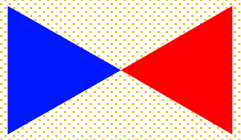


THE NON-ORIENTABLE BEHAVIOR OF THE BTZ BLACK HOLE

Pongsapol Khumma

Department of Physics, Science Faculty, Khon Kaen University

Non-orientable Manifold

Manifold is a topological space which is **smooth** and **locally like flat.**

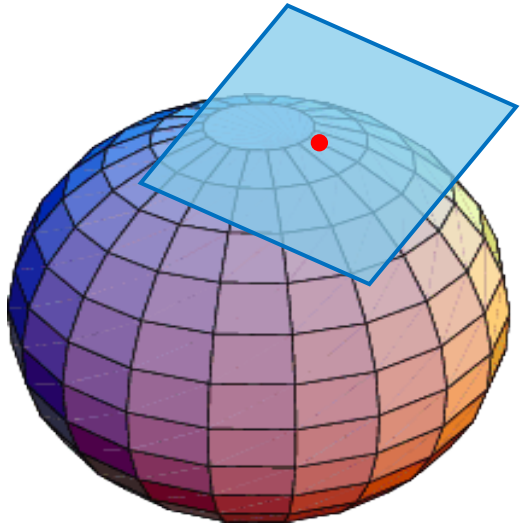


Fig 1. Manifold (Sphere)

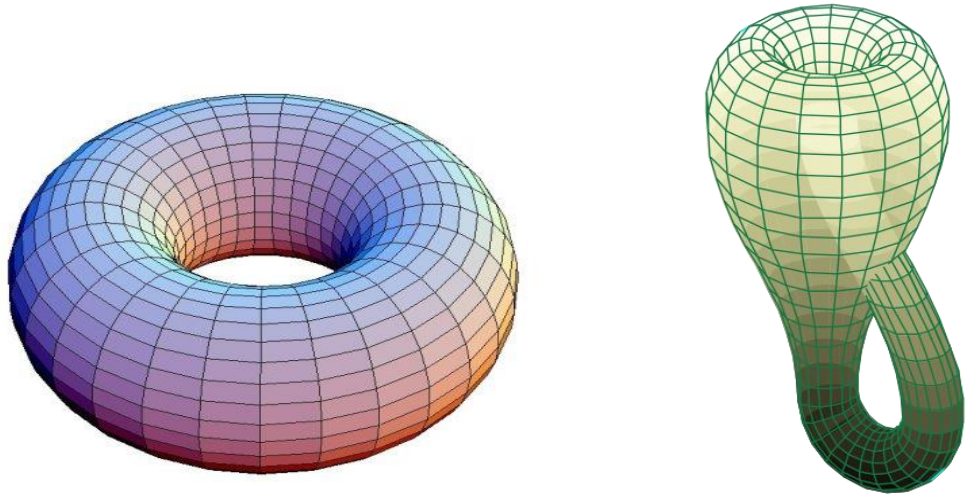


Fig 2. Torus (orientable) and Klien bottle (non-orientable)

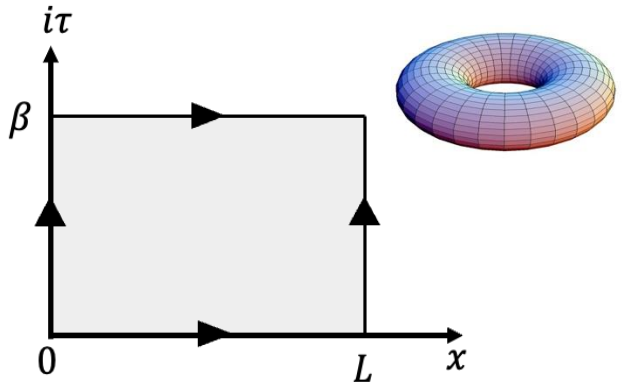
$$\tilde{e}_\alpha = \left(\frac{\partial x^\mu}{\partial y^\alpha}\right) e_\mu$$

The Jacobian is define by $J = det\left(\frac{\partial x^\mu}{\partial y^\alpha}\right)$ then

if $J > 0$, the manifold M is orientable

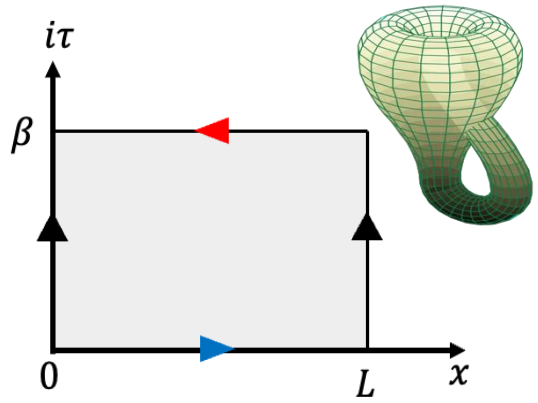
if $J < 0$, the manifold M is non-oreintable.

Partition Function on the Manifold



$$Z_T = Tr_{\mathcal{H}\otimes\bar{\mathcal{H}}}(q^{L_0-\frac{c}{24}}\,\bar{q}^{\bar{L}_0-\frac{c}{24}})$$

Torus partition function



$$Z_k = Tr_{\mathcal{H}\otimes\bar{\mathcal{H}}}(\boldsymbol{\Omega} q^{L_0-\frac{c}{24}}\,\bar{q}^{\bar{L}_0-\frac{c}{24}})$$

Klein bottle partition function

$$\Omega|\alpha,\bar{\beta}\rangle = |\beta,\bar{\alpha}\rangle$$

Fig 3. Quotient space of Torus and Klien bottle

- Let $|\alpha\rangle \in \mathcal{H}$ and $|\bar{\beta}\rangle \in \bar{\mathcal{H}}$ Then define $|\alpha,\bar{\beta}\rangle = |\alpha\rangle \otimes |\bar{\beta}\rangle$ where $|\alpha,\bar{\beta}\rangle \in \mathcal{H}_{sys}$
- Klein bottle partition function is twisted by Ω
- Twisting is the interchanging between holomorphic part and anti- holomorphic part.

Spacetime Characterization

- 3 + 1 dimension (3 space + 1time)
- Considering the point which represents the event in the spacetime
- Spacetime interval is explain via “line elements” : ds^2, e.g. Minkowski spacetime for $g_{\mu\nu}$ of signature (-,+,+,+)

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

Spacetimecan be charaterized in to 3 types

1. Timelike if $ds^2 < 0$
2. Spacelike if $ds^2 > 0$
3. Lightlike if $ds^2 = 0$.

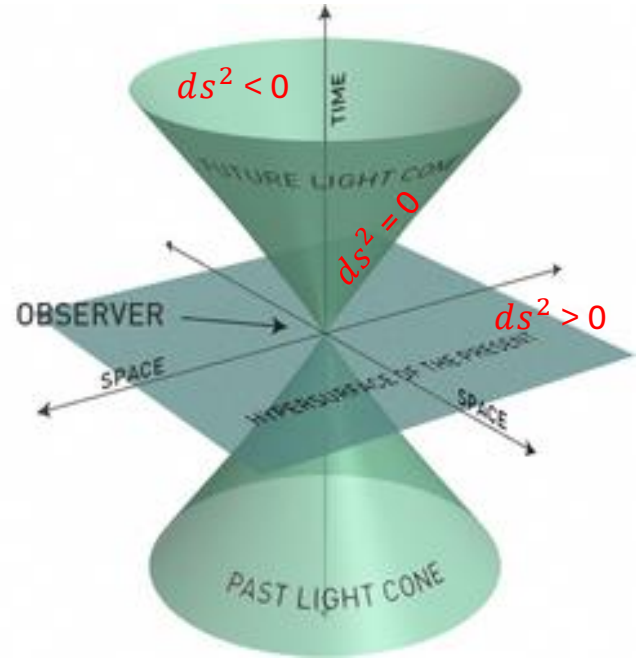


Fig 4. Light cone

Credit https://en.wiktionary.org/wiki/light_cone

BTZ Black hole solution

The model of the BTZ black hole is given by

$$S = \int d^3x\sqrt{-g}(R - 2\Lambda)$$

- The solution of BTZ black hole in (3 dimensions) is

$$ds^2 = -\left(r^2 - r_h^2\right)dt^2 + \frac{dr^2}{r^2 - r_h^2} + r^2 d\phi^2$$

- Extend to the interior solution : **Eternal black hole**
- **AdS_3 /CFT_2** Holographic to define the partition function encoded orientation of the black hole.