^4 \\
&+ (-72\alpha^2\beta\gamma^2 + 84\alpha\beta^2\gamma)B^6 + (-36\alpha\beta^2\gamma^2 + 6\beta^3\gamma)B^8 - 6\beta^3\gamma^2 B^{10}\Big) \\
&+ \frac{2 e^{-\frac{1}{2}\gamma B^2}}{3(\beta B^2+2\alpha)^4}\dot{B}^2\Big(-16\alpha^3 + (208\alpha^2\beta + 232\alpha^3\gamma)B^2 \\
&+ (-84\alpha\beta^2 - 152\alpha^3\gamma^2 + 44\alpha^2\beta\gamma)B^4 + (16\alpha^3\gamma^3 - 180\alpha^2\beta\gamma^2 - 34\alpha\beta^2\gamma)B^6 \\
&+ (24\alpha^2\beta\gamma^3 - 66\alpha\beta^2\gamma^2 + \beta^3\gamma)B^8 + (12\alpha\beta^2\gamma^3 - 7\beta^3\gamma^2)B^{10} + 2\beta^3\gamma^3 B^{12}\Big) \\
&+ \frac{2B^2 e^{-\frac{1}{2}\gamma B^2}}{3(\beta B^2+2\alpha)^4}\left(\frac{\ddot{a}}{a}\right)\times\Big(-16\alpha^3 + (-120\alpha^2\beta - 96\alpha^3\gamma)B^2 + (-44\alpha\beta^2 + 32\alpha^3\gamma^2 - 80\alpha^2\beta\gamma)B^4 \\
&+ (6\beta^3 + 48\alpha^2\beta\gamma^2 - 8\alpha\beta^2\gamma)B^6 + (24\alpha\beta^2\gamma^2 + 4\beta^3\gamma)B^8 + 4\beta^3\gamma^2 B^{10}\Big) \\
&+ \frac{2B e^{-\frac{1}{2}\gamma B^2}}{3(\beta B^2+2\alpha)^4}\ddot{B}\times\Big(-16\alpha^3 + (48\alpha^2\beta + 72\alpha^3\gamma)B^2 + (28\alpha\beta^2 - 16\alpha^3\gamma^2 + 76\alpha^2\beta\gamma)B^4 \\
&+ (-24\alpha^2\beta\gamma^2 + 22\alpha\beta^2\gamma)B^6 + (-12\alpha\beta^2\gamma^2 + \beta^3\gamma)B^8 - 2\beta^3\gamma^2 B^{10}\Big).
\end{aligned} \tag{25}
$$

Accordingly, we define the effective energy density and pressure as

$$\rho_{\text{eff}} = \rho_{NL} + \xi \rho_{NM}, \quad p_{\text{eff}} = p_{NL} + \xi p_{NM}. \tag{26}$$

With these definitions, the modified Friedmann equations formally retain the same structure as in standard cosmology, despite the presence of nonminimal couplings. By applying the covariant derivative to the field equations and invoking the identity $\nabla^\mu G_{\mu\nu} = 0$, the conservation equation for the effective energy–momentum tensor is obtained as

$$\nabla^\mu \left(T^{(NL)}_{\mu\nu} + \xi T^{(NM)}_{\mu\nu} \right) = 0. \tag{27}$$

On a homogeneous and isotropic FRW background, the spatial components of the above conservation equation are identically satisfied. The remaining temporal component yields the continuity equation for the effective cosmic fluid

$$\dot{\rho}_{\text{eff}} + 3(\rho_{\text{eff}} + p_{\text{eff}}) \left(\frac{\dot{a}}{a} \right) = 0. \tag{28}$$

Substituting the expressions for $\dot{\rho}_{\text{eff}}$, ρ_{eff} and p_{eff} into the above equation, we obtain a more detailed form of the energy-momentum conservation equation

$$\left(\dot{B} + 2B\left(\frac{\dot{a}}{a}\right)\right)\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\times$$
$$\left(1 - 2\xi\left(\frac{3B^2(-16\alpha^2\gamma - 16\alpha\beta) + 3B^4(4\alpha^2\gamma^2 - 8\alpha\beta\gamma) + 12\alpha\beta\gamma^2 B^6 + 3\beta^2\gamma^2 B^8}{-8\alpha^2 + B^2(4\alpha^2\gamma - 4\alpha\beta) + 4\alpha\beta\gamma B^4 + \beta^2\gamma B^6}\right)\right) = 0. \tag{29}$$

The energy–momentum conservation equation admits a particularly simple solution

$$\dot{B} + 2B\left(\frac{\dot{a}}{a}\right) = 0. \tag{30}$$

Solving Eq. (30), the magnetic field evolves with the scale factor as

$$B = \frac{B_0}{a^2}, \tag{31}$$

back to: Gravity and NLED with nonminimal coupling

We have the first and second Friedmann equations are given by

$$\frac{\dot{a}^2}{a^2} = \frac{\kappa^2 \rho_{\text{total}}}{3}\,, \tag{32}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}\left(\rho_{\text{total}} + 3p_{\text{total}}\right)\,, \tag{33}$$

respectively, we define $\rho_{\text{total}} = \rho_{\text{eff}} + \rho_\Lambda$ denotes the total energy density with,

$$\rho_{\text{eff}} = \frac{2a^4 B_0^2 (2a^4 + \beta B_0^2)}{(2a^4 + \beta B_0^2)^3 - 4\xi\kappa^2 a^4 B_0^2 (6a^4 - 5\beta B_0^2)}, \tag{34}$$

$$\rho_\Lambda = \frac{(2a^4 + \beta B_0^2)^3}{(2a^4 + \beta B_0^2)^3 - 4\xi\kappa^2 a^4 B_0^2 (6a^4 - 5\beta B_0^2)} \frac{\Lambda}{\kappa^2}\,, \tag{35}$$

and $p_{\text{total}} = p_{\text{eff}} + p_\Lambda$ denotes the total pressure with

$$p_{\text{eff}} = \frac{2a^4B_0^2}{3}\left[\frac{(2a^4 - 7\beta B_0^2)(2a^4 + \beta B_0^2)^3 - 4\xi\kappa^2 a^4 B_0^2(5\beta^2 B_0^4 + 68a^4\beta B_0^2 - 12a^8)}{\left[(2a^4 + \beta B_0^2)^3 - 4\xi\kappa^2 a^4 B_0^2(6a^4 - 5\beta B_0^2)\right]^2}\right], \tag{36}$$

$$p_\Lambda = -\frac{\Lambda}{3\kappa^2}(2a^4 + \beta B_0^2)^2\left[\frac{3(2a^4 + \beta B_0^2)^4 - 4\xi\kappa^2 a^4 B_0^2(5\beta^2 B_0^4 - 140a^4\beta B_0^2 + 84a^8)}{\left[(2a^4 + \beta B_0^2)^3 - 4\xi\kappa^2 a^4 B_0^2(6a^4 - 5\beta B_0^2)\right]^2}\right]. \tag{37}$$

Taking the limit for early evolution, $a(t) \longrightarrow 0$, and the limit for late evolution, $a(t) \longrightarrow \infty$, we obtain

$$\lim_{a(t)\to 0} \rho_{\text{eff}}(t) = \lim_{a(t)\to 0} p_{\text{eff}} = 0, \qquad \lim_{a(t)\to 0} \rho_\Lambda = \frac{\Lambda}{\kappa^2}, \quad \lim_{a(t)\to 0} p_\Lambda = -\frac{\Lambda}{\kappa^2}, \tag{38}$$

$$\lim_{a(t)\to \infty} \rho_{\text{eff}}(t) = \lim_{a(t)\to \infty} p_{\text{eff}} = 0, \qquad \lim_{a(t)\to \infty} \rho_\Lambda = \frac{\Lambda}{\kappa^2}, \quad \lim_{a(t)\to \infty} p_\Lambda = -\frac{\Lambda}{\kappa^2}. \tag{39}$$

Now, we could consider a case where the universe as consisting of two-component matters with the equation of state:

$$p_{\text{total}} = p_{\text{eff}} + p_{\Lambda} = w_{\text{eff}}\rho_{\text{eff}} + w_{\Lambda}\rho_{\Lambda} = w_{\text{total}}\rho_{\text{total}} \ , \tag{40}$$

We introduce the dimensional parameters x, γ and $\overline{\Lambda}$, defined as $x = a\left(2/\beta B_0^2\right)^{1/4}$, $\gamma \equiv \xi\kappa^2/\beta$ and $\overline{\Lambda} \equiv \beta\Lambda/\kappa^2$, so that the above equation becomes:

$$w_{eff} = \frac{1}{3(x^4+1)}\left[\frac{(x^4-7)(x^4+1)^3 - 2\gamma x^4(5+34x^4-3x^8)}{(x^4+1)^3 - 2\gamma x^4(3x^4-5)}\right], \tag{41}$$

$$w_{\Lambda} = -\frac{1}{3(x^4+1)}\left[\frac{3(x^4+1)^4 - 2\gamma x^4(5-70x^4+21x^8)}{(x^4+1)^3 - 2\gamma x^4(3x^4-5)}\right]. \tag{42}$$

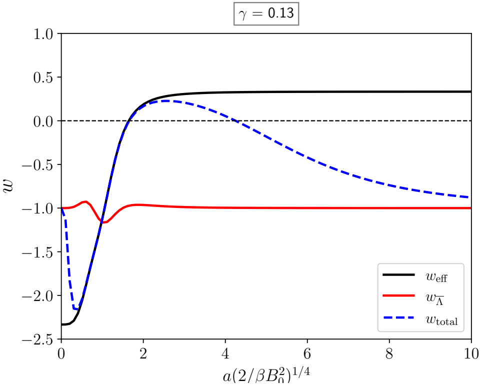


Figure: The evolution of the equation of state parameters w_{eff}, $w_{\overline{\Lambda}}$, and w_{total} versus scale factor $x = a\left(2/\beta B_0^2\right)^{\frac{1}{4}}$ for $\gamma = 0.13$. The value of w_{total} (dashed blue line) and $w_{\overline{\Lambda}}$ (solid red line) approach -1 at $x \longrightarrow 0$ and $x \longrightarrow \infty$. It is shown that the nonminimal coupling model with a cosmological constant plays a crucial role in the universe's early and late-time evolution, compared to the model with nonminimal coupling and nonlinear electrodynamics alone (solid black line). It also influenced the evolution of $w_{\overline{\Lambda}}$ (solid red line) as indicated by the fluctuations.

The Evolution of the Universe

Making use of Eqs. (34), (35) and (31) to the first Friedmann equation yields

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3}\frac{2\kappa^2 a^4 B_0^2(2a^4+\beta B_0^2)+\Lambda(2a^4+\beta B_0^2)^3}{(2a^4+\beta B_0^2)^3-4\xi\kappa^2 a^4 B_0^2(6a^4-5\beta B_0^2)}\,. \tag{43}$$

We define a dimensionless parameters, $b \equiv (\sqrt{\beta}B_0)/\sqrt{2}$ and introduce a re-scaled time variable, $\tau = (\kappa t\,|B_0|)/\sqrt{6}$, then we can write the Eq. 43 as

$$\frac{dx}{d\tau} = \frac{x}{b}\sqrt{\frac{x^4(x^4+1)+\overline{\Lambda}(x^4+1)^3}{(x^4+1)^3-2\gamma x^4(3x^4-5)}}\,, \tag{44}$$

The Eq. (44) cannot be solved exactly, therefore we will take the approximation and numerical solution of Eq. (44). The approximation to the exact solution can be calculated using the Taylor expansion series around $\overline{\Lambda} \to 0$ and $\gamma \to 0$. Hence, we obtain

$$\frac{1}{b} d\tau \approx \left[\left(\frac{1}{x^3} + x \right) + \frac{5x - 3x^5}{(1 + x^4)^2} \gamma - \frac{(1 + x^4)^3}{2x^7} \overline{\Lambda} \right] dx \, , \tag{45}$$

Integrating the equation with the initial value $x(0) = 1$, we obtain,

$$\tau = b \left[\frac{x^4 - 1}{2x^2} + \left(\frac{2x^2}{x^4 + 1} + \frac{1}{2} \arctan x^2 - 1 - \frac{\pi}{8} \right) \gamma + \left(\frac{1 + 9x^4 - 9x^8 - x^{12}}{12x^6} \right) \overline{\Lambda} \right] . \tag{46}$$

We expand the scale factor $x(\tau)$ as $x(\tau) \approx x_0(\tau) + x_{1\overline{\Lambda}}(\tau)\overline{\Lambda} + x_{1\gamma}(\tau)\gamma$, where $x_0(\tau)$ is the exact solution for $\overline{\Lambda} = \gamma = 0$, and the $x_{1\overline{\Lambda}}$ and $x_{1\gamma}$ are the first order expansion coefficients associated with $\overline{\Lambda}$ and γ, respectively.

$$
\begin{aligned}
x(\tau) \quad \approx \quad & \frac{\sqrt{\tau + \sqrt{\tau^2 + b^2}}}{\sqrt{b}} + \frac{\tau(\tau^2 + 3b^2)\sqrt{b\left(\tau + \sqrt{\tau^2 + b^2}\right)}}{3b^3\sqrt{\tau^2 + b^2}}\overline{\Lambda} \\
& + \frac{\sqrt{b\left(\tau + \sqrt{\tau^2 + b^2}\right)}}{16\sqrt{\tau^2 + b^2}}\left(8 + \pi - \frac{8b}{\sqrt{\tau^2 + b^2}} - 4\arctan\left[\frac{\tau + \sqrt{\tau^2 + b^2}}{b}\right]\right)\gamma \, .
\end{aligned} \tag{47}
$$

In the early evolution, $\tau \longrightarrow 0$, this model could explain the inflation of the universe. Meanwhile, at the late stage of the evolution, $\tau \longrightarrow \infty$, the solution (47) becomes $x \longrightarrow \sqrt{2\tau/b}\left(1 + \overline{\Lambda}\tau^2/(3b^2)\right)$ corresponding to the late-time acceleration.

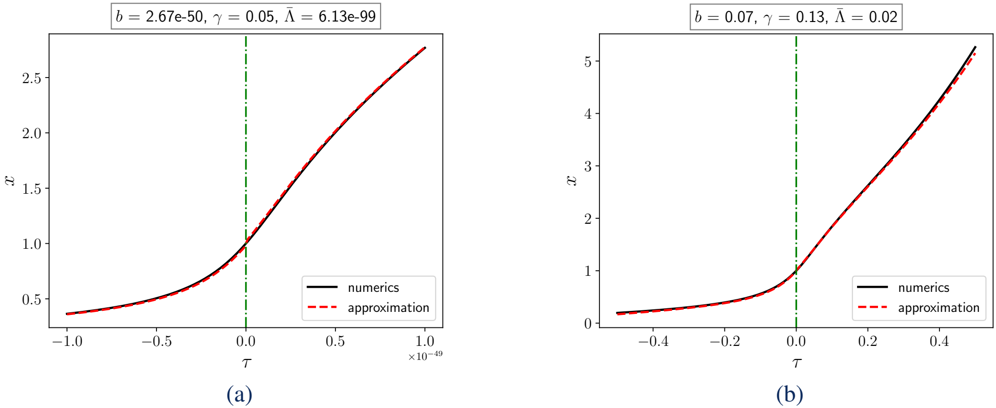


Figure: Plot x vs. τ as a approximation and numerical solution of Eq. (44) for the choice of values (a) $\gamma = 0.05$, $b = 2.67 \times 10^{-50}$, $\overline{\Lambda} = 6.13 \times 10^{-99}$, and (b) $\gamma = 0.13$, $b = 0.07$, $\overline{\Lambda} = 0.02$.

We can express the first Friedmann equation (32) in the form of a Hamiltonian as:

$$\mathcal{H} = \frac{1}{2}\dot{a}^2 + U_{\text{eff}}(a) = 0\,, \tag{48}$$

where the effective potential $U_{\text{eff}}(a)$ is given by:

$$U_{\text{eff}}(a) = -\frac{1}{6}\left[\frac{2\kappa^2 a^4 B_0^2(2a^4 + \beta B_0^2) + \Lambda(2a^4 + \beta B_0^2)^3}{(2a^4 + \beta B_0^2)^3 - 4\xi\kappa^2 a^4 B_0^2(6a^4 - 5\beta B_0^2)}\right]. \tag{49}$$

To explore the system further, we can reformulate the equation into two coupled first-order differential equations:

$$\frac{dx}{d\tau} = y, \quad \frac{dy}{d\tau} = -\frac{\partial W(x)}{\partial x}, \tag{50}$$

where $y = \dot{a}\sqrt{6}/(\kappa B_0\sqrt{b})$ and the potential function $W(x)$ is:

$$W(x) = \frac{6U_{\text{eff}}(x)}{\kappa^2 B_0^2} = -\frac{1}{2b^2}\left[\frac{x^6(x^4+1) + \overline{\Lambda}x^2(x^4+1)^3}{(x^4+1)^3 - 2\gamma x^4(3x^4-5)}\right]. \tag{51}$$

The non-trivial critical points satisfy the following polynomial equation:

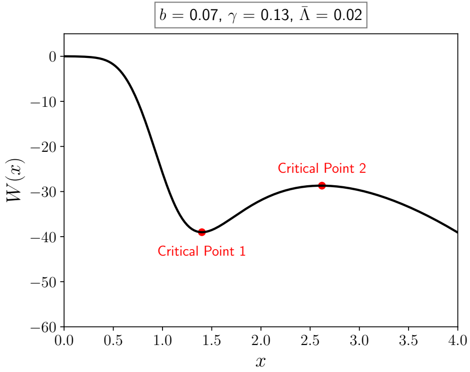
$$
\begin{aligned}
0 \quad = \quad & -\overline{\Lambda} - x^4(3 + 6\overline{\Lambda} - 10\gamma\overline{\Lambda}) - x^8(8 + 15\overline{\Lambda} + 10\gamma + 48\gamma\overline{\Lambda}) \\
& -2x^{12}(3 + 10\overline{\Lambda} + 18\gamma + 54\gamma\overline{\Lambda}) - x^{16}(15\overline{\Lambda} - 6\gamma + 32\gamma\overline{\Lambda}) \\
& +x^{20}(1 - 6\overline{\Lambda} + 18\gamma\overline{\Lambda}) - \overline{\Lambda}x^{24}. \qquad (52)
\end{aligned}
$$

$$
\overline{\Lambda}_{\text{max}} \approx 0.0547 - 0.0353\gamma + (0.0732 + 0.0279\gamma)\tan(1.214\gamma). \qquad (53)
$$

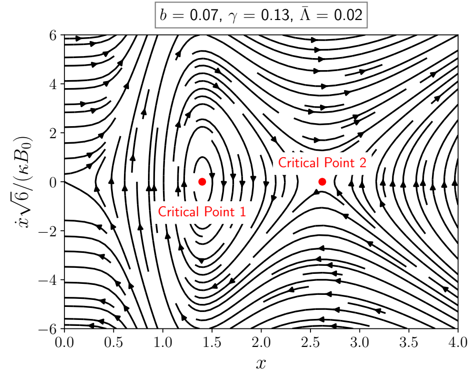
The positive real roots are given by:

$$
x_1 \approx \frac{16}{(3)^{\frac{7}{4}}}\overline{\Lambda} + \frac{3^{\frac{1}{4}}}{4}(4 + \gamma), \qquad (54)
$$

$$
x_2 \approx -\frac{3}{2}\overline{\Lambda}^{\frac{3}{4}} + \left(\overline{\Lambda}\right)^{-\frac{1}{4}}. \qquad (55)
$$



(a)



(b)

Figure: The plot of (a) effective potential $W(x)$ and (b) the phase portrait correspond to critical point 1 and 2 for the choice $b = 0.07, \gamma = 0.13$, and $\overline{\Lambda} = 0.02$.

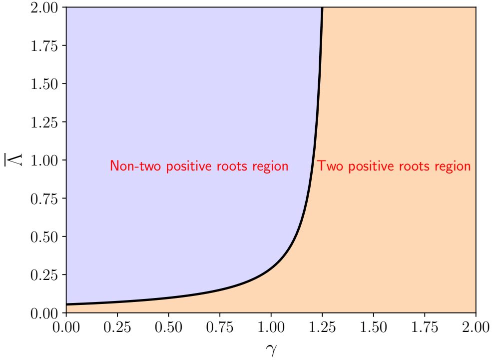


Figure: The plot represents the region in the $\overline{\Lambda}$ vs. γ parameter space where two positive roots exist and where non-two positive roots are present. The boundary between these regions is of particular interest as it helps in understanding the behavior of the model under different conditions.

The inflation phase takes place within the range $x(0) = 1$ to $x_i = x_1$. Using Eq. (46), we determine the duration of inflation in conformal time (τ_i), as follows:

$$\tau_i \approx b\left[\frac{1}{\sqrt{3}} + \frac{44}{9\sqrt{3}}\overline{\Lambda} + \gamma\left(-1 + \frac{5}{2\sqrt{3}} + \frac{\pi}{24}\right)\right]. \tag{56}$$

After the inflationary phase concludes, the universe begins to decelerate. Deceleration occurs within the range $x_i < x < x_d$, where $x_i = x_1$ and $x_d = x_2$. Therefore, the duration of deceleration in conformal time (τ_d) is given by the following:

$$\tau_d \approx b\left[-\frac{1}{\sqrt{3}} + \frac{5}{12\sqrt{\overline{\Lambda}}} - 2\sqrt{\overline{\Lambda}} - \frac{44}{9\sqrt{3}}\overline{\Lambda} + \frac{1}{12}\gamma\left(-10\sqrt{3} + \pi\right)\right]. \tag{57}$$

In terms of cosmic time, we obtain:

$$
\begin{aligned}
t_i &\approx \frac{b\sqrt{6}}{\kappa B_0}\left[\frac{1}{\sqrt{3}} + \frac{44}{9\sqrt{3}}\overline{\Lambda} + \gamma\left(-1 + \frac{5}{2\sqrt{3}} + \frac{\pi}{24}\right)\right], && (58)\\
t_d &\approx \frac{b\sqrt{6}}{\kappa B_0}\left[-\frac{1}{\sqrt{3}} + \frac{5}{12\sqrt{\overline{\Lambda}}} - 2\sqrt{\overline{\Lambda}} - \frac{44}{9\sqrt{3}}\overline{\Lambda} + \frac{1}{12}\gamma\left(-10\sqrt{3} + \pi\right)\right]. && (59)
\end{aligned}
$$

The duration of the inflation and deceleration periods depend on the values of b, γ, and $\overline{\Lambda}$. By utilizing the inflation duration $t_i \approx 10^{-32}$ s and the deceleration duration $t_d \approx 2 \times 10^{17}$ s, we can derive theoretical estimates for the parameters b, γ, and $\overline{\Lambda}$. Given that these parameters are small but, nevertheless, nonzero and positive, the appropriate values for these parameters are $b \approx 10^{-50}, \gamma \approx 10^{-1}$, and $\overline{\Lambda} \approx 10^{-99}$. We obtain the best-fit values:

$$
b = (2.67 \pm 0.5) \times 10^{-50}, \quad \gamma = (5.0 \pm 3.3) \times 10^{-2}, \quad \text{and} \quad \overline{\Lambda} = (6.13 \pm 2.9) \times 10^{-99}. \qquad (60)
$$

Using the best-fit values of the model parameters, we derive the following cosmological parameters: the nonlinear electrodynamics parameter $\beta = 1.56 \times 10^{-116}\ \mathrm{m} \cdot \mathrm{s}^2/\mathrm{GeV}$, the nonminimal coupling parameter $\xi = 2.39 \times 10^{-65}\ \mathrm{s}^2$, the current magnetic field $B_0 = 3.03 \times 10^8\ \mathrm{GeV}^{1/2}/\mathrm{m}^{1/2}/\mathrm{s}$, and the cosmological constant $\Lambda = 1.28 \times 10^{-35}\ \mathrm{s}^{-2}$. Remarkably, this value of Λ aligns well with earlier measurements by the High-Z Supernova Team and the Supernova Cosmology Project, which provided direct evidence of a non-zero cosmological constant, with $\Lambda \approx 10^{-122}$ in Planck units which gives $\Lambda = 2.03 \times 10^{-35}\ \mathrm{s}^{-2}$.

The deceleration parameter is

$$\begin{aligned} q \quad &\approx \quad -1+\frac{2\tau}{\sqrt{\tau^2+b^2}}-\frac{4\left(6\tau^5+11b^2\tau^3+3b^4\tau\right)}{3b^2\left(\tau^2+b^2\right)^{3/2}}\overline{\Lambda}+\frac{b^2}{4(\tau^2+b^2)^2}\Big[16(\tau^2-b^2)+b\pi\sqrt{\tau^2+b^2} \\ &\quad -4\sqrt{\tau^2+b^2}\left(-2b+\tau+b\arctan\left[\frac{\tau+\sqrt{\tau^2+b^2}}{b}\right]\right)\Big]\gamma. \end{aligned} \tag{61}$$

In the limit $\tau\longrightarrow\infty$, Eq. (61) simplifies to $q\longrightarrow-\frac{8\tau^2}{b^2}\overline{\Lambda}$, while for $\tau\longrightarrow 0$, we find $q\longrightarrow -1-2\gamma$. These results demonstrate that our model effectively describes both the early-time inflationary phase, driven by the nonlinear electrodynamic field, and the late-time accelerated expansion, governed by the cosmological constant.

Reference

1 Kruglov, S.: Inflation of universe due to nonlinear electrodynamics. Int. J. Mod. Phys. A 32(13), 1750071 (2017)

2 Novello, M., et al.: Cosmological effects of nonlinear electrodynamics. Class. Quant. Grav. 24(11), 3021 (2007)

3 Kruglov, S.: A model of nonlinear electrodynamics. Ann. Phys. 353, 299–306 (2015)

4 Kruglov, S.I.: Remarks on nonsingular models of Hayward and magnetized black hole with rational nonlinear electrodynamics. Grav. Cosmol. 27(1), 78–84 (2021)

5 Övgün, A., et al.: Falsifying cosmological models based on a non-linear electrodynamics. Eur. Phys. J. C 78, 1–20 (2018)

6 Asimakis, P. et al. "Modified gravity and cosmology with nonminimal direct or derivative coupling between matter and the Einstein tensor". In: Physical Review D 107.10 (2023), p. 104006.

Thank You

Acknowledgements

The Institute for Fundamental Study (IF) Naresuan University for support and guidance.