Scalar Field Dynamics in Non-Minimally Coupled Theories via Noether Symmetry and Eisenhart Lift

M.Sc. Oral Defense
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Outline

- Motivation
- Background
- Results
- Conclusion

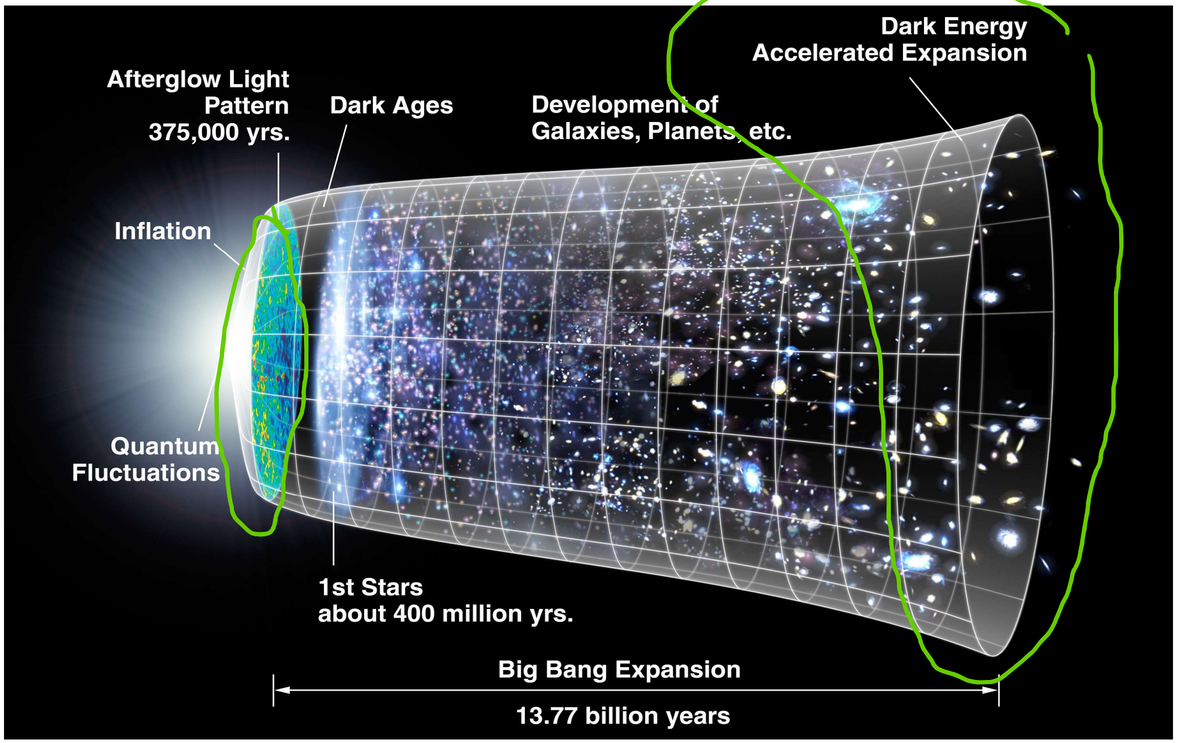

Fig.1 Timeline of the metric expansion of space, where space, including hypothetical non-observable portions of the universe, is represented at each time by the circular sections. Available from *https://en.wikipedia.org/wiki/Big_Bang*

Symmetry-Guided Strategy for NMC Cosmology

Solution to the problem

Problem / gap:
Many plausible $V(\phi)$
→ ad-hoc choices

Noether: determining equations
⇒ constraints on $V(\phi)$
⇒ closed-form families **[2,3]**

Eisenhart: lift + CK
⇒ geometric constraints yields $V(\phi)$ **[4]**

[1] Sotiriou & Faraoni, Rev. Mod. Phys. 82 (2010).
[2] Kanesom, Channuie & Kaewkhao (2021)
[3] Dolohtahe, Srikom, Channuie & Kaewkhao (2022).
[4] Finn, Karamitsos & Pilaftsis (2018).

NMC Framework (Jordan, Metric)

$$S = \int d^4x \sqrt{-g}\left[\boxed{\frac{1}{2}F(\phi)R} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$F(\phi) = 1 + \xi\phi^2$$

$$\mathcal{L} = \boxed{-6(1+\xi\phi^2)a\dot{a}^2 - 12\xi a\phi\dot{a}\dot{\phi}} + a^3\dot{\phi}^2 - 2a^3V(\phi)$$

S	$\equiv$	action
$\mathcal{L}$	$\equiv$	point-like Lagrangian
$g_{\mu\nu}$	$\equiv$	Jordan-frame FLRW metric
$\sqrt{-g}$	$\equiv$	determinant of the metric
R	$\equiv$	Ricci scalar
$\phi(t)$	$\equiv$	homogeneous scalar field
$V(\phi)$	$\equiv$	scalar field potential
$\mathrm{F}(\phi)$	$\equiv$	NMC function
ξ	$\equiv$	coupling constant

Frame: Jordan for derivations; EF readout only, $g_E = F(\phi)g$ **[5]**

Observables: slow-roll (ϵ,η) later in (Observables section)

[5] Bezrukov & Shaposhnikov (2008)

Noether Symmetry: Operator, Integrals, and Selection

Operator

Generator $X = \alpha(a, \phi)\, \partial_a + \varphi(a, \phi)\, \partial_\phi$

- optional **gauge** $G(a, \phi)$.

Rund-Trautman Identity[2,3]

$$\dot{G} = X^{[1]}_{NGS}\mathcal{L} + \mathcal{L}\dot{\tau}$$

Series of PDEs:

constraint equations

Selection outcome

Determining system ⇒ **constraints on** $V(\phi)$ ⇒**closed-form families** (Exact solution)

[2] Kanesom, Channuie & Kaewkhao (2021).
[3] Dolohtahe, Srikom, Channuie & Kaewkhao (2022).

Eisenhart–Duval Lift: Geodesics, CK/Killing, Payoff

$$\mathcal{L}_{\mathrm{Lift}} = \frac{1}{2}\, G_{\mathrm{AB}}\dot{\Phi}^{\mathrm{A}}\dot{\Phi}^{\mathrm{B}} - \cancel{V(\Phi^{\mathrm{A}})} \qquad G_{\mathrm{AB}} = \begin{bmatrix} -12a(1+\xi\phi^2) & -12a^2\xi\phi & 0 \\ -12a^2\xi\phi & 2a^3 & 0 \\ 0 & 0 & -\frac{2}{a^3 V(\phi)} \end{bmatrix}.$$

- **Lifted metric** on extended space → **dynamics ≡ geodesics**
- Potential absorbed into kinetic term
- **Killing / CK structure** ⇒ constraints (via V’/V)

[4] Finn, Karamitsos & Pilaftsis (2018).

Branch-to-(n_s, r) Map (Indicative, Not Fitted)

Branch	EF-U Geometry	Viability	Slow-roll at	Exit
Noether quadratic $V = V_0(1+\xi\phi^2)$	Hilltop, then exp	Top – yes Tail – no	Small-field (near peak)	Top – natural Tails – no
Noether Higgs-like $V = V_0(1+\xi\phi^2)^2$	flat plateau (dS)	Need slight tilt	Eternal	None
EGR (-) wings $V = V_0\left(C_1 e^{\rho\phi} - C_2 e^{-\rho\phi}\right)^p$	Single slope exp	no (high r)	Power-law if shallow	None
EGR (+) hilltop, (p<0) $V = V_0\left(C_1 e^{\rho\phi} + C_2 e^{-\rho\phi}\right)^p$	Hilltop, then exp	Typically, yes	Small-field around maximum	Natural at minimum
Eisenhart NMC SC $V = V_0\phi^{8/3}$	Fixed-slope exp	No (huge r)	Power-law	none

EF slow-roll:

$$\epsilon_V = \frac{1}{2}(U'/U)^2$$
$$\eta_V = U''/U$$
$$n_s \approx 1 - 6\epsilon_V + 2\eta_V$$
$$\mathrm{r} \sim 16\epsilon_V$$

$$r = 8(1-n_s)$$

Thankyou

Q&A's time