Cosmology of gravity theories beyond General Relativity

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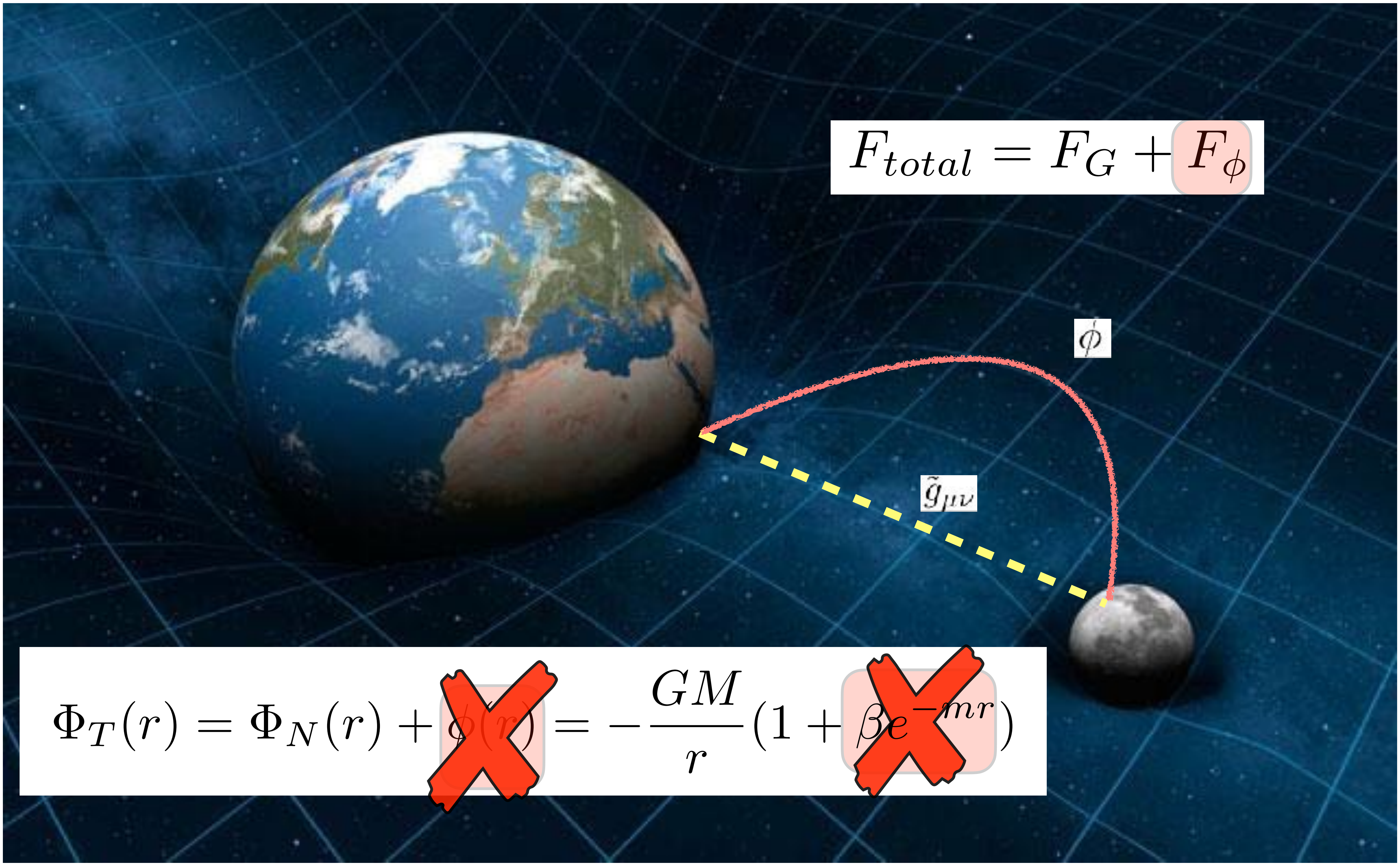
LECTURE 2

Screening Mechanisms

Extending General Relativity

extra degree of freedom driving acceleration



Screening Mechanisms in Scalar-Tensor Gravity

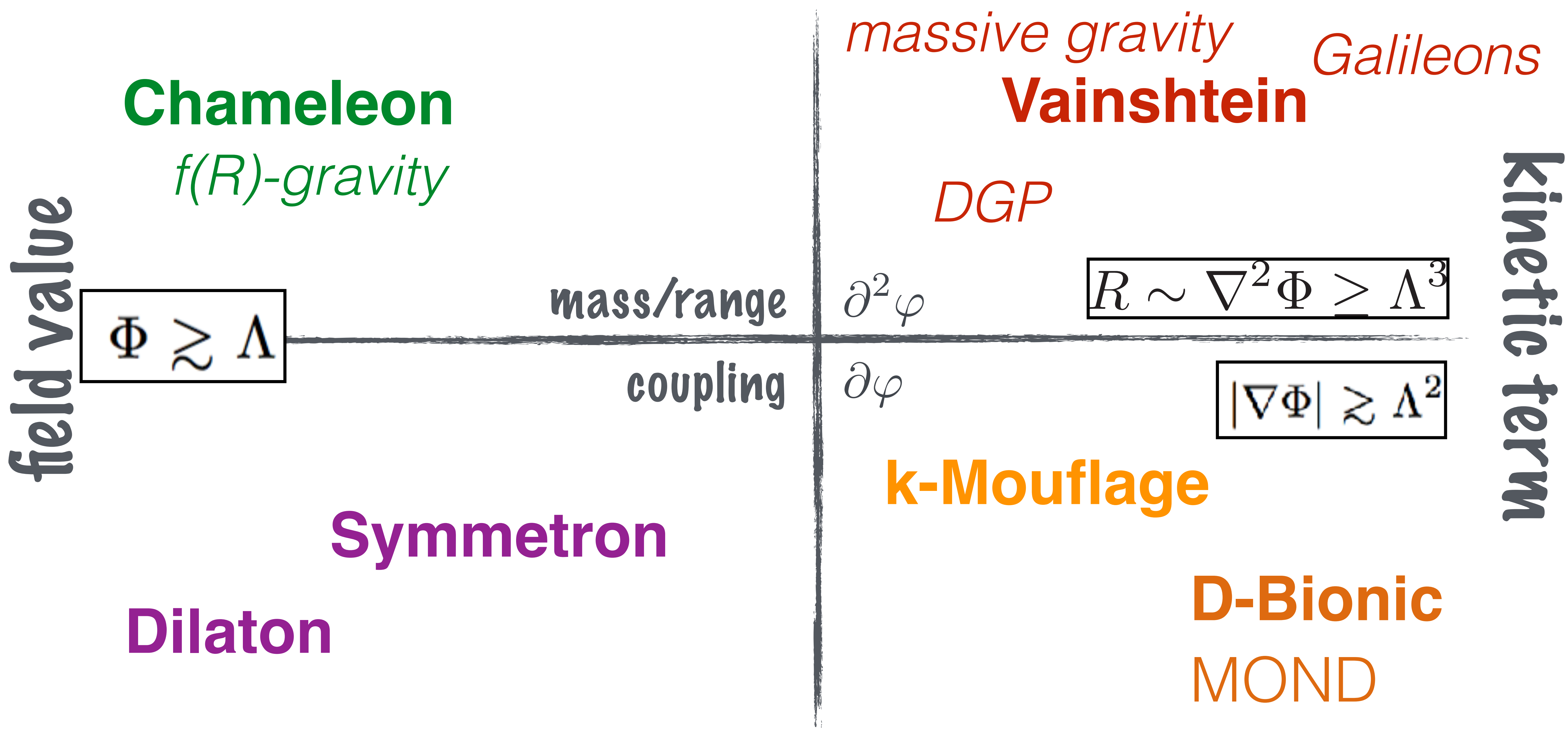
$$\Phi_T(r) = -\frac{GM}{r}(1 + \cancel{\boxed{\beta e^{-mr}}})$$

$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \ldots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T^\mu_\mu$$

Screening mechanisms classification

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \ldots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T^\mu_\mu$$

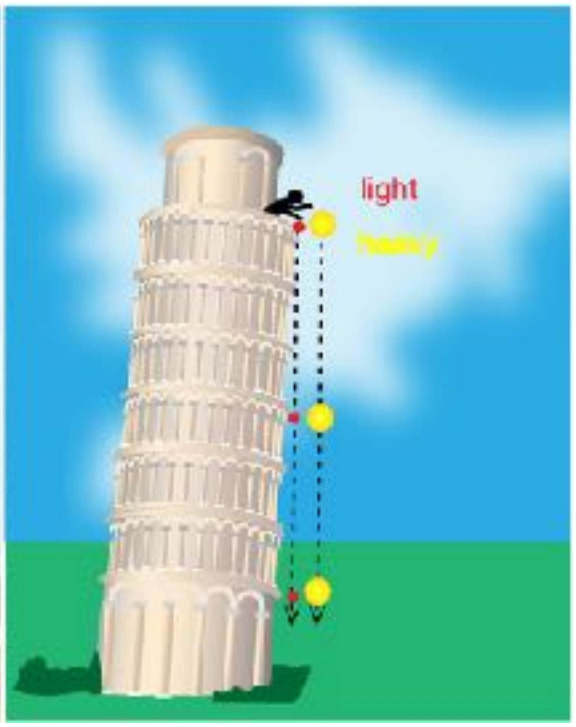
$$V(r) = -\frac{g^2(\bar\phi)}{Z(\bar\phi)c_s^2(\bar\phi)} \frac{e^{-\frac{m(\bar\phi)}{\sqrt{Z(\bar\phi)}c_s(\bar\phi)}r}}{4\pi r}\mathcal{M}$$

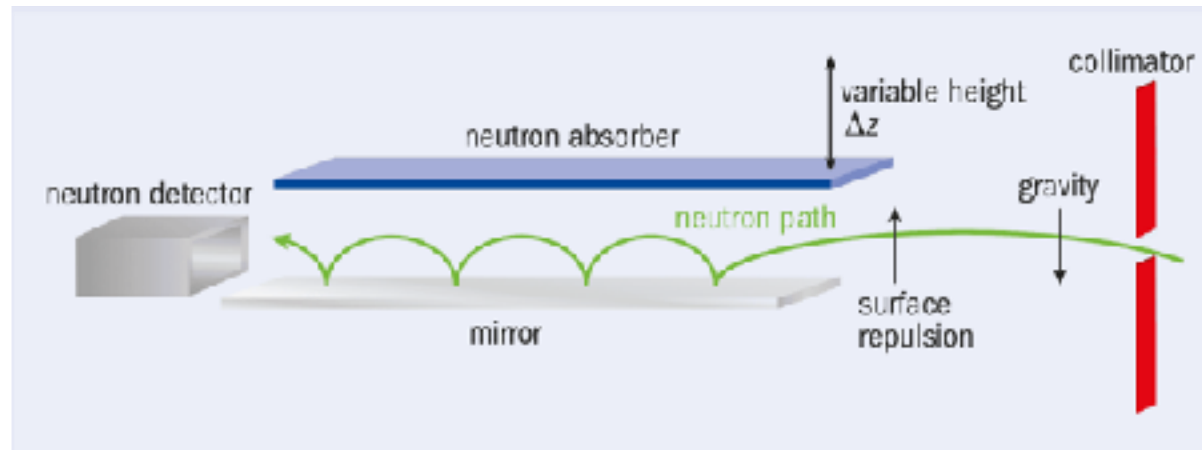


Why and where do we need screening mechanisms?

Laboratory tests

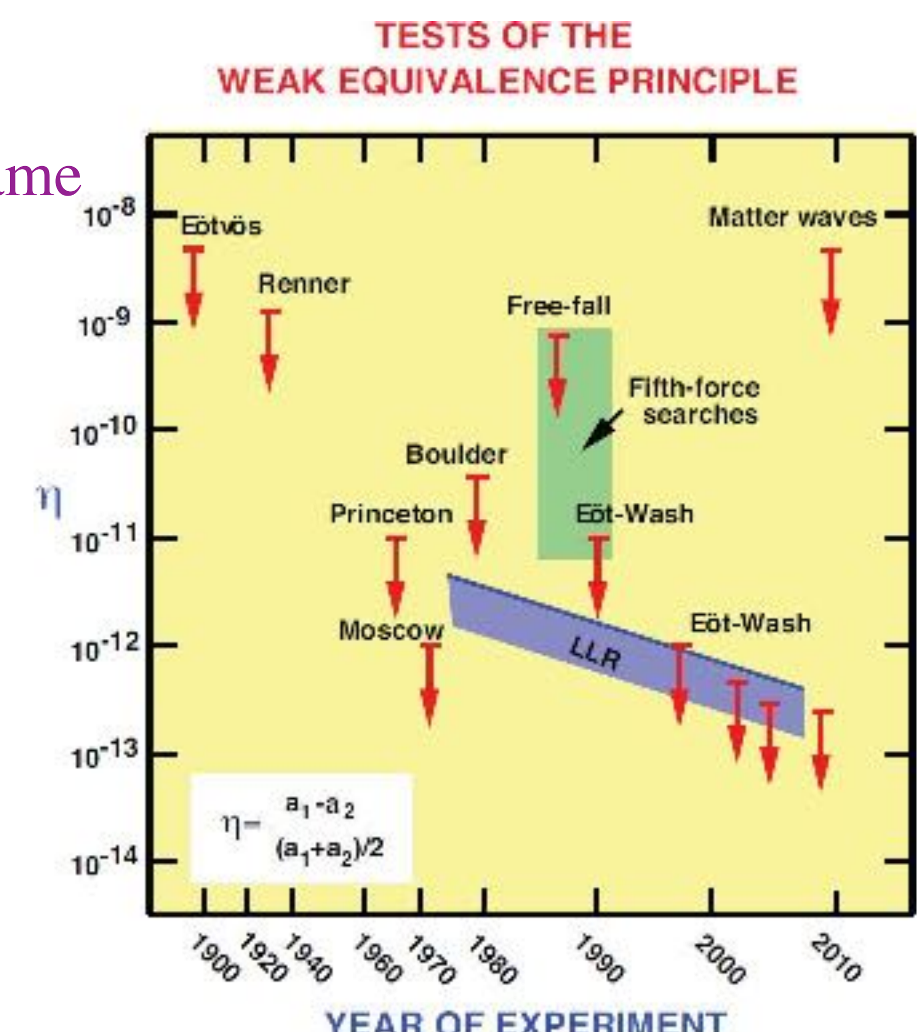
Robustly confirm the predictions of General Relativity

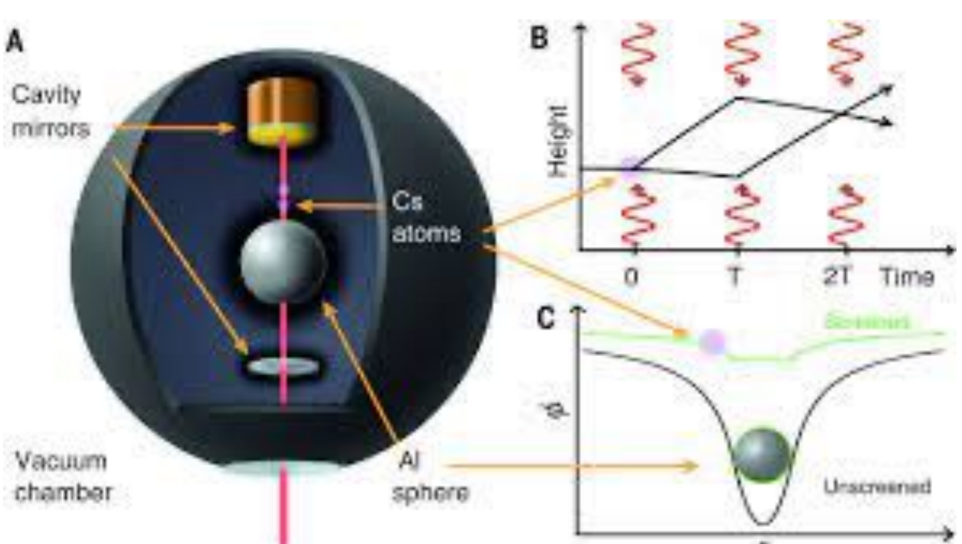




Equivalence Principle:

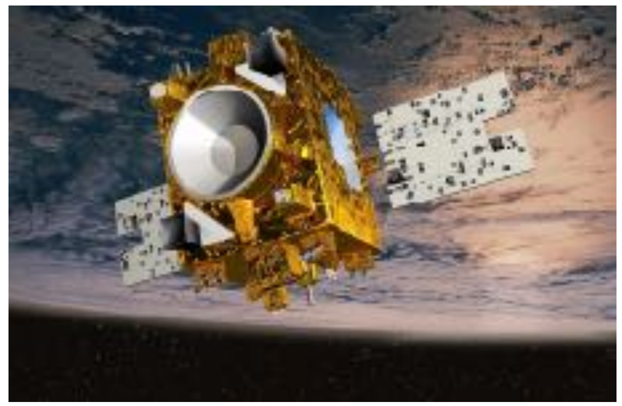
All bodies fall in gravitational field with same acceleration regardless of mass or internal structure

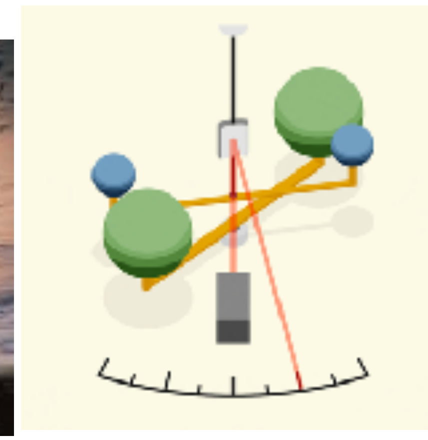




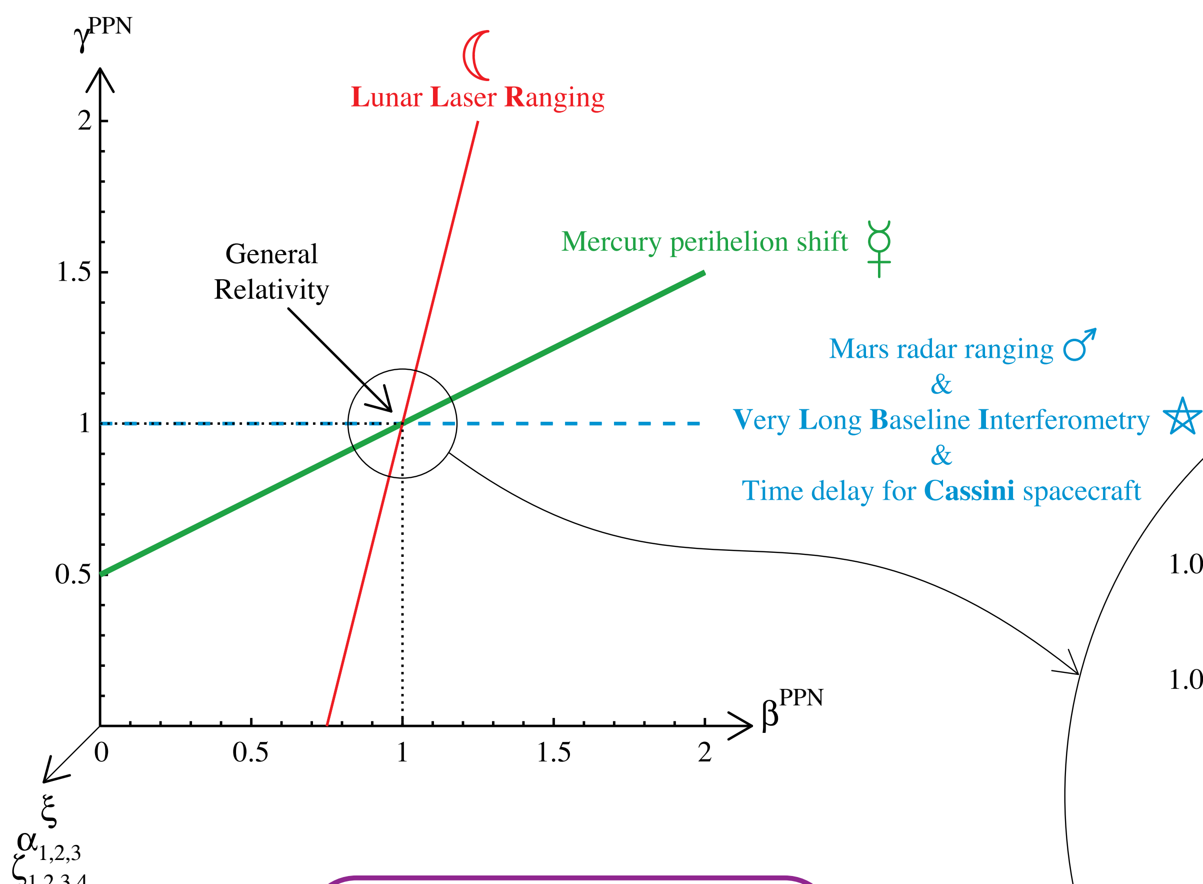
Solar System tests
Confirm General Relativity

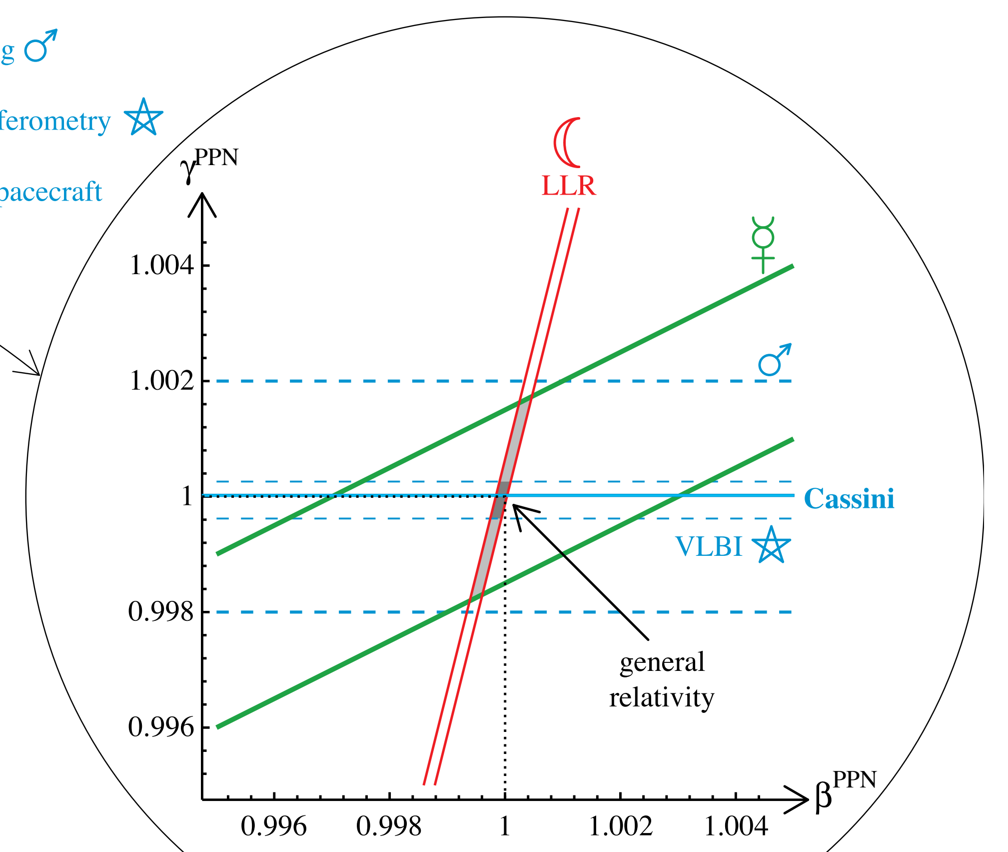
MICROSCOPE



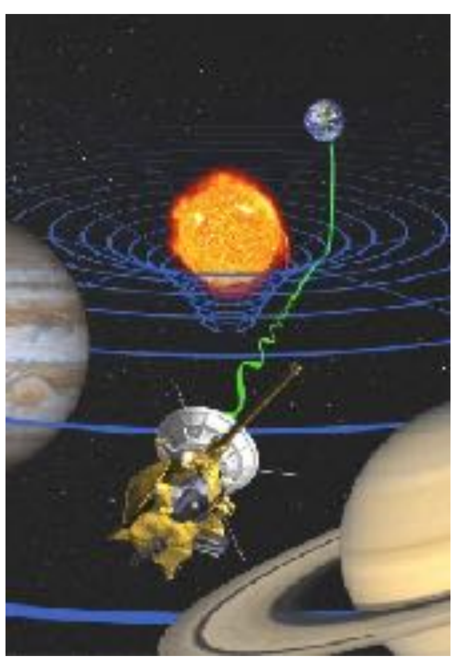


Conclusion of experimental tests in the **P**arametrized **P**ost-**N**ewtonian formalism





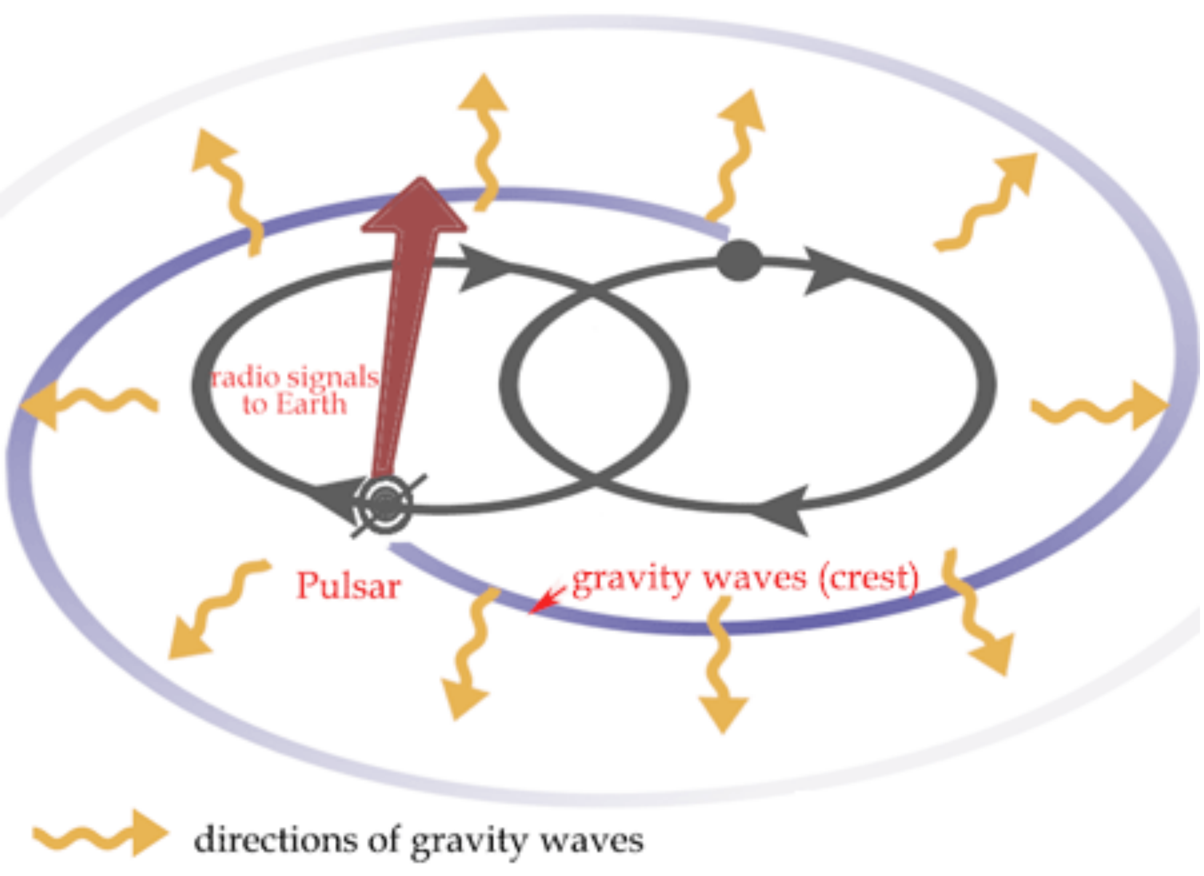
GENERAL RELATIVITY
is essentially the only
theory consistent with
weak-field experiments

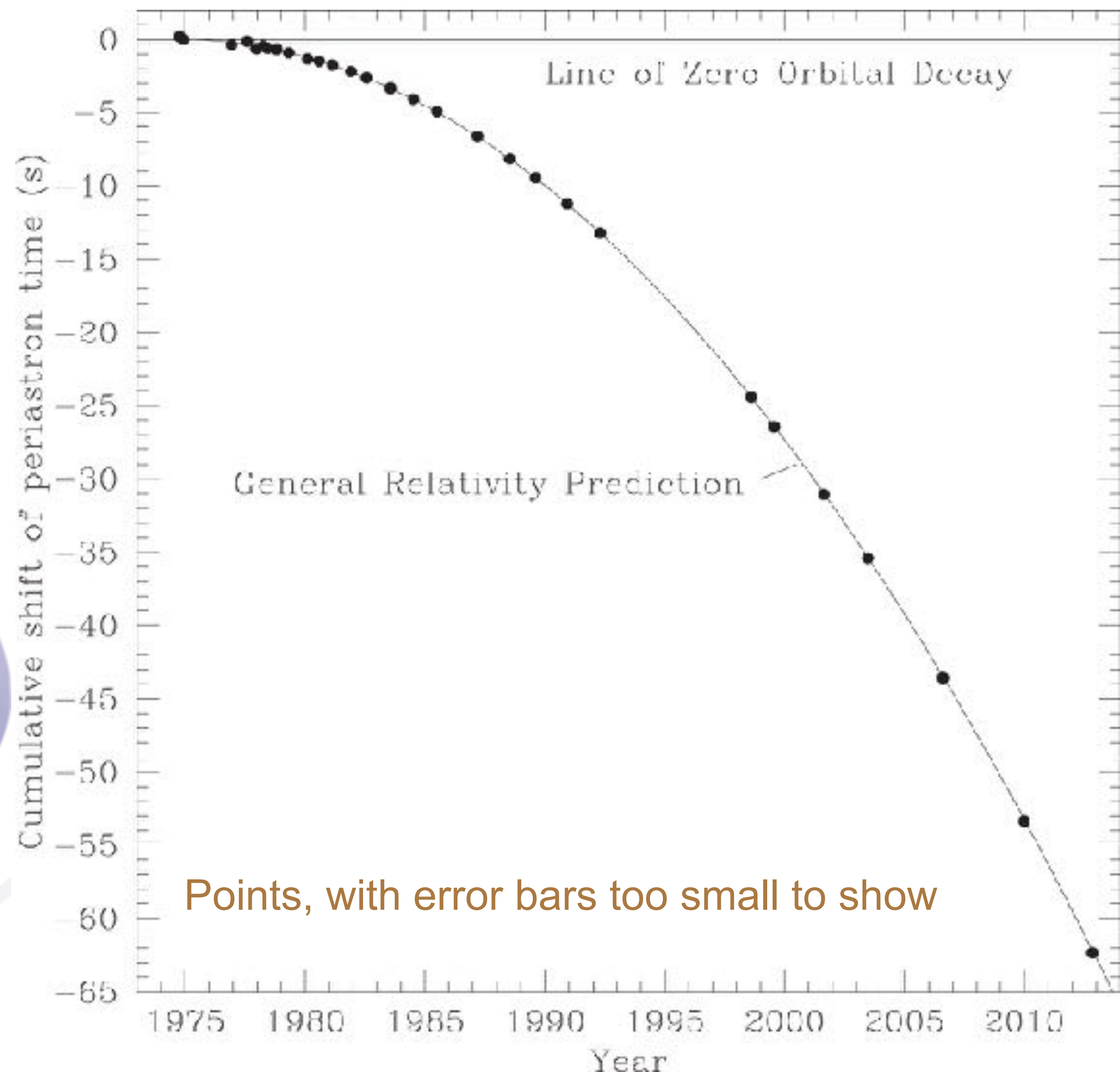


CASSINI

Astrophysical Observations
Robustly confirm the predictions of General Relativity

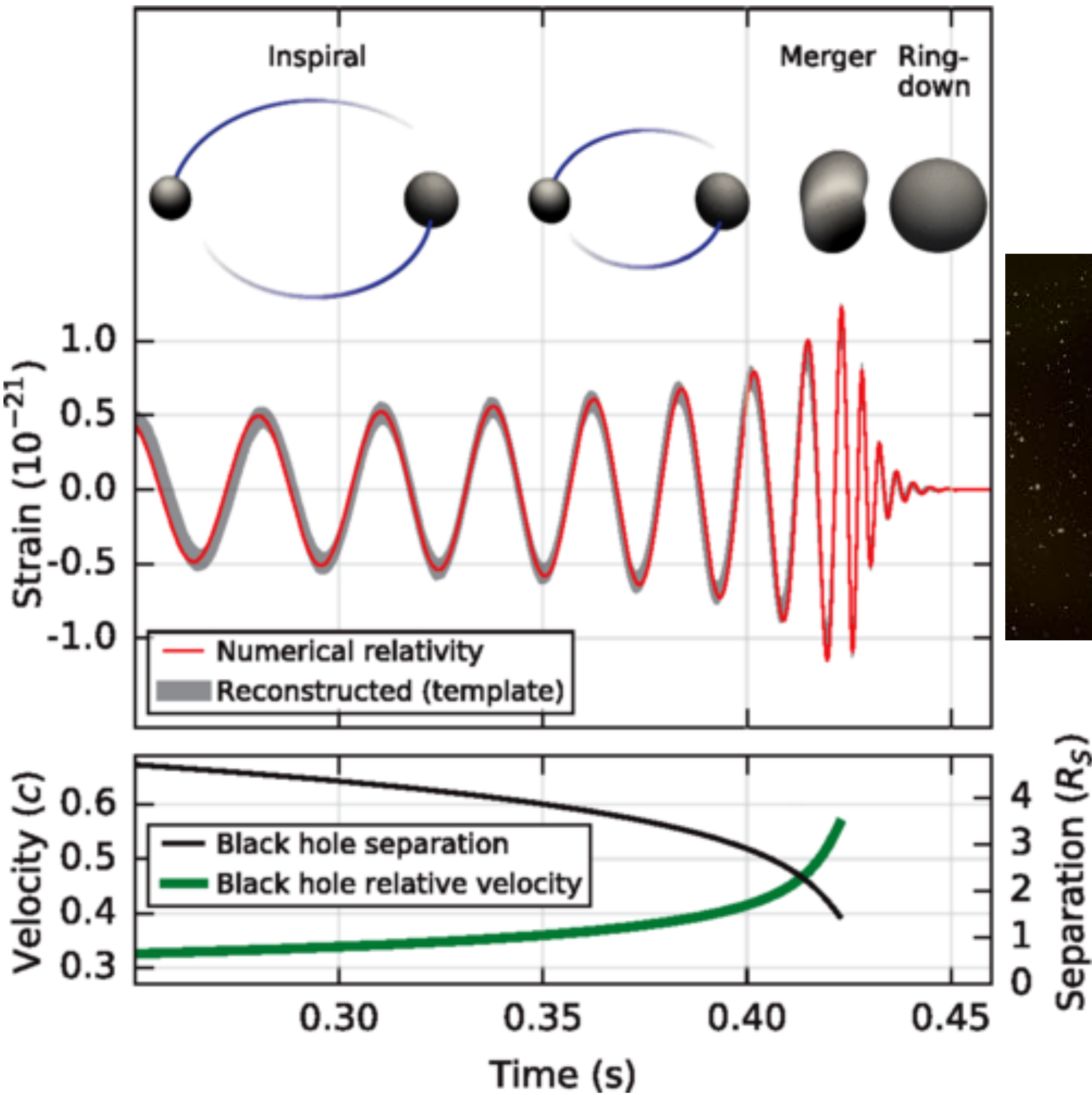
Pulsar (neutron star emitting EM 17/sec)
Mass 1.4 M_{sun} and 1.3 M_{sun}
Period 8 hours
Decay 75 microsec per orbit

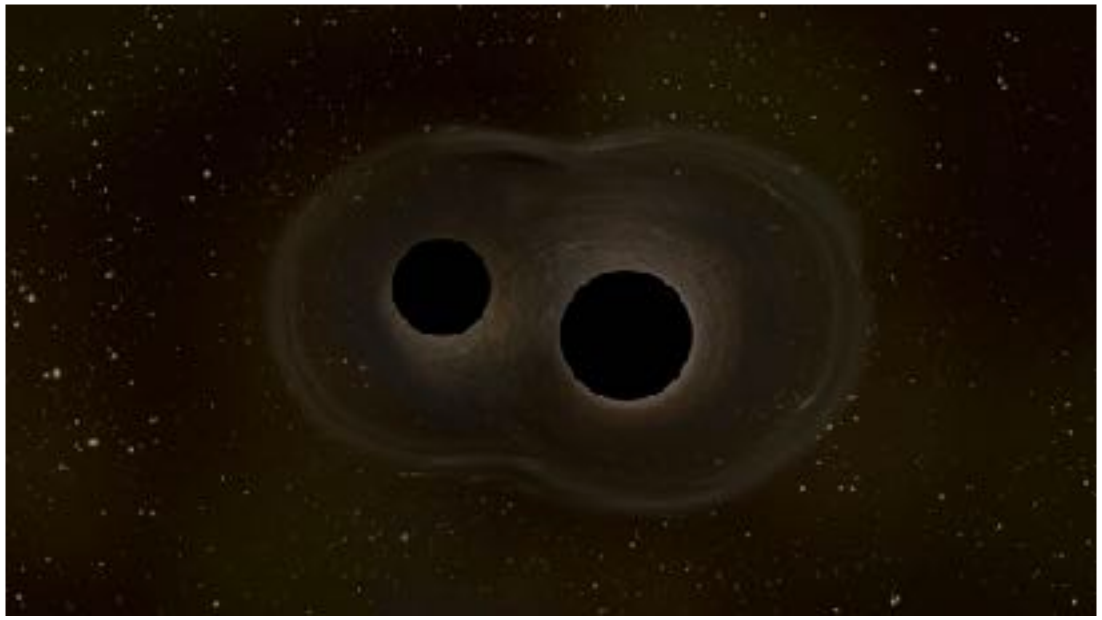




Points, with error bars too small to show

Compact Objects
Properties according to General Relativity



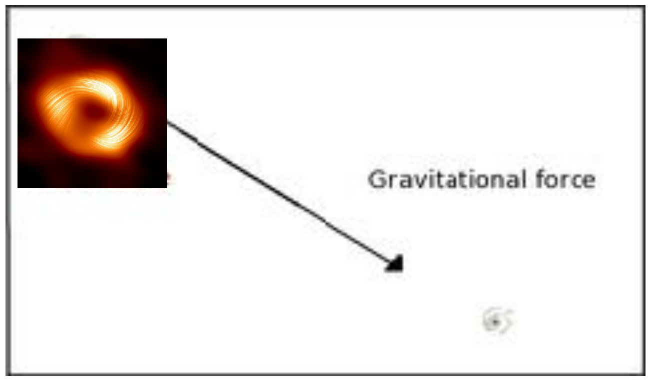


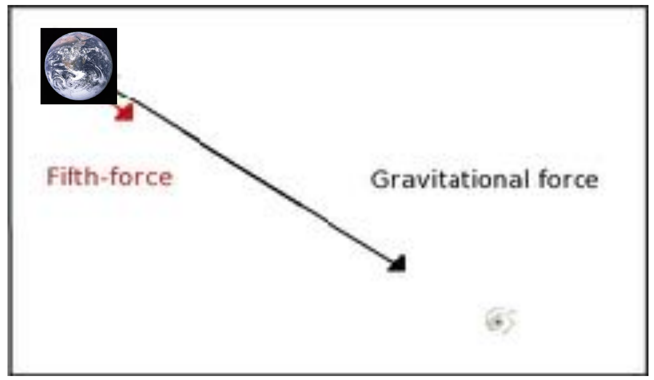
Why and where do we need screening mechanisms?

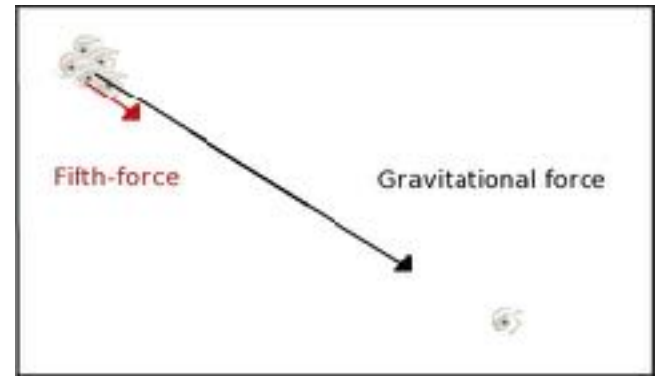
Modified Gravity must recover
General Relativity in high curvature regimes

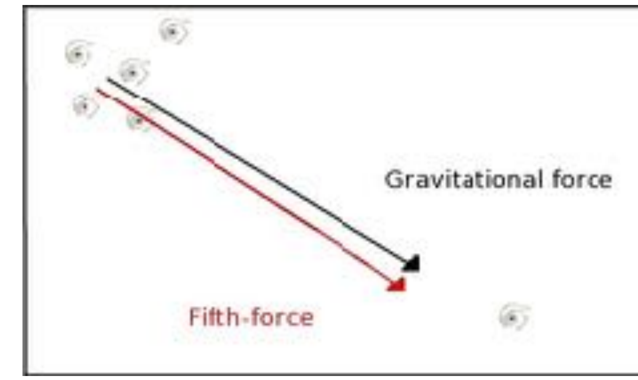
Extra-degree of freedom screened in high curvature

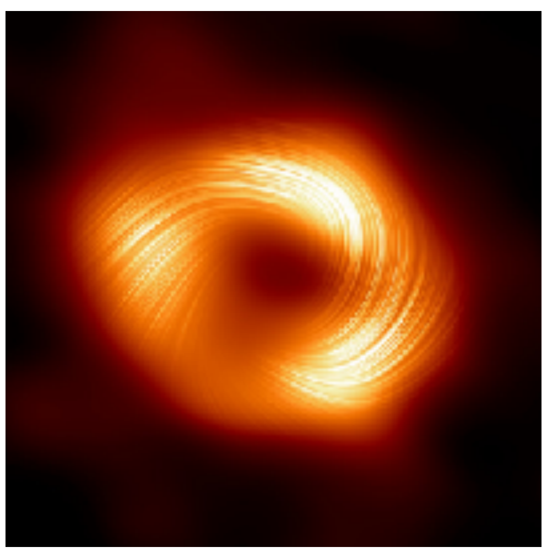
(In most gravity theories equivalent to high density)











Fifth forces are described by it range and strength

(and as you will see they will have a “onset scale”!)

Chameleon Mechanism

Idea: Make the mass of the scalar field depend on the local matter density.

Action

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] + \mathcal{L}_{\rm m}[\tilde{g}, \psi]$$

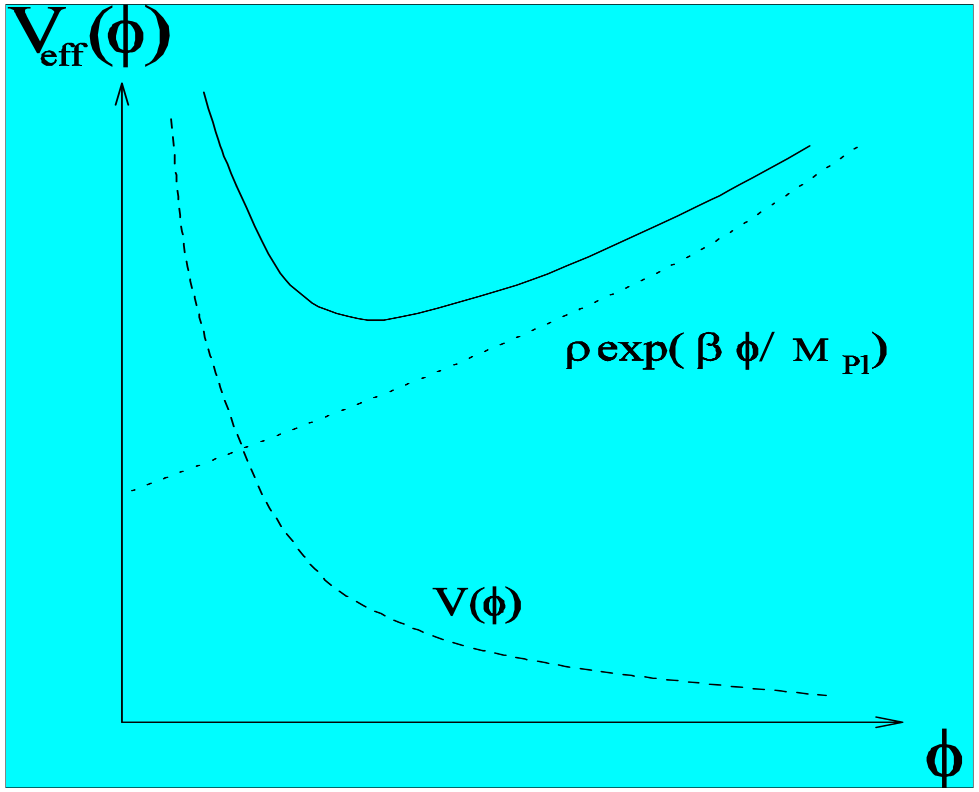
- Matter fields ψ
- Minimal coupling of ψ to the metric: $\tilde{g}_{\mu\nu} = A^2(\phi)\, g_{\mu\nu}$

Chameleon Mechanism: Effective Potential

Around a spherical body: spherical symmetry, static, flat space

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi}{\mathrm{d}r} = V_{,\phi} + A_{,\phi}\,\rho$$

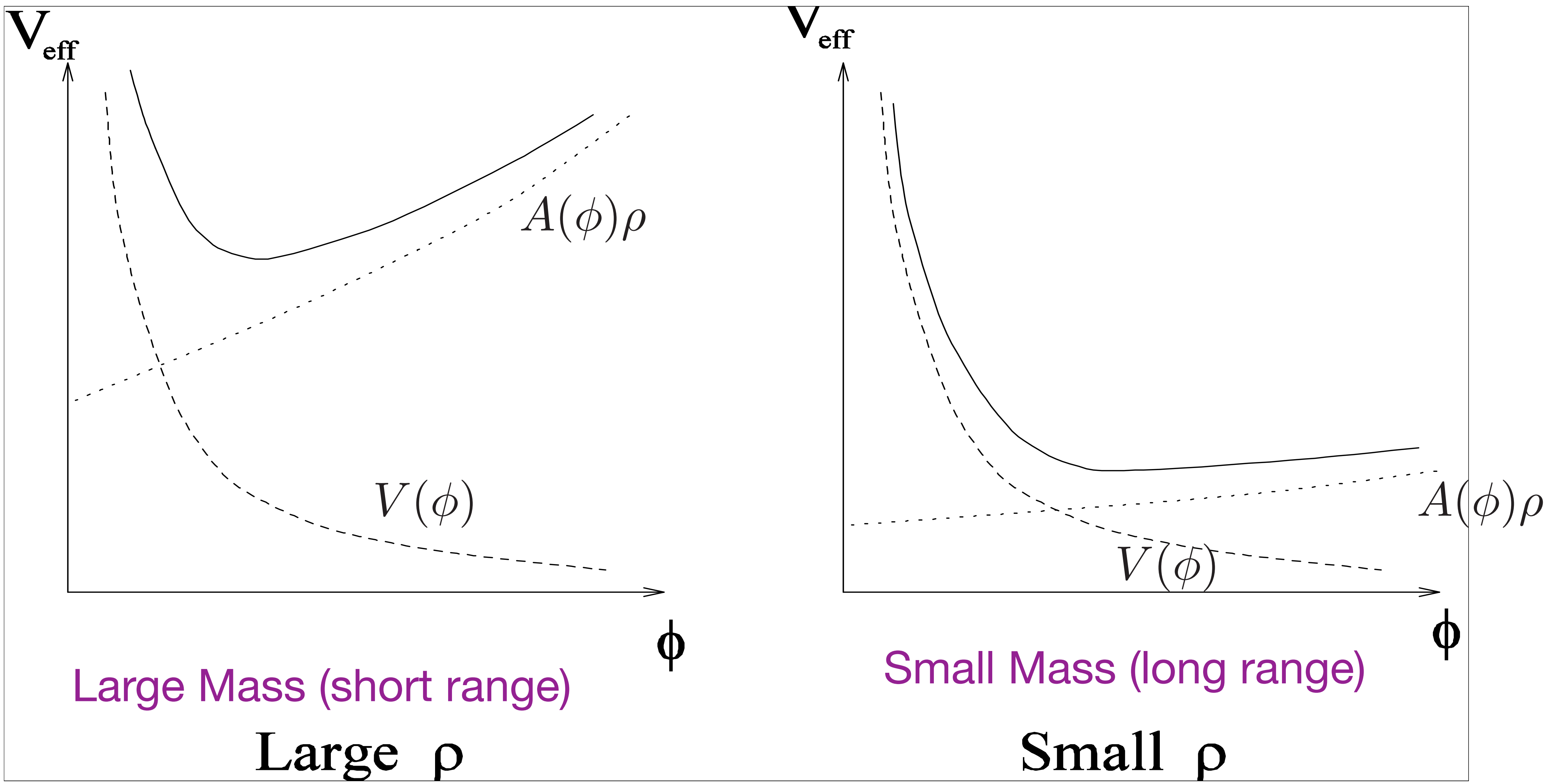
where $\rho = A^3\tilde{\rho}$ is the density conserved in the Einstein frame.



Chameleon Mass Not Constant

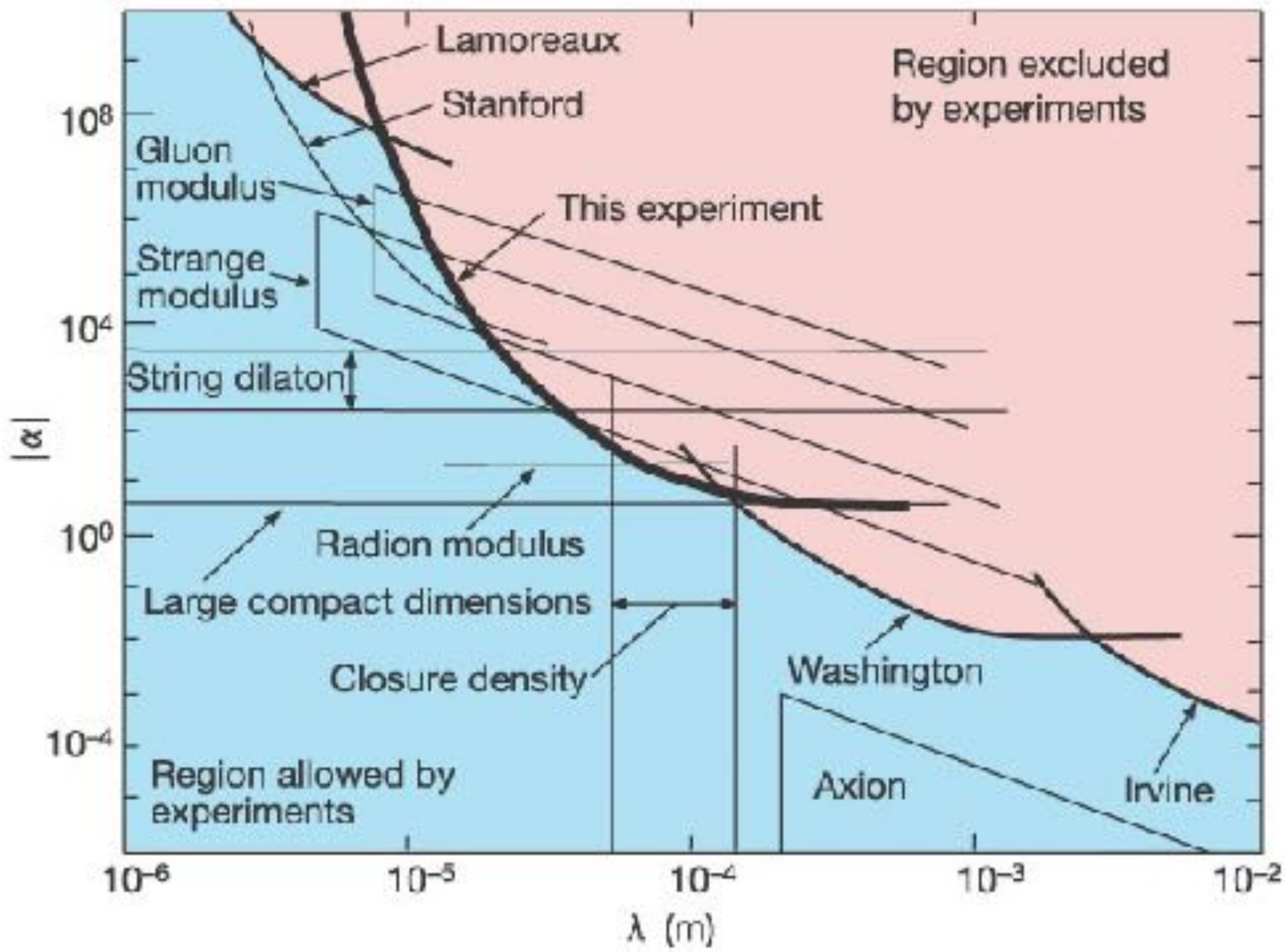
A chameleon field has mass: $m_\phi = \sqrt{V''_{eff}}$
Not constant but changes with local matter density

$$V_{eff}(\phi) = V(\phi) + \rho_m A(\phi)$$



Bounds on Fields Coupled to Matter

Long et al. (2004)



$$V(r) = -\frac{m_1 m_2}{8\pi M_{Pl}^2}[1 + \alpha \frac{e^{-r/\lambda}}{r}]$$

$$\alpha \sim 1/M_{Pl} \to 10^{-4} m$$

Modified Gravity either:

1) strong coupling and very short range

2) very weak coupling and long range

Chameleon Screening

(Khoury & Weltman 2004)

Mass/Range of field depends on local density

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{\mathrm{matter}}\left[A^2(\phi)g_{\mu\nu}, \psi\right]$$

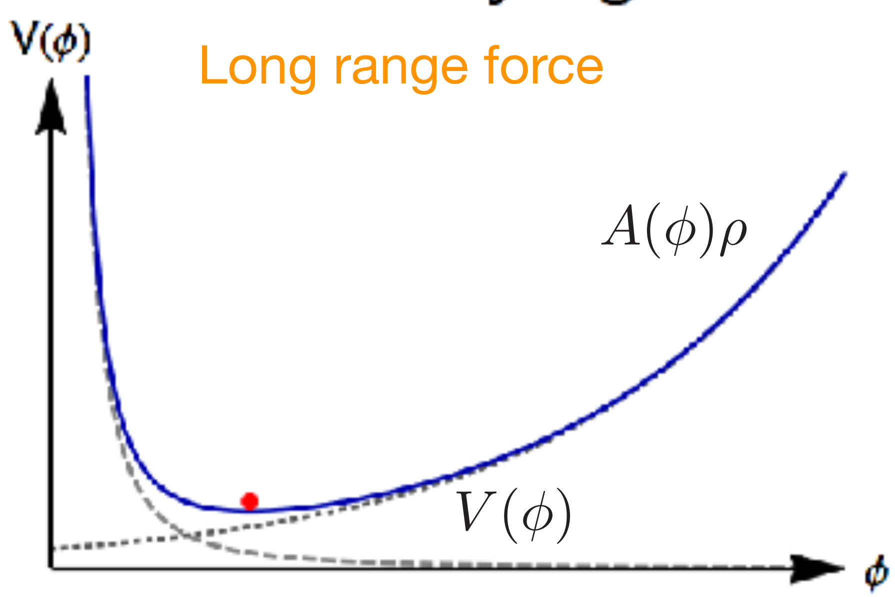
$$A(\phi) \simeq 1 + \xi\frac{\phi}{M_{\mathrm{Pl}}} \qquad V(\phi) = \frac{M^{4+n}}{\phi^n}$$

$$V = -\frac{GM}{r}\left(1 + \alpha e^{-mr}\right)$$

$$m_{\mathrm{eff}}^2(\bar\phi) = V^{\mathrm{eff}}_{,\phi\phi}(\bar\phi) = V_{,\phi\phi}(\bar\phi) + A_{,\phi\phi}(\bar\phi)\rho$$

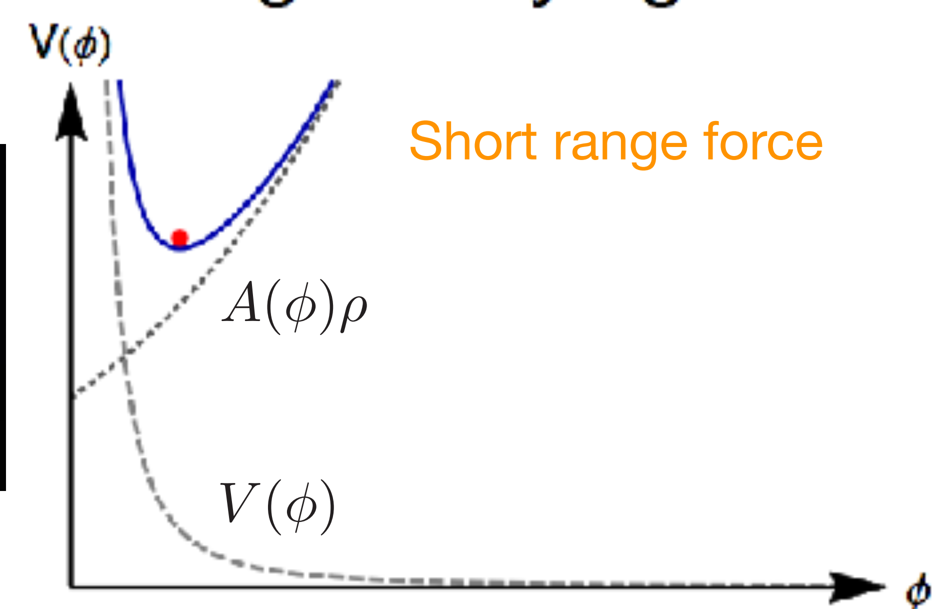
Mass

Low density region



$V''(\phi) \ll 1 \rightarrow$ Unscreened

High density region

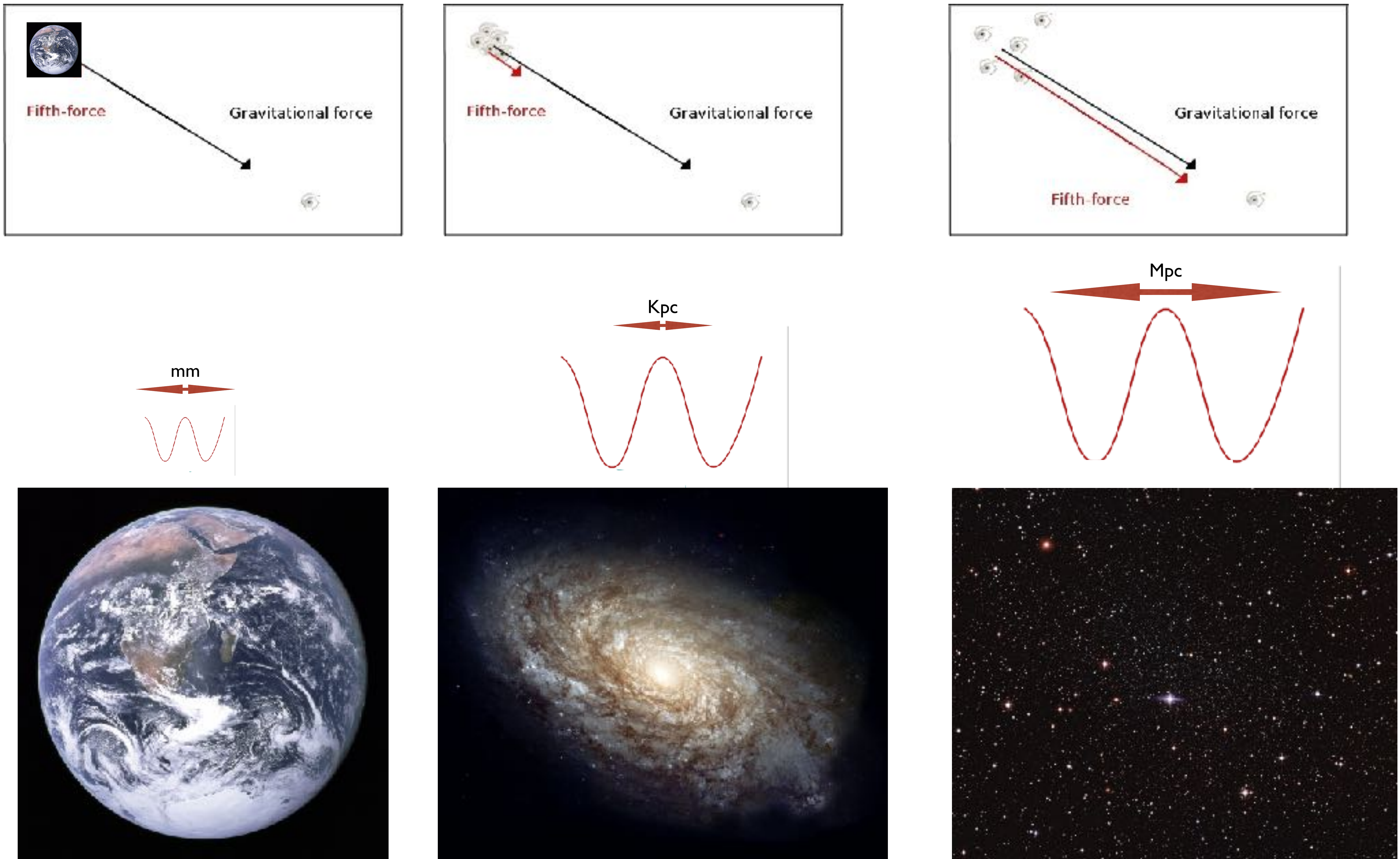


$V''(\phi) \gg 1 \rightarrow$ Screened

Chameleon mechanism

(Khoury & Weltman 2004)

Range of dark force depends on local environment



Symmetron Screening

Strength of fifth force depends on local density

$$S=\int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right)+S_{\rm matter}\left[A^2(\phi)g_{\mu\nu},\psi\right]$$

$$V=-\frac{GM}{r}(1+\alpha e^{-mr})$$

$$A(\phi)=1+\frac{1}{2M^2}\phi^2$$

coupling

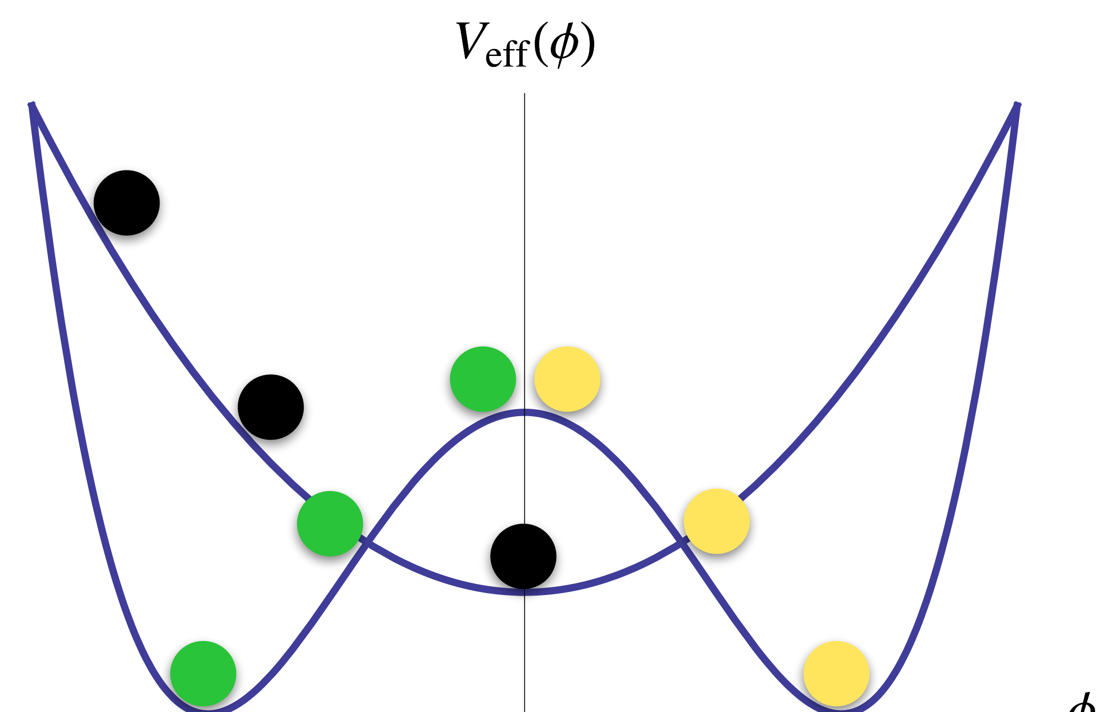
$$V_{\rm eff}(\phi)=\frac{1}{2}\left(\frac{\rho}{M^2}-\mu^2\right)\phi^2+\frac{1}{4}\lambda\phi^4$$

(High density)

No coupling!

$$A(\phi)=1$$

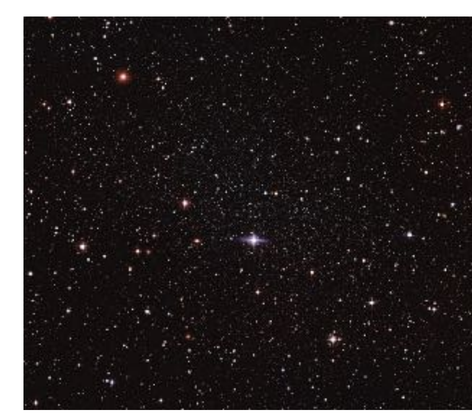
$$G_{eff}=G$$



(Low density)

coupled!

$$A(\phi)=1+\frac{1}{2M^2}\phi_{\rm vev}^2$$



$$G_{eff}=G(\phi)$$

Symmetron mechanism

Strength of fifth force depends on local environment (Hinterbicheler & Khoury 2010)

F(R) Gravity

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]$$

F(R) Gravity

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)

Modified Einstein Equation

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved

$$G_{\alpha\beta} + f_R R_{\alpha\beta} - \left(\frac{f}{2} - \Box f_R \right) g_{\alpha\beta} - \nabla_\alpha \nabla_\beta f_R = 8\pi G \; T_{\alpha\beta}$$

Modified Einstein Equation

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved

$$G_{\alpha\beta} + f_R R_{\alpha\beta} - \left(\frac{f}{2} - \Box f_R\right) g_{\alpha\beta} - \nabla_\alpha \nabla_\beta f_R = 8\pi G\, T_{\alpha\beta}$$

- Trace can be interpreted as a scalar field equation for f_R with a density-dependent effective potential ($p = 0$)

$$3\Box f_R + f_R R - 2f = R - 8\pi G \rho$$

- For small deviations, $|f_R| \ll 1$ and $|f/R| \ll 1$,

$$\Box f_R \approx \frac{1}{3}(R - 8\pi G \rho)$$

f(R) gravity conformally equivalent to scalar-tensor

$f(R)$ modifies GR by replacing the Ricci scalar, R, in the gravitational action with a more general function $f(R)$,

$$S = \int \mathrm{d}^4x \sqrt{-g}\,\frac{M_{\rm pl}}{2}\, f(R) \,+\, S_m.$$

f(R) gravity conformally equivalent to scalar-tensor

The effective potential is then $V(\varphi)$,

$$V(f_R) = \frac{M_{\rm pl}}{2} \frac{R\dfrac{\partial f(R)}{\partial R} - f(R)}{\dfrac{\partial f(R)}{\partial R}}.$$

By specifying the form of $f(R)$, the effective potential will trap the field in regions of high density.

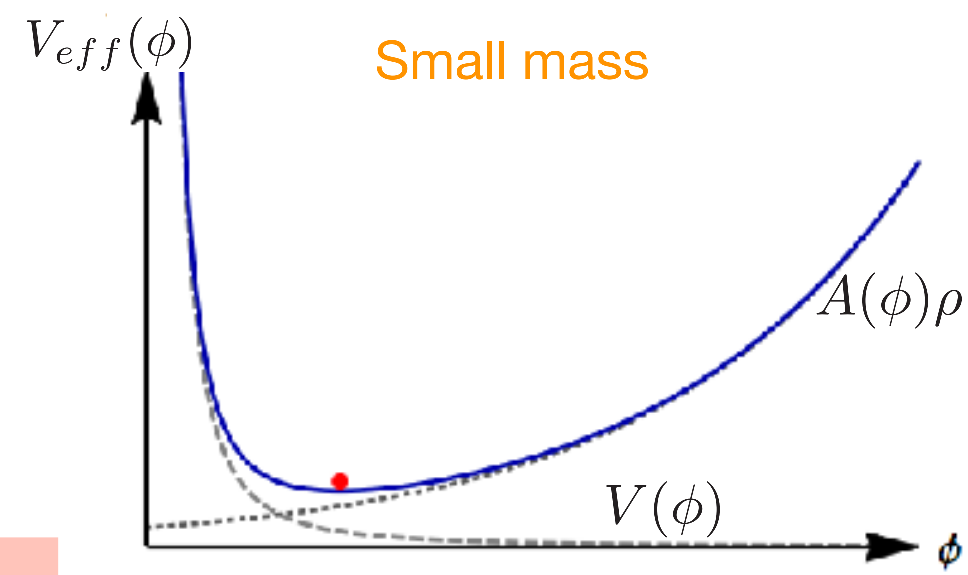
f(R) gravity with Chameleon screening

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{\rm matter}\left[A^2(\phi)g_{\mu\nu}, \psi\right]$$

$$m_{\rm eff}^2(\bar\phi) = V_{,\phi\phi}^{\rm eff}(\bar\phi) = V_{,\phi\phi}(\bar\phi) + A_{,\phi\phi}(\bar\phi)\rho$$

$$A(\phi) \simeq 1 + \xi\frac{\phi}{M_{\rm Pl}} \qquad V(\phi) = \frac{M^{4+n}}{\phi^n}$$

Low density region



$V''(\phi) \ll 1 \rightarrow$ Unscreened

Chameleon f(R)-gravity

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\Big(R + f(R)\Big) + S_{\rm matter}[g_{\mu\nu}, \psi]$$

$$f(R) = -m^2\frac{c_1\left(\dfrac{R}{m^2}\right)^n}{c_2\left(\dfrac{R}{m^2}\right)^n + 1},$$

Gravity beyond GR:
Cosmological Perturbations Theory

Cosmological Perturbation Theory

f(R) Gravity

- f(R) gravity $$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + L_m \right]$$

Gravitational Split!

Hu-Sawicki model

- No cosmological constant $f(R) \to 0, R \to 0$ (in the future)

Drives acceleration nowadays!

Background Cosmology

- Background cosmology Ω_Λ, f_{R0}

Modified Friedmann equation:

$$H^2 - f_R(HH' + H^2) + \frac{1}{6} f + H^2 f_{RR} R' = \frac{8\pi G}{3}\rho \qquad ' = \frac{d}{d\ln a}$$

Structure formation in GR

$$ds^2 = a^2(1 + 2\Phi)d\eta^2 - (1 - 2\Psi)d\vec{x}^2$$

$$\begin{aligned} \delta R &= -8\pi G \delta\rho \\ \nabla^2 \Phi &= 4\pi G a^2 \delta\rho \end{aligned}$$

Structure formation in f(R)

$$ds^2 = a^2(1 + 2\Phi)d\eta^2 - (1 - 2\Psi)d\vec{x}^2$$

$$\delta R = -8\pi G\delta\rho - \frac{3\nabla^2\delta f_R}{a^2}$$

$$\nabla^2\Phi = 4\pi G a^2\delta\rho + \left(\frac{4\pi G a^2\delta\rho}{3} + \frac{a^2}{6}\delta R\right)$$

$$= 4\pi G a^2\delta\rho_{\mathrm{eff}}$$

Linear and large scale effects

- Linear perturbations

Large scale matter clustering

- Growth of density perturbations

Parameter $|f_{R0}|$ $\quad f_R \equiv \frac{\mathrm{d}f(R)}{\mathrm{d}R}$

- Compton wavelength

For a larger $|f_{R0}|$,

the Compton wavelength

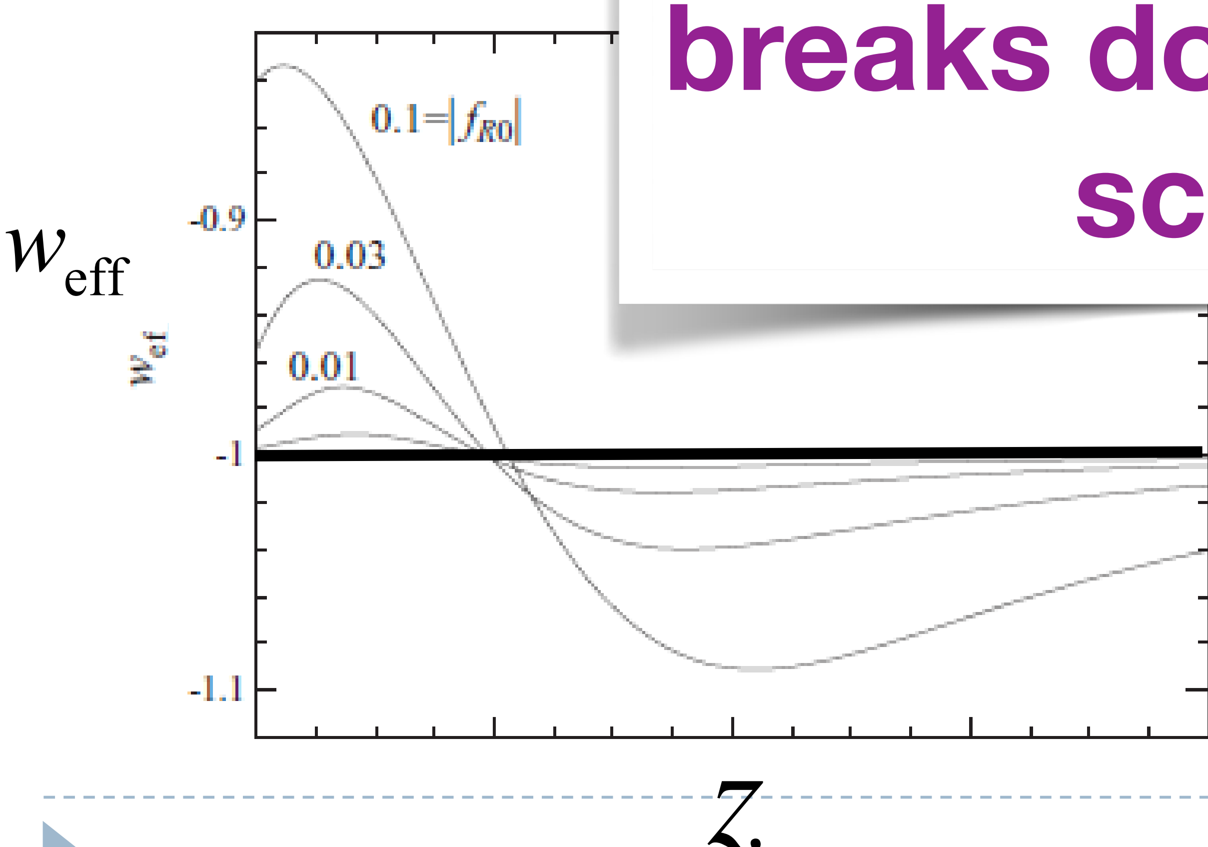
is longer

Range of the fifth force
(1/mass of the field)
changes in time:
Chameleon mechanism

Summary of the model

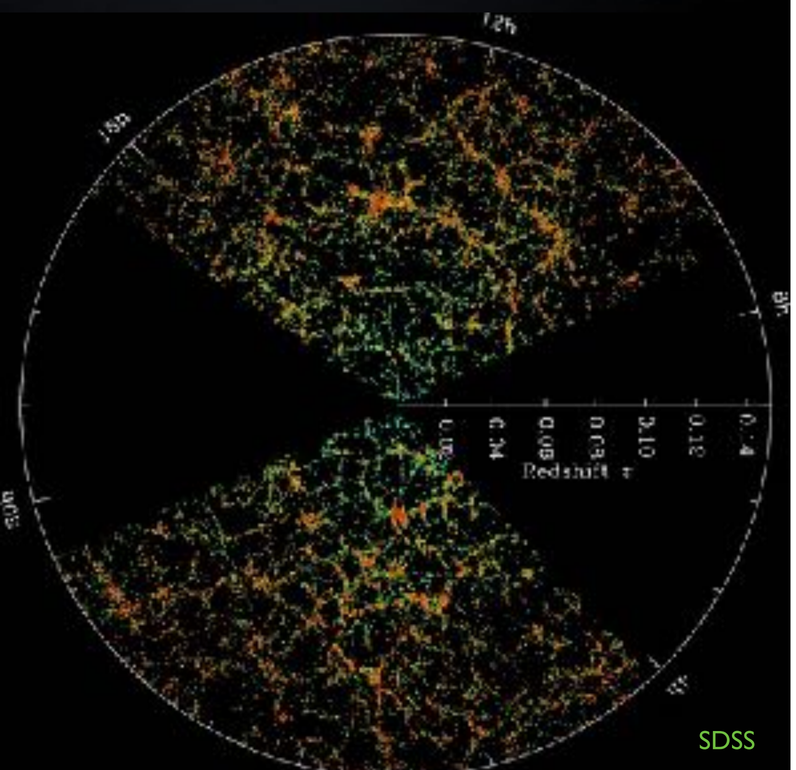
- Background

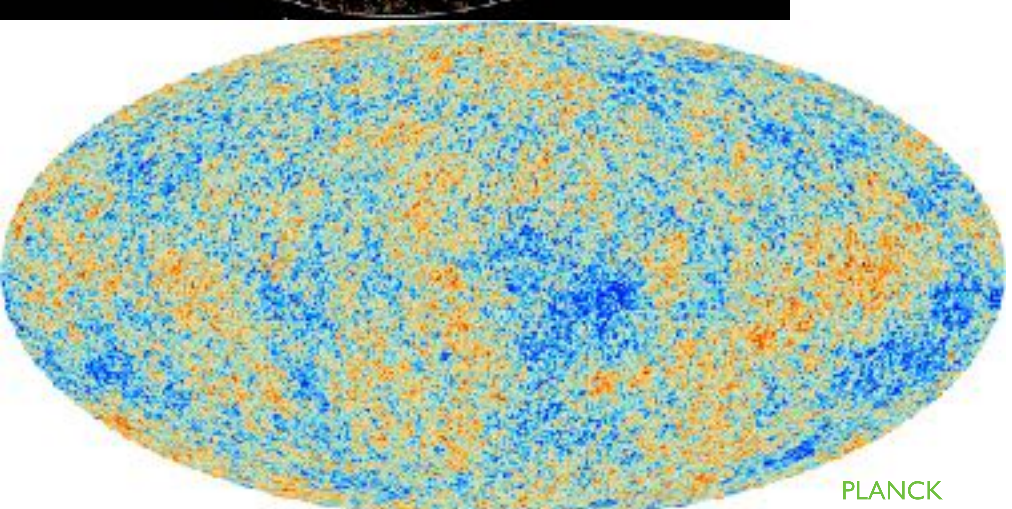
  Indistinguishable from LCDM if the solar system constraints are satisfied $|f_{R0}| \leq 10^{-5}, \quad f_{R0} \equiv \bar{f}_R(t_0)$



Linear approximation breaks down at small scales!

Background Constraints on Modified Gravity





$$\mathcal{L}_{ModGrav} = R - \alpha R^{\beta}$$

Supernovae + Large Scale Structures + CMBR + Baryon Oscillations