Exercises on Review of constraint analysis ISHEAP2026 (13th-20th August 2026)

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1. In the constraint analysis of Maxwell theory, show that requiring $\dot{\phi}_2 \approx 0$ (where $\phi_2 = \partial_i \pi_i$ is the secondary constraint) does not give rise to further constraint. You may recall that the primary constraint is $\phi_1 = \pi^0$ and total Hamiltonian is

$$H_T = \int d^3\vec{x} \ \left(\frac{1}{2}\pi_i \pi_i + \frac{1}{4} F_{ij} F_{ij} + \pi_i \partial_i A_0 + u\pi^0 \right). \tag{1}$$

2. In Maxwell's theory, show that if we introduce extra constraints

$$\chi_1 = A_0, \qquad \chi_2 = \partial_i A_i, \tag{2}$$

then the consistency requirements $\dot{\chi}_1 \approx 0, \dot{\chi}_2 \approx 0$ do not introduce further constraints.

3. In Maxwell's theory, show that if we introduce only one extra constraint

$$\chi_2 = \partial_i A_i, \tag{3}$$

then the consistency requirement $\dot{\chi}_2 \approx 0$ would lead, after appropriate boundary condition at spatial infinity, to $\chi_1 = A_0 \approx 0$.

4. For constraint analysis of Einstein's gravity theory, show that consistency requirements $\dot{\Pi}_0 \approx 0, \dot{\Pi}_i \approx 0$ (where Π_0 is the conjugate momentum corresponding to the lapse function N while Π_i are the conjugate momenta corresponding to the shift vectors N^i) leads to constraints $\mathcal{H}_0 \approx 0$, $\mathcal{H}_k \approx 0$.