SHEAP School 2026

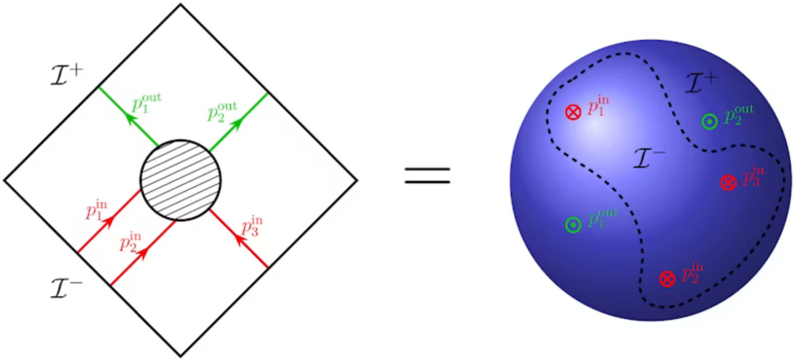
Intro. to Asymptotic Symmetries and Celestial Holography

Pongwit Srisangyingcharoen

The Institute for Fundamental Study

17 August 2026

Big picture



4D S-matrix $\quad = \quad$ 2D correlation function

Penrose diagram of Minkowski space

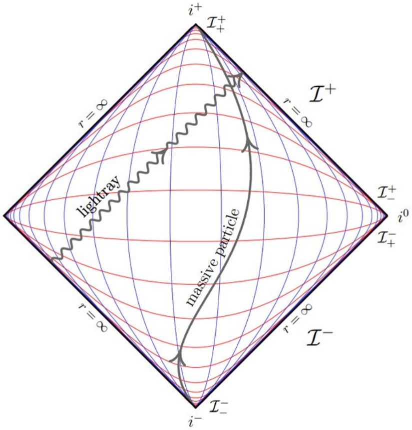


Figure: Penrose diagram of Minkowski space

Consider Minkowski metric in Bondi coordinates

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2.$$

We can represent the whole space in the Penrose diagram. It is useful to introduce the retarded and advanced time coordinates

$$u = t - r \quad v = t + r.$$

- Massive particle trajectories begin at past timelike infinity i^- and end at future timelike infinity i^+. These timelike points can be reached by keeping r fixed and taking $t \to -\infty$ for i^- and $t \to \infty$ for i^+.
- Massless particles start at past null infinity $\mathcal{I}^-$ (take $r \to \infty$ while keeping v fix) and end at future null infinity $\mathcal{I}^+$ (take $r \to \infty$ while keeping u fix).

Exact Minkowski Spacetime ($\mathbb{M}^4$)

Enjoys exact finite-dimensional **Poincaré Symmetry**:

$$\text{Poincaré} = \text{Translations } (\mathbb{R}^4) \rtimes \text{Lorentz SO}(1,3)$$

- Exactly **10 generators** (4 spacetime translations $+6$ Lorentz boosts/rotations).

What are symmetries for Asymptotically Flat Spacetime (AFS)? Does it reduce back to Poincaré symmetry?

The Discovery (Bondi-Metzner-Sachs -1962)

Surprising Result: Asymptotically flat spacetime does **NOT** fall back to Poincaré!

- Enjoys an **infinite-dimensional** symmetry group:

$$\text{BMS}_4 = \textbf{Supertranslations } (\mathcal{S}) \rtimes \text{Lorentz SO}(1,3)$$

- Angle-dependent translations on the celestial sphere: $\alpha(\theta, \phi)$.

Asymptotically Flat Spacetime near $\mathscr{I}^+$

Bondi Metric Expansion in Complex Coordinates $(z, \bar{z})$

$$
\begin{aligned}
ds^2 = & -du^2 - 2\,du\,dr + 2r^2\gamma_{z\bar{z}}\,dz\,d\bar{z} \\
& + \frac{2m_{\mathrm{B}}}{r}du^2 + rC_{zz}\,dz^2 + \left[D^z C_{zz} + \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_{\mathrm{B}}\right) - \frac{1}{4}D_z\left(C_{zz}C^{zz}\right)\right)\right] du\,dz + \text{c.c.} \\
& + \dots
\end{aligned}
$$

$m_{\mathrm{B}}(u, z, \bar{z})$: Bondi Mass Aspect

Local energy density per unit solid angle at $\mathscr{I}^+$. Integrating over the 2-sphere gives the total Bondi mass $M_{\mathrm{B}}(u) = \frac{1}{4\pi}\int_{S^2} m_{\mathrm{B}}\,d\Omega$.

C_{AB} (C_{zz}): Gravitational Data

Symmetric, traceless tensor encoding the radiation degrees of freedom.

N_A (N_z): Angular Momentum Aspect

Encodes the local angular momentum density and momentum flux across the celestial sphere.

m_B and N_A is constrained through Einstein field equation.

Asymptotic Symmetries

Exact Symmetries $\Longrightarrow$ Isometries of Minkowski Metric $\eta_{\mu\nu}$

A vector field $\xi = \xi^\mu \partial_\mu$ generates an exact symmetry (isometry) if it preserves the metric:

$$\mathcal{L}_\xi \eta_{\mu\nu} = \xi^\rho \underbrace{\partial_\rho \eta_{\mu\nu}}_{=0} + \eta_{\rho\nu}\partial_\mu \xi^\rho + \eta_{\mu\rho}\partial_\nu \xi^\rho = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0 \quad \text{(Killing Equation)}$$

Taking a derivative $\partial_\rho(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu) = 0$ implies $\partial_\mu \partial_\nu \xi_\rho = 0 \Longrightarrow \xi^\mu(x)$ is at most **linear** in x^ν.

General Solution: The Poincaré Vector Field

$$\xi^\mu(x) = a^\mu + \omega^\mu{}_\nu x^\nu \quad \text{where} \quad \omega_{\mu\nu} \equiv \eta_{\mu\rho}\omega^\rho{}_\nu = -\omega_{\nu\mu}$$

4 Global Spacetime Translations (a^μ)

- Constant vector $a^\mu \in \mathbb{R}^{1,3}$ (4 parameters).

6 Lorentz Transformations ($\omega_{\mu\nu}$)

- Antisymmetric tensor $\omega_{\mu\nu} = -\omega_{\nu\mu} \Longrightarrow \binom{4}{2} = 6$ parameters.

Instead of exact isometry ($\mathcal{L}_\xi g_{\mu\nu} = 0$), we ask for ξ that obey the Asymptotic Killing equation:

$$\mathcal{L}_\xi g_{\mu\nu} \approx 0 \quad \text{as } r \to \infty$$

consistent with asymptotic flatness near $\mathcal{I}^+$.
Solving the gauge-fixing and fall-off conditions yields the general vector field $\xi = \xi^\mu \partial_\mu$:

$$\xi = \left(f + \frac{u}{2}\psi\right)\partial_u + \left(-\frac{r}{2}\psi + \frac{1}{2}D^2 f\right)\partial_r + \left(Y^z - \frac{1}{r}D^z f\right)\partial_z + \left(Y^{\bar{z}} - \frac{1}{r}D^{\bar{z}} f\right)\partial_{\bar{z}} + \mathcal{O}(r^{-1})$$

- **Supertranslations:** $f(z, \bar{z})$ is an *arbitrary real function* on the celestial sphere S^2.
- **Lorentz / Superrotations:** $Y^z(z)$ is a conformal Killing vector field on S^2 (with $\psi \equiv D_z Y^z + D_{\bar{z}} Y^{\bar{z}}$).

$\text{BMS}_4 = \text{Supertranslations} \rtimes \text{Lorentz } \text{SO}(1,3)$

Expanding $f(z, \bar{z})$ in spherical harmonics $Y_{\ell m}(\theta, \phi)$:

- $\ell = \mathbf{0}, \mathbf{1}$ (4 modes): Standard global spacetime translations P_μ (1 time +3 space).
- $\ell \geq \mathbf{2}$ (∞-many modes): **Supertranslations** — angle-dependent time shifts $u \to u + f(z, \bar{z})$.

Connection to Soft Theorems: BMS Ward Identities

Weinberg's Soft Graviton Theorem (1965)

Inserting an outgoing soft graviton with momentum $q = (\omega, \vec{q})$ gives a universal pole as $\omega \to 0$:

$$\lim_{\omega \to 0} \omega \, \mathcal{A}_{n+1} = \left(\sum_{i=1}^{n} \frac{p_i^{\mu} p_i^{\nu} \epsilon_{\mu\nu}}{p_i \cdot q} \right) \mathcal{A}_n$$

where $\mathcal{A}_n = \langle \text{out} | \mathcal{S} | \text{in} \rangle$ is the n-point hard scattering amplitude.

At the quantum level, BMS supertranslation symmetry parametrized by $f(z, \bar{z})$ requires the S-matrix to obey a **Ward-Takahashi identity**:

$$\langle \text{out} \,|\, [Q[f], \mathcal{S}] \,|\, \text{in} \rangle = 0 \implies \langle \text{out} \,|\, Q_{\text{out}}[f] \mathcal{S} - \mathcal{S} \, Q_{\text{in}}[f] \,|\, \text{in} \rangle = 0$$

He,Lysov,Mitra,Strominger (2015)

BMS Supertranslation Ward Identity $\Longleftrightarrow$ Weinberg's Soft Graviton Theorem

BMS Symmetry & Gravitational Memory Effect

The Displacement Gravitational Memory Effect

When a burst of gravitational radiation passes through two freely falling test masses, it leaves behind a **permanent physical displacement**:

$$\Delta x^A = \frac{1}{2r}\Delta C^{AB} x_0^B \quad \text{where} \quad \Delta C_{AB} \equiv C_{AB}(u=+\infty) - C_{AB}(u=-\infty) = \int_{-\infty}^{+\infty} N_{AB}\, du$$

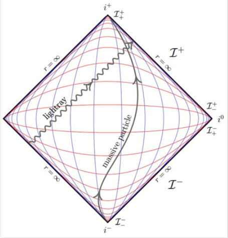
The Memory Observable (ΔC_{AB})

A change, ΔC_{AB} is generated by supertranslations via

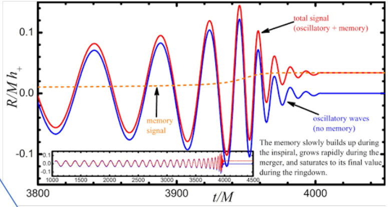
$$\Delta C_{AB} = -2D_A D_B f + \gamma_{AB} D^2 f\,.$$

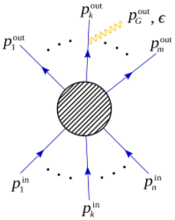
In quantum picture, gravitational memory refer to as a **vacuum transition** between degenerate BMS vacua

The infrared triangle [Strominger'18]



Asymptotic symmetry





Soft theorem

Memory effect

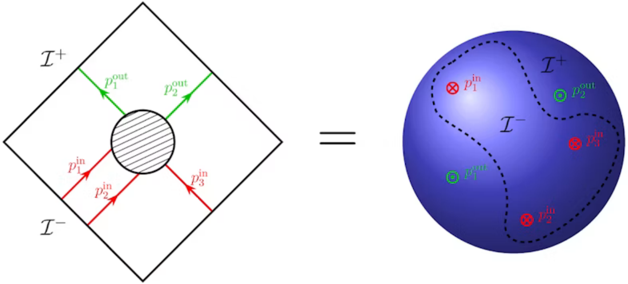
Review of Celestial holography

Celestial holography: the 4D S-matrix in asymptotically flat spacetime is encoded in a 2D celestial conformal field theory [Pasterski,Shao,Strominger'16]

The boundary of the flat space is null infinity $\mathcal{I}^{\pm} = \mathbb{R} \times S^2$. The 4D

Lorentz group $SO^+(1,3)$ is isomorphic to $PSL(2,\mathbb{C})$, The action of the Lorentz group on the celestial sphere is realized as Möbius transformations

$$z \to \frac{az+b}{cz+d} \quad \text{with } ad - bc = 1.$$



4D S-matrix = 2D correlation function

Mapping 4D Amplitudes to 2D Celestial Correlators

A massless 4D momentum p^μ ($p^2 = 0$) is factorized into an overall energy ω and a reference vector $q^\mu(z, \bar{z})$ pointing to a point $(z, \bar{z})$ on the celestial sphere S^2:

$$p^\mu = \omega\, q^\mu(z, \bar{z}) \quad \text{with} \quad q^\mu(z, \bar{z}) = (1 + z\bar{z},\ z + \bar{z},\ -i(z - \bar{z}),\ 1 - z\bar{z})\,.$$

Under Lorentz boosts $SL(2, \mathbb{C})$, energy scales as $\omega \to |cz + d|^2 \omega$.

Change of Basis: The Mellin Transform

A **Mellin transform** on the scattering states:

$$\phi_{\Delta,J}(z, \bar{z}, X) = \int_0^\infty d\omega\, \omega^{\Delta - 1}\, a_J(\omega, z, \bar{z}, X)$$

This transforms 4D momentum eigenstates (plane waves $e^{\pm i p \cdot X}$) into **Conformal Primary Wavefunctions** $\phi_{\Delta,J}(z, \bar{z}, X)$.

The Celestial Holographic Dictionary

$$\mathcal{A}_n^{\text{celestial}}(\Delta_i, z_i, \bar{z}_i) = \left(\prod_{i=1}^{n} \int_0^{\infty} d\omega_i\, \omega_i^{\Delta_i - 1}\right) \mathcal{A}_n(\omega_i, z_i, \bar{z}_i) = \left\langle \mathcal{O}_{\Delta_1, J_1}(z_1, \bar{z}_1) \cdots \mathcal{O}_{\Delta_n, J_n}(z_n, \bar{z}_n) \right\rangle_{\text{CCFT}_2}$$

where $\Delta \in 1 + i\mathbb{R}$ [Pasterski, Shao, Strominger '17].

Under a 4D Lorentz transformation, the celestial coordinates undergo Möbius transformations $z \to \frac{az+b}{cz+d}$ (with $ad - bc = 1$).

Each operator $\mathcal{O}_{\Delta,J}$ transforms as a **2D conformal primary field** with weights $(h, \bar{h}) = \left(\frac{\Delta+J}{2}, \frac{\Delta-J}{2}\right)$:

$$\mathcal{O}_{\Delta,J}(z, \bar{z}) \longrightarrow \mathcal{O}_{\Delta,J}\left(\frac{az+b}{cz+d}, \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}\right) = (cz+d)^{2h}(\bar{c}\bar{z}+\bar{d})^{2\bar{h}}\, \mathcal{O}_{\Delta,J}(z, \bar{z})$$

Consequently,

$$\mathcal{A}_n^{\text{celestial}}\left(\Delta_i, \frac{az_i+b}{cz_i+d}, \frac{\bar{a}\bar{z}_i+\bar{b}}{\bar{c}\bar{z}_i+\bar{d}}\right) = \prod_{i=1}^{n}(cz_i+d)^{2h_i}(\bar{c}\bar{z}_i+\bar{d})^{2\bar{h}_i}\, \mathcal{A}_n^{\text{celestial}}(\Delta_i, z_i, \bar{z}_i)$$

Summary

- **Asymptotic Symmetries of Flat Spacetime**
  - Asymptotically flat spacetime enjoys infinite-dim enhanced Poincaré symmetry, namely **BMS group**:

$$\mathrm{BMS}_4 = \textbf{Supertranslations } (\mathcal{S}) \rtimes \text{Lorentz } SO(1,3)$$

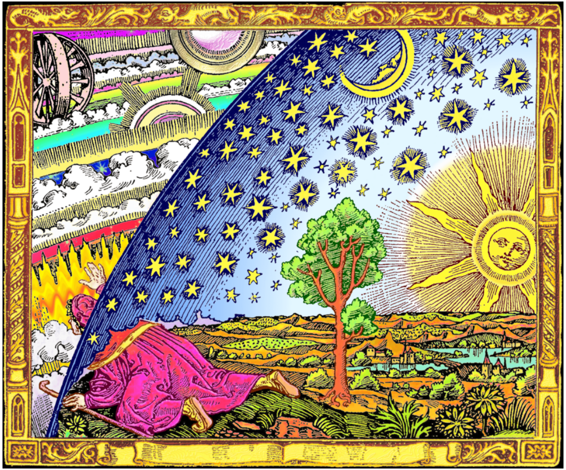
- **Strominger's Infrared Triangle**
  - A fundamental tripartite equivalence connects three distinct areas of physics:

BMS Ward Identities $\iff$ **Weinberg's Soft Theorem** $\iff$ **Gravitational Memory**

- **Celestial Holography Dictionary**
  - Performing a **Mellin transform** ($\omega \to \Delta \in 1 + i\mathbb{R}$) maps 4D flat-space scattering amplitudes into 2D conformal correlation functions on the celestial sphere:

$$\mathcal{A}_n^{4D}(\omega_i, z_i, \bar{z}_i) \xrightarrow[\text{Transform}]{\text{Mellin}} \left\langle \mathcal{O}_{\Delta_1, J_1}(z_1, \bar{z}_1) \cdots \mathcal{O}_{\Delta_n, J_n}(z_n, \bar{z}_n) \right\rangle_{\mathrm{CCFT}_2}$$

Thank You!
Questions & Discussion



Picture of French astronomer Camille Flammarion's 1888 book L'atmosphère: météorologie populaire. It symbolizes humanity's quest to cross boundaries, seek hidden truths, and discover knowledge