Black Hole Phase Transitions in Massive Gravity with Power Yang–Mills Fields

Hieu H. Phung

Phenikaa Institute for Advanced Study
Phenikaa University, Hanoi 12116, Vietnam

hieu.phunghuy@phenikaa-uni.edu.vn

Outline

Motivation

- Black Holes are not only gravitational objects. They also behave as thermodynamic system.
- In extended place cosmological constant can be interpreted as thermodynamic pressure: $\Lambda \to P$.
- This allow me to study interesting phenomena such as critical points and phase transitions.
- I also consider two modification of the standard black-hole system: massive gravity and Yang- Mills field.

The main question I want to investigate is: How do these modifications affect the thermodynamic behavior and phase transitions of black holes?

Theoretical framework

The action of the model is given by

$$\mathcal{S} = \frac{1}{16\pi}\int d^n x\sqrt{-g}\left[R - 2\Lambda + m_g^2\sum_{i=1}^{4} c_i\mathcal{U}_i - (\mathcal{F})^s\right]. \qquad (1)$$

- Λ: cosmoslogical constant
- m_g: gravition mass
- $\mathcal{U}_i$: dRGT massive potentials
- s : Power Yang–Mills nonlinearity

Black Hole solution

- By consider a static and spherically metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{n-2}^2. \tag{2}$$

- The metric function is defined as

$$f(r) = k - \frac{m_0}{r^{n-3}} - \frac{2\Lambda r^2}{(n-1)(n-2)} + f_{YM}(r) + f_{mg}(r). \tag{3}$$

- The event horizon is determined by

$$f(r_+) = 0, \tag{4}$$

where r_+ is the horizon radius.

$$
\begin{aligned}
f(r) = \; & k - \frac{m_0}{r^{n-3}} - \frac{2\Lambda r^2}{(n-1)(n-2)} - \\
& - \frac{(n-2)^{s-1}(n-3)^s q^{2s}}{n-4s-1}\, r^{2-4s} + \\
& + \frac{c_0 m_g^2 r}{n-2}\left[c_1 + \frac{c_0 c_2 (n-2)}{r} + \frac{c_0^2 c_3 (n-2)(n-3)}{r^2} + \right. \\
& \left. \qquad + \frac{c_0^3 c_4 (n-2)(n-3)(n-4)}{r^3}\right].
\end{aligned}
\tag{5}
$$

Black Hole thermodynamics

- The Hawking temperature

$$T(r_+) = \frac{f'(r_+)}{4\pi}. \tag{6}$$

- Entropy

$$S = \frac{\omega_{n-2} r_+^{n-2}}{4}, \tag{7}$$

where $\omega = \frac{2\pi^{\frac{n-1}{2}}}{\Gamma(\frac{n-1}{2})}$ is the unit volume.

- In the extended space phase the pressure is defined as

$$P = -\frac{\Lambda}{8\pi}. \tag{8}$$

- As the result the black hole temperature can be rewritten as an equation of state

$$P = P(T, r_+, q, s, m_g, c_i). \tag{9}$$

Critical behavior

- The critcal value can be determined by solving

$$\frac{\partial P}{\partial r_+} = 0 \equiv \frac{\partial^2 P}{\partial r_+^2}, \tag{10}$$

  these conditions determine the critical radius, critcal temperature, and critical pressure.

- Due to the complexity, critical values can only be obtained numerically. But we have some simple case like $s = 1$ and $s = 2$.

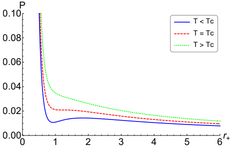

Figure 1. $s = 1$.

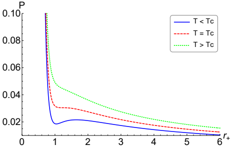

Figure 2. $s = 2$.

- These diagrams display the characteristic behavior of a Van der Waals–like behavior.

Phase transition

- In order to analyze the global phase structure, we define the Gibbs free energy

$$G = M - TS, \tag{11}$$

where M is the Arnowitt-Deser-Misner mass.

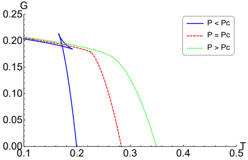

Figure 5. Gibbs free energy for $s = 1$.

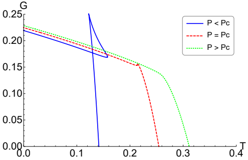

Figure 6. Gibbs free energy for $s = 2$.

- The nonlinearity parameter s plays a crucial role in modifying the characteristic **swallow-tail behavior**, signaling a first-order small-to-large black hole phase transition.

Parameters effect

- The Yang–Mills charge q changes the electromagnetic or gauge-field contribution to the geometry and therefore modifies the critical behavior.
- The parameter s controls the nonlinearity of the Yang–Mills field. Changing s can significantly modify the equation of state and the location of the critical point.
- At the same time, the graviton mass m_g introduces additional contributions to the black-hole geometry and thermodynamics.
- Therefore, the phase structure is determined from the interplay between the Yang–Mills nonlinearity, the gauge charge, and the massive-gravity sector.

Conclusion

- To conclude, i have studied higher- dimensional black holes in dRGT massive gravity coupled to a PYM fields.
- The extended phase space allows me to interpret the cosmological constant as pressure and construct an equation of state for the black hole.
- The system can exhibit Van der Waals-like critical behavior and a first-order phase transition between small and large black holes.
- The critical structure is affected by the Yang–Mills charge, the nonlinear parameter s, and the graviton mass.
- This study shows that modified gravity and nonlinear gauge fields provide useful ways to explore richer thermodynamic behavior of black holes.

THANK YOU FOR YOUR ATTENTION!