

Lecture 3: U-DHOST Theories and the Shadowy Mode

Spatially Covariant Gravity and Maple Implementation

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The Unitary Gauge

- In scalar-tensor theories, the scalar field ϕ often has a timelike gradient.
- **Unitary gauge:** choose the time coordinate such that $\phi = \phi(t)$.
- The action can be built from geometric quantities of the 3D spatial hypersurfaces Σ_t .
- If a theory is designed to be degenerate *only* in the unitary gauge, it propagates 3 degrees of freedom (2 tensor + 1 scalar).
- What happens if we perform a general coordinate transformation?

Apparent Ghost in General Gauge

- In a general gauge, ϕ acquires spatial dependence.
- A naive analysis might suggest the kinetic matrix is no longer degenerate, seemingly reviving the Ostrogradsky ghost.
- **Resolution:** for a regular timelike scalar foliation, the apparent higher-derivative mode carries no independent initial data.
- It is a “shadowy” response fixed by an elliptic equation and spatial boundary conditions; it is not a pure-gauge artifact.
- U-DHOST theories are covariant representations of this spatially covariant construction.

The Stückelberg Trick

- Procedure to covariantize a 3D spatially invariant action into a fully 4D diffeomorphism-invariant theory.
- Consider a simple 3D action with extrinsic curvature K_{ij} and 3D Ricci $R^{(3)}$:

$$S = \int d^4x \sqrt{h} N \left[\alpha_1 K_{ij} K^{ij} + \alpha_2 K^2 + \alpha_3 R^{(3)} \right] \quad (1)$$

- The coefficients α_i depend on t .
- Crucially, there are **no time derivatives of the lapse** N ($\dot{N}$ is absent).
- The momentum conjugate to the lapse vanishes, $\pi_N = 0$.

Restoring 4D Covariance

- Introduce $\phi(x^\mu)$ and promote $\alpha_i(t) \rightarrow \alpha_i(\phi)$.
- The unit normal to constant- ϕ hypersurfaces is:

$$n_\mu = -\frac{\nabla_\mu \phi}{\sqrt{-X}}, \quad X = g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \quad (2)$$

- In the unitary coordinate choice $t = \phi$, $n_\mu dx^\mu = -N dt$ gives:

$$N = \frac{1}{\sqrt{-X}} \quad (3)$$

Covariantizing the Trace of Extrinsic Curvature

- Calculate the trace $K = g^{\mu\nu} K_{\mu\nu}$:

$$K = \nabla_\mu n^\mu = -\frac{\square\phi}{\sqrt{-X}} - \frac{n^\mu \nabla_\mu X}{2X} \quad (4)$$

- The trace K is directly linked to $\square\phi$.
- Squaring K yields terms proportional to $(\square\phi)^2$, equivalent to the $L_2^{(2)}$ operator in DHOST.
- Similarly, $K_{\mu\nu} K^{\mu\nu}$ generates $\phi_{\mu\nu} \phi^{\mu\nu}$ (the $L_1^{(2)}$ operator).

Covariantizing the 3D Ricci Scalar

- Use the Gauss equation:

$$R^{(3)} = R + K^2 - K_{\mu\nu}K^{\mu\nu} + 2\nabla_\mu(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu) \quad (5)$$

- By substituting these into the 3D action, we obtain a 4D scalar-tensor theory.
- Because the original 3D action lacked $\dot{N}$, the resulting 4D action satisfies U-degeneracy conditions **by construction!**

Breaking Horndeski Relations

- U-DHOST theories can break standard Horndeski relations while remaining viable.
- Class-I DHOST may have $\alpha_H \neq 0$; luminal models additionally impose $c_T^2 = 1$.
- Spatially Covariant Gravity allows operators like $D_i N D^i N$:

$$D_i N D^i N \longrightarrow \frac{1}{4(-X)^3} h^{\mu\nu} \nabla_\mu X \nabla_\nu X \quad (6)$$

- Modifies clustering of matter and effective Newton's constant on cosmological scales.

Covariant vs Unitary Degeneracy

- DHOST theories are degenerate *before* choosing a gauge.
- U-DHOST theories are degenerate *only after* choosing the unitary gauge.
- On the unitary foliation ($\partial_i \phi = 0$):

$$\hat{A}_i = D_i \phi = 0, \quad A_* = \frac{\dot{\phi}}{N}, \quad X = -A_*^2. \quad (7)$$

- Hence $\mathcal{V}_{ij} = v_1 h_{ij}$: the $v_2 \hat{A}_i \hat{A}_j$ direction disappears and the scalar kernel test becomes 2×2 .

Which Combination of D_0 , D_1 , D_2 Survives?

- Evaluate the covariant determinant polynomial on $A_*^2 = -X$:

$$D_U(X) = D_0(X) - XD_1(X) + X^2D_2(X) = 0.$$

Do not confuse two restrictions

$A_* = 0$ extracts the coefficient D_0 ; unitary gauge instead sets $\hat{A}_i = 0$ and has $A_*^2 = -X \neq 0$ for a non-trivial timelike scalar.

- U-DHOST imposes this one linear combination; covariant DHOST imposes $D_0 = D_1 = D_2 = 0$ separately.

The U-Degeneracy Condition

- Requiring a null direction only on the unitary foliation gives one condition:

$$\begin{aligned} &4[X(A_1 + 3A_2) + 2F][A_1 + A_2 + X(A_3 + A_4) + X^2A_5] \\ &= 3X(2A_2 + XA_3 + 4F_X)^2 \end{aligned} \tag{8}$$

- Every** quadratic DHOST theory satisfies this (covariant degeneracy implies gauge degeneracy).
- A **U-DHOST** theory satisfies this, but *not* all three covariant DHOST conditions $D_0 = D_1 = D_2 = 0$.
- After removing a totally U-degenerate block, the following frames reduce this equation to one 2×2 determinant.

Notation Used in the Scordatura Paper

De Felice–Mukohyama–Takahashi (arXiv:2204.02032) define

$$c_{D1} = X(A_1 + A_2),$$

$$\Upsilon = -\frac{X(4F_X + 2A_2 + XA_3)}{2\mathcal{H}_D},$$

$$c_{D2} = X^2 A_4 + 4X\Upsilon F_X - 2\mathcal{G}_T + 2(1 - \Upsilon)\tilde{\mathcal{G}}_T,$$

$$c_U = -X^3 A_5 - X^2(A_3 + A_4) - c_{D1} + 3\Upsilon^2 \mathcal{H}_D.$$

$$\mathcal{G}_T = F - XA_1,$$

$$\tilde{\mathcal{G}}_T = (1 - \Upsilon)F - 2XF_X,$$

$$\mathcal{H}_D = 2\mathcal{G}_T + 3c_{D1},$$

The similar subscripts are not a dictionary

$c_{D1} \neq D_1$ and $c_{D2} \neq D_2$ in general. The D_i are coefficients of the covariant determinant polynomial; the c 's are combinations adapted to unitary-gauge perturbations.

Connecting the Two Degeneracy Tests

Let $\mathcal{C}_{mU}$ denote the left-hand side minus the right-hand side of the full U-degeneracy condition. On the regular sector,

$$D_{mU} = \mathcal{G}_T \mathcal{C}_{mU}, \quad c_U = -\frac{X}{4\mathcal{H}_D} \mathcal{C}_{mU} = -\frac{X}{4\mathcal{H}_D \mathcal{G}_T} D_{mU}.$$

Hence $c_U = 0$ and $D_{mU} = 0$ have the same regular zero locus, but are not the same coefficient.

Hierarchy of conditions

regular class-I DHOST: $c_{D1} = c_{D2} = c_U = 0$,

genuine U-DHOST: $c_U = 0$, $(c_{D1}, c_{D2}) \neq (0, 0)$.

Totally U-degenerate Building Blocks

- We can define an action whose entire kinetic sector vanishes in unitary gauge.

$$\begin{aligned} L_{\text{tUd}}[F, \gamma] = & FR + \frac{F}{X}(L_1^{(2)} - L_2^{(2)}) + \frac{2(F - 2XF_X)}{X^2}(L_3^{(2)} - L_4^{(2)}) \\ & - \gamma(XL_4^{(2)} - L_5^{(2)}) \end{aligned} \quad (9)$$

- Direct ADM decomposition yields $\mathcal{A}_U = \mathcal{B}_U^{ij} = \mathcal{K}_U^{ij,kl} = 0$.
- Spatial derivatives and lower-derivative terms remain.
- Useful because adding it does not rotate the null vector on the unitary foliation.

α_I Are Residual Coefficients

$$FR + \sum_{I=1}^5 A_I L_I^{(2)} = L_{\text{tUd}}[F, \gamma] + \sum_{I=1}^5 \alpha_I L_I^{(2)}.$$

With $C_F \equiv 2(F - 2XF_X)/X^2$, coefficient matching gives

$$\alpha_1 = A_1 - \frac{F}{X}, \quad \alpha_2 = A_2 + \frac{F}{X}, \quad \alpha_3 = A_3 - C_F,$$
$$\alpha_4 = A_4 + C_F + \gamma X, \quad \alpha_5 = A_5 - \gamma.$$

The determinant combinations are split-independent:

$$B = \frac{X(A_1 + 3A_2) + 2F}{X}, \quad T = 2A_2 + XA_3 + 4F_X,$$
$$S = A_1 + A_2 + X(A_3 + A_4) + X^2 A_5.$$

The function γ cancels; $\alpha_I = A_I$ only for $F = \gamma = 0$.

Trace and Traceless Kinetic Blocks

After removing the totally U-degenerate curvature block, define

$$V_* \equiv \frac{\dot{A}_* - N^i D_i A_*}{N}, \quad K_{ij} = \tilde{K}_{ij} + \frac{1}{3} K h_{ij}, \quad h^{ij} \tilde{K}_{ij} = 0,$$

and

$$B = \alpha_1 + 3\alpha_2, \quad T = 2\alpha_2 + X\alpha_3, \quad S = \alpha_1 + \alpha_2 + X(\alpha_3 + \alpha_4) + X^2\alpha_5.$$

The unitary-gauge kinetic terms are

$$L_{\text{kin}}^{\text{U}} = -X\alpha_1 \tilde{K}_{ij} \tilde{K}^{ij} + S V_*^2 + A_* T K V_* - \frac{X}{3} B K^2.$$

Why the first term separates

Trace and traceless tensors are orthogonal. Hence this is an invertible block when $-X\alpha_1 \neq 0$; it affects regularity and stability, but it cannot create the scalar null direction.

One 2×2 Determinant Is Enough

$$M_U^{\text{scal}} = \begin{pmatrix} S & \frac{1}{2}A_*T \\ \frac{1}{2}A_*T & -\frac{X}{3}B \end{pmatrix}, \quad \det M_U^{\text{scal}} = \frac{X}{12} (3T^2 - 4BS).$$

Since unitary gauge requires $X = -A_*^2 < 0$, degeneracy gives

$$4BS = 3T^2.$$

For $B \neq 0$, the scalar sector becomes

$$-\frac{XB}{3} \left(K + \frac{3T}{2A_*B} V_* \right)^2, \quad (v_0, \mathcal{V}_{ij}) \propto \left(1, -\frac{T}{2A_*B} h_{ij} \right).$$

Exceptional branches

$\alpha_1 = 0$ adds a traceless degeneracy; $B = 0$ must be analyzed before dividing by B .

A Solvable 1 + 1 Model

- Consider a two-dimensional Minkowski spacetime with signature $(-, +)$.
- Toy model action:

$$L = -\frac{1}{2}X - \mu \left(XL_4^{(2)} - L_5^{(2)} \right), \quad \mu > 0 \quad (10)$$

- The second term is totally U-degenerate (with $F = 0$).
- The first term supplies one ordinary scalar mode.

Background Aligned with Unitary Time

- Let $\bar{\phi} = t$, and $\phi = t + \chi(t, x)$.
- Quadratic Lagrangian:

$$\mathcal{L}_0^{(2)} = \frac{1}{2}(\dot{\chi}^2 - \chi'^2) + \mu\dot{\chi}'^2 \quad (11)$$

- Notice the absence of $\ddot{\chi}$.
- Dispersion relation ($\chi \propto \exp(-i\omega t + ikx)$):

$$\omega_{\pm} = \pm \frac{k}{\sqrt{1 + 2\mu k^2}} \quad (12)$$

- No high- k gradient instability for $\mu > 0$. Group velocity tends to zero as $k \rightarrow \infty$.

A Tilted Background

- Now take $\bar{\phi} = t + \alpha x$, with $|\alpha| < 1$.
- Gradient is timelike but *not* aligned with t .
- The Lagrangian $\mathcal{L}_\alpha^{(2)}$ now contains $\ddot{\chi}^2$ and mixed derivatives.
- The dispersion relation becomes a quartic polynomial in ω :

$$\begin{aligned} 0 = & 2\alpha^2 \mu \omega^4 + 4\alpha(1 + \alpha^2) k_\mu \omega^3 \\ & + \left[1 + 2(\alpha^4 + 4\alpha^2 + 1) k_\mu^2 \right] \omega^2 \\ & + 4\alpha(1 + \alpha^2) k_\mu^3 \omega + 2\alpha^2 k_\mu^4 - k^2 \end{aligned} \tag{13}$$

Apparent Instability?

- For generic nonzero tilt, the quartic equation has two real roots and a complex-conjugate pair.
- If we interpret all four as propagating modes with arbitrary initial amplitudes, the theory appears unstable!
- **However**, this misses the boundary-value nature of the problem.

The Boost to Unitary Frame

- Since $|\alpha| < 1$, apply a Lorentz boost to align coordinates with the scalar gradient:

$$\tilde{t} = \frac{t + \alpha x}{\sqrt{1 - \alpha^2}}, \quad \tilde{x} = \frac{x + \alpha t}{\sqrt{1 - \alpha^2}} \quad (14)$$

- The equation of motion becomes second order in physical time $\tilde{t}$:

$$\left[1 - 2\mu(1 - \alpha^2)^2 \partial_{\tilde{x}}^2 \right] \partial_{\tilde{t}}^2 \chi - \partial_{\tilde{x}}^2 \chi = 0 \quad (15)$$

Exposing the Elliptic Equation

- Introduce an auxiliary field ψ and $\kappa = [\sqrt{2\mu}(1 - \alpha^2)]^{-1}$.
- The system splits into:

$$(\partial_t^2 - \partial_x^2)\chi = \psi \quad (\text{Hyperbolic}) \quad (16)$$

$$(\partial_x^2 - \kappa^2)\psi = -\partial_x^4 \chi \quad (\text{Elliptic}) \quad (17)$$

- The second equation is an elliptic constraint on each constant- $\tilde{t}$ slice.
- It requires **spatial boundary conditions**, not initial data ($\dot{\psi}$).

The Meaning of “Shadowy”

- The Green function for the elliptic operator decays exponentially over $\ell_{\text{sh}} = \kappa^{-1}$, the “length of the shadow”.
- What about the complex roots? Their mode functions diverge at spatial infinity!
- Spatial boundary conditions set their amplitudes to zero.

Key Idea

The shadowy mode is *not* an additional wave carrying independent initial data. It is a spatially nonlocal response fixed on each preferred-time slice by an elliptic constraint and boundary conditions.

A Cosmological U-DHOST Model

- A simple 4D model isolating the shadowy structure:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X) + Q(X) \square \phi - \xi \left(X \phi^\mu \phi_{\mu\nu} \phi^{\nu\lambda} \phi_\lambda - (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2 \right) \right] \quad (18)$$

- Here $A_4 = -\xi X$ and $A_5 = \xi$.
- Satisfies the U-degeneracy condition, but breaks covariant DHOST if $\xi \neq 0$.

Background Blindness

- On a homogeneous FLRW background, the ξ operator vanishes identically!
- The background equations are identical to Kinetic Gravity Braiding (KGB).
- **Pitfall:** A background-only code cannot distinguish this U-DHOST model from its $\xi = 0$ DHOST limit.
- The U-DHOST character first appears in inhomogeneous perturbations.

Perturbations and Phase Space

- In unitary spatial gauge, perturbed lapse α , shift χ , and spatial curvature ζ .
- The momenta of α and χ vanish (primary constraints).
- Preservation produces two secondary constraints.
- Phase space count: $N_{\text{dof}} = \frac{1}{2}(6 - 4) = 1$ scalar mode.

The Elliptic Constraint

- The lapse secondary constraint has the form:

$$\left[2a\xi \frac{\dot{\phi}^6}{N^6} \partial^2 - 2a^3 \left(\Sigma + \frac{3\Theta^2}{M_{\text{Pl}}^2} \right) \right] \alpha \approx -2M_{\text{Pl}}^2 a \partial^2 \zeta - \frac{\Theta}{M_{\text{Pl}}^2} p_\zeta \quad (19)$$

- For $\xi = 0$ (DHOST), lapse is fixed algebraically.
- For $\xi \neq 0$ (U-DHOST), it is fixed by a **Poisson-type (elliptic) equation**.
- The homogeneous solution to this equation is the cosmological shadowy mode.

Maple Setup for U-DHOST

- The following workflow is implemented in `../Maple_code/udhost_opt.mpl` using DifferentialGeometry and Tensor.
- It treats the general shift-symmetric quadratic perturbation problem; it does not reproduce the $1 + 1$ toy dispersion relation above.

```
restart:
with(DifferentialGeometry):
with(Tensor):
DGsetup([t,x,y,z],M4):
# Define the perturbed metric in unitary gauge
g:=evalDG(-N(t)^2*(1+2*epsilon*alpha(t,x))*(dt &t dt)
+ N(t)*epsilon*D[2](chi)(t,x)*((dt &t dx)+(dx &t dt))
+ a(t)^2*(1+2*epsilon*zeta(t,x))*((dx &t dx)+(dy &t dy)+(dz &t dz))):
```

Constructing Building Blocks

- We expand the scalar field $\phi = \phi(t) + \epsilon \delta \phi(t, x)$.
- Calculate covariant derivatives and invariants ($X, \square \phi, L_1$ to L_5).

```
phi_sc := phi(t) + epsilon*dphi(t,x);
D_phi_d := CovariantDerivative(phi_sc, Cf1);
ginv := InverseMetric(g);
D_phi_u := RaiseLowerIndices(ginv, D_phi_d, [1]);
X_sc := ContractIndices(D_phi_u, D_phi_d, [[1,1]]);
D2_phi_dd := CovariantDerivative(D_phi_d, Cf1);
L1 := ContractIndices(D2_phi_dd, D2_phi_uu, [[1,1],[2,2]]);
# ... similarly for L2, L3, L4, L5
```

The Full Lagrangian

- Construct the general quadratic action:

```
RST := RicciScalar(g):  
detg := LinearAlgebra[Determinant](convert(g,DGArray)):  
  
actio := sqrt(-detg) * (P(X_sc) + Q(X_sc)*Box_phi + F(X_sc)*RST  
    + A1(X_sc)*L1 + A2(X_sc)*L2 + A3(X_sc)*L3  
    + A4(X_sc)*L4 + A5(X_sc)*L5):
```


Checking the U-Degeneracy Condition

- The U-DHOST condition derived in class can be written in Maple:

```
deg_cond := 4*(X_sc*(A1(X_sc)+3*A2(X_sc))+2*F(X_sc)) *  
            (A1(X_sc)+A2(X_sc)+X_sc*(A3(X_sc)+A4(X_sc))+X_sc^2*A5(X_sc))  
            - 3*X_sc*(2*A2(X_sc)+X_sc*A3(X_sc)+4*D(F)(X_sc))^2:
```

- We expand this condition to the background order (ϵ^0):

```
deg_cond_BKG := simplify(convert(series(deg_cond, epsilon, 1), polynom)):
```

Background Equations of Motion

- By extracting the $\mathcal{O}(\epsilon)$ part of the action, we vary it with respect to α, ζ , and $\delta\phi$ to find background equations.

```
as1 := simplify(coeff(convert(series(actio,epsilon,2),polynom),epsilon)):

eom_alpha := simplify(subs(tt1=t, xx1=x,
                           convert(Physics[Fundiff](Physics[Intc](as1,t,x), alpha(tt1,xx1)
                           ),D))) :
# Similarly for eom_zet, eom_dphi
```

- The displayed residuals should be checked explicitly against the chosen background equations.

Quadratic Action and Kinetic Matrix

- To extract the kinetic matrix, we take the $\mathcal{O}(\epsilon^2)$ part of the action.
- We use heavy integration by parts routines to isolate $\dot{\zeta}^2, \dot{\alpha}^2, \dot{\zeta}\dot{\alpha}$:

```
az1 := simplify(coeff(convert(series(actio,epsilon,3),polynom),epsilon^2)):
# ... extensive by-parts routines (e.g. by_parts_D2_BG)

K11 := simplify(coeff(az3, D[1](zet)(t,x)^2)):
K22 := simplify(coeff(az3, D[1](alpha)(t,x)^2)):
K12 := simplify(1/2*coeff(coeff(az3, D[1](zet)(t,x)), D[1](alpha)(t,x))):
```

Verifying Degeneracy in Maple

- We verify that the kinetic determinant vanishes exactly when the U-degeneracy condition is met:

```
simplify(K11*K22 - K12^2 - 1/N(t)^4*(-(3*a(t)^6*D(phi)(t)^2)/4)*deg_cond_BKG);  
# Output: 0
```

- This means $K_{22} = K_{12}^2/K_{11}$. We perform a field redefinition to diagonalize the kinetic sector:

```
zet := unapply(zet2(t,x) - K12/K11*alpha(t,x), t, x):
```

Constraint Equations and the Shadowy Mode

- After diagonalizing, the $\dot{\alpha}^2$ coefficient is directly proportional to `deg_cond_BKG`.
- Imposing U-degeneracy sets this term to zero: no dynamics for the lapse.
- We then integrate by parts to isolate coefficients for spatial derivatives like $\partial_x^2 \alpha$.

```
B1_sz := simplify(coeff(az5, D[2,2](chi)(t,x)^2)):
B2_sz := simplify(coeff(az5, D[2](alpha)(t,x)^2)):
# ... B2_sz contains the Poisson constraint term.
```

- A nonzero spatial-gradient coefficient is the ingredient that turns the lapse equation into an elliptic constraint; the script does not by itself choose its boundary conditions.

What `udhost_opt.mpl` Reduces

After U-degeneracy, a field redefinition, and integrations by parts, the worksheet rewrites the quadratic action as

$$L^{(2)} = B_1(\partial^2\chi)^2 + B_2(\partial\alpha)^2 + B_3\alpha\partial^2\zeta_2 + B_4\alpha\partial^2\chi + B_5\alpha\dot{\zeta}_2 + B_6(\partial^2\chi)\dot{\zeta}_2 \\ + B_7(\partial^2\chi)\zeta_2 + B_8\alpha^2 + B_9\dot{\zeta}_2^2 + B_{10}(\partial\zeta_2)^2 + B_{11}(\partial\chi)^2.$$

- Explicit zero tests justify $B_7 = B_{11} = 0$ and $B_4 = B_5B_6/(2B_9)$ on the selected background branch.
- The code checks these relations before substituting them.

Reduced Action, No Ghost, and Propagation Speed

- 1 Fourier map: $(\alpha, \chi, \zeta_2) \rightarrow (AL, CH, ZZ2)$ and $\partial^2 \rightarrow -k^2$.
- 2 Solve the two auxiliary equations, first for AL and then for CH; substitute into the action.
- 3 Integrate $\zeta_k \dot{\zeta}_k$ by parts and read

$$L_{\text{red}}^{(2)} = \frac{a^3}{N} Q_s \dot{\zeta}_k^2 - Na Q_s c_s^2 k^2 \zeta_k^2 + \dots$$

$$Q_s^{\text{UV}} > 0 \quad (\text{no ghost}), \quad (c_s^2)^{\text{UV}} > 0 \quad (\text{no UV gradient instability}).$$

A rational dependence on k^2 is the Fourier image of the inverse elliptic operator; denominators and boundary zero modes must also be checked.

Summary of U-DHOST Analysis

- **Theory:** U-DHOST breaks covariant degeneracy but maintains it in unitary gauge.
- **Result:** No Ostrogradsky ghost! The apparent extra mode is a constrained "shadowy" mode fixed by elliptic boundary conditions.
- **Cosmology:** Background is often indistinguishable from standard DHOST/KGB, but perturbations reveal modified spatial constraints.
- **Practice:** Maple automates the ADM decomposition and integrations by parts; a trustworthy run must also display and inspect the zero residuals and assumptions introduced by `solve`.