How Spontaneous Scalarization Arises in Scalar-Tensor Gravity

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There is no reason not to couple scalar to the curvature R(g)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{g}\left[f(\phi)R(g) - 2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2{m_\phi}^2\phi^2\right] + S_m[\psi_m, g_{\mu\nu}]$$

(Jordan frame)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{g^*}\left[R(g^*) - 2g_*^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2m_\phi^2\phi^2\right] + S_m[\psi_m, A^2(\phi)g^*_{\mu\nu}]$$

(Einstein frame)

Consider coupling function

$$f(\phi) = A^2(\phi) \text{ where } A(\phi) = e^{\beta\phi^2/2}$$

Scalar field equation

$$\Box_{g^*}\phi = -4\pi\alpha(\phi)T^* + m_\phi^2\phi \qquad \text{(Einstein frame)}$$

$$\Box_{g}\phi = -4\pi\alpha(\phi)A^4T + m_\phi^2\phi \qquad \text{(Jordan frame)}$$

$$= -4\pi\beta\phi A^4T + m_\phi^2\phi$$

$$\equiv m_{\text{eff}}^2\phi$$

$$\alpha \equiv \frac{\partial \ln A}{\partial\phi} = \beta\phi$$

Small perturbation about the GR solution

$$\phi = \phi + \delta\phi \qquad (\phi = 0)$$

$$\phi = \delta\phi$$

$$\begin{aligned} m_{\text{eff}}^2 &\equiv -4\pi\beta A^4T + m_\phi^2 \\ &= -4\pi\beta e^{2\beta\phi^2}T + m_\phi^2 \\ &\approx -4\pi\beta T + m_\phi^2 \\ &\approx 4\pi\beta\rho + m_\phi^2 \end{aligned}$$

$$\Box_g \delta\phi = m_{\text{eff}}^2 \delta\phi$$

$$\delta\phi(x,t) \sim e^{i(\vec{k}\vec{x}-\omega t)}$$

Dispersion relation

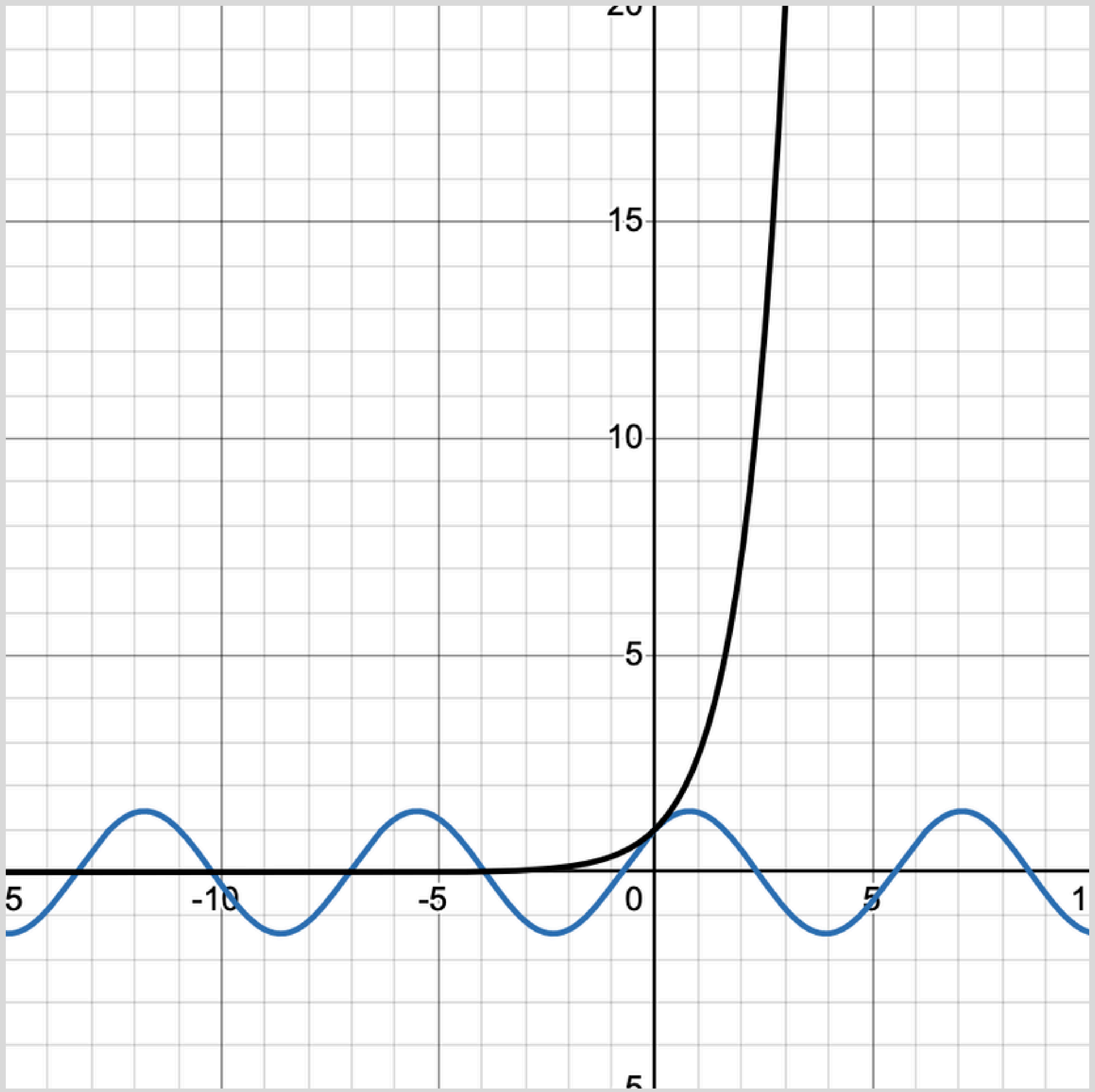
$$\omega \sim \sqrt{k^2 + m_{\text{eff}}^2}$$

(Note: $\beta < 0$)

For $m_{\text{eff}}^2 < 0$ and low enough k,

ω is imaginary

Solution grows exponentially!



$$
\begin{aligned}
m_{\text{eff}}^2 &\equiv -4\pi\beta A^4 T + m_\phi^2 \\
&= -4\pi\beta e^{2\beta\phi^2} T + m_\phi^2
\end{aligned}
$$

Instability is suppressed as the field grows, leading to a stable scalar cloud

References

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2. Tuna, S., Ünlütürk, K. İ., & Ramazanoğlu, F. M. (2022). Constraining scalar-tensor theories using neutron star mass and radius measurements. Physical Review. D/Physical Review. D., 105(12). https://doi.org/10.1103/physrevd.105.124070