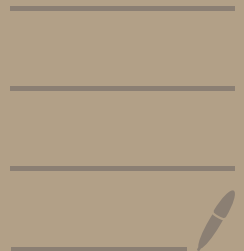


EFT of Inflation / DE

1. Review dynamics of inflation.
2. Construction of unitary-gauge EFT
3. Applications for inflation
4. Applications for DE



1. Quick review on Inflation

- Since we are going to construct the EFT of perturbations during inflation / dark energy, it's better to give you a quick review on a canonical single-field inflation.
- The same idea can be applied to dark energy
[The differences are (1) energy scales
(2) matter contents]
E.g. For dark energy, we usually take into account the mixing with matters $\Rightarrow$ screening mechanism
(consistent with solar system tests)
- Without going through the motivations like solving the horizon problem and the flatness problem, we begin by saying that inflation is an accelerating phase in the early universe, that provides initial conditions to the Hot Big Bang.

- Inflation = Quasi de Sitter

↳ It has to stop at some point.

- We will see that some of the de Sitter isometries remain to be good approximation
e.g. scale invariance

- In the single-field model, inflation is driven by $\bar{\phi}(t)$

$$\ddot{\bar{\phi}} + 3H \dot{\bar{\phi}} + V' = 0$$

$$H^2 = \frac{\rho_{\bar{\phi}}}{3M_{Pl}^2} = \frac{1}{3M_{Pl}^2} \left(\frac{1}{2} \dot{\bar{\phi}}^2 + V(\bar{\phi}) \right)$$

⇒ We obtain the dynamics of $\bar{\phi}(t)$ and $a(t)$

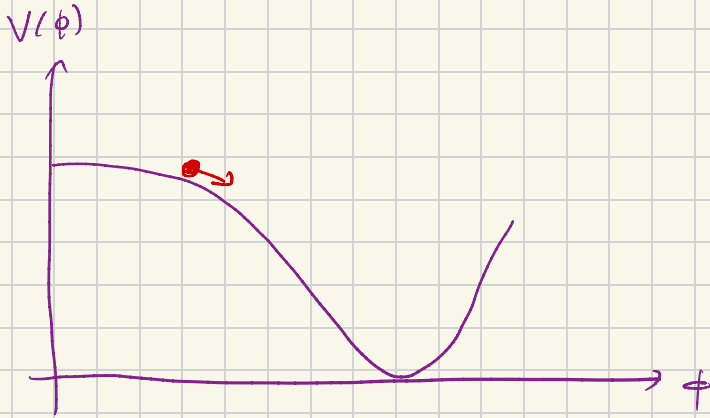
$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

- Additionally, one assumes the slow-roll conditions

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1 \quad \text{and} \quad \eta = -\frac{\ddot{\bar{\phi}}}{H\dot{\bar{\phi}}} \ll 1$$

$$\Rightarrow \dot{\phi} \simeq \frac{V'}{3H} \quad \text{and} \quad H^2 \simeq \frac{V}{3M_{pl}^2} \simeq \text{const}$$

$$a \sim e^{3Ht}$$



- Now we are going to study the EFT of perturbations, i.e.

$$\phi = \bar{\phi}(t) + \delta\phi(t, \vec{x})$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, \vec{x})$$

Under $x^\mu \rightarrow x^\mu + \xi^\mu$

$$\Rightarrow \delta\phi \rightarrow \delta\phi - \xi^0 \dot{\bar{\phi}} \quad ; \quad \delta g_{\mu\nu} \rightarrow \delta g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu$$

$$\Rightarrow \text{By choosing } \xi^0 = \frac{\delta\phi}{\dot{\bar{\phi}}} \Rightarrow \delta\phi = 0 : \text{Unitary gauge}$$

In other word, the preena $\bar{\Phi}(t)$ spontaneously breaks time diffeomorphism.

↳ The action is not invariant under $t \rightarrow t + \xi^0$

[This choice of unitary gauge is similar to choosing the NG boson $\pi = 0$ and A_μ becomes manifestly massive]

↳ Breaking of the gauge symmetry

However, setting $\delta\phi = 0$ (or $\pi = 0$)

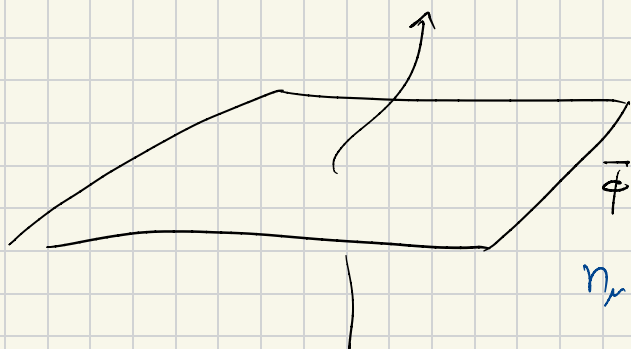
, it means that the metric (the gauge field A_μ) has eaten this scalar d.o.f.

⇒ Inside the metric, there are 11 indep. d.o.f

⇒ Once fixing ξ^0 such that 3 $\Downarrow$ propagating d.o.f
 $\delta\phi = 0$, $\delta g_{\mu\nu}$ now contains scalar perturbation

Now we choose $\delta\phi = 0$

↳ This corresponds to identifying t with $\bar{\phi}(t)$
 $\Rightarrow \bar{\phi}(t)$ behaves as a clock field


$$X = (\partial\phi)^2$$
$$\eta_\mu \eta^\mu = -1$$
$$\eta_\mu = - \frac{\partial_\mu \phi}{\sqrt{-X}} \rightarrow - \frac{\delta_\mu^0}{\sqrt{-g^{00}}}$$

We are going to write down the EFT in unitary gauge
compatible with the 3d diff. $X^i \rightarrow X^i + \xi^i$
 $\Rightarrow$ The EFT can be constructed from 3d
covariant quantities such N , N^i , K_{ij} , etc.

Recall the ADM form:

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt)(dx^j + N^j dt)$$

$$g^{00} = -\frac{1}{N^2}, \quad g^{i0} = \frac{N^i}{N^2}, \quad g^{ij} = h^{ij} - \frac{N^i N^j}{N^2}$$

$$g_{00} = -N^2 + h_{ij} N^i N^j, \quad g_{0i} = h_{ij} N^j \equiv N_i, \quad g_{ij} = h_{ij}$$

• What terms we can add in the EFT?

1.) Terms invariant under 4d diff.

$$R, \text{Riem}^2, R^2, \text{etc.}$$

[Here we are not interested in higher curvature corrections]

2.) 3d diff. covariant quantities:

$$g^{00}, K_{ij}, {}^{(3)}R_{ij}$$

$$\Rightarrow \tilde{g}^{00} = \frac{\partial \tilde{t}}{\partial X^\mu} \frac{\partial \tilde{t}}{\partial X^\nu} g^{\mu\nu} = \delta_\mu^0 \delta_\nu^0 g^{\mu\nu} = g^{00}$$

$\Rightarrow g^{00}$ is a scalar under 3d diff.

We can include polynomials of g^{00} in the EFT

• Extrinsic curvature

$$K_{\mu\nu} = h_{\nu}^{\rho} \nabla_{\rho} n_{\mu} \quad n_{\mu} = - \frac{\nabla_{\mu} \phi}{\sqrt{-X}}$$

$$h_{\mu\nu} = g_{\mu\nu} + n_{\mu} n_{\nu}$$

↳ Induced metric on a ϕ -const. surface

$$K_{ij} = \frac{1}{2N} (\dot{h}_{ij} - D_i N_j - D_j N_i), \quad K = h^{ij} K_{ij}$$

↳ covariant derivative wrt h_{ij}

- Also the action can depend on t explicitly since the time diff has been broken
- The most general EFT action in unitary gauge is

$$S_{\text{EFT}} = \int d^4x \sqrt{-g} \mathcal{L}(R_{\mu\nu\rho\sigma}, g^{\alpha\beta}, K_\mu, \nabla_\mu t)$$

- We can expand this order by order in perturbations

$$Sg^{\alpha\beta} = g^{\alpha\beta} - \bar{g}^{\alpha\beta} = g^{\alpha\beta} + 1$$

"-1 on FRW"

$$SK^\mu{}_\nu = K^\mu{}_\nu - \bar{K}^\mu{}_\nu = K^\mu{}_\nu - H h^\mu{}_\nu$$

"-H h^\mu{}_\nu on FRW"

$$(SK = K - 3H)$$

- Note that ${}^{(3)}R$ or ${}^{(3)}R_{ij}$ is already at first-order in perturbations since on the flat FRW, ${}^{(3)}R_{ij}$ vanish.

We call $\{Sg^{\alpha\beta}, SK, {}^{(3)}R \dots\}$ as EFT building blocks.

We write the unitary-gauge EFT action as

$$S_{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{Pl}}^2 R - \Lambda(t) - C(t) g^{00} \right. \\ + \frac{1}{2!} M_2^4(t) (g^{00})^2 + \frac{1}{3!} M_3^4(t) (g^{00})^3 + \dots \\ - \frac{\overline{M}_1^3(t)}{2} g^{00} \delta K - \frac{\overline{M}_2^2}{2} \delta K^2 - \frac{\overline{M}_3^2(t)}{2} \delta K^\mu{}_\nu \delta K^\nu{}_\mu \\ \left. + \dots \right]$$

* The first three terms contain terms linear in perturbations $\Rightarrow$ Tadpole cancellation conditions

||
Background equations of motion

$\Rightarrow$ $\Lambda(t)$ and $C(t)$ will be fixed by the background dynamics; i.e. FRW evolution.

$\Rightarrow$ Other linear terms besides the ones we wrote will contain derivatives, and they can be integrated by parts to give a combination of those three terms

- Other terms in the EFT start at quadratic or higher $\Rightarrow$ They contribute to the dynamics of perturbations around flat FLRW metric
- The unperturbed universe fixes $c(t)$ and $\Lambda(t)$, while the difference among different models will be encoded into higher-order terms.

We take the EFT up to tadpole terms:

$$S_{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \Lambda(t) - C(t) g^{00} \right]$$

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu$$

$$T_{\mu\nu} = - \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = (C - \Lambda) \bar{g}_{\mu\nu} + 2C \delta_\mu^0 \delta_\nu^0$$

Exercise :

$$H^2 = \frac{1}{3M_{\text{pl}}^2} (C + \Lambda)$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = - \frac{1}{3M_{\text{pl}}^2} (2C - \Lambda)$$

One can solve $C(t)$ and $\Lambda(t)$

$$C = -M_{\text{pl}}^2 \dot{H} \Rightarrow \text{since } \dot{H} < 0 \text{ (NEC)}$$

$$\Lambda = M_{\text{pl}}^2 (3H^2 + \dot{H}) \quad \left(\overset{\uparrow}{C} > 0 \right)$$

We thus have

$$\begin{aligned} S_{\text{EFT}} = \int d^4x \sqrt{-g} & \left[\frac{1}{2} M_{\text{pl}}^2 R + M_{\text{pl}}^2 \dot{H} g^{00} \right. \\ & - M_{\text{pl}}^2 (3H^2 + \dot{H}) + \frac{1}{2!} M_2^4(t) (8g^{00})^2 + \frac{1}{3!} M_3^4(t) (8g^{00})^3 \\ & + \dots - \frac{\overline{M}_2^3(t)}{2} 8g^{00} \delta K - \frac{\overline{M}_2^2(t)}{2} \delta K^2 - \frac{\overline{M}_3^2(t)}{2} \delta K^\mu_\nu \delta K^\nu_\mu \\ & \left. + \dots \right] \end{aligned}$$

* EFT coefficients can be functions of time

$\Rightarrow$ They depend via ϕ -dependence in the covariant theories $\Rightarrow$ shift symmetry of ϕ imposes non-trivial relations among EFT parameters

But since H and $\dot{H}$ do not vary much in one Hubble time during inflation, one can approximately neglect their time dependence.

⚠ If the time shift (shift symmetry of ϕ) becomes exact, one needs to be careful

since $\dot{H} \rightarrow 0 \Rightarrow$ The sound speed of scalar fluctuation becomes zero

$$c_s^2 \rightarrow 0 \Rightarrow \text{Ghost condensation}$$

/ Ghost inflation
(0312100, 0312099)

⚠ Comments :

- This time shift has nothing to do with the breaking of the time diffeo.
- Invariance under time translation in the unitary gauge ($\delta\phi = 0$) is implied by the shift sym. $\phi \rightarrow \phi + c$
- This shift sym. and the time translation sym. of the ϕ Lagrangian are broken down to the diagonal subgroup by the background.
- This residual sym. is the time shift in the unitary-gauge EFT

Before breaking

$$\mathcal{L}(x) \rightarrow \mathcal{L}(x)$$

$$t \rightarrow t + \delta^0$$

$$\text{and } \phi \rightarrow \phi + C$$

After breaking

choose $\phi = t$ (unitary gauge)

we are left with
time-shift symmetry

"Shift-symmetric EFT" (Fradetti et al. 1802.01580)

(We will come back to this later)

Simple example

Let us start with a simple model of inflation,
a canonical single field inflation:

$$S_\phi = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

↖ slow-roll potential

(obviously no shift symmetry)

⇒ In unitary gauge $\phi = \phi_0(t)$

⇒ $(\partial\phi)^2 \rightarrow g^{00} \dot{\phi}_0^2$ and $V(\phi) \rightarrow V(\phi_0(t))$

$$\Rightarrow \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

↓

$$\int d^4x \sqrt{-g} \left[-\frac{\dot{\phi}_0^2}{2} g^{00} - V(\phi_0(t)) \right]$$

$$\Rightarrow C = \frac{\dot{\phi}_0^2}{2} \quad \text{and} \quad \Lambda = V(t)$$

Since $C = -2M_{pl}^2 \dot{H}$ and $\Lambda = M_{pl}^2 (3H^2 + \dot{H})$, we obtain the usual Friedman eqs.

$$\dot{H} = -\frac{\dot{\phi}_0^2}{2M_{pl}^2} \quad \text{and} \quad 3M_{pl}^2 H^2 = V - \dot{H} M_{pl}^2$$

$$3M_{pl}^2 H^2 = \underbrace{\frac{1}{2} \dot{\phi}_0^2 + V}_{P_\phi}$$

Now we can generalize to the k-essence case

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R + P(X, \phi) \right]$$

$X = (\partial\phi)^2$ [Same applies to (beyond) Horndeski;
or DHOST or HOST or U-DHOST]

⇒ In unitary gauge $\phi = \phi_0(t)$

$$E, \quad X = g^{\mu\nu} \dot{\phi}_0^2$$

$$\text{Therefore, } S \rightarrow \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R + P(g^{\mu\nu} \dot{\phi}_0^2, \phi_0(t)) \right]$$

To match with the EFT action, we expand

$$g^{\mu\nu} = \bar{g}^{\mu\nu} + \delta g^{\mu\nu} = -1 + \delta g^{\mu\nu}$$

Recall

$$S_{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \Lambda - C g^{\mu\nu} + \frac{1}{2} M_2^4 (\delta g^{\mu\nu})^2 + \frac{1}{3!} M_3^4 (\delta g^{\mu\nu})^3 + \dots + \frac{1}{n!} M_n^4 (\delta g^{\mu\nu})^n \right]$$

$$M_n^4(t) = \dot{\phi}_0^{2n} \frac{\partial^n P}{\partial X^n} \bigg|_{\phi_0(t)}$$

$$C = - \dot{\phi}_0^2 \frac{\partial P}{\partial X} \bigg|_{\phi_0(t)} = - \dot{\phi}_0^2 \bar{P}_X$$

$$\Lambda = - \bar{P} - \dot{\phi}_0^2 \bar{P}_X$$

Note that from the Friedmann eq.

$$3M_{pl}^2 H^2 = \Lambda + C$$

$$3M_{pl}^2 H^2 = -\bar{P} - 2\dot{\phi}_0^2 \bar{P}_X$$

and $2M_{pl}^2 \dot{H} = +2\dot{\phi}_0^2 \bar{P}_X$

$$\rho = -\bar{P} - 2\dot{\phi}_0^2 \bar{P}_X$$

and $\rho + P = -2\dot{\phi}_0^2 \bar{P}_X$

$$\Rightarrow P = \bar{P}(\eta^{\mu\nu}\dot{\phi}_0^2, \phi_0(t))$$

$\Rightarrow$ The function $P(X, \phi)$ evaluated on the background is simply the pressure due to the scalar field

$$\Rightarrow \text{If } \bar{P}_X = 0 \Rightarrow \rho = -\bar{P} = -P$$

$\hookrightarrow$ One condition to realize ghost condensate

This effectively behaves as a cosmological constant, even without adding Λ_c explicitly.

Action for the Goldstone boson

Similar to the case of massive gauge field,
here we introduce the Goldstone boson π
by performing the Stückelberg trick:

$$t \rightarrow t + \pi(t, \vec{x})$$

$$g^{\mu\nu} \rightarrow \frac{\partial(t+\pi)}{\partial x^\mu} \frac{\partial(t+\pi)}{\partial x^\nu} g^{\mu\nu}$$

$$g^{\mu\nu} \rightarrow g^{\mu\nu} + 2g^{\mu\lambda} \partial_\lambda \pi + g^{\lambda\nu} \partial_\lambda \pi \partial_\nu \pi$$

$$\Rightarrow \int d^4x \sqrt{-g} [A(t) + B(t) g^{\mu\nu}]$$

$$\rightarrow \underbrace{\int d^4x \sqrt{-g}}_{4d \text{ diff. inv.}} [A(t+\pi) + B(t+\pi) (g^{\mu\nu} + 2g^{\mu\lambda} \partial_\lambda \pi + g^{\lambda\nu} \partial_\lambda \pi \partial_\nu \pi)]$$

$\Rightarrow$ No π here

* You can check that $\tilde{g}^{\mu\nu} = g^{\mu\nu} + 2g^{\mu\lambda} \partial_\lambda \pi + g^{\lambda\nu} \partial_\lambda \pi \partial_\nu \pi$
invariant under $t \rightarrow t + \xi^0$ and $\pi \rightarrow \pi - \xi^0$

Then, applying this trick to the EFT of inflation we obtain.

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - M_{pl}^2 (3H^2(t+\pi) + \dot{H}(t+\pi)) \right. \\ \left. + M_{pl}^2 \dot{H}(t+\pi) (g^{00} + 2\partial_r \pi g^{0r} + g^{rr} \partial_r \pi \partial_r \pi) \right. \\ \left. + \sum_{n=2}^{\infty} \frac{M_n^4}{n!} (1 + g^{00} + 2g^{0r} \partial_r \pi + g^{rr} \partial_r \pi \partial_r \pi)^n \right]$$

Note the Stückelberg trick for ADM variables are

$$N \rightarrow N \left(1 - \dot{\pi} + N^i \partial_i \pi + \dot{\pi}^2 + \frac{1}{2} N^2 h^{ij} \partial_i \pi \partial_j \pi \right) + \mathcal{O}(3)$$

$$N^i \rightarrow N^i (1 - \dot{\pi}) + N^2 h^{ij} (1 - 2\dot{\pi}) \partial_j \pi + \mathcal{O}(3)$$

$$h_{ij} \rightarrow h_{ij} - N_i \partial_j \pi - N_j \partial_i \pi - N^2 \partial_i \pi \partial_j \pi + \mathcal{O}(3)$$

Note $\partial_0 \rightarrow (1 - \dot{\pi} + \dot{\pi}^2) \partial_0 + \mathcal{O}(3)$

$$\partial_i \rightarrow \partial_i - (1 - \dot{\pi}) \partial_i \pi \partial_0 + \mathcal{O}(3)$$

Thus, up to first order in π we have

$$\delta K \rightarrow \delta K - \dot{\bar{K}} \pi - h^{ij} N \partial_i \partial_j \pi - 2 h^{ij} \partial_i N \partial_j \pi - \frac{1}{2N} N^e h^{ij} \dot{h}_{ij} \partial_e \pi$$

$$\begin{aligned} \delta K^i_j \rightarrow \delta K^i_j - \dot{\bar{K}}^i_j \pi - h^{ik} N \partial_k \partial_j \pi \\ - 2 h^{ik} \partial_k N \partial_j \pi + \frac{1}{2N} (h^{ek} N^i - h^{ie} N^k) \dot{h}_{ek} \partial_j \pi \\ - \frac{1}{2N} h^{ie} N^k \dot{h}_{ke} \partial_j \pi \end{aligned}$$

$${}^{(3)}R \rightarrow {}^{(3)}R + \frac{1}{2} h^{ek} \dot{h}_{ek} h^{ij} \partial_i \partial_j \pi - \frac{1}{2} h^{ek} h^{ij} \dot{h}_{ki} \partial_e \partial_j \pi$$

Applications in inflation

& the decoupling limit

- Let us analyze the action for $\pi(x)$ in the decoupling limit, i.e. $M_{\text{pl}} \rightarrow \infty$
 $\dot{H} \rightarrow 0$, so that $M_{\text{pl}}^2 \dot{H} = \text{const.}$
($\epsilon \rightarrow 0$)

↳ π decouples from metric perturbations

⇒ we can study the dynamics of π + compute its observable on a fixed background metric

Let us estimate when we can neglect the mixing with gravity

$$\Rightarrow \text{set } M_2 = M_3 = \dots = M_n = 0$$

$$\Rightarrow \frac{1}{2} M_{\text{pl}}^2 R - M_{\text{pl}}^2 \left(3H^2(t+\pi) + \dot{H}(t+\pi) \right) + M_{\text{pl}}^2 \dot{H}(t+\pi) \left(g^{00} + 2\partial_\mu \pi g^{0\mu} + g^{\mu\nu} \partial_\mu \pi \partial_\nu \pi \right)$$

This also includes the standard slow-roll inflation.

* Typically the mixing terms contain fewer derivatives than the kinetic term of π

↳ At sufficiently high energy, we can safely neglect the mixing.

⇒ In this case ($M_2 = \dots = M_n = 0$)

The leading mixing term comes from

↳ • Kinetic term $\sim M_{pe}^2 \dot{H} \delta g^{00} \rightarrow M_{pe}^2 \dot{H} \dot{\pi}^2$

• Mixing term $\sim M_{pe}^2 \dot{H} \delta g^{0i} \rightarrow M_{pe}^2 \dot{H} \partial_i \pi \underbrace{\delta g^{0i}}$

↙
constrained
variable

At short distance;

$$M_{pe}^2 \partial_j^i \delta g^{0i} \sim M_{pe}^2 \dot{H} \partial_i \pi$$

$$\Rightarrow \delta g^{0i} \sim \dot{H} \frac{\partial_i \pi}{\nabla^2}$$

Thus

$$\frac{\text{Mixing}}{\text{Kinetic}} \sim \frac{\delta g^{0i} \partial_i \pi}{\dot{\pi}} \sim \frac{\dot{H} \partial_i \pi \frac{\partial_i \pi}{\nabla^2}}{\dot{\pi}}$$

$$\frac{\text{Mixing}}{\text{Kinetic}} \sim \frac{\dot{H}\pi}{\ddot{\pi}} \sim \frac{\dot{H}}{EH} \ll 1 \Rightarrow E \gg EH$$

where E is a typical energy scale of π

$$\dot{\pi} \sim E\pi$$

Therefore, for $E \gg EH$, the mixing terms are negligible (GR equivalence principle)

Of course, the actual mixing scale E_{mix} depends on the actual operators, but typically if $E \gg E_{\text{mix}}$ we can neglect the mixing terms.

In the regime $E \gg E_{\text{mix}}$, we have

$$S_{\pi} = \int d^4x \sqrt{-g} \left[\underbrace{\frac{1}{2} M_{\text{pl}}^2 R - M_{\text{pl}}^2 \dot{\pi} \left(\dot{\pi} - \frac{(\partial_i \pi)^2}{a^2} \right)}_{\text{standard kinetic term}} + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \frac{\dot{\pi}}{a^2} (\partial_i \pi)^2 \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 + \dots \right]$$

* This action may not be relevant for cosmological observations at late times. However, as for standard single-field slow-roll inflation, one can prove that f is constant outside horizon.

$$S_{\pi} = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{pl}}^2 R - M_{\text{pl}}^2 \dot{H} \left(\dot{\pi} - \frac{(\partial_i \pi)^2}{a^2} \right) \right]$$

slow-roll inflation

$$+ \sum_{n=2}^{\infty} \frac{M_n^4}{n!} \underbrace{\left(-2\dot{\pi} + (\partial_r \pi)^2 \right)^n}_{P(x), \text{ DBI}, \dots} \Big]$$

$P(x)$, DBI, ...

Suppose we keep up to M_2^4 :

$$\begin{aligned} \mathcal{L}_{\pi}^{(2)} &= M_{\text{pl}}^2 \dot{H} (\partial_r \pi)^2 + 2M_2^4 \dot{\pi}^2 \\ &= \frac{M_{\text{pl}}^2 \dot{H}}{C_s^2} \left(\dot{\pi}^2 - \frac{C_s^2}{a^2} (\partial_i \pi)^2 \right) \end{aligned}$$

$\Rightarrow$ we introduce a non-trivial speed of sound C_s^2

$$C_s^2 \equiv \frac{M_{\text{pl}}^2 \dot{H}}{M_{\text{pl}}^2 \dot{H} - 2M_2^4} \leq 1 \quad \text{subluminal} \quad \Rightarrow M_2^4 > 0$$

$[C_s^2 > 1 \Rightarrow \text{No Lorentz-inv. UV completion}]$

$\left\{ \begin{array}{l} \hookrightarrow \text{For } M_2 = 0 \Rightarrow C_s^2 \rightarrow 1 \text{ (Lorentz-invariance)} \\ \hookrightarrow \text{In the exact } \dot{H} = 0 \text{ limit (Ghost condensate)} \\ \Rightarrow C_s^2 = 0 \Rightarrow \text{Strong-coupling problem!} \end{array} \right.$

To explicitly see the symmetry breaking scale,
(Lorentz breaking scale)

we rescale

$$x^i \rightarrow \tilde{x}^i \equiv C_s^{-1} x^i$$

$$\Rightarrow \tilde{\mathcal{L}}_{\pi}^{(2)} \equiv C_s^3 \mathcal{L}_{\pi}^{(2)} = \frac{f_{\pi}^4}{2} \left(\dot{\pi}^2 - \frac{(\tilde{\partial}_c \pi)^2}{a^2} \right) \\ = -\frac{1}{2} (\tilde{\partial}_r \pi_c)^2$$

where $f_{\pi}^4 \equiv 2M_{\text{pl}}^2 |\dot{H}| C_s$: symmetry breaking scale

and $\pi_c = f_{\pi}^2 \pi$: canonically normalized $\pi(x)$

[We neglect time dependence of f_{π}]

$$\left[\dot{\pi} = (\pi_c / f_{\pi}^2)' = \frac{1}{f_{\pi}^2} \dot{\pi}_c - \frac{2\pi_c}{f_{\pi}^3} \dot{f}_{\pi} \right]$$

• Power spectrum

During inflation, we often introduce "curvature perturbation"

$$\delta \text{ through } g_{ij} = a^2 e^{2\delta} \delta_{ij}$$

[δ is conserved outside horizon for single-field inflation at any order in perturbation theory]

- When neglecting the mixing with gravity, the relation between π and δ is

$$a(t+\delta t)^2 = a^2(1+H\delta t)$$

$$\delta(\vec{x}, t) = -H(t) \pi(\vec{x}, t)$$

$$\left[\begin{array}{l} \Rightarrow \text{This can be obtained by simply performing} \\ \text{a time diff. } t \rightarrow t + \delta t \text{ to } h_{ij} = a^2 \delta_{ij} \\ \text{in } \pi\text{-gauge to } \delta\text{-gauge: } h_{ij} = a^2 e^{2\delta} \delta_{ij} \\ \Rightarrow \text{The time-diff parameter } \delta t = \pi \end{array} \right]$$

Note for $a^2 h_{ij} = 2C a^2 \delta_{ij}$

under time-diff : $C \rightarrow C - H\delta t$

$$\left. \begin{array}{l} \pi\text{-gauge} \\ C = 0 \end{array} \right\} \left. \begin{array}{l} \delta\text{-gauge} \\ C = \delta \end{array} \right\} C \rightarrow C' = C - H\pi$$

$$\boxed{\delta = -H\pi}$$

$\Rightarrow$ Therefore, from the decoupling-limit action of π we can compute $\langle \pi \dots \pi \rangle$ and use $\delta = -H\pi$ to obtain $\langle \delta \dots \delta \rangle$ which directly relate to observations such as CMB.

⇒ Let us compute $\langle g^2 \rangle$

⇒ We can perform the computation in different gauges, e.g. π -gauge or j -gauge

In π -gauge: $\dot{H} < 0$ during SR inflation

$$\mathcal{L}_{\pi}^{(2)} = - \frac{M_{pl}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - \frac{c_s^2}{a^2} (\partial_i \pi)^2 \right)$$

Solve for $\pi_k^{cl}(t)$

$$\frac{d}{dt} \left(- \frac{a^3 \dot{H}}{c_s^2} \dot{\pi}_k^{cl} \right) + \dot{H} c_s^2 a k^2 \pi_k(t) = 0$$

Write $\hat{\pi}_k(t) = \pi_k^{cl}(t) \hat{a}_k^\dagger + \pi_k^{cl*}(t) \hat{a}_{-k}$

$$\pi_k^{cl}(\eta) = - \frac{1}{(2\epsilon)^{1/2} M_{pl}} \frac{1}{(2k)^{3/2}} \underbrace{(1 - i k \eta) e^{i k \eta}}_{\sim}$$

In the limit $k\eta \rightarrow -\infty$
⇒ $\pi^{cl} \sim e^{i k \eta}$
(Mode function in Minkowski)

$$\Rightarrow \delta_k^{cl}(\gamma) = \frac{H}{(2G)^{1/2} M_{pl}} \frac{1}{(2k)^{3/2}} (1 - ik\gamma) e^{ik\gamma}$$

Then, the 2-point function is

$$\begin{aligned} \langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \rangle &= (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2) \cdot \frac{H_*^4}{4M_{pl}^2 C_s |\dot{H}_*|} \frac{1}{k_1^3} \\ &= (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2) \cdot \frac{H_*^2}{4\epsilon_* M_{pl}^2 C_s} \frac{1}{k_1^3} \end{aligned}$$

* : evaluated at Hubble crossing

$$\Rightarrow n_s = \frac{d}{d \log k} \frac{H_*^4}{|\dot{H}_*| C_s} = \frac{4 \dot{H}_*}{H_*^2} - \frac{\ddot{H}_*}{H_* \dot{H}_*} - \frac{\dot{C}_s}{C_s H_*}$$

This spectral index now depends on $\dot{C}_s(t)$

$\Rightarrow$ In a small C_s limit $\Rightarrow$ large non-Gaussianities
e.g. $\langle \delta^3 \rangle$

- Estimate non-Gaussianities

Recap : we start from the EFT of Inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - M_{\text{pl}}^2 (3H^2 + \ddot{H}) + M_{\text{pl}}^2 \dot{H} g^{00} + \frac{1}{2!} M_2^4 (8g^{00})^2 + \frac{1}{3!} M_3^4 (8g^{00})^3 + \dots \right]$$

- Let's extract π^3 by performing $t \rightarrow t + \pi$ and we fix gravity (neglect mixings with gravity)

$$S_\pi = \int d^4x \sqrt{-g} \left[-\frac{M_{\text{pl}}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) + M_{\text{pl}}^2 \dot{H} \left(1 - \frac{1}{c_s^2} \right) \left(\dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 + \dots \right]$$

$E \gg E_{\text{mix}} \approx E_H$

Note $c_s^{-2} = 1 - \frac{2M_2^4}{M_{\text{pl}}^2 \dot{H}}$

$\Downarrow$
 $\langle \pi^3 \rangle$

⇒ To estimate the size of non-Gaussianities,

ie compare $\mathcal{L}_3^\pi / \mathcal{L}_2^\pi$ at $\omega \sim H$

(these derivative couplings are less important when going outside the horizon)

• Between $\dot{\pi}^3$ and $\dot{\pi} (\partial_i \pi)^2$

For $C_s \ll 1 \Rightarrow \dot{\pi} (\partial_i \pi)^2$ wins

The spatial derivatives get enhanced $\frac{k}{a} \sim H/C_s$ wrt. time derivatives.

Note

$$H \sim \omega \sim C_s \frac{k}{a} \Rightarrow \frac{k}{a_*} \sim \frac{H}{C_s} \gg H$$

$$\Rightarrow \frac{\mathcal{L}_3^\pi (\partial_i \pi)^2}{\mathcal{L}_2^\pi} \sim \frac{H \pi \left(\frac{H}{C_s} \pi \right)^2}{(H \pi)^2} ; \dot{\pi} \sim H \pi$$

$$\sim \frac{H}{C_s^2} \pi \sim \frac{1}{C_s^2} \delta ; \delta \sim H \pi$$

Typically we introduce f_{NL} via

$$\frac{\mathcal{L}_{\dot{\pi}(\partial,\pi)^2}}{\mathcal{L}_2} \sim f_{NL} \mathcal{F} = \frac{1}{C_s^2} \mathcal{F} \quad \mathcal{F} \sim 10^{-5}$$

$\Rightarrow f_{NL}^{equi} \rightarrow$ equilateral or orthogonal form

$$f_{NL}^{equi}, \dot{\pi}(\partial,\pi)^2 \sim \frac{1}{C_s^2}$$

This derivative coupling only peaks at the horizon crossing

$\hookrightarrow$ The correlation $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ is only among modes with comparable wavelength.

Note For $\dot{\pi}^3$, we can check that

$$f_{NL}^{equi}, \dot{\pi}^3 \sim 1 - \frac{4}{3} \frac{M_3^4}{M_{pe}^2 |\dot{H}| C_s^{-2}}$$

• Cutoff of the EFT

⇒ For $C_s < 1$, S_π contains non-renormalizable interactions

↳ At certain energy scale, the theory becomes strongly coupled.

⇒ We can estimate this cutoff Λ_{cutoff}

1. Use the partial wave decomposition to obtain the unitarity bound
2. Loop corrections become relevant.

The diagram shows two terms. The first term is a tree-level exchange of a pion between two external pions, represented by two horizontal dashed lines connected by a vertical dashed line. The second term is a loop correction diagram where a pion loop connects the two vertices, represented by two horizontal dashed lines connected by a loop of dashed lines.

$$\Lambda_{\text{cutoff}}^4 \simeq 16\pi^2 M_\pi^4 \frac{C_s^2}{(1 - C_s^2)^2} \simeq 16\pi^2 M_{\text{Pl}}^2 |H| \frac{C_s^5}{1 - C_s^2}$$

↳ This is energy scale where loop corrections to $\langle \pi\pi \rangle$ become relevant.

$$\Lambda_{\text{cutoff}}^4 = 16\pi^2 M_{\text{pl}}^2 |H| \frac{C_s^5}{1 - C_s^2}$$

(1) $C_s \rightarrow 0 \Rightarrow \Lambda_{\text{cutoff}}^4$ decreases

and when $\Lambda_{\text{cutoff}}^4 \rightarrow 0$ the theory becomes infinitely strongly coupled

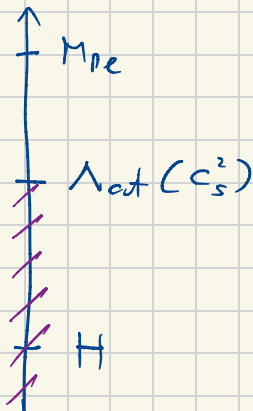
$\Rightarrow$ All perturbative calculations do not make sense

(2) $C_s \rightarrow 1$ (Lorentz limit)

$\Rightarrow \Lambda_{\text{cutoff}}^4$ becomes higher and higher

$\hookrightarrow$ In this limit, the theory contains no non-renormalizable interactions

and $\Lambda_{\text{cut}} \rightarrow M_{\text{pl}}$



(1) $C_s \rightarrow 0 ; \Lambda_{\text{cut}} \rightarrow 0$
Nothing makes sense.

(2) $C_s \rightarrow 1 ; \Lambda_{\text{cut}} \rightarrow M_{\text{pl}}$
The EFT remains weakly coupled until M_{pl} .

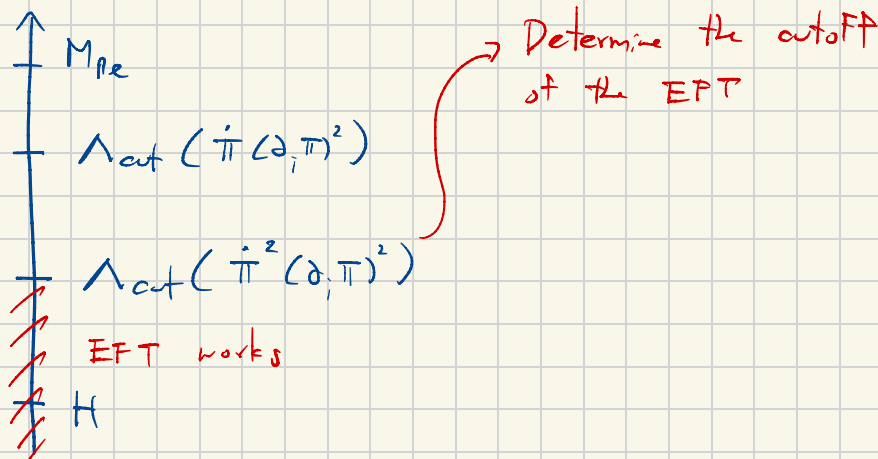
Warnings!

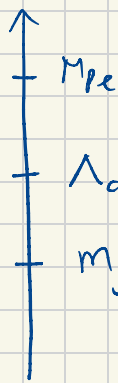
- This Λ_{cut} is obtained by only considering $(sg'')^2$

It could be that other independent operators give rise to lower cutoff

- This Λ_{cut} only indicates the maximum energy scale at which the theory is weakly coupled (perturbation theory makes sense)

But any new physics (other degrees of freedom) can appear even much below Λ_{cut}





In a specific theory
with concrete UV completion

$$\mathcal{L}(\pi, p)$$

Integrate out p

This implies that
new physics (p)

must be included even below

the unitarity cutoff of $\mathcal{L}_\pi$

$$\mathcal{L}_\pi = \dot{\pi}^2 - \frac{c_s^2 (\partial_i \pi)^2}{a^2} + \alpha \dot{\pi} (\partial_i \pi)^2 + \dots$$

The physical observables $\langle \pi^n \rangle$
computed in the EFT (only π) are
potentially affected by heavy modes

$\Rightarrow$ One has to check case-by-case
whether p affects the EFT observables of π or not
e.g. $U(1) \oplus$ with Higgs potential

$\hookrightarrow \Lambda_{UV}$ of $\mathcal{L}_{IR}(\pi)$ is above m_h

- One might ask what C_s^2 we have to use such that non-Gaussianities becomes $O(1)$ at $\omega \sim H$

Since $H \ll \Lambda_{\text{cut}}$ The formula for Λ_{cut} in $C_s \ll 1$

$$\Rightarrow H^4 \ll M_{\text{pl}}^2 |\dot{H}| C_s^5$$

and since $P_g \sim \frac{H^4}{M_{\text{pl}}^2 |\dot{H}| C_s}$; $P_g \sim 10^{-10}$ on CMB

$$\Rightarrow P_g \ll C_s^4$$

or $C_s \gg P_g^{1/4} \simeq 0.003$

(If your C_s is lower than this, the theory becomes strongly coupled at $E \simeq H$)

Looking back at $\frac{\mathcal{L}_{\pi(\partial, \pi)^2}}{\mathcal{L}_2} \sim \frac{1}{C_s^2} \int \sim 1$

$\Rightarrow C_s \sim 0.003 \Rightarrow$ This is C_s that gives $O(1)$ non-Gaussianities

⇒ When the theory becomes strongly coupled at $E \simeq H$ ($C_s \sim 0.003$), it also leads to $\mathcal{O}(1)$ non-Gaussianities in observations

↳ Actually this makes sense since one typically expects large non-Gaussianities when the theory becomes strongly coupled

(There are some exceptions e.g. resonant inflation where the theory is quite Gaussian even though one considers $E \sim \Lambda_{\text{cut-off}}$)



(1) Large n effects

(2) Revised unitarity bound

Comments on Ghost condensate

(0312100, 0312099)

$\Rightarrow$ We saw that in the limit $c_s \rightarrow 0$ the theory becomes strongly coupled.

To resolve this issue, one adds

$$S = \int d^4x \sqrt{-g} \left(-\frac{\bar{M}_2^2}{2} \delta K^2 - \frac{\bar{M}_3^2}{2} \delta K^\mu_\nu \delta K^\nu_\mu \right)$$

$$\supset \int d^4x \sqrt{-g} \left[-\frac{\bar{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 \right]$$

"Scordatura term"

$$\bar{M}^2 = \bar{M}_2^2 + \bar{M}_3^2$$

Non-rel dispersion relation
of π

$$\omega^2 = \cancel{c_s^2 k^2} + \frac{k^4}{M^2}$$

This is a leading term when $c_s \rightarrow 0$

$\Rightarrow$ No strong coupling since the cutoff is now given by M

$\Rightarrow$ Naturally this model leads to large non-Gaussianities

Applications for DE

then we switch our gear to Dark Energy which is a late-time universe.

- We still use the EFT formulation of scalar and metric perturbations on FRW

↳ Only 2 physical D.O.Fs.

- Basically, the background dynamics is the essentially the same as inflation except
 - we don't have to assume slow-roll
 - $H_0 \sim 10^{-33}$ eV
 - Couplings with matters

In unitary gauge $\bar{\Phi} = \bar{\Phi}(t)$

$$(\delta\Phi = 0)$$

We write

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \Lambda(t) - c g^{00} \right. \\ \left. + \frac{1}{2} m_2^4 (\delta g^{00})^2 + \frac{1}{2} m_3^3 \delta g^{00} \delta K \right. \\ \left. - m_4^2 (\delta K^2 - \delta K^\mu_\nu \delta K^\nu_\mu) + \dots \right]$$

$$\delta g^{00} = g^{00} + 1, \quad \delta K^\mu_\nu = K^\mu_\nu - H \delta^\mu_\nu$$

$$\delta K = K - 3H$$

- Here for simplicity we are interested in the propagation of GWs, γ_{ij}

$$h_{ij} = a^2(e^{\gamma})_{ij}$$

$$\gamma^i_i = 0 \quad \partial_i \gamma_{ij} = 0 \quad \text{TT gauge.}$$

From GW 170817 / GRB 170817A

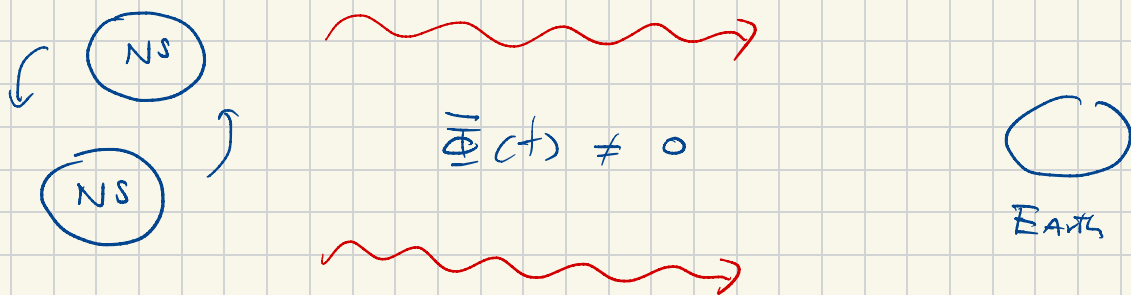
$$|c_T^2 - 1| < 10^{-15}$$

$$\omega \sim \Lambda_{\text{LIGO}} \sim 10^{-13} \text{ eV}$$

(100 Hz)

From the point of view of EFT of DE,

$\Rightarrow$ GWs travel in a Lorentz-violating medium of $\bar{\Phi}(t)$ or DE



$$c_T^2 \neq 1$$

$\Rightarrow$ Same as light traveling in a medium like water.

The operator $m_4^2 (\delta K^2 - \delta K^\mu_\nu \delta K^\nu_\mu)$ gives

$$S_T^{(2)} = \frac{M^2}{8} \int d^4x a^3 \left(\dot{\gamma}_{ij}^2 - \frac{C_T^2}{a^2} (\partial_k \gamma_{ij})^2 \right)$$

$$1 - C_T^2 = \frac{2m_4^2}{M^2 f} \quad 1710.05877$$

$\Rightarrow$ This implies that for $C_T^2 = 1$
one has to set $m_4^2 = 0$

$$\begin{aligned} S = \int d^4x \sqrt{-g} & \left[\frac{M^2}{2} f R - \Lambda - C g^{00} + \frac{m_2^2}{2} (\delta g^{00})^2 \right. \\ & - \frac{m_3^2}{2} \delta K \delta g^{00} - m_4^2 \delta K_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} \delta R \\ & - \frac{m_5^2}{2} \delta g^{00} \delta K_2 - \frac{m_6}{3} \delta K_3 - \tilde{m}_6 \delta g^{00} \delta G_2 \\ & \left. - \frac{m_7}{3} \delta g^{00} \delta K_3 \right] \end{aligned}$$

= corresponds to Beyond Horndeski up to $\alpha(3)$

$$\Rightarrow \text{We set } m_4 = 0 \Rightarrow C_T^2 = 1$$

But we have to make sure that this condition is stable under the changes of FRW backgrounds

↳ (1) change $H(t)$

(2) change $\rho(t)$

$$\Rightarrow \delta g^{00} \rightarrow \delta g^{00} + \delta g_{bg}^{00}$$

$$\delta K \rightarrow \delta K + \delta K_{bg}$$

↗ other background

→ m_4^2 receives additional contributions e.g.

$$\delta m_4^2 = \frac{m_5^2}{2} \delta g_{bg}^{00} \quad \text{or}$$

$$\delta m_4^2 = \delta H_{bg} \cdot m_6$$

⋮

We need to set all of them to zero

$$\Rightarrow \tilde{m}_4^2 = m_5^2, \quad m_6 = \tilde{m}_6 = m_7 = 0$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f R - \Lambda - c g^{00} \right. \\ \left. + \frac{m_2^4}{2} (g^{00})^2 - \frac{m_3^3}{2} g^{00} \delta K \right. \\ \left. + \frac{\tilde{m}_4^2}{2} g^{00} ({}^{(1)}R - \delta K_2) \right]$$

$$\delta K_2 = \delta K^2 - \delta K^\mu{}_\nu \delta K^\nu{}_\mu$$

$$\delta G_2 = \delta K^\mu{}_\nu {}^{(3)}R^\nu{}_\mu - \delta K {}^{(3)}R/2$$

$$\delta K_3 = \delta K^3 - 3 \delta K \delta K^\mu{}_\nu \delta K^\nu{}_\mu + 2 \delta K^\mu{}_\nu \delta K^\nu{}_\rho \delta K^\rho{}_\mu$$

$$\equiv G_2(\phi, x) + G_3(\phi, x) \square \phi + B_4(\phi, x) R$$

$$- \frac{4}{x} B_{4x} (\phi^\mu \phi^\nu \phi_\mu \square \phi - \phi^\mu \phi_\mu \phi_\nu \phi^\nu)$$

$$B_4 \equiv G_4 + \frac{x}{2} G_{5\phi} \quad ; \quad \phi_\mu \equiv \nabla_\mu \phi$$

! Cveet 1806.09417

" If $\Lambda_{\text{cut-off}}$ of the EFT of DE is lower than Λ_{LIGO} , the bound on C_T^2 does not apply to the EFT and those operators still survive "

↳ But we cannot arbitrarily lower $\Lambda_{\text{cut-off}}$ because it is controlled by radiative corrections.

⇒ This bound on C_T^2 at LIGO is very robust.

⇒ No-decay of GW at LIGO

$\gamma \rightarrow \pi\pi$ and π is stable at non linear level

$$\Rightarrow \alpha_H \lesssim 10^{-10} \quad \text{and} \quad |\alpha_B| \lesssim 10^{-2}$$

$$\alpha_H \equiv \frac{2\tilde{m}_\phi^2}{M_{\text{Pl}}^2} \quad \text{and} \quad \alpha_B = -\frac{m_3^3}{2M_{\text{Pl}}^2 H}$$

• More on decay $\sigma \rightarrow \pi\pi$

• Assume $C_T^2 = 1$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (g^{00})^2 - \frac{m_3^3}{2} g^{00} \delta K + \frac{\tilde{m}_4^2}{2} g^{00} ({}^{(3)}R - \delta K_2) \right]$$

(1) Let's consider

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt)(dx^j + N^j dt)$$

$$\text{with } \left. \begin{aligned} N^2 &= 1 + 2\Phi, & N_i &= 0 \\ h_{ij} &= a^2 (1 - 2\Phi) (e^{\gamma})_{ij} \end{aligned} \right\} \begin{array}{l} \text{Newtonian} \\ \text{gauge} \end{array}$$

$$\gamma_{00} = \partial_j \gamma_{ij} = 0$$

(2) Perform the Stückelberg trick

$$g^{00} \rightarrow g^{00} + 2g^{0\mu} \partial_\mu \pi + g^{\mu\nu} \partial_\mu \pi \partial_\nu \pi$$

$$\delta K_j^i \rightarrow \delta K_j^i - \dot{H} \pi \delta_j^i - \frac{1}{a^2} \partial_i \partial_j \pi + \mathcal{O}(2)$$

$${}^{(3)}R \rightarrow {}^{(3)}R + \frac{4}{a^2} H \partial^2 \pi + \mathcal{O}(2)$$

(3) Vary the action wrt. Φ :

$$2 M_{pe}^2 \partial^2 \Phi + m_3^2 \partial^2 \pi + 4 \tilde{m}_4^2 \partial^2 (\Phi + H \pi) = 0$$

→ In subhorizon limit ($H\pi \ll \dot{\pi}$)

$$\Phi = - \frac{m_3^2 + 4 \tilde{m}_4^2 H}{2 (M_{pe}^2 + 2 \tilde{m}_4^2)} \pi$$

Vary the action wrt. Φ :

$$M_{pe}^2 \partial^2 (\Phi - \Phi) + 2 \tilde{m}_4^2 \partial^2 (\Phi - \dot{\pi}) = 0$$

$$\Phi = \frac{2 \tilde{m}_4^2}{M_{pe}^2 + 2 \tilde{m}_4^2} \dot{\pi}$$

(4) Extract the coupling $\gamma \pi \pi$ with highest number of derivatives

4-derivatives

$$S_{\gamma \pi \pi} = \frac{M_{pe}^2 \tilde{m}_4^2}{M_{pe}^2 + 2 \tilde{m}_4^2} \int d^4 x \, \ddot{\gamma}_{ij} \partial_i \pi \partial_j \pi$$

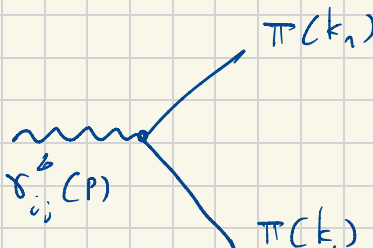
(take $a = 1$)

⇒ Canonical normalize :

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{\Lambda_\pi^3} \ddot{\gamma}_{ij}^{(\sigma)} \partial_i \pi^{(c)} \partial_j \pi^{(c)}$$

where $\gamma_{ij}^{(\sigma)} = \frac{M_{\rho\pi}}{\sqrt{2}} \gamma_{ij}$

$$\pi^{(c)} = \frac{M_{\rho\pi} [3m_\pi^2 + 4M_{\rho\pi}^2(C + 2m_\pi^2)]^{1/2}}{\sqrt{2} (M_{\rho\pi}^2 + 2\tilde{m}_\pi^2)} \pi$$



$$= 2 \cdot \frac{1}{\Lambda_\pi^3} P^2 k_{2i} k_{2n} \left[\frac{1}{2} (\delta_{in} \delta_{jn} + \delta_{in} \delta_{jm}) - \frac{1}{3} \delta_{ij} \delta_{mn} \right]$$

$$\Gamma_{\sigma \rightarrow \pi\pi} = \frac{1}{2} \frac{1}{2E_p} \int \frac{d^3k_1}{(2\pi)^3 2E_{k_1}} \frac{d^3k_2}{(2\pi)^3 2E_{k_2}} (2\pi)^4 \delta^{(4)}(P - k_1 - k_2) |\mathcal{M}|^2$$

$$\mathcal{M} = \frac{2i}{\Lambda_\pi^3} P^2 k_{2i} k_{2j} \epsilon_{ij}^{*b}(\vec{P})$$

$\Rightarrow |\mathcal{M}|^2 = \frac{2P^4}{\Lambda_\pi^6} k_1^4 (1 - \Omega^2)^2$

 $\nearrow$ Solid angle
 $\Omega \equiv \cos\theta$

Then

$$\Gamma_{\gamma \rightarrow \pi\pi} = \frac{P^7 (1 - C_s^2)^2}{480 \pi C_s^7 \Lambda_*^6}$$

$$\Rightarrow \frac{\Gamma_{\gamma \rightarrow \pi\pi}}{H_0} = \left(\frac{P}{\Lambda_*} \right)^6 \frac{(1 - C_s^2)^2}{480 \pi C_s^7} \frac{P}{H_0}$$

for $P \sim \Lambda_{\text{LIGO}} \sim \Lambda_3 \sim 10^{-13} \text{ eV}$
 $H_0 \sim 10^{-33} \text{ eV}$

$$\Rightarrow \frac{\Gamma_{\gamma \rightarrow \pi\pi}}{H_0} \sim 10^{20} \left(\frac{\Lambda_3}{\Lambda_*} \right)^6 \frac{(1 - C_s^2)^2}{480 \pi C_s^7}$$

Since $\left(\frac{\Lambda_3}{\Lambda_*} \right)^3 = \frac{\alpha_H (1 + \alpha_H)}{\sqrt{2} \alpha}$

For $\left. \frac{\Gamma_{\gamma \rightarrow \pi\pi}}{H_0} \right|_{\text{LIGO}} \ll 1 \Rightarrow 10^{20} \alpha_H^2 \frac{(1 - C_s^2)^2}{480 \pi C_s^7} \ll 1$
 $0 < C_s < 1$

No GW decay at
LIGO

$$\alpha_H \ll 10^{-10}$$

Beyond Horndeski
is highly constrained

- Scalar-tensor theories that survive all constraints are

$$\mathcal{L} = G_2(\phi, X) + C(\phi, X) R + \frac{6 C_X(\phi, X)^2}{C(\phi, X)} \phi^\mu \phi_{,\mu} \phi_\nu \phi^{,\nu}$$

↳ Subclass of DHOST theories.

Final remarks

- Similar EFT applies to BH perturbations w/ time-like scalar (2204.00228, ...) w/ space-like scalar (1810.07706, 2507.02066)
- Bounds on GWs only apply on cosmological scales $\Rightarrow$ Deviations from GR can be present in BH regime.