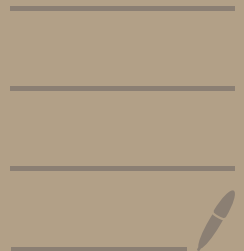


Lecture 1

- Introduction to EFT
 - : top-down & bottom-up
- Simple examples.



Lecture : EFT (of modified gravity)

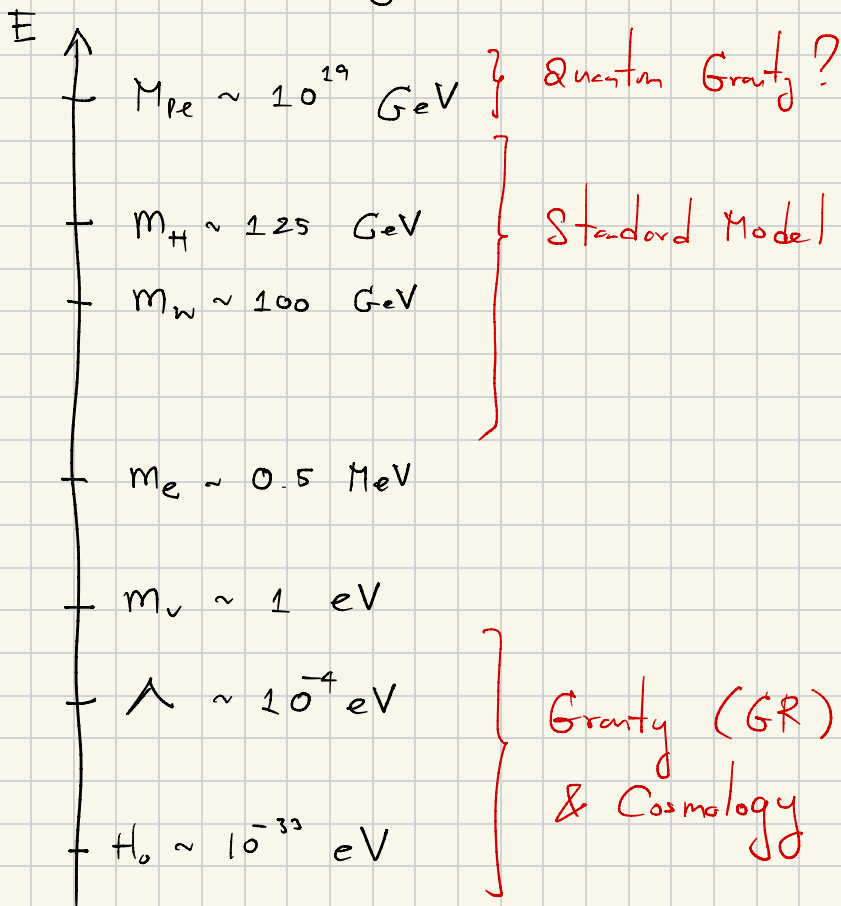
Structure

1. Introduction to EFT
2. Theory of Goldstone
3. EFT of Inflation / Dark Energy
4. Applications & Outlook

- Refs
- M. Serone, Notes on AFT I
 - C. Cheung et al. EFT of Inflation
0709.0293
 - L. Senatore, Lectures on Inflation
1609.00916
 - Piazza & Vernizzi 1307.4350
EFT of cosmo. perturbations
 - Gubitosi et al. EFT of DE
1210.0201

1. Introduction to EFT

- The goal of this lecture is to give you a generic tool of how to describe a physical system within some scales.
- Of course, we know that our nature comes with many scales



- What we see here is that "physics depends on scales"

[Of course, ultimately we want to have a single theory that describes physics at all scales at once]

You are able to write a set of equations of motion that is able to make physical predictions

↳ Consistent with observations.

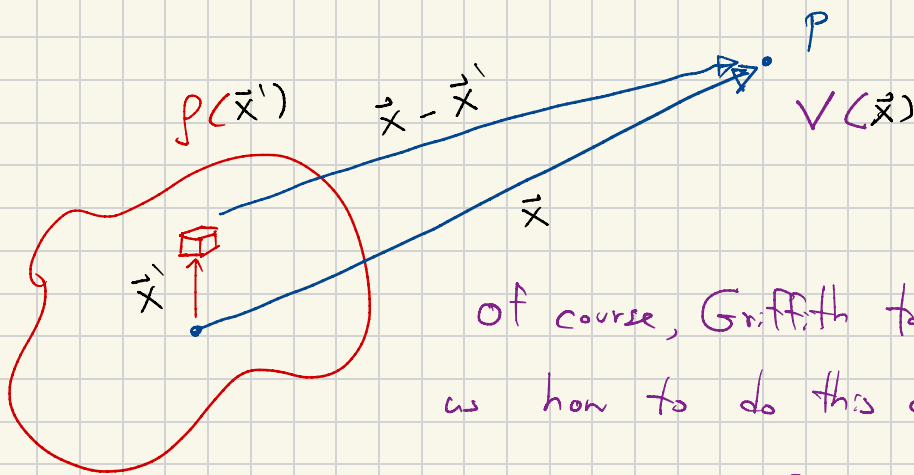
- But the idea of the EFT comes when you start realizing that not all physics is relevant to the one you are interested in.

- Simple examples

1. Multipole expansion in Electrostatic

this is perhaps the simplest effective theory you have encountered in your E&M lecture

(But maybe no one tells you this is indeed EFT)



of course, Griffith taught us how to do this calculation

using

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3\vec{x}'$$

$$Q = \int \rho(\vec{x}') d^3\vec{x}'$$

For $r \gg R$:

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0 r} \sum_{l=0}^{\infty} \sum_{m=-l}^l \left(\frac{R}{r}\right)^l Q_{lm} Y_{lm}(\theta, \phi)$$

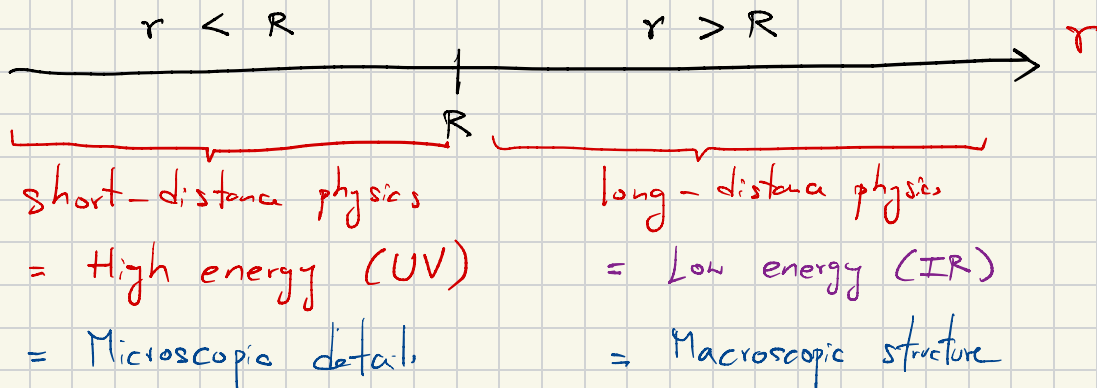
R : typical length scale of the charge distribution.

• But here I want you to guess the form of $V(\vec{r})$ without doing any calculation!

(Of course, you can take the formula of $V(\vec{x})$ and compute to obtain the exact result)

Starting point : We are interested in the electric potential at $\vec{x}$ far away from the charge distribution :

$$r = |\vec{x}| \quad : \quad r \gg R$$



Our goal is to write down $V(\vec{x})$, focussing on "long-distance physics"

↳ We hide those microscopic details into some parameters.

To do so, we organize terms in $V(\vec{x})$ using the fact that $\epsilon \equiv R/r \ll 1$

↳ Power counting / derivative expansion.
(So far, no symmetry has been imposed)

With this in mind, we write

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \left[\underbrace{\frac{Q}{r}}_{\text{Monopole}} + \underbrace{\frac{P_i n_i}{r^2}}_{\text{Dipole}} + \underbrace{\frac{Q_{ij} n_i n_j}{2 r^3}}_{\text{Quadrupole}} + \dots \right]$$

$$\hat{n} = \vec{x}/r$$

$$P_i : \text{Dipole moment} : P_i = \int d^3x' x'_i \rho(\vec{x}') \sim Q x'_i$$

$$Q_{ij} : \text{Quadrupole moment} : Q_{ij} = \int d^3x' \left(x'_i x'_j - \frac{1}{3} \delta_{ij} x'^2 \right) \rho \sim Q x'_i x'_j$$

- These multipole moments encode all short-distance information, which can be measured at long distance.

[The more terms you add, the more accuracy you will get]

Note this multipole expansion essentially comes from the expansion of $\frac{1}{|\vec{x} - \vec{x}'|}$ about $\vec{x}' = 0$

$$\frac{1}{|\vec{x} - \vec{x}'|} = \frac{1}{r} + x'_0 \partial_0 \frac{1}{r} + \frac{1}{2} x'_0 x'_0 \partial_0 \partial_0 \frac{1}{r} + \dots$$

$$|\vec{x}'| \leq R \ll r$$

The more $\vec{x}'$ you go, the more derivatives you'll have

⇒ More suppression by $1/r$ in the regime $r \gg R$

! Terms that are suppressed at long distance (low energy) are called "Irrelevant terms"

Therefore, if we are only interested in the limit $r \gg R$, only a few terms in the multipole expansion are enough

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r} + \frac{p_i n_i}{r^2} + \frac{Q_{ij} n_i n_j}{r^3} + \dots \right]$$

(Wilson coefficients)

Very suppressed at long distance

(We can safely neglect them) $\leftarrow$

In field-theory language, we call those multipole moment as "Wilson coefficients"

- Of course, this form of $V(\vec{x})$ starts to break down as $r \rightarrow R$. And we cannot trust the IR expression (EFT) when $r \sim R$

We call R as the cutoff scale

- As long as $r \gg R$, the effective $V(\vec{x})$ is valid. For $r \sim R$, we have to use the exact formula

Note : the Wilson coefficients can be determined by either observations/experiment, or

matching with UV physics

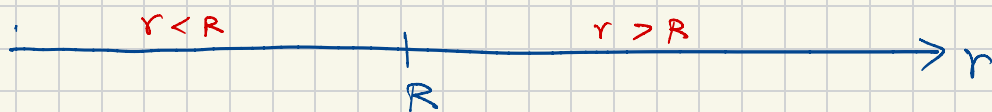
↳ In this case, UV physics is

when you use the exact formula for $V(\vec{x})$

[$\Rightarrow$ You see all the fine structure of the charge distribution]

UV physics is known, by getting rid of short-distance physics we go to IR with a few terms

UV $\longrightarrow$ IR : Top-down



UV $\longleftarrow$ IR : Bottom-Up

UV physics is not known, we need to add more terms to describe IR physics.

What about Symmetry?

So far, we have not used the symmetry in our EFT.

Symmetry essentially tells you which terms are allowed or not allowed in your action or Lagrangian.

Ex. In multipole expansion, if we impose spherical symmetry $\Rightarrow$ Only monopole term survives

Or if we impose azimuthal symmetry, only terms with $m=0$ survive

$\Rightarrow$ Symmetry constrains the form of EFT operators

- What have we learned so far?

EFT : - low-energy description (long-distance physics)

- Separation of scales with the cutoff
- Terms are organized by derivative expansion ($\partial/\Lambda \ll 1$)
- EFT coupling constants can be determined by matching with the UV or by observations.
- Symmetry constraints.

Additional constraints : this is positivity program.

Assume nice properties of the UV : Local, causality, unitarity, Lorentz inv.

What are the implications on the EFT parameters?

- Lorentz-inv. case 06.02.176
- Non-Lorentz inv. case 22.07.14224

2. Two-field model

Here we study the simplest EFT model of two scalar-field model

- We are entering into the effective description in the context of field theory.
- To be able to distinguish the importance of an effective operator at a given energy scale, we need to know some terminology of field theory.
- Let us consider the following Lagrangian:

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 \right.$$

$$- \lambda_3 \phi^3 - \lambda_4 \phi^4 + \dots \quad \text{"Non-derivative"}$$

$$- \alpha_4 (\partial\phi)^4 - \alpha_5 (\partial\phi)^6 + \dots \quad \left. \begin{array}{l} \text{Derivative} \\ \text{couplings.} \end{array} \right\}$$

$$- \beta_3 \phi (\partial\phi)^2 - \beta_4 \phi^2 (\partial\phi)^2 + \dots$$

- Let's consider non-derivative terms.

$$\mathcal{L}_\phi = -\frac{1}{2} m^2 \phi^2 - \lambda_3 \phi^3 - \lambda_4 \phi^4 - \lambda_5 \phi^5 + \dots$$

In 4D : $[\phi] = 1$

$$\Rightarrow [m] = 1, [\lambda_3] = 1$$

$\Rightarrow$ We must compare λ_3 with some energy scale

$$\underbrace{\frac{\lambda_3}{E}}_{\hat{\lambda}_3} E \phi^3 \Rightarrow \text{The dimensionless coupling } \hat{\lambda}_3 \text{ is becoming less and less important at high energy}$$

$$E \gg \lambda_3$$

But it becomes more and more important at low energy $\Rightarrow$ Relevant operators

- This applies also to the mass term.

Typically, in QFT we require the theory to be weakly coupled, i.e. we treat all interactions as perturbative corrections to the quadratic terms (otherwise we cannot do any calculation)

$$\hookrightarrow \frac{\mathcal{L}_3}{\mathcal{L}_2} < 1$$

$$\frac{\lambda_3 \phi^3}{(\partial\phi)^2} \sim \frac{\lambda_3 \phi^3}{p^2 \phi^2} = \frac{\lambda_3}{p} \frac{\phi}{p}$$

$$\text{Let } \lambda_3 = \hat{\lambda}_3 \Lambda \quad \phi = \hat{\phi} \mu$$

$$\frac{\mathcal{L}_3}{\mathcal{L}_2} \sim \underbrace{\hat{\lambda}_3 \hat{\phi}}_{\sim \mathcal{O}(1)} \left(\frac{\Lambda}{p} \right) \left(\frac{\mu}{p} \right) < 1$$

Perturbation theory.

$$\sqrt{\Lambda \mu} < p$$

If $\mu \sim p \Rightarrow \boxed{p > \Lambda}$ such that $\lambda_3 \phi^3$ can be treated as perturbative corrections.

- For $\lambda_4 \phi^4 \Rightarrow [\lambda_4] = 0$

$\Rightarrow$ No scale associated with this interaction

$\Rightarrow$ Marginal operators.

$\Rightarrow$ Perturbation theory requires that

$$\frac{\lambda_4 \phi^4}{(\partial\phi)^2} \sim \lambda_4 \left(\frac{\mu}{p} \right)^2 \hat{\phi}^2$$

If $\mu \sim p \Rightarrow \lambda_4 < 1$ Perturbation theory

[$\exists$ cases where μ can be larger than p or larger than a typical value, that signals non-perturbative effects]

$\hookrightarrow \lambda_4 \left(\frac{\mu}{p} \right)^2 \sim 1$: Breakdown of perturbative theory.

- $\lambda_n \phi^n$; $n \geq 5$

All of these couplings come with negative power in energy

$$\lambda_5 \phi^5 : [\lambda_5] = -1$$

$$\lambda_6 \phi^6 : [\lambda_6] = -2$$

$$\Rightarrow \lambda_5 \phi^5 = \underbrace{(\lambda_5 E)}_{\hat{\lambda}_5} \frac{\phi^5}{E}$$

At low energy, it becomes less and less important

$\Rightarrow$ Irrelevant operator

$$\Rightarrow \frac{\mathcal{L}_5}{\mathcal{L}_2} \sim \frac{\lambda_5 \phi^5}{(\partial \phi)^2} \sim \frac{\hat{\lambda}_5}{\Lambda} \cdot \frac{\phi^3}{p^2}$$

$$\text{If } \phi \sim p \Rightarrow \frac{\mathcal{L}_5}{\mathcal{L}_2} \sim \frac{p}{\Lambda} < 1$$

$$\hat{\lambda}_5 \sim \mathcal{O}(1)$$

$$\Rightarrow p < \Lambda = \Lambda_{\text{cutoff}}$$

For irrelevant operators, as long as we are sufficiently low energy, we can do calculations using perturbation theory.

$$\mathcal{L}_\phi = \underbrace{(\partial\phi)^2}_{\text{quadratic}} - \underbrace{m^2\phi^2}_{\text{Relevant}} - \underbrace{\lambda_3\phi^3}_{\text{Marginal}} - \underbrace{\sum_{n=5}^{\infty} \lambda_n\phi^n}_{\text{Irrelevant.}}$$

! Marginal + Relevant terms are renormalizable
[Only finite counterterms are needed to get rid of infinity]

! Irrelevant terms are non-renormalizable
[infinite counterterms]

⇒ The theory is not complete [but it doesn't mean that you cannot compute anything], it simply means that this theory is expected to break down at some energy scale [it breaks down at $M \sim 1$ unitarity bound] ⇒ We need new physics after that
⇒ It naturally leads to the EFT idea.

- Derivative couplings

Perhaps, you have seen these couplings in other lectures, like in Galilean theories.

- What terms can we add? $\phi \rightarrow \phi + C + b_\mu x^\mu$

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} (\partial\phi)^2 + \alpha_1 (\partial\phi)^4 + \alpha_2 (\partial\phi)^6 \\ & + \beta_1 \phi (\partial\phi)^2 + \beta_2 \phi^2 (\partial\phi)^2 + \dots \\ & + \gamma_1 (\partial\phi)^2 \square \phi + \dots \end{aligned}$$

- The more derivatives you have, the more you pay the price $\Rightarrow$ the more it gets suppressed at low energy.
- Derivative couplings are very common in cosmology and gravity $\rightarrow$ Higher curvature corrections.
- All derivative couplings are irrelevant operators, i.e. they are not important at low energy.

Examples

Suppose the UV theory is

$$\begin{aligned} \mathcal{L}_{UV} = & -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 \\ & -\frac{1}{2}(\partial\chi)^2 - \frac{1}{2}M^2\chi^2 \\ & - \underbrace{g\chi\phi^2/2}_{\text{Break } \chi \rightarrow -\chi} \end{aligned}$$

with $m \ll M$

Idea: if we are interested in $E \ll M$, we can "integrate out" the field χ

Formally we write

$$e^{iS_{\text{eff}}[\phi]} = \int \mathcal{D}\chi \, e^{iS[\phi, \chi]}$$

↓

Effective action only for ϕ (light field)

⇒ We will then write down $S_{\text{eff}}[\phi]$ and fix the IR parameters using the matching conditions at the cutoff

[This only works because in this case we know the UV theory]

We write $\mathcal{L}_{\text{IR}}(\phi)$ as

$$\mathcal{L}_{\text{IR}} = -\frac{1}{2} Z (\partial \phi)^2 - \frac{1}{2} \tilde{m}^2 \phi^2 - \frac{\lambda}{4!} \phi^4 + \text{Higher-order terms.}$$

At tree-level, we have

$$\text{EOM of } \chi : -\partial_\mu \chi \partial^\mu \chi - M^2 \chi \chi - \frac{g}{2} \phi^2 = 0$$

$$(\square + M^2) \chi = -\frac{g}{2} \phi^2$$

$$\chi = -\frac{g}{2} \frac{1}{\square + M^2} \phi^2$$

For $E \ll M^2$

$$\chi = - \frac{g}{2M^2} \frac{1}{1 + \frac{\square}{M^2}} \phi^2$$

$$\chi = - \frac{g\phi^2}{2M^2} - \underbrace{\frac{g}{2M^2} \sum_{n=1}^{\infty} \left(\frac{\square}{M^2}\right)^n}_{\text{Derivative terms suppressed by } E^2/M^2} \phi^2$$

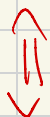
Here, at tree-level
we kept only
the leading term

Derivative terms suppressed
by E^2/M^2

Plug it back to $\mathcal{L}_{\text{eff}}$, we obtain

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 \\ &\quad - \frac{1}{2} \left[\partial \left(-\frac{g}{2} \frac{\phi^2}{M^2} \right) \right]^2 - \frac{1}{2} M^2 \frac{g^2}{4M^4} \phi^4 \\ &\quad - \frac{g}{2} \left(-\frac{g}{2} \frac{\phi^2}{M^2} \right) \phi^2 + \dots\end{aligned}$$

$$\mathcal{L}_{\text{IR}}(\phi) = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{g^2}{8M^2}\phi^4 - \frac{g^2}{2M^4}(\phi\partial_\mu\phi)^2$$



$$\mathcal{L}_{\text{IR}}(\phi) = -\frac{1}{2}Z(\partial\phi)^2 - \frac{1}{2}\tilde{m}^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \dots$$

Matching at tree-level gives

$$Z = 1 + \mathcal{O}(g^2)$$

$$\tilde{m}^2 = m^2 + \mathcal{O}(g^2)$$

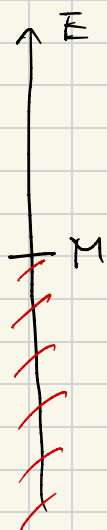
} The effects of χ appear at $\mathcal{O}(g^2)$

$$\lambda = -\frac{3g^2}{M^2} + \mathcal{O}(g^4)$$



χ already enters at tree-level.

- This $\mathcal{L}_{IR}$ only valid up to M



Once $E \sim M$, one needs to take into account new physics (heavy mode)

$\mathcal{L}_{IR}(\phi)$

* It could be that

the so-called unitarity cutoff

is much higher than M

e.g. London pole in QED :

Λ_L : the scale at which unitarity breaks down or the theory becomes strongly-coupled.

[Naively ; 1-loop $\sim$ tree-level]

But $\Lambda_L \simeq 10^{277} \text{ GeV} \gg M_{Pl}$

and we know that from $m_e \rightarrow M_{Pl}$, there are a lot of heavy particles/interactions, for which QED cannot explain.

To obtain the full $S_{\text{eff}}[\phi]$, one uses

$$e^{iS_{\text{eff}}[\phi]} = \int D\chi e^{iS[\phi, \chi]}$$

$\chi = \chi_{\text{cl}} + \chi_1$ and send $\hbar \rightarrow 0$
[semi-classical approximation]

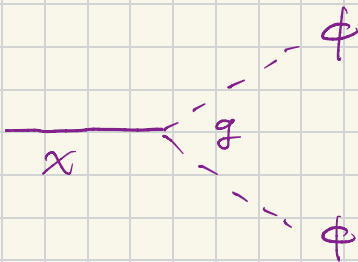
$$e^{iS_{\text{eff}}[\phi]} = e^{iS[\phi, \chi_{\text{cl}}]} \times \int D\chi_1 e^{\frac{i}{2} \frac{\delta^2 S}{\delta \chi_1^2} \chi_1^2 + \dots}$$

$$e^{iS_{\text{eff}}[\phi]} \simeq \frac{1}{\det\left(\frac{\delta^2 S}{\delta \chi_1^2}\right)} e^{iS[\phi, \chi_{\text{cl}}]}$$
$$= e^{iS[\phi, \chi_{\text{cl}}]} + \log \det\left(\frac{\delta^2 S}{\delta \chi_1^2}\right)$$

$\Rightarrow$ Loop contributions are those corresponding to prefactor of the path integral in semiclassical limit.

Diagrammatically :

From $\mathcal{L}_{UV} \supset -\frac{g}{2} \chi \phi^2$



• Contributions in Z or $\tilde{m}^2 \Rightarrow \langle \phi \phi \rangle$

$\langle \phi \phi \rangle^{UV} =$ $+ \dots$

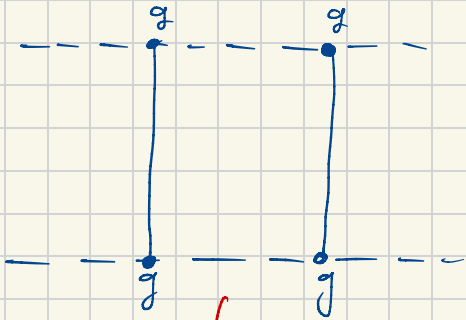
$IR =$ $= \frac{1}{Zp^2 - m^2}$

$\langle \phi^4 \rangle^{UV} =$ $+ \dots$

$IR \equiv$ λ

$\hookrightarrow \lambda \sim g^4 + g^2 + \dots$

For example,



$$= \frac{6ig^4}{8\pi^2 M^4} \left(1 + \frac{1}{2} \log \frac{m^2}{M^2} \right)$$

↳ [UV finite - no divergence]

$$\lambda(\mu) = -\frac{3g^2(\mu)}{M^2} - \frac{3g^4}{8\pi^2 M^4} \left(3 + 2 \log \frac{\mu^2}{M^2} \right)$$

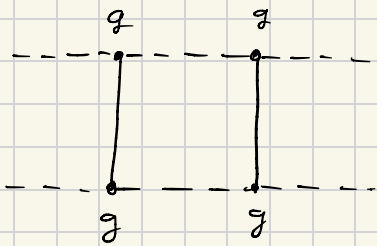
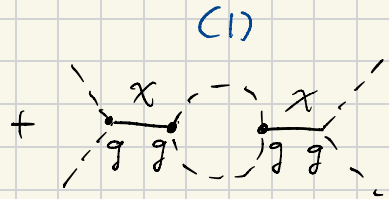
$\downarrow$
 renormalization scale

At $\mu = M$

$$\lambda(M) = -\frac{3g^2(M)}{M^2} - \frac{9g^4}{8\pi^2 M^4}$$

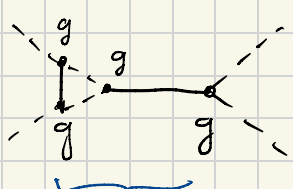
- We see that the idea of EFT naturally applies to field theory.
- IR theory is valid up to the cutoff and all the terms in $\mathcal{L}_{\text{IR}}$ are organized by derivative expansion.
- Matching conditions at the cutoff helps connect UV & IR only when the UV theory is known.

Other diagrams for $\langle \phi^4 \rangle$ at g^4

$$\langle \phi^4 \rangle =$$

 $+$


(1)

$\Downarrow$
log. div.

$$+$$


(2)

UV Finite

(UV finite
= Absence of the
sliding scale μ)

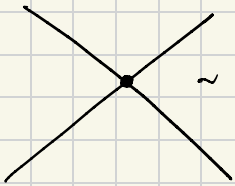
$$(1) = \frac{3ig^4}{32\pi^2 M^4} \log \frac{m^2}{M^2}$$

$$(2) = \frac{3ig^4}{8\pi^2 M^4} \left(1 + \log \frac{m^2}{M^2} \right)$$

Other examples of EFT

(1) Fermi's theory as an EFT of SM

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} \bar{\psi}_p \gamma^\mu \psi_n \bar{\psi}_e \gamma_\mu \psi_{\bar{\nu}_e} + \text{h.c.}$$

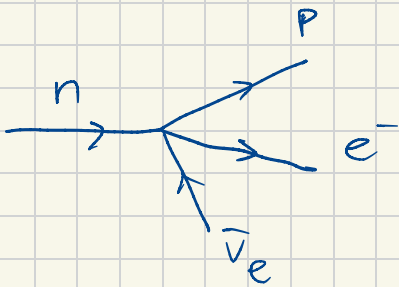


$\sim G_F$

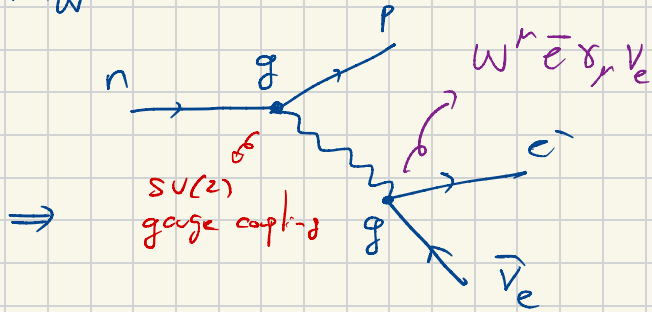
It describes β -decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

The cutoff is $\sim m_W$



IR ; $E < m_W$



UV , $E > m_W$

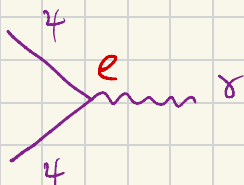
Tree-level matching : $G_F = \frac{g^2}{2m_W^2} \sim 10^{-5} \text{ GeV}^{-2}$

(1) Euler-Heisenberg : EFT of QED

when integrating out e^-

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \gamma^\mu (i\partial_\mu - m_e) \psi + e A_\mu \bar{\psi} \gamma^\mu \psi$$

Note no self interactions of A_μ in QED!

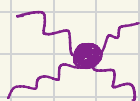


• In EFT ($E \ll m_e$) we write the EFT of photon

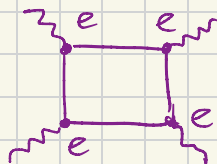
$$\mathcal{L}_{\text{IR}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A (\underbrace{F_{\mu\nu} F^{\mu\nu}}_{\text{wavy}}) + B (\underbrace{F_{\mu\nu} \tilde{F}^{\mu\nu}}_{\text{wavy}})^2$$

$$\tilde{F}_{\mu\nu} = \frac{1}{4} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

Self-couplings of A_μ



EFT



+ ...

$$\alpha = \frac{e^2}{4\pi}$$

$$A, B \sim \frac{ce^4}{m_e^4} \sim \frac{c\alpha^2}{m_e^4}$$

From matching at $E = m_e$

$$A = \frac{\alpha^2}{96m_e^4}, \quad B = \frac{7\alpha^2}{360m_e^4}$$

■ Summary of EFT approach :

- (1) Identify your energy scale of interest
- (2) Include all relevant d.o.f.s
- (3) Symmetries of the EFT
- (4) Write down all operators compatible with the symmetries
- (5) Organize terms with field / derivative expansion
- (6) Compute observables
- (7) Measure parameters / put bounds on EFT parameters

+ Theoretical constraints on the IR parameters coming from positivity programme.