δ-CDM: a controlled deviation from ΛCDM

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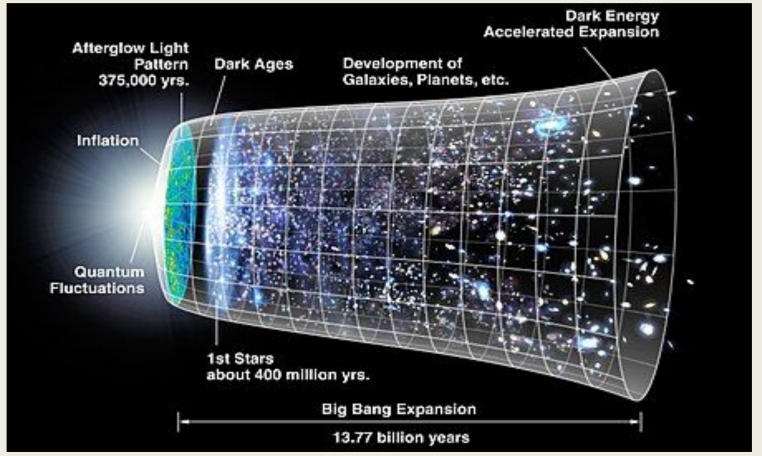
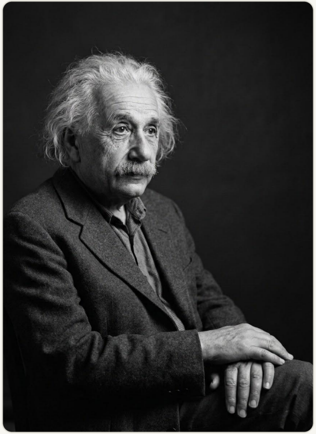
The Universe is accelerating



The cosmological constant returned to centre stage



$$R\mu\nu \ -1/2\ R\ g\mu\nu + \Lambda\ g\mu\nu = (\ 8\pi G\ /\ c4\ )\ T\mu\nu$$

1917 — The Static Universe

First introduced by Einstein to achieve a static universe by counterbalancing the attraction of gravity. He abandoned the idea in 1931 after Hubble's discovery of cosmic expansion.

1998 — Accelerated Expansion

Discovered by the Supernova Cosmology Project and the High-Z Supernova Search Team using distant Type Ia supernovae. This breakthrough revived the cosmological constant as an active research field.

Why Einstein introduced Λ

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4 x \, \sqrt{-g} \, (R - 2\Lambda) + S_\mathrm{m}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \, T_{\mu\nu}.$$

- General relativity initially predicted a dynamical Universe.
- A positive Λ was added to permit a static cosmological solution.
- In modern cosmology, the same term acts as a constant vacuum-energy density and accelerate the universe.

Modern interpretation

$\rho_\Lambda = \Lambda/(8\pi G)$ has pressure $p_\Lambda = -\rho_\Lambda$, hence $w_\Lambda = -1$.

Einstein's static Universe is unstable

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3},$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}.$$

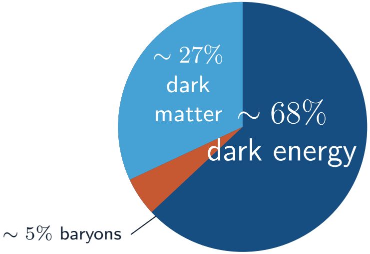
For dust, $p = 0$, and a static state, $H = \ddot{a} = 0$:

$$\rho = \frac{\Lambda}{4\pi G}, \qquad \frac{K}{a^2} = \Lambda.$$

Instability

A small fluctuation can either expand of collapse the universe very rapidly. The equilibrium does not self-correct.

ΛCDM became the concordance model



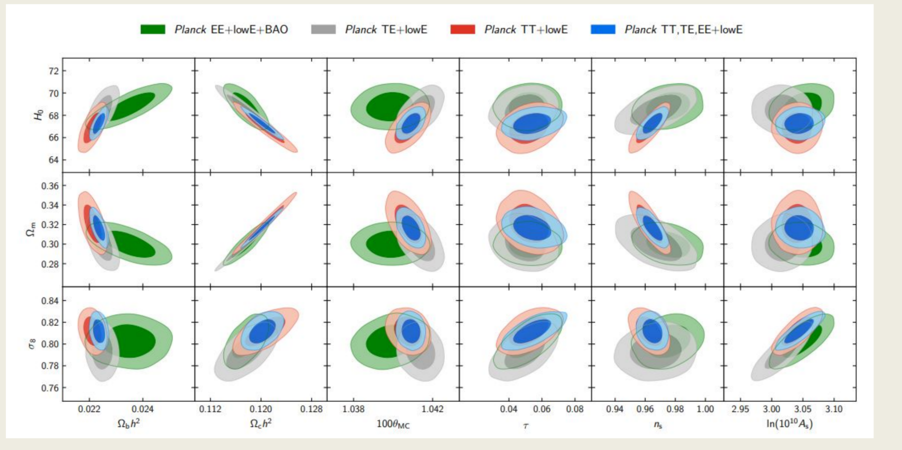
- A spatially flat, six-parameter model fits CMB anisotropies with remarkable precision.
- Cold dark matter builds structure; Λ drives the recent accelerated expansion.
- BAO, supernovae, lensing, and large-scale structure broadly support the same background history.

Its success makes every proposed deviation answer to a very high bar.

Planck fixes the six-parameter model tightly

Parameter	Plik best fit	Plik [1]	CamSpec [2]	$([2]-[1])/\sigma_1$	Combined
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015	0.02229 ± 0.00015	-0.5	0.02233 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012	0.1197 ± 0.0012	-0.3	0.1198 ± 0.0012
$100\theta_{MC}$	1.040909	1.04092 ± 0.00031	1.04087 ± 0.00031	-0.2	1.04089 ± 0.00031
τ	0.0543	0.0544 ± 0.0073	$0.0536^{+0.0069}_{-0.0077}$	-0.1	0.0540 ± 0.0074
$\ln(10^{10} A_s)$	3.0448	3.044 ± 0.014	3.041 ± 0.015	-0.3	3.043 ± 0.014
n_s	0.96605	0.9649 ± 0.0042	0.9656 ± 0.0042	$+0.2$	0.9652 ± 0.0042
$\Omega_m h^2$	0.14314	0.1430 ± 0.0011	0.1426 ± 0.0011	-0.3	0.1428 ± 0.0011
$H_0\,[\mathrm{km\,s^{-1}\,Mpc^{-1}}]$	67.32	67.36 ± 0.54	67.39 ± 0.54	$+0.1$	67.37 ± 0.54
Ω_m	0.3158	0.3153 ± 0.0073	0.3142 ± 0.0074	-0.2	0.3147 ± 0.0074
Age [Gyr]	13.7971	13.797 ± 0.023	13.805 ± 0.023	$+0.4$	13.801 ± 0.024
σ_8	0.8120	0.8111 ± 0.0060	0.8091 ± 0.0060	-0.3	0.8101 ± 0.0061
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.8331	0.832 ± 0.013	0.828 ± 0.013	-0.3	0.830 ± 0.013
z_{re}	7.68	7.67 ± 0.73	7.61 ± 0.75	-0.1	7.64 ± 0.74
$100\theta_*$	1.041085	1.04110 ± 0.00031	1.04106 ± 0.00031	-0.1	1.04108 ± 0.00031
r_{drag} [Mpc]	147.049	147.09 ± 0.26	147.26 ± 0.28	$+0.6$	147.18 ± 0.29

Independent Planck spectra give compatible constraints



Precision success does not remove the open questions

Theoretical puzzles

- Why is the observed vacuum energy so small?
- Why are matter and dark-energy densities comparable today?
- Is Λ a fundamental constant or an effective description?

Observational tensions

- Early- and late-Universe estimates of H_0 disagree.
- Some weak-lensing surveys prefer lower clustering amplitude.
- Recent BAO and supernova combinations admit evolving dark energy.

The question is not whether ΛCDM fits, but whether the residual patterns contain new physics.

The vacuum-energy estimate misses by an enormous factor

$$\frac{\rho_{\rm vac}^{\rm QFT}}{\rho_{\Lambda}^{\rm obs}} \sim 10^{121}.$$

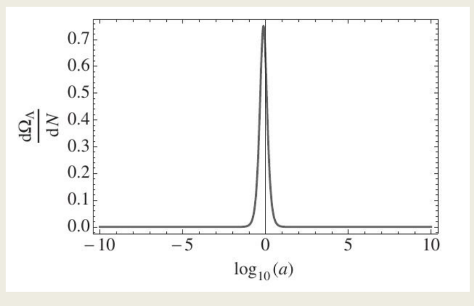
Why is the cosmological constant non-zero, yet so tiny?

Why do matter and dark energy coincide now?

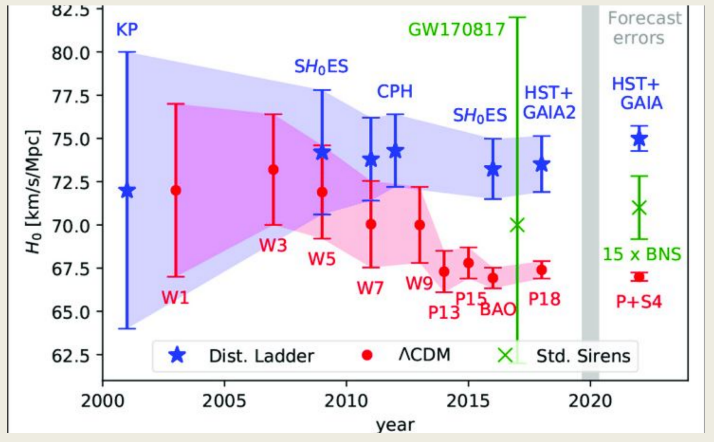
- Matter dilutes as $\rho_m \propto a^{-3}$.
- A cosmological constant remains fixed: $\rho_\Lambda = \text{constant}$.
- Their comparable values today therefore occupy a narrow interval of cosmic history.

Coincidence problem

Why do observers live so close to the transition from matter domination to dark-energy domination?



The Hubble tension compares two cosmic calibrations



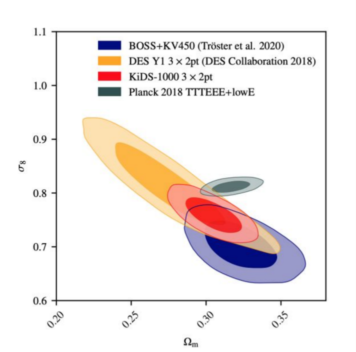
Early Universe

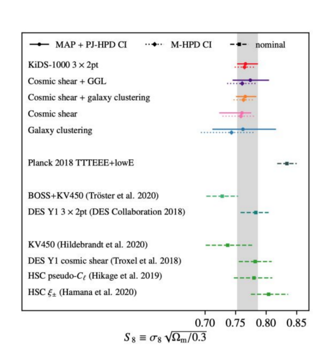
CMB plus a ΛCDM calibration gives $H_0 \simeq 67$–68.5 km s^{-1} Mpc^{-1}.

Distance ladder

Calibrated local supernovae give $H_0 \simeq 73$–74 km s^{-1} Mpc^{-1}.

Weak lensing tests the amplitude of structure growth

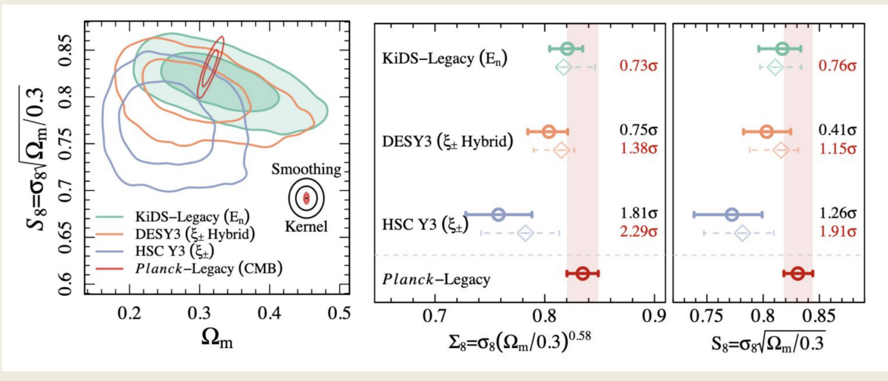




Apart from the Hubble tension, another tension between the Planck data with the weak lensing and the redshift surveys has been reported.

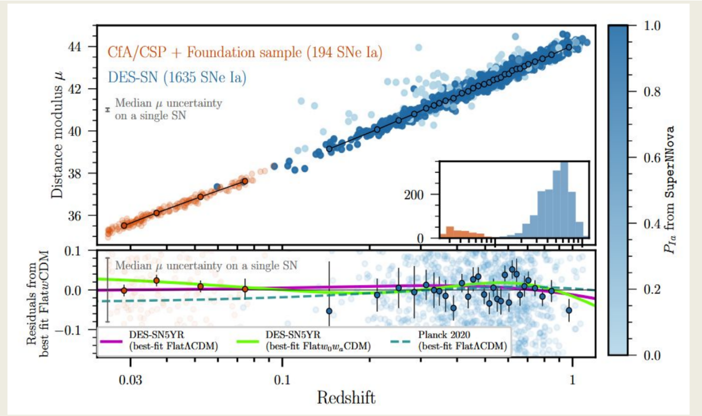
$S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$ combines clustering amplitude and matter density along the principal lensing degeneracy.

The updated S_8 picture is survey dependent



New analyses reduce some discrepancies, but calibration, nonlinear modelling, and survey combinations remain important.

DES extends the supernova Hubble diagram



- The five-year DES sample contains 1,635 high-quality transients.
- 1,499 have a Type-Ia probability above 50%.
- The enlarged high-redshift sample sharpens tests of the recent expansion history.

DES compares nested expansion histories

Cosmological Model	Friedmann Equation: $\mathbf{E(z) = H(z)/H_0} =$	Fit Parameters Θ
Flat-ΛCDM	$\left[\Omega_M(1+z)^3 + (1-\Omega_M)\right]^{1/2}$	Ω_M
ΛCDM	$\left[\Omega_M(1+z)^3 + \Omega_\Lambda + (1-\Omega_M-\Omega_\Lambda)(1+z)^2\right]^{1/2}$	Ω_M, Ω_Λ
Flat-wCDM	$\left[\Omega_M(1+z)^3 + (1-\Omega_M)(1+z)^{3(1+w)}\right]^{1/2}$	Ω_M, w
Flat-w_0w_aCDM	$\left[\Omega_M(1+z)^3 + (1-\Omega_M)(1+z)^{3(1+w_0+w_a)}e^{-3w_a z/(1+z)}\right]^{1/2}$	Ω_M, w_0, w_a

The comparison ranges from a fixed cosmological constant to constant-w and time-dependent dark-energy descriptions.

DES mildly shifts the preferred constant w above -1

	Ω_M	Ω_K	w_0	w_a	χ^2/dof
DES-SN5YR (no external priors)					
Flat-ΛCDM	0.352 ± 0.017	-	-	-	1649/1734=0.951
ΛCDM	$0.291^{+0.063}_{-0.065}$	0.16 ± 0.16	-	-	1648/1733=0.951
Flat-wCDM	$0.264^{+0.074}_{-0.096}$	-	$-0.80^{+0.14}_{-0.16}$	-	1648/1733=0.951
Flat-w_0w_aCDM	$0.495^{+0.033}_{-0.043}$	-	$-0.36^{+0.36}_{-0.30}$	$-8.8^{+3.7}_{-4.5}$	1641/1732=0.948
DES-SN					
Flat-ΛCD					.952
ΛCDM					.950
Flat-wCD					.951
Flat-w_0w_a					.951

5.1.2. *Is dark energy a cosmological constant?*

As seen in Sec 4.1, a cosmological constant is a good fit to our data, but not the best fit. Our best fit equation of state parameter is slightly (more than 1σ) higher than the cosmological constant value of $w = -1$ (both for SNe alone and in combination with Planck or BAO+3×2pt). Our result agrees with the recent result from the UNION3 compilation analyzed with the UNITY framework (Rubin et al. 2023) (which appeared while this paper was under internal review). The Pantheon+ result (Brout et al. 2022a) is within 1σ of $w = -1$, but also on the high side ($w = -0.90 \pm 0.14$).

- A constant equation of state remains an adequate description.
- The DES best fit lies slightly on the $w > -1$ side.

DESI DR2 changes the late-time evidence base

~ 14 million
galaxies and
quasars

- The three-year DESI dataset maps BAO over a broad redshift range.
- Multiple tracer populations provide radial and transverse distance information.
- Combined with CMB and supernovae, the data sharpen tests of evolving dark energy.

Late-time dynamics is now tested by a genuinely multi-probe distance ladder.

CPL is the standard two-parameter benchmark

$$w_{\mathrm{de}}(z) = w_0 + w_a \frac{z}{1+z}$$

w_0
present-day value

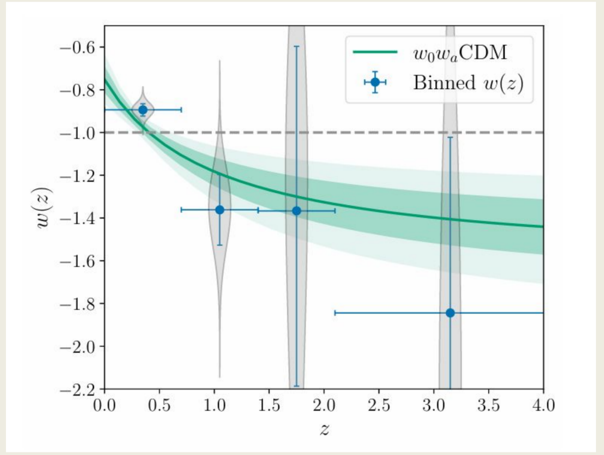
w_a
evolution amplitude

$(-1, 0)$
ΛCDM limit

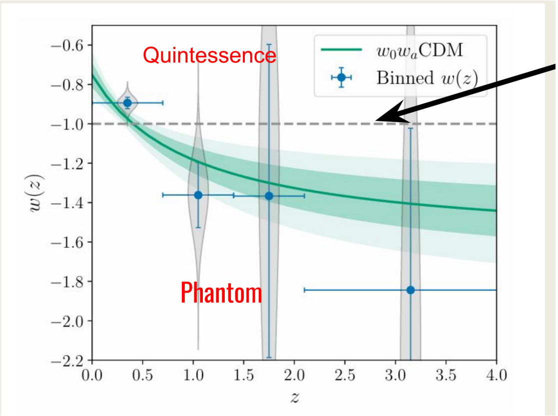
Useful but phenomenological

CPL is flexible enough to cross $w = -1$, but its parameters do not by themselves specify an inline physical theory.

The reconstructed $w(z)$ suggests possible evolution



Phantom-barrier crossing motivates a controlled alternative



- $w > -1$: quintessence-like behaviour.
- $w = -1$: cosmological-constant boundary.
- $w < -1$: phantom-like behaviour.

Question

Can we parameterize the departure from -1 directly, retain the standard model as a strict limit, and follow the perturbations consistently?

The δ-CDM definition makes the ΛCDM limit explicit

$$w_{\mathrm{de}}(z) = -1 + \delta(z)$$

$$\delta(z) = 0$$

Exactly ΛCDM

$$\delta(z) > 0$$

$w > -1$: quintessence-like

$$\delta(z) < 0$$

$w < -1$: phantom-like

Why this parametrization?

It turns "is dark energy dynamical?" into a reconstruction problem for a single deviation function, with the standard model nested at $\delta = 0$.

Baisri et al., Sec. II.B.

Expansion history connects the effective fluid to observables

$$E^2(z) = \Omega_{m0}(1+z)^3 + \Omega_{\mathrm{de},0}\, \exp\left[3\int_0^z \frac{\delta(z')}{1+z'}\mathrm{d}z'\right],$$

$$E(z) \equiv H(z)/H_0.$$

- The entire late-time modification enters through the integral of $\delta(z)$.
- Setting $\delta = 0$ immediately recovers the familiar matter plus constant-Λ expansion history.

$$\delta(z) \xrightarrow{\text{continuity equation}} E(z) \xrightarrow{\text{distances \& spectra}} \text{BAO, SNe, CMB}$$

Baisri et al., Sec. II.B.

A unified scalar field gives a physical reconstruction route

$$\rho_\phi = \frac{\epsilon}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \frac{\epsilon}{2}\dot{\phi}^2 - V(\phi),$$

$$\epsilon = +1 \quad \text{(quintessence)}, \qquad \epsilon = -1 \quad \text{(phantom)}.$$

- One switching parameter carries both signs of the kinetic term.

scalar field ϕ → effective fluid → $\delta(z)$

same formalism, two scalar field realizations

Baisri et al., Sec. II.A.

The reconstruction links field motion directly to $\delta(z)$

$$\frac{\mathrm{d}\phi}{\mathrm{d}z} = \frac{\sqrt{3}}{E(1+z)} \left[\frac{\left(E^2 - \Omega_{m0}(1+z)^3\right)\delta(z)}{\epsilon} \right]^{1/2},$$

$$V(z) = 3H_0^2\left(E^2 - \Omega_{m0}(1+z)^3\right)\left(1 - \frac{\delta(z)}{2}\right).$$

Interpretation

For a specified expansion history, $\delta(z)$ determines both the field trajectory and its reconstructed potential.

The key move: do not postulate an arbitrary $w(z)$; derive its allowed form from a thawing-field approximation.

Geometry, Friedmann equations, and field dynamics

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2),$$

$$H^2 = \frac{1}{3}(\rho_m + \rho_\phi), \qquad \dot{H} = -\frac{1}{2}(\rho_m + \rho_\phi + p_\phi),$$

$$\rho_\phi = \frac{\epsilon}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \frac{\epsilon}{2}\dot{\phi}^2 - V(\phi),$$

$$\epsilon\ddot{\phi} + 3\epsilon H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0.$$

First algebraic step

Combining the acceleration equation with the scalar density and pressure cancels V: $\rho_\phi + p_\phi = \epsilon\dot{\phi}^2$. This produces the kinetic-energy relation used for reconstruction.

Baisri et al., Sec. II.A.

Redshift reconstruction and the effective EoS

$$2\dot{H} = -\frac{\rho_{m0}}{a^3} - \epsilon\dot{\phi}^2, \qquad a\frac{\mathrm{d}(H^2)}{\mathrm{d}a} + \frac{\rho_{m0}}{a^3} = -\epsilon\dot{\phi}^2,$$

$$\dot{\phi} = aH\frac{\mathrm{d}\phi}{\mathrm{d}a}, \qquad \frac{\mathrm{d}\phi}{\mathrm{d}z} = \left[\frac{2E\,\mathrm{d}E/\mathrm{d}z - 3\Omega_{m0}(1+z)^2}{\epsilon E^2(1+z)}\right]^{1/2},$$

$$\frac{V(z)}{3H_0^2} = -\frac{(1+z)}{3}E\frac{\mathrm{d}E}{\mathrm{d}z} + E^2 - \frac{1}{2}\Omega_{m0}(1+z)^3,$$

$$\Omega_m(z) = \frac{\Omega_{m0}(1+z)^3}{E^2}, \qquad \Omega_\phi(z) = 1 - \Omega_m(z),$$

$$w_\phi(z) = \frac{2(1+z)E\,\mathrm{d}E/\mathrm{d}z - 3E^2}{3\left[E^2 - \Omega_{m0}(1+z)^3\right]}.$$

Defining the deformation and its expansion history

$$
\begin{aligned}
w_{\rm de}(z) &= -1 + \delta(z), \\
\delta(z) &= \frac{1+z}{3\Omega_\phi E^2}\left[2E\frac{\mathrm{d}E}{\mathrm{d}z} - 3\Omega_{m0}(1+z)^2\right], \\
E^2(z) &= \Omega_{m0}(1+z)^3 + \Omega_{\rm de,0}\exp\left[3\int_0^z \frac{\delta(z')}{1+z'}\mathrm{d}z'\right], \\
\frac{\mathrm{d}\phi}{\mathrm{d}z} &= \frac{\sqrt{3}}{E(1+z)}\left[\frac{\left(E^2 - \Omega_{m0}(1+z)^3\right)\delta(z)}{\epsilon}\right]^{1/2}, \\
\frac{\mathrm{d}\phi}{\mathrm{d}z} &\simeq \frac{\sqrt{3}}{E(1+z)}\left[\frac{\Omega_{\rm de,0}\delta(z)}{\epsilon}\right]^{1/2}, \\
V(z) &= 3H_0^2\left(E^2 - \Omega_{m0}(1+z)^3\right)\left(1 - \frac{\delta(z)}{2}\right), \qquad V(z) = A + B\delta(z).
\end{aligned}
$$

Baisri et al., Sec. II.B–C.

Thawing closure gives the two-parameter EoS

$$\frac{\mathrm{d}\delta}{\mathrm{d}z} - \frac{\widetilde{w}_0}{1+z}\delta = 0,$$

$$\delta(z) = \widetilde{w}_a(1+z)^{\widetilde{w}_0},$$

$$\widetilde{w}_0 = \frac{6}{1 + \frac{2B}{3H_0^2\Omega_{\mathrm{de},0}}},$$

$$w_{\mathrm{de}}(z) = -1 + \widetilde{w}_a(1+z)^{\widetilde{w}_0}.$$

1. Insert the low-z thawing approximation and the effective potential into the Klein–Gordon equation.
2. Solve the separable ODE. $\widetilde{w}_0 < 0$ ensures $w_{\mathrm{de}} \to -1$ at high redshift.
3. The sign parameter ϵ cancels: the effective EoS is independent of the scalar-field realization.

Baisri et al., Sec. II.C.

Linear Perturbation

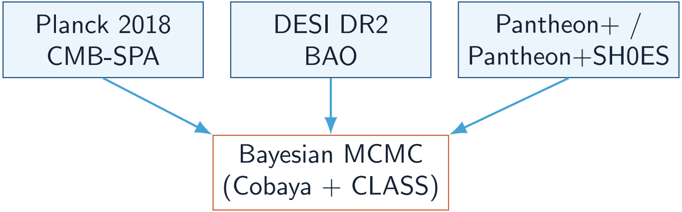
$$
\begin{aligned}
\mathrm{d}s^2 &= a^2(\eta)\left[-(1+2\psi)\mathrm{d}\eta^2 + (1-2\phi)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right],\\
\delta' &= -(1+w)(\theta - 3\phi') - 3\mathcal{H}\left(\frac{\delta p}{\delta\rho} - w\right)\delta,\\
\theta' &= -\mathcal{H}(1-3w)\theta - \frac{w'}{1+w}\theta + \frac{\delta p/\delta\rho}{1+w}k^2\delta + k^2\psi,\\
\frac{\delta p}{\delta\rho} &= c_s^2 + 3\mathcal{H}(1+w)(c_s^2 - c_a^2)\frac{\theta}{k^2},\\
c_a^2 &= w - \frac{a}{3(1+w)}\frac{\mathrm{d}w}{\mathrm{d}a}.
\end{aligned}
$$

Why PPF is needed

The $1/(1+w)$ terms are numerically delicate near $w=-1$. CLASS uses PPF to evolve perturbations stably through this regime.

Baisri et al., Sec. III.

The test combines early- and late-universe distance information



Fit parameters: standard cosmological parameters plus $\widetilde{w}_0, \widetilde{w}_a$.

Comparisons: ΛCDM, reconstructed δ-CDM, and a $w_0 w_a$ model.

Baisri et al., Sec. IV.

Model comparison asks two different questions

Goodness of fit

$$\Delta\chi^2_{\rm min}$$

Does the more flexible model describe the data more closely?

Parsimony-aware comparison

$$\mathrm{AIC} = \chi^2_{\rm min} + 2k$$

Is the improvement large enough to justify the extra parameters?

$$\boxed{\text{better fit}} \xrightarrow{\text{penalize complexity}} \boxed{\text{preferred model}}$$

Important: a negative $\Delta\chi^2$ alone is not evidence that the extension is favored.

Baisri et al., Secs. IV–V.

Standard-parameter constraints

Model	Dataset	H_0	$\Omega_b h^2$	$\Omega_c h^2$	Ω_m	σ_8
	CMB-SPA	67.32 ± 0.38	0.022397 ± 0.000095	0.12042 ± 0.00093	0.3166 ± 0.0055	0.8147 ± 0.0039
	CMB-SPA+PP+DESI	68.16 ± 0.24	0.022466 ± 0.000089	0.11836 ± 0.00060	0.3045 ± 0.0033	0.8133 ± 0.0039
ΛCDM	CMB-SPA+PPS+DESI	68.46 ± 0.24	0.022518 ± 0.000093	0.11772 ± 0.00057	0.3006 ± 0.0031	0.8128 ± 0.0038
	PP+DESI	72^{+10}_{-4}	$0.0270^{+0.013}_{-0.0046}$	$0.134^{+0.032}_{-0.020}$	0.3047 ± 0.0080	$0.79^{+0.16}_{-0.36}$
	PPS+DESI	73.8 ± 1.0	0.0285 ± 0.0015	$0.1365^{+0.0050}_{-0.0059}$	0.3042 ± 0.0078	$0.80^{+0.19}_{-0.36}$
	CMB-SPA	77^{+10}_{-9}	0.022435 ± 0.000097	0.11959 ± 0.00097	$0.251^{+0.032}_{-0.085}$	$0.896^{+0.097}_{-0.064}$
	CMB-SPA+PP+DESI	67.71 ± 0.59	0.022429 ± 0.000092	0.11948 ± 0.00078	0.3110 ± 0.0055	0.8157 ± 0.0074
$w_0 w_a$	CMB-SPA+PPS+DESI	69.04 ± 0.54	0.022447 ± 0.000094	0.11964 ± 0.00076	0.2995 ± 0.0049	0.8283 ± 0.0071
	PP+DESI	70^{+10}_{-6}	$0.0262^{+0.013}_{-0.0053}$	0.125 ± 0.027	$0.305^{+0.018}_{-0.012}$	$0.74^{+0.17}_{-0.35}$
	PPS+DESI	73.4 ± 1.0	$0.0308^{+0.0024}_{-0.0036}$	$0.132^{+0.012}_{-0.0090}$	$0.304^{+0.018}_{-0.013}$	$0.75^{+0.17}_{-0.35}$
	CMB-SPA	$74.5^{+5.8}_{-9.7}$	0.022404 ± 0.000097	0.12031 ± 0.00098	$0.267^{+0.051}_{-0.061}$	$0.874^{+0.052}_{-0.078}$
	CMB-SPA+PP+DESI	67.32 ± 0.58	0.022475 ± 0.000090	0.11813 ± 0.00063	0.3118 ± 0.0056	0.8052 ± 0.0065
δ-CDM	CMB-SPA+PPS+DESI	$68.83^{+0.58}_{-0.66}$	0.022514 ± 0.000092	0.11792 ± 0.00066	0.2978 ± 0.0054	$0.8171^{+0.0068}_{-0.0080}$
	PP+DESI	70^{+10}_{-5}	$0.0260^{+0.014}_{-0.0053}$	$0.125^{+0.030}_{-0.021}$	0.3061 ± 0.0083	$0.76^{+0.17}_{-0.35}$
	PPS+DESI	73.4 ± 1.0	0.0303 ± 0.0017	0.1337 ± 0.0057	0.3058 ± 0.0083	$0.77^{+0.16}_{-0.35}$

The five rows in each block are the five dataset combinations; quoted intervals are 68% confidence intervals.

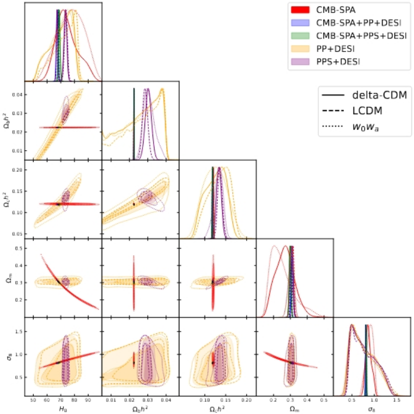
Dark-energy posteriors and model comparison

Model	Dataset	$w_0/\tilde{w}_0$	$w_a/\tilde{w}_a$	$\chi^2_{\rm min}$	$\Delta\chi^2_{\rm min}$	ΔAIC
ΛCDM	CMB-SPA	-	-	613.5 ± 4.2	0	0
	CMB-SPA+PP+DESI	-	-	2036.5 ± 4.6	0	0
	CMB-SPA+PPS+DESI	-	-	2116.2 ± 4.9	0	0
	PP+DESI	-	-	1418.1 ± 1.9	0	0
	PPS+DESI	-	-	1468.4 ± 2.5	0	0
$w_0 w_a$	CMB-SPA	< -0.596	< -2.31	610.6 ± 4.3	-2.9	1.1
	CMB-SPA+PP+DESI	-0.837 ± 0.054	$-0.63^{+0.21}_{-0.18}$	2028.2 ± 4.7	-8.3	-4.3
	CMB-SPA+PPS+DESI	-0.873 ± 0.053	-0.68 ± 0.21	2103.6 ± 4.7	-12.6	-8.6
	PP+DESI	$-0.882^{+0.055}_{-0.065}$	$-0.28^{+0.46}_{-0.37}$	1415.2 ± 2.7	-2.9	1.1
	PPS+DESI	$-0.887^{+0.053}_{-0.062}$	$-0.24^{+0.46}_{-0.34}$	1465.2 ± 2.9	-3.2	0.8
δ-CDM	CMB-SPA	> -5.89	< -0.272	612.8 ± 4.4	-0.7	3.3
	CMB-SPA+PP+DESI	< -5.62	$0.116^{+0.065}_{-0.094}$	2035.4 ± 4.9	-1.1	2.9
	CMB-SPA+PPS+DESI	—	$-0.028^{+0.047}_{-0.061}$	2117.1 ± 5.3	0.9	4.9
	PP+DESI	< -4.08	$0.149^{+0.067}_{-0.097}$	1414.9 ± 2.5	-3.2	0.8
	PPS+DESI	< -4.13	$0.156^{+0.068}_{-0.092}$	1464.9 ± 2.8	-3.5	0.5

$\Delta\chi^2_{\rm min}$ measures fit improvement; ΔAIC includes the penalty for two additional dark-energy parameters.

Baisri et al., dark-energy and model-selection summary.

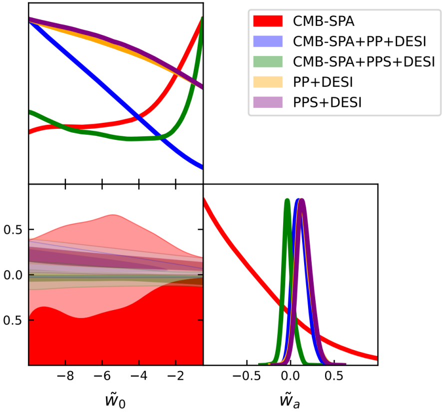
Posterior constraints



Solid: δ-CDM; dashed: ΛCDM; dotted: $w_0 w_a$; contours: 68% and 95%.

Baisri et al., Fig. 1.

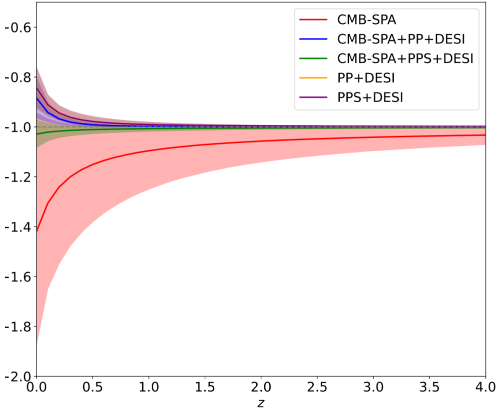
Posterior of the reconstructed dark-energy parameters



DESI DR2 and supernovae contract the $\widetilde{w}_0$–$\widetilde{w}_a$ degeneracy; CMB-SPA alone remains broad.

Baisri et al., Fig. 2 and Sec. V.

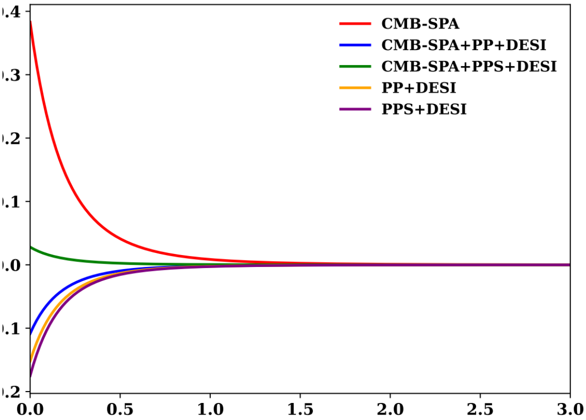
Reconstructed equation of state: a small, late-time deviation



$w \to -1$ at high redshift; no phantom-divide crossing occurs in this thawing subclass.

Baisri et al., Fig. 3.

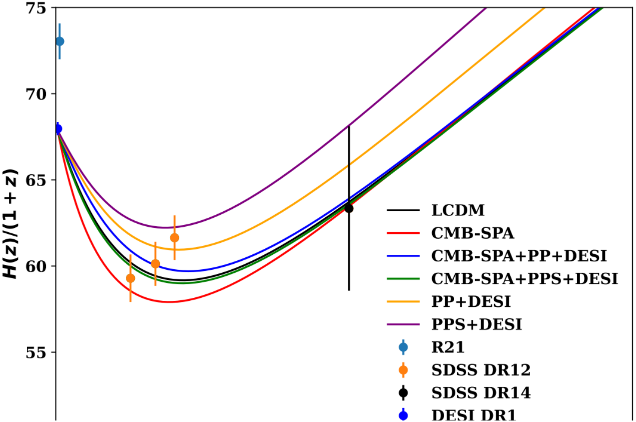
Reconstructed scalar-field potential



$V(z) = B\delta(z)$ with $A = 0$; phantom-like and quintessence-like fits have opposite late-time trends.

Baisri et al., Fig. 4 and Sec. II.C.

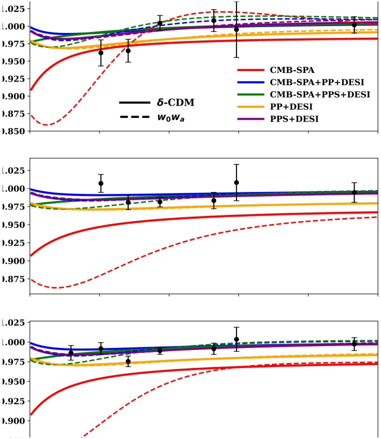
Expansion history: $H(z)/(1+z)$



CMB-containing combinations track ΛCDM; low-redshift-only fits depart more visibly at $z \lesssim 1.5$.

Baisri et al., Fig. 5.

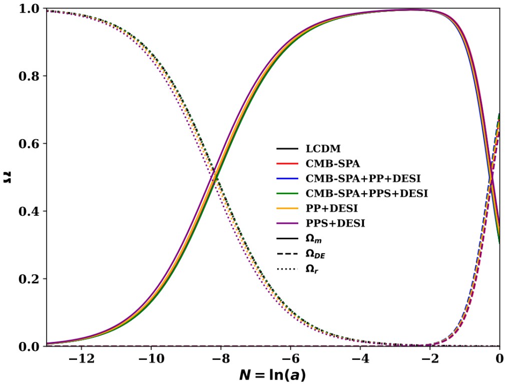
DESI DR2 BAO distances: where the fit is tested directly



Top to bottom: $\Delta(D_H/r_d)$, $\Delta(D_M/r_d)$, $\Delta(D_V/r_d)$; black points are DESI DR2.

Baisri et al., Fig. 6.

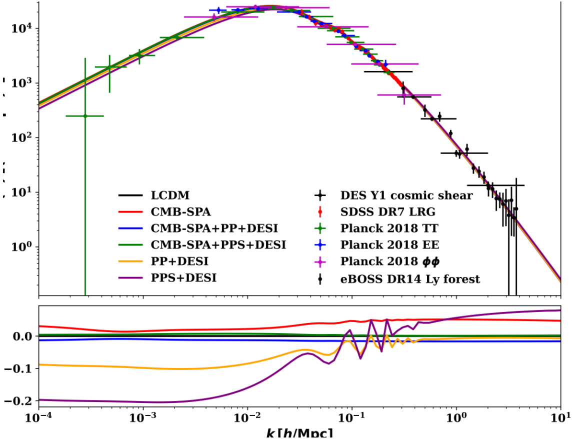
Background cosmic history stays close to standard evolution



Radiation, matter, and dark-energy domination occur in the conventional sequence in every fit.

Baisri et al., Fig. 7.

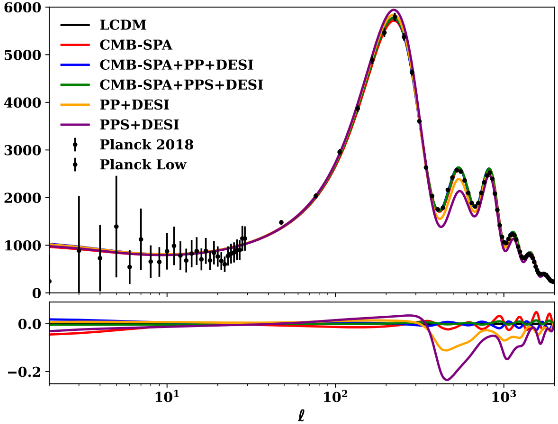
Matter power spectrum: deviations are small and scale dependent



Largest fractional changes: low k for PP+DESI and PPS+DESI, the combinations without CMB.

Baisri et al., Fig. 8.

CMB temperature spectrum: consistency with Planck is retained



Best-fit curves remain close to ΛCDM; late-time-only fits create the larger fractional deviations.

Baisri et al., Fig. 9.

Take-home message

1. δ-CDM is a minimal deformation because it makes departures from $w = -1$ explicit and keeps ΛCDM inside the model.
2. A thawing scalar-field approximation reconstructs

$$w_{\rm de}(z) = -1 + \widetilde{w}_a (1+z)^{\widetilde{w}_0}, \qquad \widetilde{w}_0 < 0.$$

3. Current observations allow small late-time departures, but complexity-aware model selection does not yet displace ΛCDM.

Where the framework can go next

δCDM formalism → Other dark-energy / modified-gravity models → Higher-precision surveys

- Generalize the reconstruction while retaining $\delta(z) = 0$ as the unambiguous ΛCDM anchor.
- Test whether future BAO, supernova, weak-lensing, and CMB data can discriminate functional forms rather than merely fit them.
- This is an example how we can build fluid description of dark energy from inline physical theory and it can be applied to other dark energy or modified gravity models.

References and discussion

P. Baisri, N. Roy, P. Sahoo, S. Chakrabarti, and J. L. Said, *δ-CDM: A Minimal Deformation of ΛCDM with Scalar Field Reconstruction*, arXiv:2606.08383 (2026).

Questions?