Cosmology of gravity theories beyond General Relativity

David F. Mota

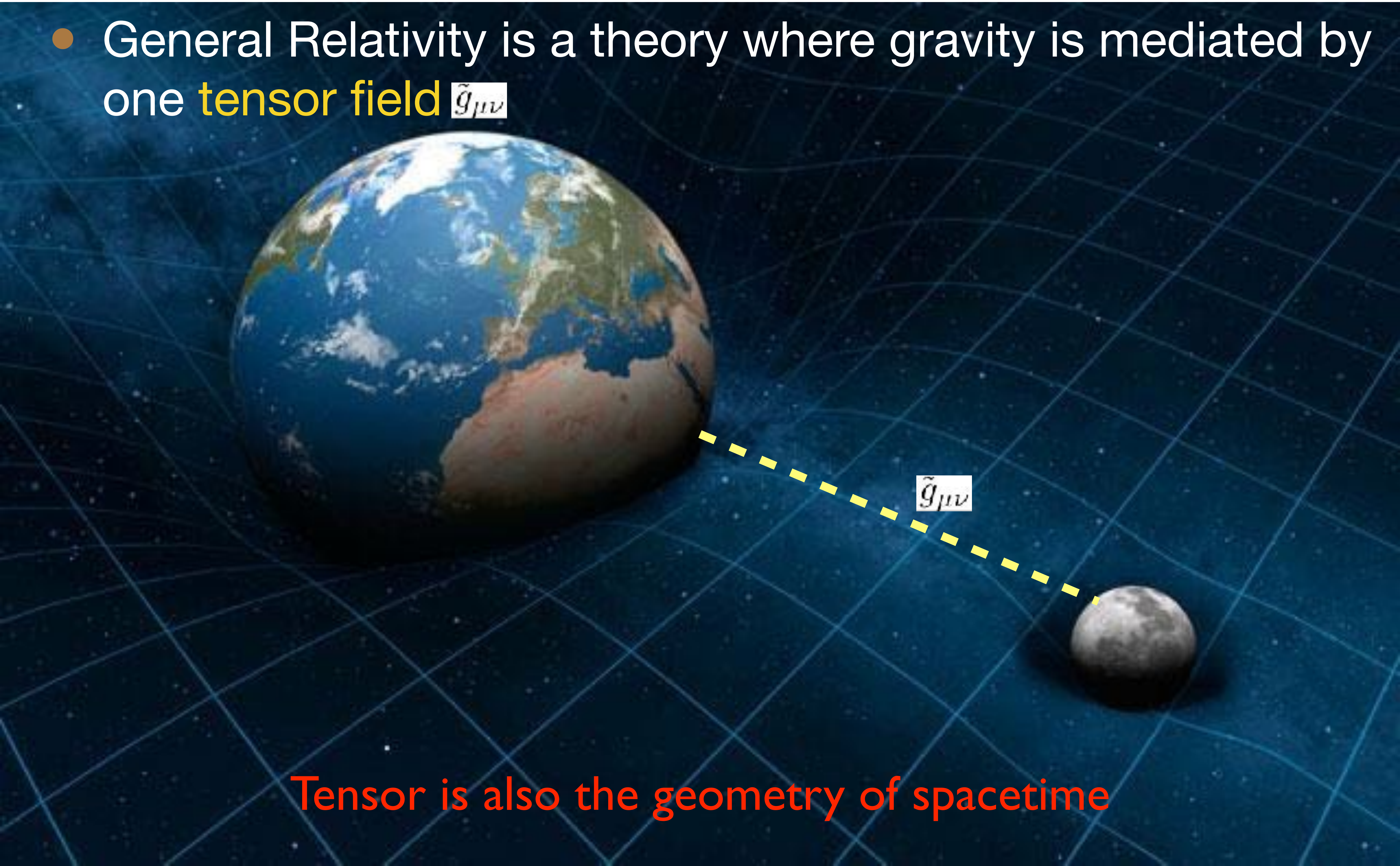
UiO : Institute of Theoretical Astrophysics
University of Oslo

LECTURE 1

Gravity beyond General Relativity

What is General Relativity?

- General Relativity is a theory where gravity is mediated by one tensor field $\tilde{g}_{\mu\nu}$

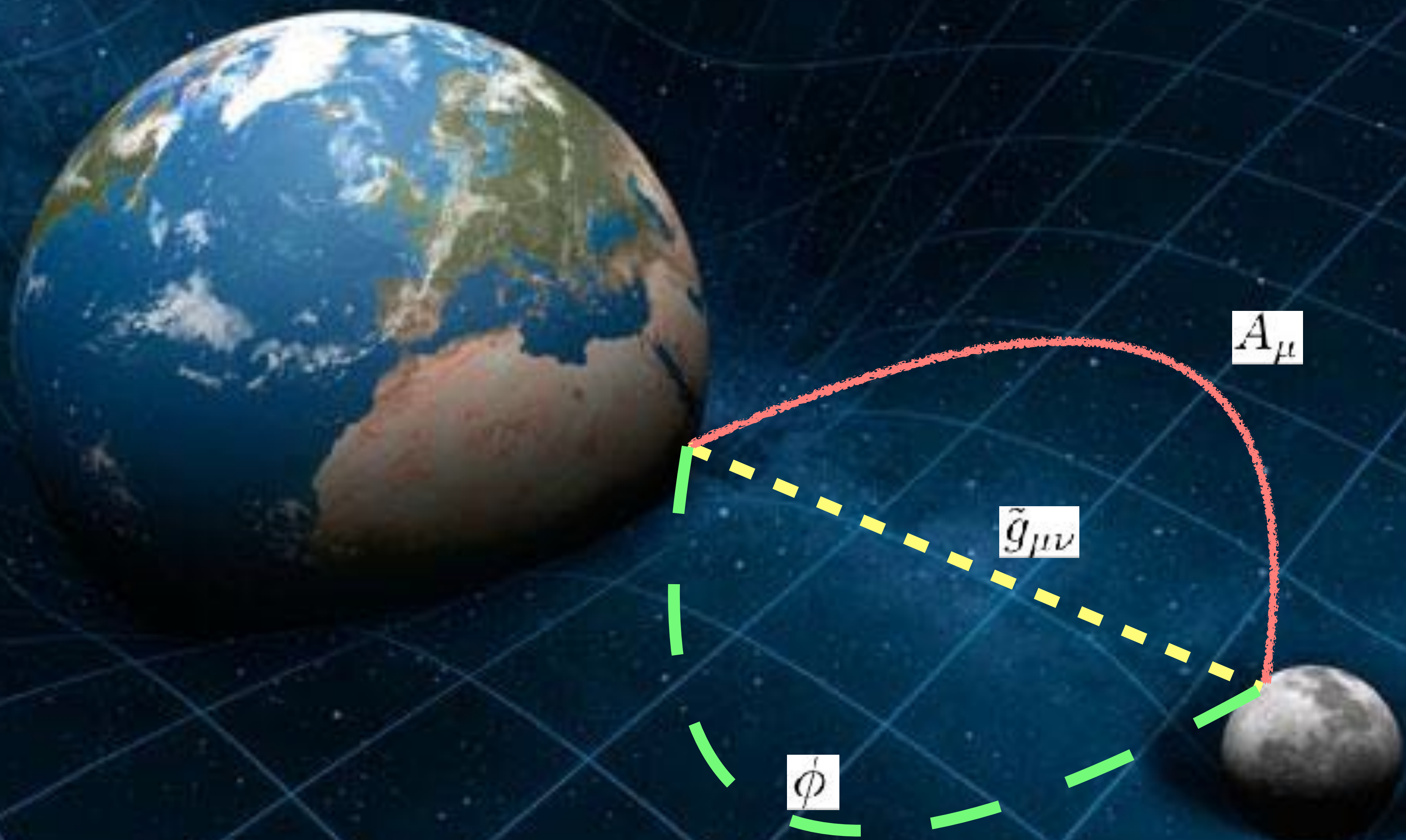


Tensor is also the geometry of spacetime

Other Theories of Gravity

Extra carriers of interaction

- Gravity may be mediated by a tensor field $\tilde{g}_{\mu\nu}$, and/or a vector field A_μ , and/or a scalar field ϕ



- Spacetime geometry may be a combination of all:

$$g_{\mu\nu} = e^{-2\phi}(\tilde{g}_{\mu\nu} + A_\mu A_\nu) - e^{2\phi} A_\mu A_\nu .$$

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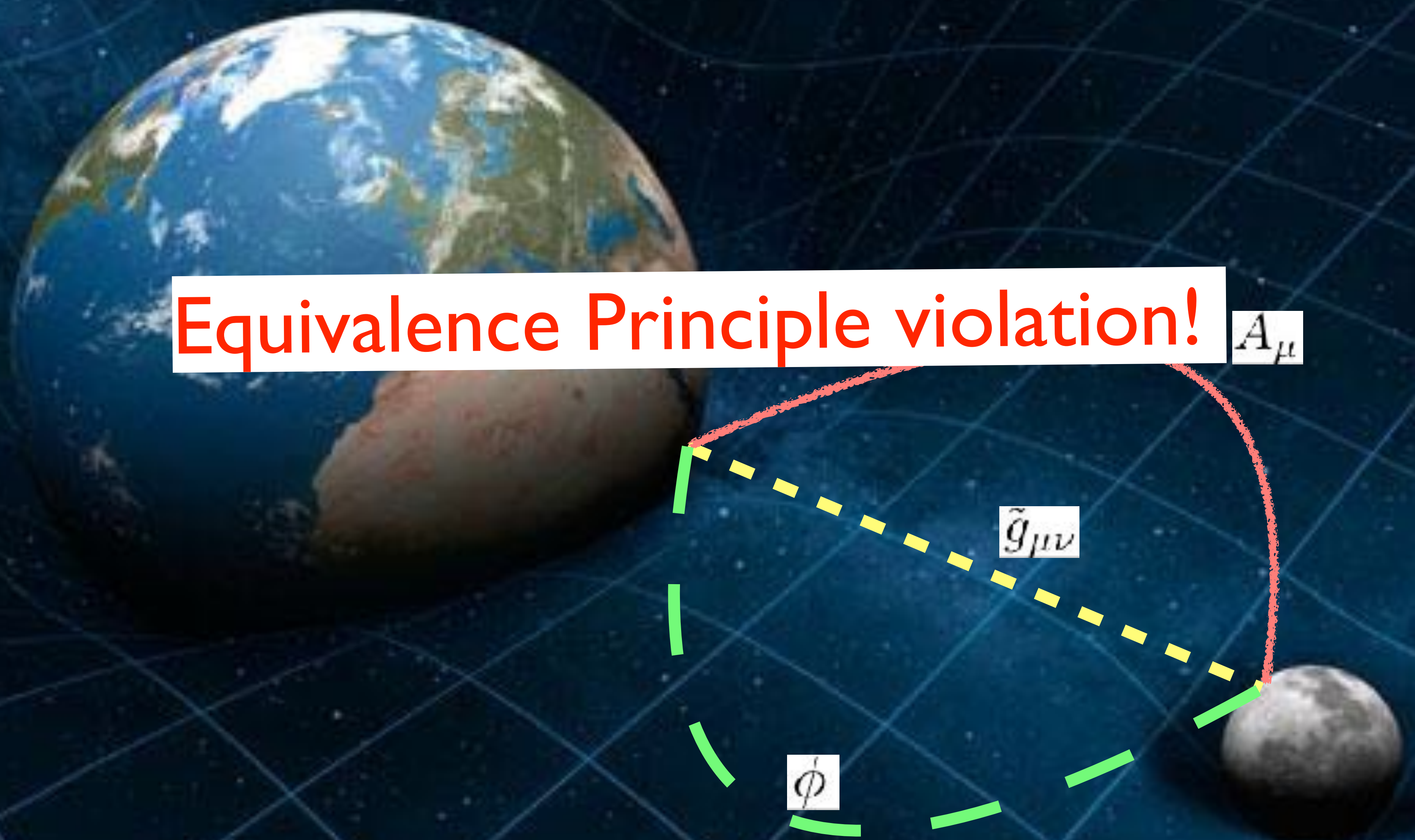

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$$g_{\mu\nu} = e^{-2\phi}(\tilde{g}_{\mu\nu} + A_\mu A_\nu) - e^{2\phi} A_\mu A_\nu .$$

GR in a nutshell

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance

Physical metric

$$S_{matter}(\psi, g_{\mu\nu})$$

Newtonian limit of Equivalence principle and test particles

Action of a test mass:

$$S = -\int mc\sqrt{-g_{\mu\nu}v^\mu v^\nu}dt \qquad \text{with} \qquad \begin{aligned} v^\mu &= dx^\mu/dt \\ u^\mu &= dx^\mu/d\tau \end{aligned}$$

Pure geometry!

Newtonian vs. Poisson field

Non-relativistic gravity theories

Definition of Newtonian potential : $m_i \vec{a} = -\nabla \Phi_N$

What about Gravity from matter distribution?

Definition of Poisson potential : $\nabla^2 \Phi_P = 4\pi G_N \rho$

simplest theory : Newtonian gravity $\Phi_P = \Phi_N$

Metric theories of gravity

One can use two metrics :

$g_{\mu\nu}$ Physical/Jordan matter metric : matter follows geodesics of $g_{\mu\nu}$ (analogue of $m_i\vec{a} = -\nabla\Phi_N$)

$\tilde{g}_{\mu\nu}$ Einstein type metric : Action for $\tilde{g}_{\mu\nu}$ is Einstein-Hilbert. (analogue of $\nabla^2\Phi_P = 4\pi G_N\rho$)

Need a relation between them ($\Phi_P = \Phi_N$)

Einstein Gravity: $g_{\mu\nu} = \tilde{g}_{\mu\nu}$

Conformal relation: $g_{\mu\nu} = e^{2\phi}\tilde{g}_{\mu\nu}$

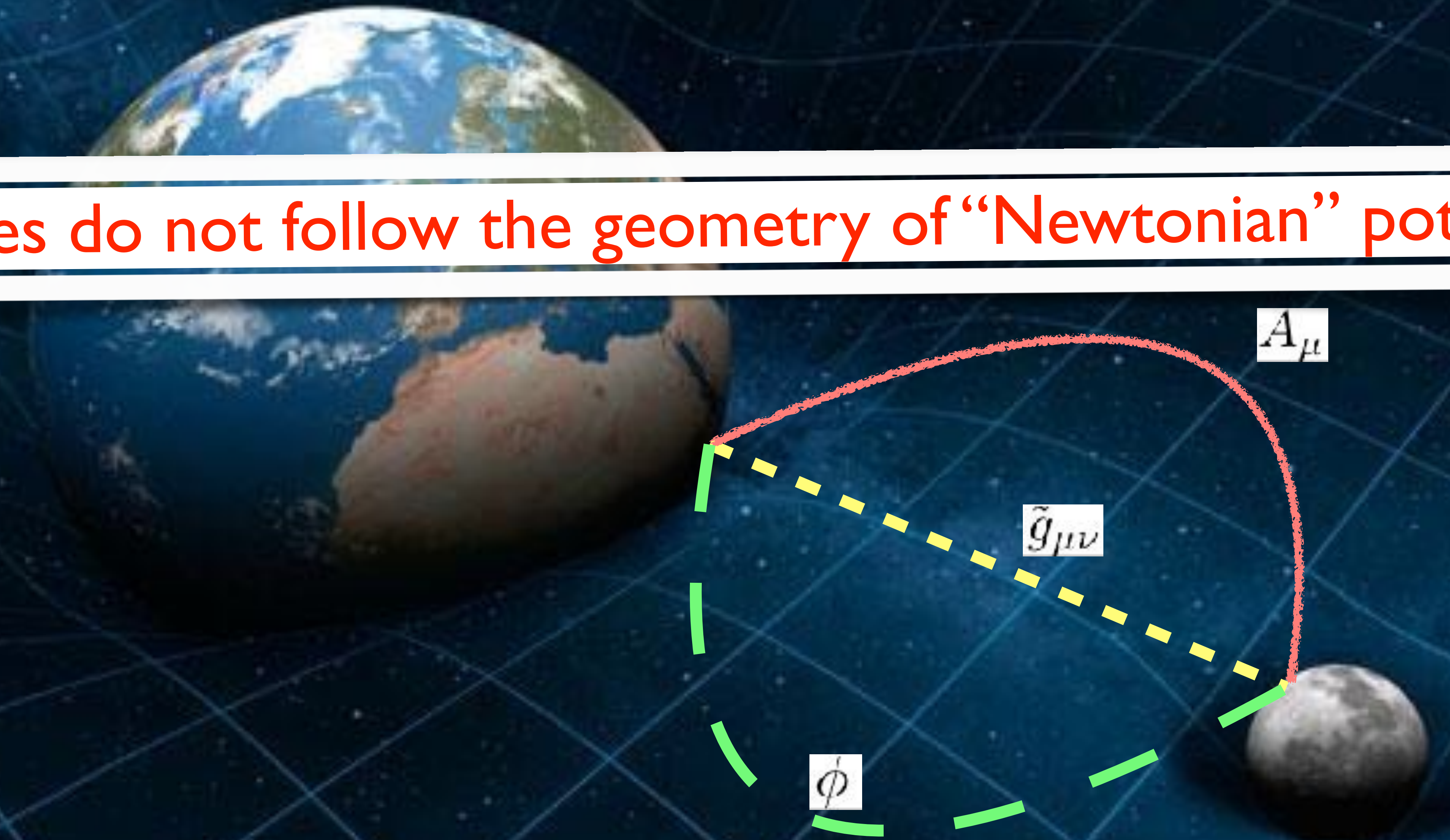
Disformal relations: $g_{\mu\nu} = e^{-2\phi}(\tilde{g}_{\mu\nu} + A_\mu A_\nu) - e^{2\phi}A_\mu A_\nu.$

Modified Gravity

Extra carriers of interaction

- Gravity may be mediated by a tensor field $\tilde{g}_{\mu\nu}$, and/or a vector field A_μ , and/or a scalar field ϕ

Particles do not follow the geometry of “Newtonian” potential



- Spacetime geometry may be a combination of all:

$$g_{\mu\nu} = e^{-2\phi}(\tilde{g}_{\mu\nu} + A_\mu A_\nu) - e^{2\phi} A_\mu A_\nu .$$

General Relativity is quite unique

Curvature

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}R(g) + \int d^4x\sqrt{-g}\mathcal{L}(g,\mathrm{matter})$$

Metric of space time

Lovelock's theorem (1971) :*"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."*

LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

$$S_{\mathrm{grav}} = \frac{M_{Pl}^2}{2}\int\sqrt{-g}\; d^4x\, [\ R\ ]$$

Five options:

LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

$$S_{\rm grav} = \frac{M_{Pl}^2}{2}\int\sqrt{-g}\, d^4x \left[\phi R - \frac{\omega(\phi)}{\phi}(\nabla\phi)^2 - 2V(\phi)\right]$$

Five options:

1. Add new field content.

LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

$$S_{\mathrm{grav}} = \frac{{M_D}^2}{2} \int \sqrt{-\gamma \, d^D x} \, [\, \mathcal{R} + \alpha \, \mathcal{G} \,]$$

Five options:

1. Add new field content.

2. Higher dimensions.

LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

$$S_{\mathrm{grav}} = \frac{M_{Pl}^2}{2}\int\sqrt{-g}\, d^4x \left[R + \beta_1 R\nabla_\mu\nabla^\mu R \;+\; \beta_2\nabla_\mu R_{\beta\gamma}\nabla^\mu R^{\beta\gamma}\right]$$

Five options:

1. Add new field content.

2. Higher dimensions.

3. > 2nd order derivatives in the field equations.

LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

$$S_{\mathrm{grav}} = \frac{M_{Pl}^2}{2} \int \sqrt{-g}\, d^4x \left[R + f\!\left(\frac{1}{\Box} R \right) \right]$$

Five options:

1. Add new field content.

2. Higher dimensions.

3. > 2nd order derivatives in the field equations.

4. Non-locality.

LOVELOCK'S THEOREM (1971)

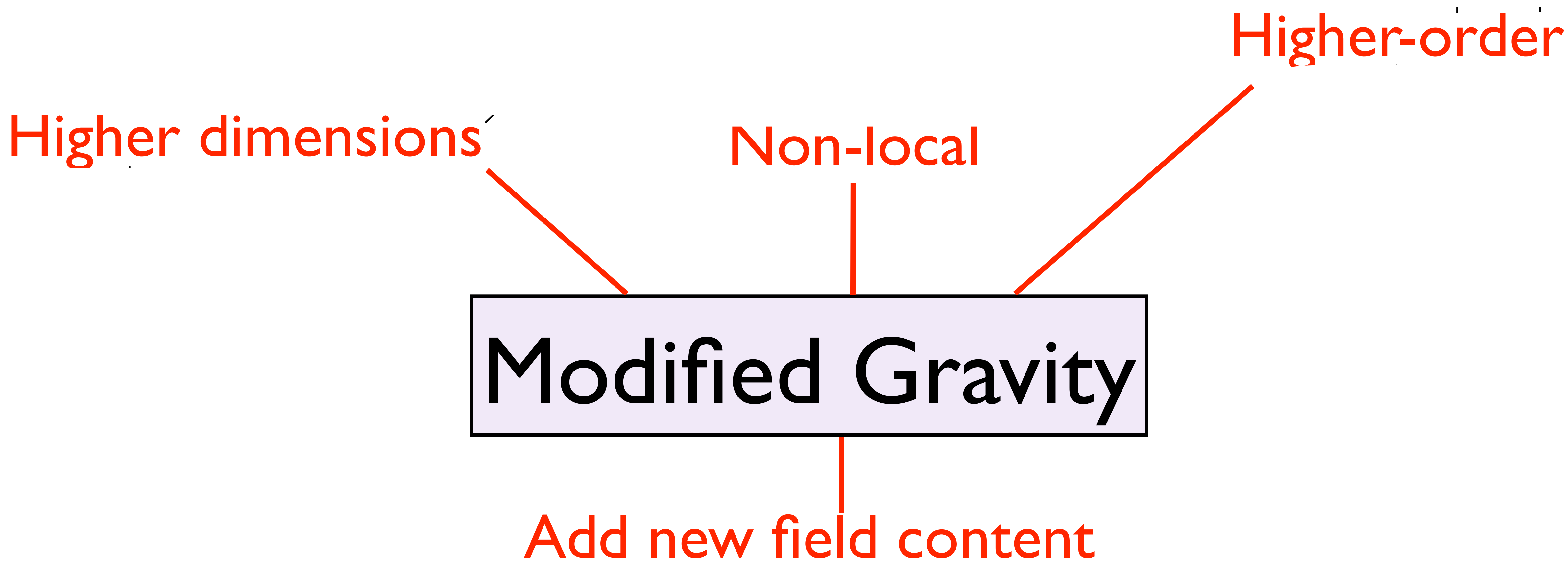
"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

? ? ? ? ? ? ?

Five options:

1. Add new field content.
2. Higher dimensions.
3. > 2nd order derivatives in the field equations.
4. Non-locality.
5. Radically change our action principle (emergence).

Tessa Baker


Higher dimensions
Non-local
Higher-order
Modified Gravity
Add new field content

Extra degrees of freedom in Scalar-Tensor Gravity

Fifth Force

Tensor field: $g^{\mu\nu}$
of General Relativity

+

Scalar field: Φ
Giving rise to a **5th force**

Action: $S = \int d^4x \sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] + \int d^4x \sqrt{-\tilde{g}}\mathcal{L}_m(\Omega_{(i)}^2(\phi)g_{\mu\nu}, \psi_m^{(i)})$

Kinetic term

Self-interacting potential
(bare potential)

coupling

Extra degrees of freedom in Scalar-Tensor Gravity

Fifth Force

Tensor field: $g^{\mu\nu}$
of General Relativity

➕

Scalar field: Φ
Giving rise to a **5^{th} force**

Action: $S = \int d^4x\,\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] + \int d^4x\,\sqrt{-\tilde{g}}\mathcal{L}_m(\Omega^2_{(i)}(\phi)g_{\mu\nu}, \psi_m^{(i)})$

Strength
force

5^{th} force in the Newtonian limit: $\boldsymbol{F}_\phi = -m\frac{\partial\ln\Omega_{(i)}}{\partial\phi}\boldsymbol{\nabla}\phi$

Gradient of the field

Extra degrees of freedom in Scalar-Tensor Gravity

Fifth Force

Tensor field: $g^{\mu\nu}$
of General Relativity

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Scalar field: Φ
Giving rise to a **5^{th} force**

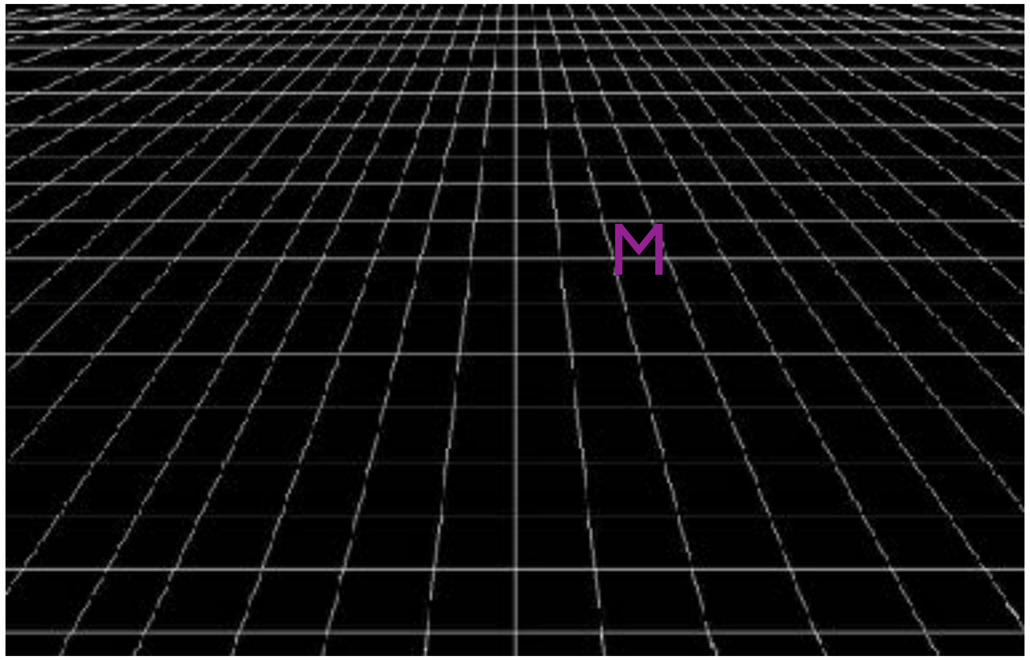
Action: $S = \int d^4x \sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] + \int d^4x\sqrt{-\tilde{g}}\mathcal{L}_m(\Omega^2_{(i)}(\phi)g_{\mu\nu}, \psi_m^{(i)})$

5^{th} force in the Newtonian limit: $\boldsymbol{F}_\phi = -m\frac{\partial \ln\Omega_{(i)}}{\partial\phi}\boldsymbol{\nabla}\phi$

Properties :
- Coupling Strength
- Range
- Activation epoch /environment

Gradient of scalar field
(Newtonian potential example)

Assuming: spherical symmetry, quasi-static: $$\Phi_P \sim -\frac{M}{r}$$



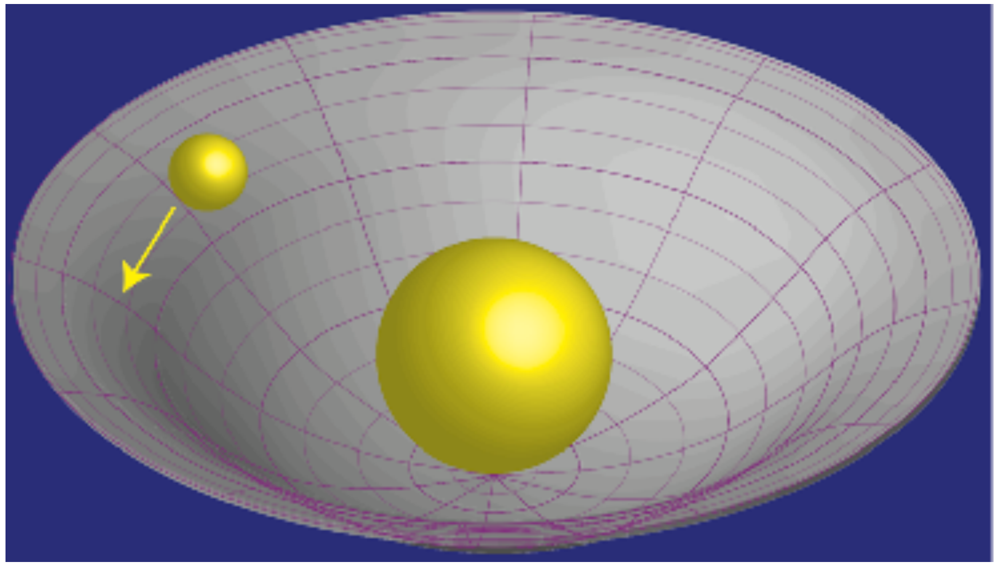
Equation of motion for Poisson field

$$\nabla^2 \Phi_P = 4\pi G_N \rho$$

Matter sources a potential

What is the acceleration of a test particle?

Newtonian Equivalence principle



Recall that:

Definition of Newtonian potential : $\vec{a} = -\nabla \Phi_N$

Definition of Poisson potential : $\nabla^2 \Phi_P = 4\pi G_N \rho$

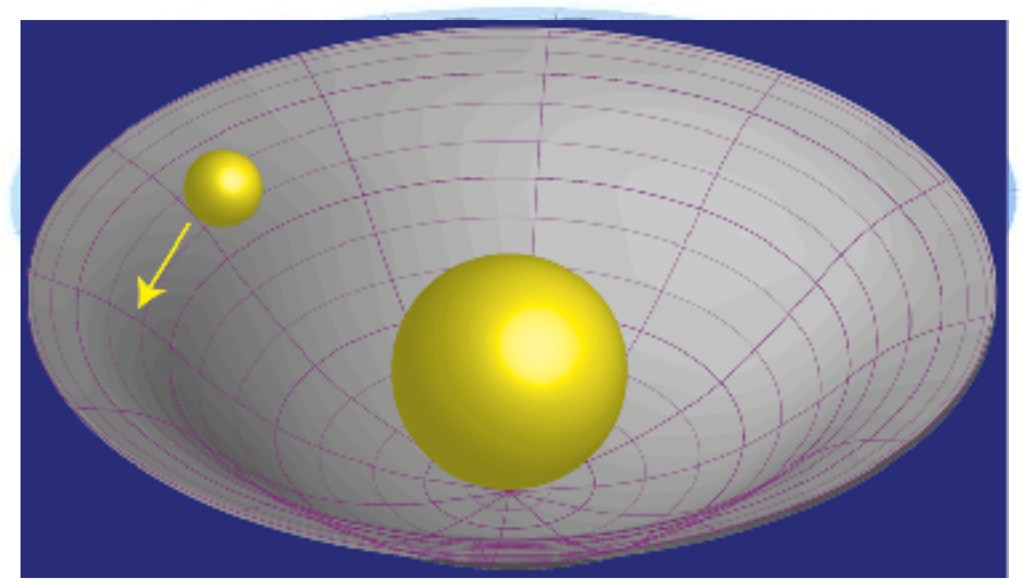
Newton's Constant

matter density

simplest theory : Newtonian gravity $\Phi_P = \Phi_N$

Gradient of scalar field
(Newtonian potential example)

$$\Phi_P \sim -\frac{M}{r}$$



Equation of motion for Poisson field

$$\nabla^2 \Phi_P = 4\pi G_N \rho$$

Matter sources a potential

What is the acceleration of a test particle?

$$\vec{a} = -\nabla \Phi_N = -\nabla \Phi_P \sim -\frac{M}{r^2}$$

Fifth force potential / scalar field

Bare Potential

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] - \int d^4x\sqrt{-g}\mathcal{L}_m(\psi^{(i)}, g_{\mu\nu}^{(i)})$$

Matter fields

$$g_{\mu\nu}^{(i)} = e^{2\beta_i\phi/M_{Pl}} g_{\mu\nu}$$

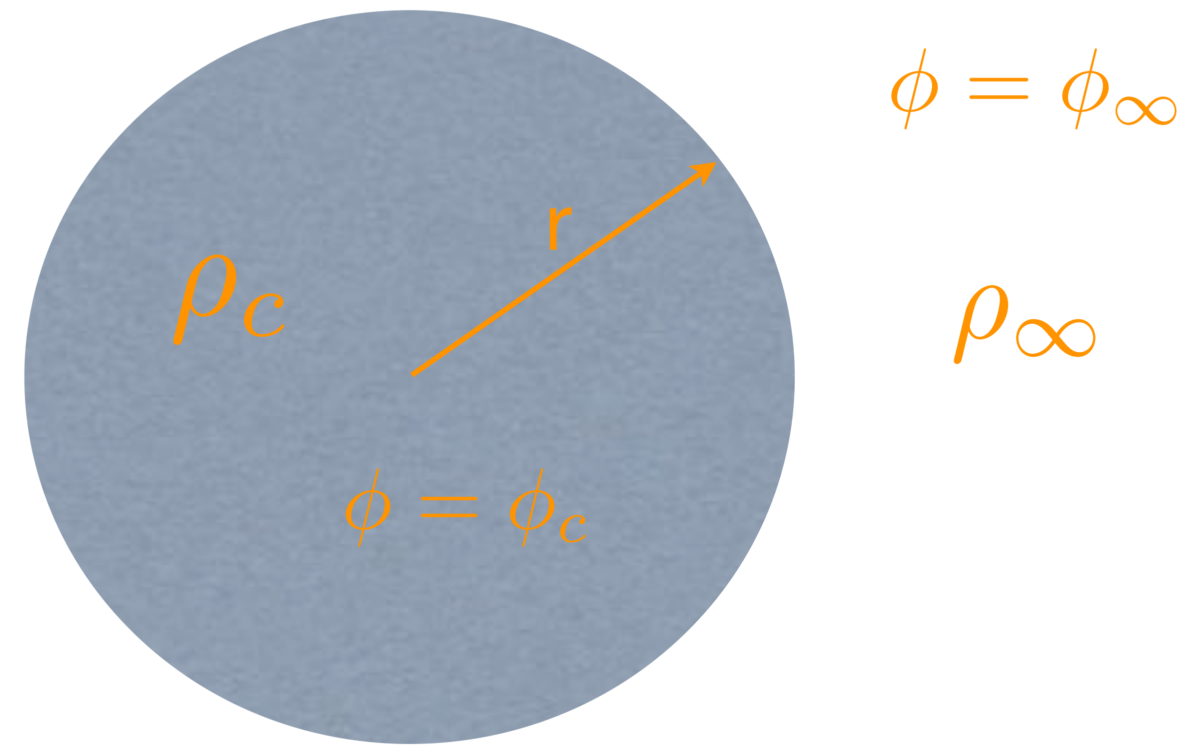
Conformally Coupled

$$\Box\phi = V_{,\phi} - \sum \frac{\beta_i}{M_{Pl}} e^{4\beta_i\phi/M_{Pl}} g_{(i)}^{\mu\nu} T_{\mu\nu}^{(i)}$$

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

Solutions near massive objects

- Assumptions:
  - Flat
  - Static
  - Spherical Symmetry
  - Homogeneous density



Equation of motion:

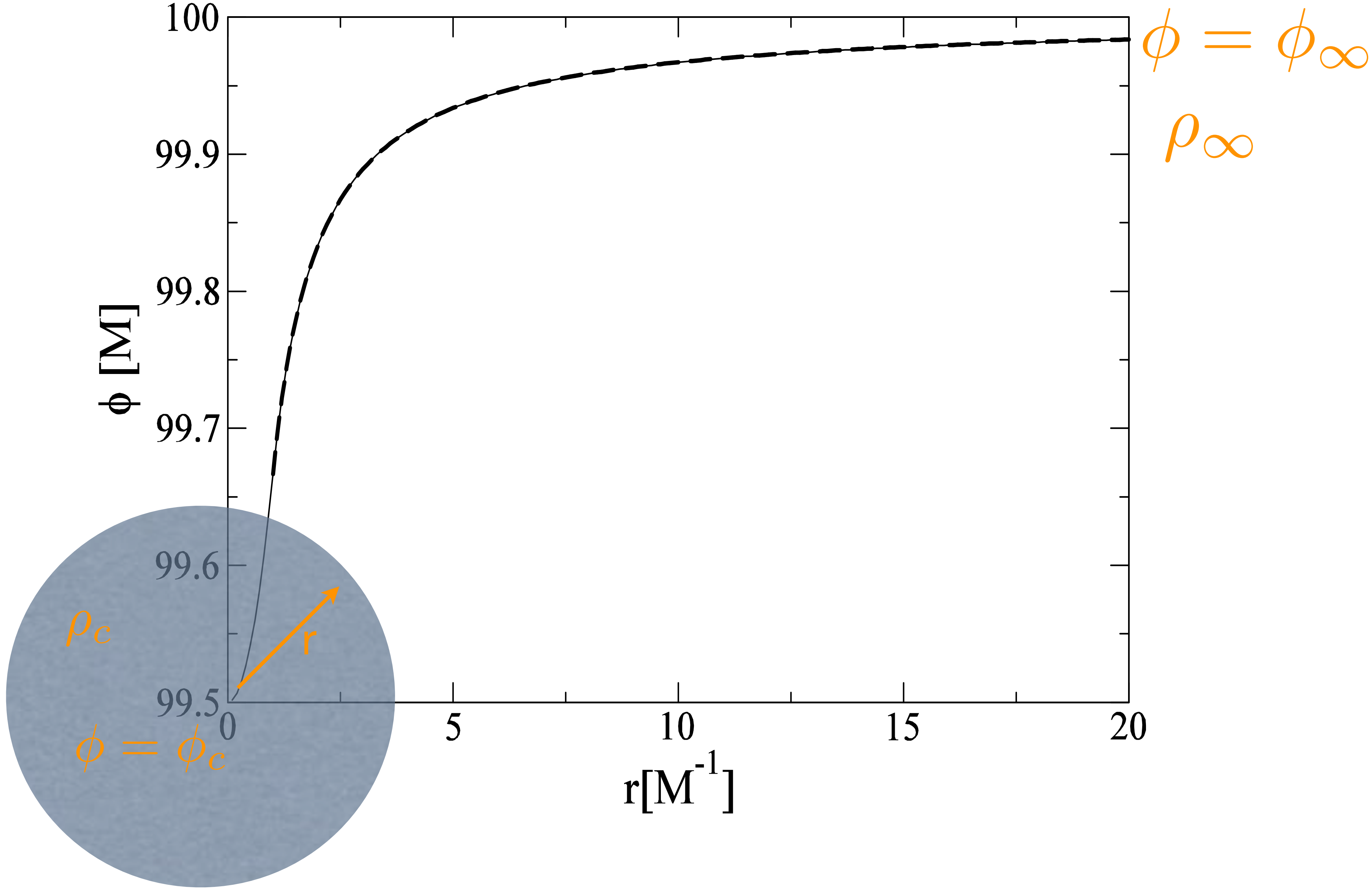
$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = V_{,\phi} + \frac{\beta}{M_{Pl}}\rho(r)e^{\beta\phi/M_{Pl}}$$

Boundary Conditions:

$$\frac{d\phi}{dr} = 0 \qquad at \qquad r = 0$$

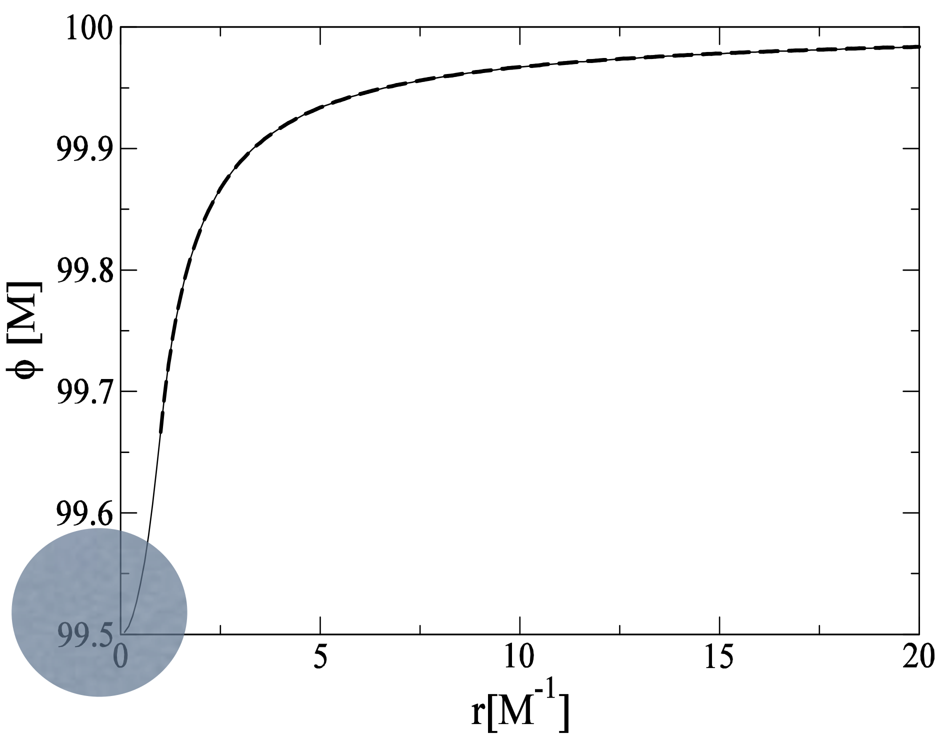
$$\phi \to \phi_\infty \qquad as \qquad r \to \infty$$

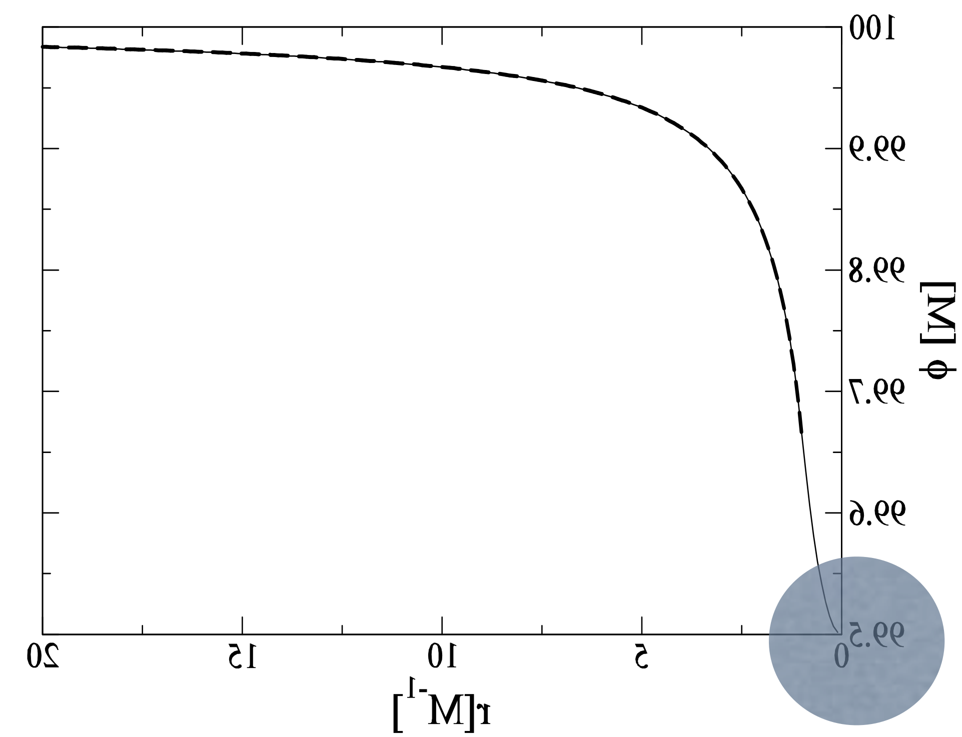
Scalar field Profile: Yukawa solution



$$\phi(r) \approx \phi_\infty - \frac{\beta}{4\pi M_{Pl}} \frac{M e^{-m_\infty r}}{r}$$

Fifth force between particles



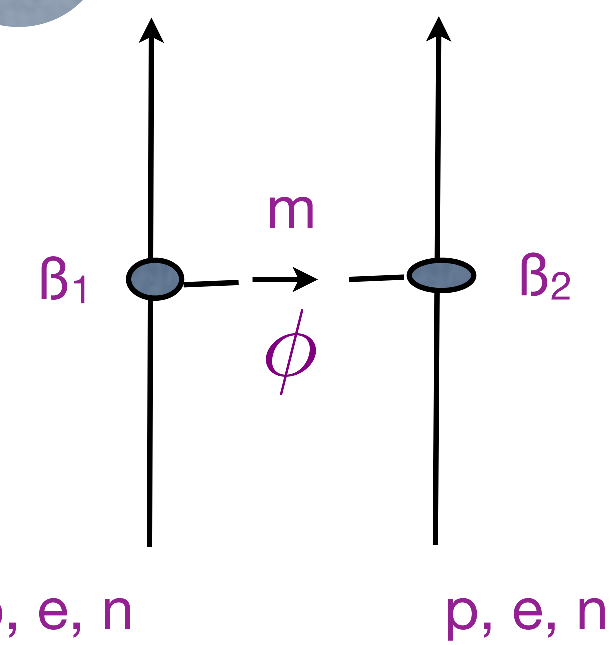


$$\vec{F}_\phi = -Gm_1m_2\nabla\frac{\beta_1\beta_2 e^{-mr}}{r}$$

Newtonian potential

Yukawa

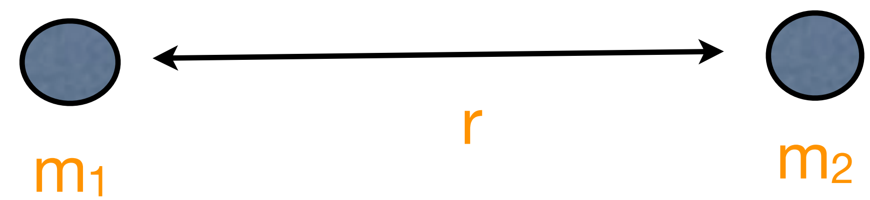
$$V(r) = -G\frac{m_1m_2}{r}(1+\beta_1\beta_2 e^{-r/\lambda})$$



<u>*Does not vary as $1/r^2$*</u>

<u>*Not independent of nature of 1 or 2 (EP Violating)!*</u>

Newtonian Gravity



Newtonian Gravity

$$V(r) = -G\frac{m_1 m_2}{r}$$

$$\vec{F}(r) = -G\frac{\bar{m}_1 m_2}{r^2}\hat{r} = \bar{m}_1 \vec{a}_1$$

$$\vec{a}_1 = -\frac{G m_2}{r^2}\hat{r}$$

Independent of nature of m_1

(Equivalence Principle)

Varies as $1/r^2$

Scalar-tensor Gravity

$$V(r) = -G\frac{m_1 m_2}{r}(1 + \alpha_{12} e^{-r/\lambda})$$

$$F(r) = -G(r)\frac{m_1 m_2}{r^2}\hat{r}$$

$$G(r) = G[1 + \alpha_{12} e^{-r/\lambda}(1 + r/\lambda)]$$

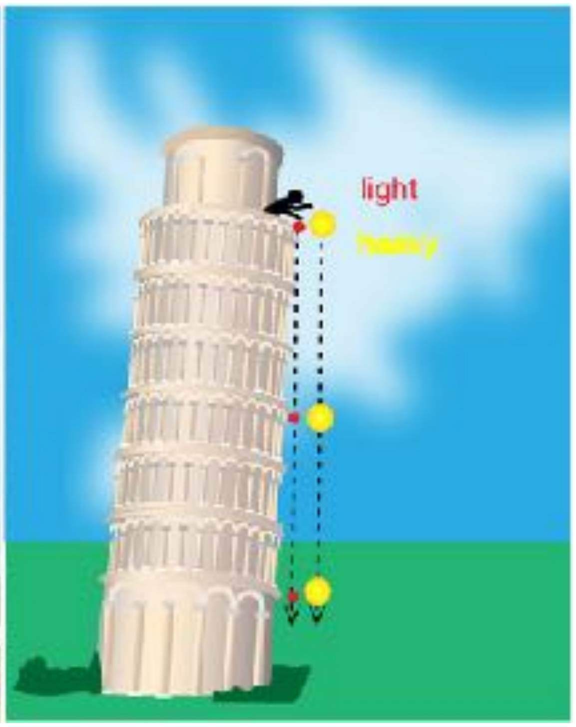
Not independent of 1 or 2!

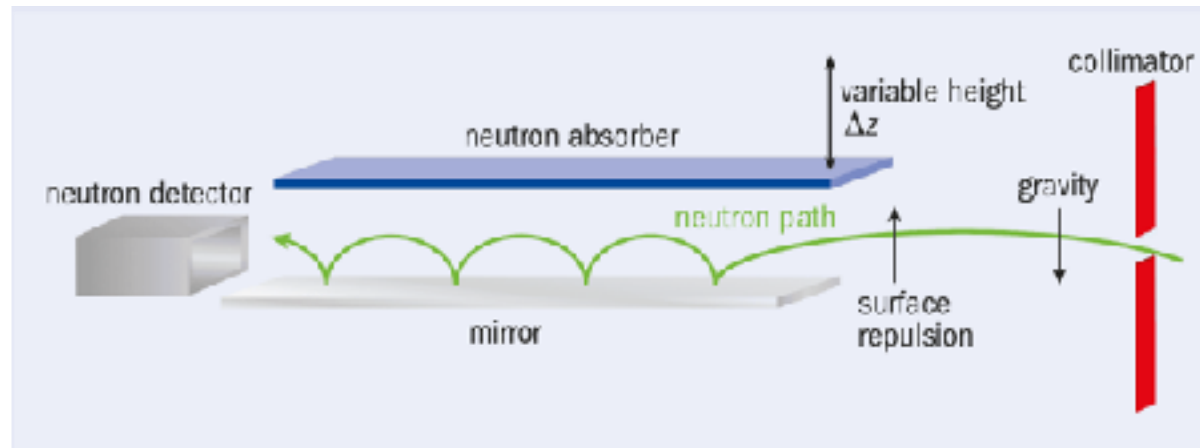
Does not vary as $1/r^2$

Evidence for either points out to a new fundamental force in nature!

Laboratory tests

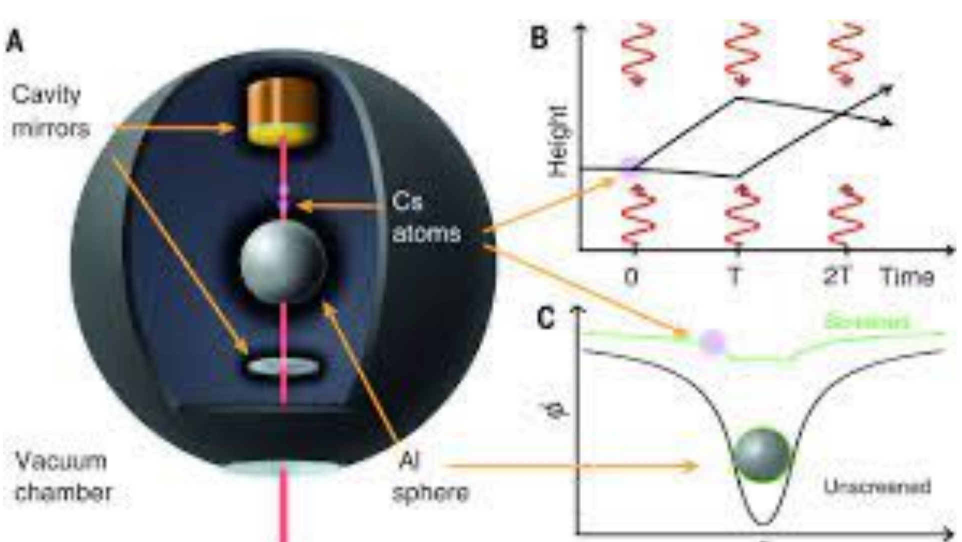
Robustly confirm the predictions of General Relativity

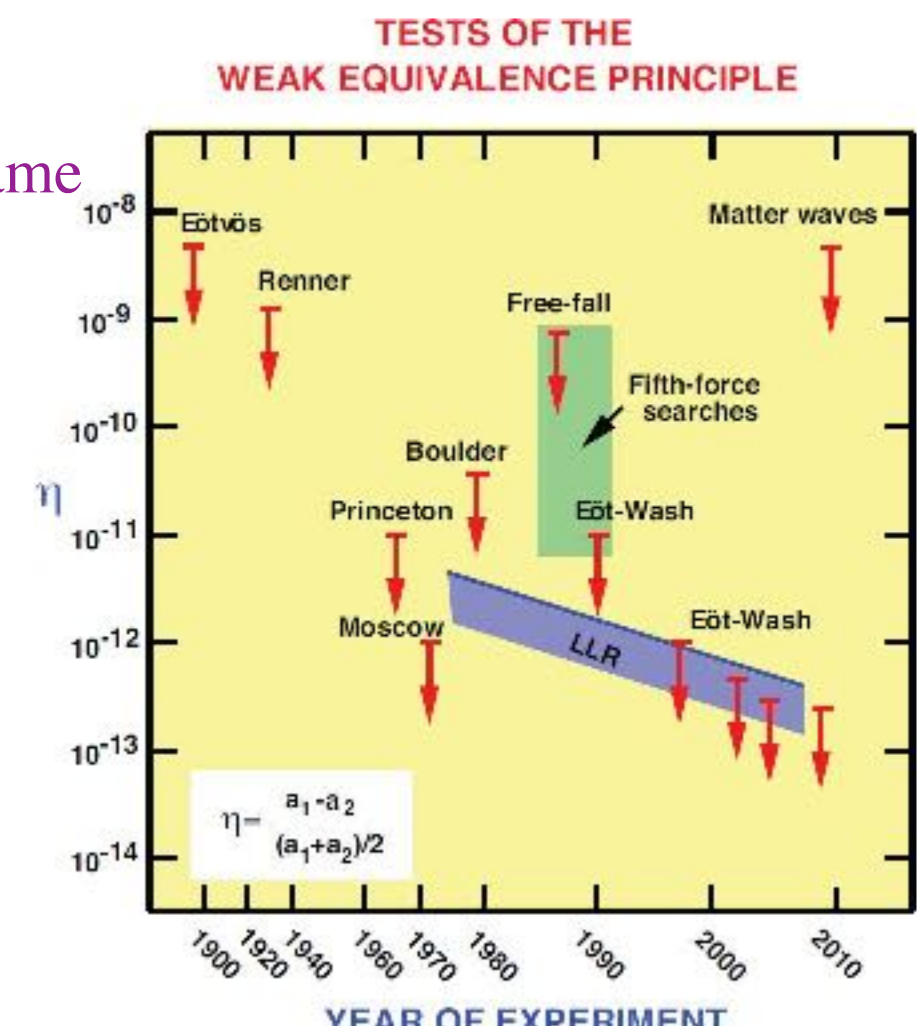




Equivalence Principle:

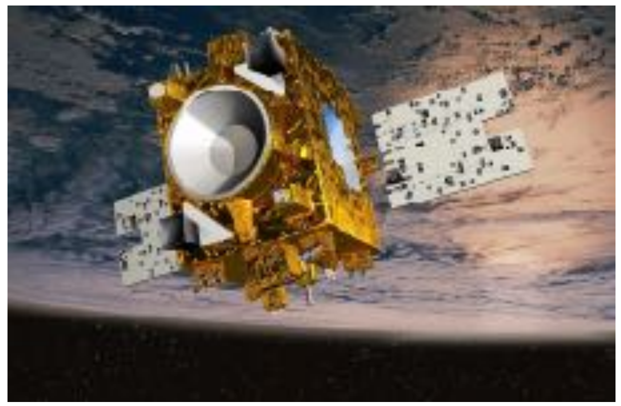
All bodies fall in gravitational field with same acceleration regardless of mass or internal structure

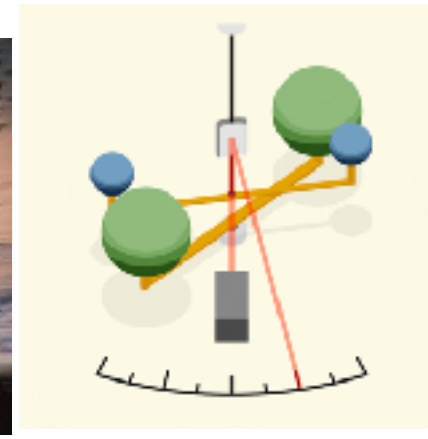




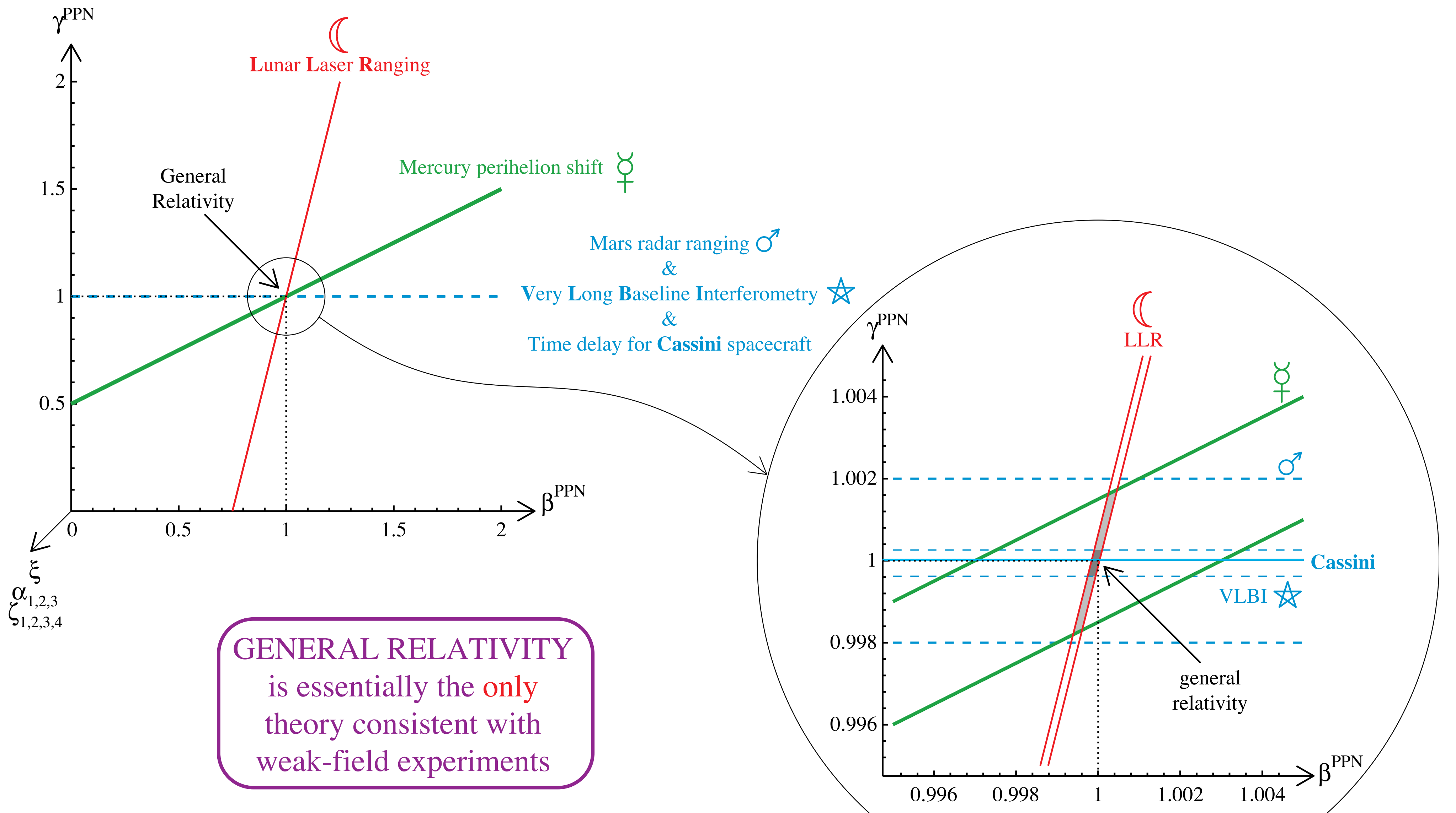
Solar System
General Relativity performs flawlessly

MICROSCOPE

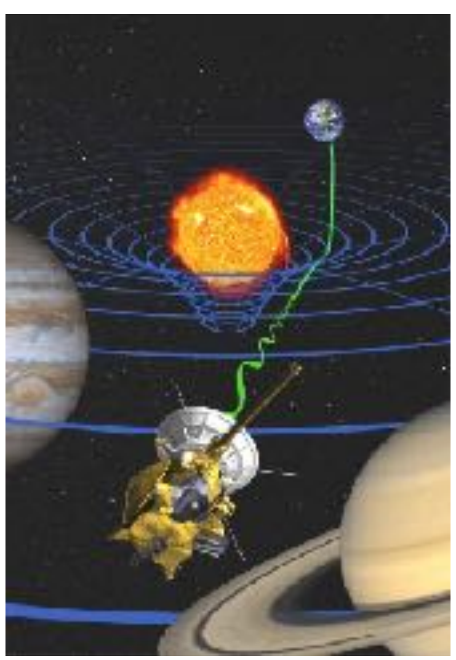




Conclusion of experimental tests in the **P**arametrized **P**ost-**N**ewtonian formalism



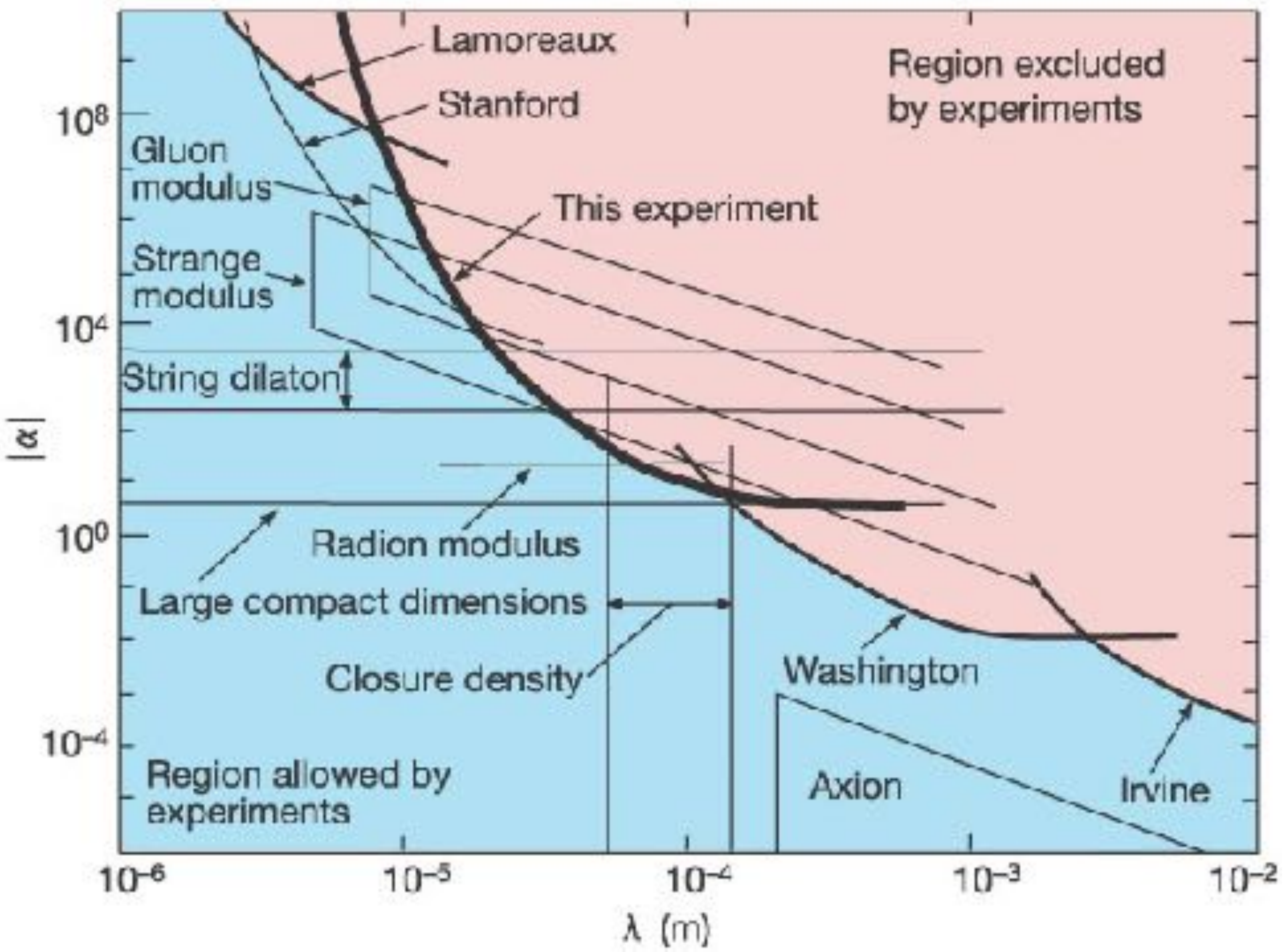
GENERAL RELATIVITY
is essentially the only
theory consistent with
weak-field experiments



CASSINI

Bounds on Fields Coupled to Matter

Long et al. (2004)



$$V(r) = -\frac{m_1 m_2}{8\pi M_{Pl}^2}[1+\alpha\frac{e^{-r/\lambda}}{r}]$$

$$\alpha \sim 1/M_{Pl} \to 10^{-4}m$$

Modified Gravity either:

1) strong coupling and very short range

2) very weak coupling and long range

Range and Strength of Fifth Force

$$S = \int dx^4 \sqrt{-g}\left[\frac{R}{2}M_{\rm pl}^2 - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] + S_m(A^2(\phi)g_{\mu\nu}, \psi_i)$$

$$\Psi(r) = -\frac{GM}{r}(1+\alpha e^{-r/\lambda})$$

Strength

$$\Box\phi = -\frac{dV_{eff}(\phi)}{d\phi}$$

range

$$V_{eff}(\phi) = V_{bare}(\phi) + Interactions(A(\phi))$$

$$m_\phi = \sqrt{V''_{eff}(\phi_c)}$$

$$\lambda = \frac{\hbar}{mc}$$