Emulating Interacting Vacuum Energy with CosmoPower

Chakkrit Kaeonikhom

Based on arXiv:2210.05363

Motivation: Markov Chain Monte Carlo (MCMC) analyses require evaluating the modified background and perturbation equations in Boltzmann solvers such as CLASS **millions of times** — a severe computational bottleneck for interacting dark-sector models.

Mathematical Model of Interacting Vacuum Energy

In these models, energy is exchanged between the Dark Matter (c) and Vacuum (V) sectors, modifying their individual conservation equations while their sum remains conserved.

$$\dot{\rho}_c + 3H\rho_c = Q$$

$$\dot{\rho}_V = -Q$$

The interaction kernel Q introduces new free parameters, enlarging the parameter space and making the background/perturbation integration numerically **stiffer**.

3.1 Energy continuity and Euler equations

The continuity equations for first-order perturbations of the baryons, dark matter and vacuum energy read [17, 42]

$$\dot{\delta\rho_b} + 3H\delta\rho_b + 3\rho_b\dot{C} + \rho_b\frac{\nabla^2}{a^2}\left(\theta_b + \sigma\right) = 0\,, \tag{3.7}$$

$$\dot{\delta\rho_c} + 3H\delta\rho_c + 3\rho_c\dot{C} + \rho_c\frac{\nabla^2}{a^2}\left(\theta_c + \sigma\right) = -\delta Q - QA\,, \tag{3.8}$$

$$\delta\dot{V} = \delta Q + QA\,, \tag{3.9}$$

while momentum conservation requires

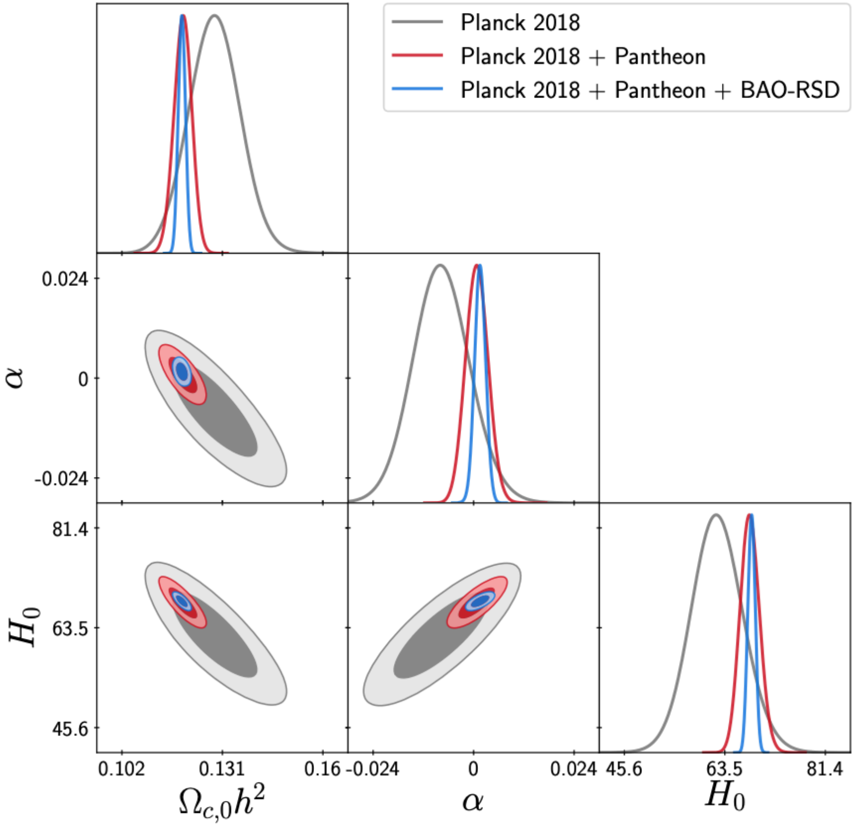
$$\rho_b\left(\dot{\theta}_b + A\right) = 0\,, \tag{3.10}$$

$$\rho_c\left(\dot{\theta}_c + A\right) - Q\theta_c = -f - Q\theta\,, \tag{3.11}$$

$$-\delta V = f + Q\theta\,, \tag{3.12}$$

Bottle Neck: Boltzmann Solver

To get this contour plot, we need to evaluate the Boltzmann solver (CLASS) for each point in the parameter space. This is computationally expensive, especially for interacting dark-sector models. This takes 1.5 Months on HPC (32 Cores)



Dimensionality Reduction for CMB Spectra (PCA)

Emulating a full C_ℓ spectrum directly is hard: the **TE** cross-spectrum changes sign across ℓ, so its zero-crossings make a simple log-scaling of the spectrum invalid.

Principal Component Analysis (PCA) instead compresses each spectrum into a small number of coefficients:

$$C_\ell^{TE}(\theta) \approx \bar{C}_\ell^{TE} + \sum_{k=1}^{n_{pc}} \alpha_k(\theta) M_{k,\ell}$$

where $\bar{C}_\ell$ is the mean spectrum, $M_{k,\ell}$ are the principal components, and α_k are the PCA coefficients.

Neural Network Mapping (CosmoPower)

A neural network learns the map from cosmological parameters θ to the PCA coefficients α:

$$\hat{\alpha} = f_{NN}(\theta; W, b)$$

trained by minimizing the mean squared error between predicted and true coefficients:

$$\mathcal{L}(W, b) = \frac{1}{N} \sum_{i=1}^{N} | \ \alpha_{true}^{(i)} - f_{NN}(\theta^{(i)}; W, b)|^2$$

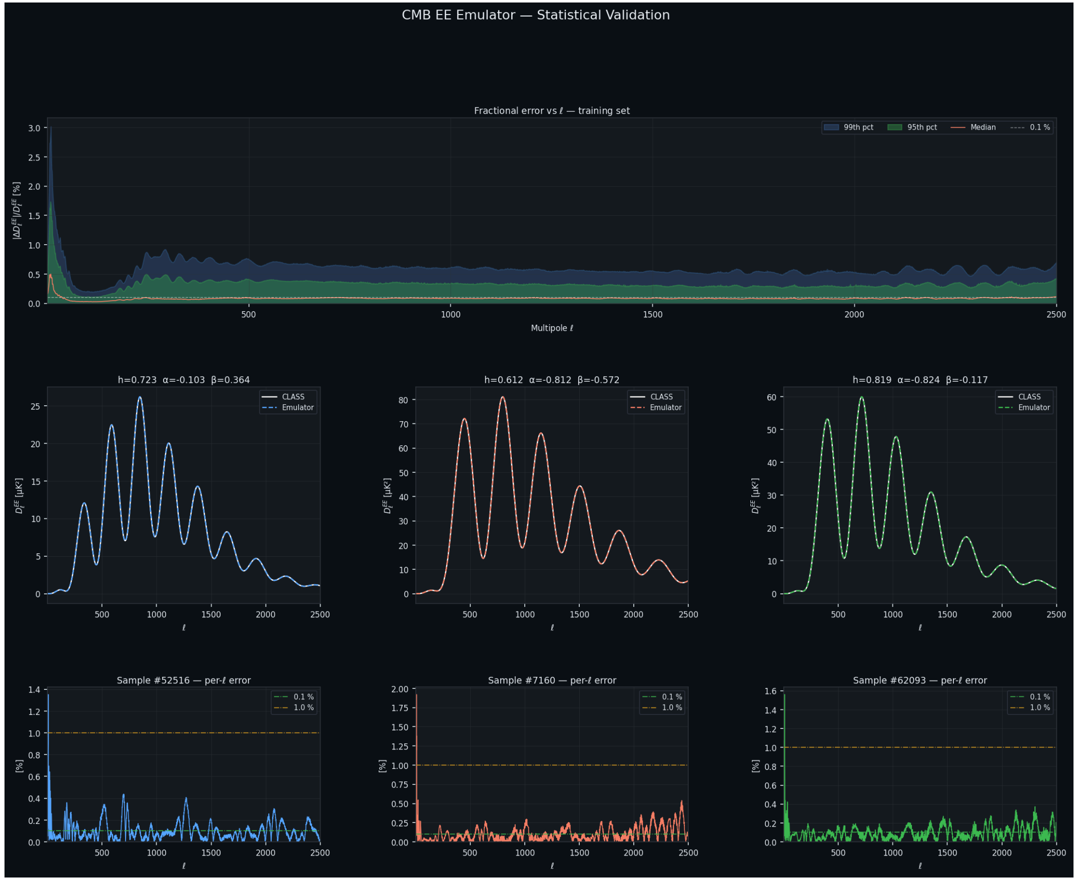
Conclusion & Performance

$$\theta \to f_{NN} \to \hat{\alpha} \to \hat{C}_\ell$$

Replacing the Boltzmann solver with the trained emulator reduces the per-evaluation cost from **seconds** (CLASS) to **milliseconds** (CosmoPower) —

enabling highly efficient MCMC sampling even for complex, multi-parameter dark-sector interaction models.

Some results – EE Spectrum validation



Some results – TE Spectrum validation